AERO-222: Introduction to Aerospace Computation
Fall 2021. Homework #4. **Due date: November 15, 2021**

Show all work and justify your answers!

Instructions

- *This homework contains both handwritten and coding problems and shall be submitted according to the following guidelines.*
- *Hardcopy:*
  - *Due on CANVAS at 11:59 PM on the day of the deadline.*
  - *Shall include screenshots of any hand-written work.*
  - *For coding problems, the hardcopy shall include any relevant derivations and emphasize the final results (i.e. boxed, highlighted, etc.). INCLUDE ALL CODING RESULTS (including plots, final values) IN THE HARDCOPY.*
  - *Shall be submitted as a single file according to the provided template with the following naming scheme:* "LastnameHW#.pdf"
- *Coding Submission:*
  - *Due on CANVAS at 11:59 PM on the day of the deadline.*
  - *Shall be submitted as a single file according to the provided template with the following naming scheme:* "LastnameHW#.py"
  - *The script shall print out all outputs asked for in the problem.*
- *Late submissions will be accepted with a 10 point deduction per day late.*

1. **Numerical Differentiation (20 pts) Code.** The derivative of the function,

$$f(x) = 5\,\cos(10\,x) + x^3 - 2\,x^2 - 6\,x + 10$$

is the function, $f'(x) = -50\,\sin(10\,x) + 3\,x^2 - 4\,x - 6$. Discretize the function $f(x)$ using $N = 100$ points, $[x_k, f_k]$, that are uniformly distributed in $x \in [0, 4]$. Evaluate the first numerical derivative using the 5-point difference formula. Plot the absolute error between true and numerical derivatives. Pay attention to the two extremes: you cannot always use the 5-point **central** difference formula.

2. **Richardson Extrapolation (20 pts) By hand.** Using the function given in problem #1, compute the accuracy gain using Richardson extrapolation to compute the first derivative using the central seven points formula at $x = 2$, using $h_1 = 0.02$ and $h_2 = 0.03$. Compute also the absolute error with respect to the true solution.

3. **Numerical Differentiation (20 pts) By hand** Show all steps to derive the second derivative, f_i'', using using all of the following points

$$f_{i-4}, \qquad f_{i-3}, \qquad f_{i-2}, \qquad f_{i-1}, \qquad f_{i+1},$$

via matrix inversion.

4. **Numerical Differentiation (20 pts) Code.** Consider the function $f(x) = x + \sin x$ and a step size $h = \pi/8$.

   (a) Compare the three-point forward, backward, and central finite-difference approximations of the second derivative of $f(x)$ with respect to the exact solution at $x = 1$. Provide a table of values.

   (b) Compute the absolute error of the 3-point forward, backward, and central approximations for a step size range $\Delta x \in [-0.5, +0.5]$ broken into $1,000$ values. Provide a single semi-logarithmic plot of the absolute errors versus the given x-range. Include proper axes labels and a reference legend.

5. **An Aerospace Application (20 pts) Code.** An airfoil is placed in a wind tunnel and heated to 80°C before lowering the surface temperature to 20.7°C using convective cooling. A thermocouple is placed on the surface of the airfoil and measures the surface temperature at discrete time steps. The following temperatures are recorded,

t (sec)	0	5	10	15	20	25
T (°C)	80	44.5	30	24.1	21.7	20.7

Use numerical differentiation to compute the first derivative $T'(t_k)$ at each time step given in the table above. Specifically, apply the three-point forward, backward or central finite-difference method wherever appropriate in order to estimate the airfoil's temperature gradient, $T'(t_k)$. Provide a table of values.