AERO-222: Introduction to Aerospace Computation – Fall 2021
Homework #3 – Due Date: October 20, 2021

Show all work and justify your answers!

Instructions

- *This homework contains both handwritten and coding problems and shall be submitted according to the following guidelines.*
- *Hardcopy:*
  - *Due on CANVAS at 11:59 PM on the day of the deadline.*
  - *Shall include screenshots of any hand-written work.*
  - *For coding problems, the hardcopy shall include any relevant derivations and emphasize the final results (i.e. boxed, highlighted, etc.). INCLUDE ALL CODING RESULTS (including plots, final values) IN THE HARDCOPY.*
  - *Shall be submitted as a single file according to the provided template with the following naming scheme:* "LastnameHW#.pdf"
- *Coding Submission:*
  - *Due on CANVAS at 11:59 PM on the day of the deadline.*
  - *Shall be submitted as a single file according to the provided template with the following naming scheme:* "LastnameHW#.py"
  - *The script shall print out all outputs asked for in the problem.*
- *Late submissions will be accepted with a 10 point deduction per day late.*

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1. **Least-squares (20 pts) Code.** Create a set of $N = 100$ points, x_k, uniformly distributed in the $[-1, +1]$ range. Compute the function $f(x) = \sin(3x)$ at the x_k data points. Corrupt the $f(x_k)$ values by adding a Gaussian noise with standard deviation $\sigma = 0.08$. Plot $f(x_k)$ and $f_c(x_k)$. Perform the least-squares of $f_c(x_k)$ using the fitting functions,

$$f_1(x) = c_1 + c_2\, x + c_3\,(2x^2 - 1) + c_4\,(4x^3 - 3x)$$
$$f_2(x) = c_1\, \sin x^2 + c_2\,(1 - \cos x) + c_3\,(\cos x\, \sin x) + c_4\, \frac{2 - x}{2 + x}$$

   by computing the c_k coefficients for each function. Plot the estimated functions, $f_1(x)$ and $f_2(x)$, and compute the 1-norm, the 2-norm, and the ∞-norm of the two residuals vectors.

2. **Interpolation (20 pts) by hand.** This problem compare Lagrange interpolation with interpolation using linear combination of different functions. Consider the points,

x_k	1	2	3	5
y_k	-1	1	0	1

Perform the interpolation at $x = 4$ using Lagrange polynomials and using the function,

$$f(x) = c_1 \sin x + c_2 \, (1 - x^2) + 5 \, c_3 \, \frac{\cos x}{x + 1} + 3 \, c_4$$

3. **System of nonlinear equations (20 pts) Code.** Solve the following nonlinear system,

$$\begin{cases} 3\,z - \sin(xy) - 1/8 = 0 \\ x^2 - 19\,(y + 0.1)^3 + \cos z = 0 \\ e^{-y} + 20\,x\,z^2 + \pi/4 = 0 \end{cases}$$

using $\boldsymbol{v}_0 = \{x_0, y_0, z_0\}^{\mathrm{T}} = \{0.1, 0.1, -0.1\}^{\mathrm{T}}$ as initial guess. Iterates until $\|\boldsymbol{v}_{k+1} - \boldsymbol{v}_k\|_2 < 10^{-9}$. Generate a plot of $\|\boldsymbol{v}_{k+1} - \boldsymbol{v}_k\|_2$ as a function of the number of iterations.

4. **System of nonlinear equations (20 pts) Code.** Solve the following nonlinear system using iterative Newton,

$$\begin{cases} f_1(x, y) = x^2 - y^2 - 1 = 0 \\ f_2(x, y) = -x^2 + 2\,y^3 - e^{-x\,y} + 1 = 0 \end{cases}$$

using $x_0 = 3$ and $y_0 = 4$ as initial guess. 1) How many iterations do you need to obtain the 2-norm of $\begin{Bmatrix} x_k - x_{k-1} \\ y_k - y_{k-1} \end{Bmatrix} < 10^{-6}$. 2) Plot the following table for 10 iterations

iteration	x_k	y_k	$f_1(x_k, y_k)$	$f_2(x_k, y_k)$
1	???	???	???	???
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
10	???	???	???	???

5. **Least-squares (20 pts) by hand.** Show how to estimate φ, ξ, and λ by linear least-squares using the n set of data points, $[x_k, y_k]$, and the function,

$$y(x) = \sin(3x + \varphi) - x + x^2 - x^3 + \xi \, \cos(2x - \lambda)$$