AERO-222: Introduction to Aerospace Computation
Fall 2021 – Exam #3 – Wednesday, December 8, 2021

This is a closed-note exam. The exam consists of six problems: one Python programming problem, two multiple choice problems, and 3 problems by hand. You can receive up to 120/100 points ... if all problems are correct. Good luck!

1. Coding (20 pts)

Generate a psuedo-code to integrate the following initial value problem using Euler's method:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y}{x^2}, \quad \text{subject to: } y(3) = 2$$

over the interval $x = [3, 6]$ using 50 intervals.

2. Gauss-Legendre (20 pts, multiple choice)

Estimate the integral,

$$I = \int_{-1}^{+1} (x-1) \sin x^2 \, \mathrm{d}x$$

using the 3-point Gauss-Legendre method where: $x_1 = -x_3 = \sqrt{3/5}$, $x_2 = 0$, $w_1 = w_3 = 5/9$, and $w_2 = 8/9$.

$$(A)\ -0.6274, \quad (B)\ -0.7246, \quad (C)\ -2.4670, \quad (D)\ -0.2764, \quad (E)\ -0.4276$$

3. Richardson Extrapolation (20 pts, by hand)

Code. The 5-point, central, 2nd derivative, $\hat{f}''$, has precision $\mathcal{O}(h^3)$. Using $h_1 = 0.35$ the formula provides $\hat{f}''(h_1) = 1.7033$ while using $h_2 = 0.51$ the formula provides $\hat{f}''(h_2) = 1.8544$. Compute the 2nd derivative using Richardson extrapolation, f''_{RE}.

4. Integral using trapezoid method (20 pts, multiple choice)

Compute the integral using the trapezoid method using the data given in the table.

k	0	1	2	3	4
x_k	0	2	4	6	8
y_k	3	7	11	9	3

(A) 64.0, (B) 48.0, (C) 54.0, (D) 60.0, (E) 36.0

5. ODE using improved Euler (20 pts, by hand)

Consider the 1st order Ordinary Differential Equation (ODE),

$$y' + 2y = x^3\, e^{-2x} \qquad \text{subject to:} \qquad y(0) = 1.$$

Use the improved Euler **with average derivative** to perform a single iteration with step $h = 0.1$. Tip: write the ODE as $y' = f(x, y)$.

6. Derivative using Lagrange polynomials (20 pts, by hand)

Using the data provided in the table, compute the first derivative at $f'(2)$ interpolating with Lagrange polynomials

k	1	2	3
x_k	0	2	4
y_k	3	7	11