AERO-222: Introduction to Aerospace Computation - Fall 2021
Homework #5 (last one!) - Due date: December 3, 2021

Show all work and justify your answers!

Instructions

- *This homework contains both handwritten and coding problems and shall be submitted according to the following guidelines.*
- *Hardcopy:*
  - *Due on CANVAS at 11:59 PM on the day of the deadline.*
  - *Shall include screenshots of any hand-written work.*
  - *For coding problems, the hardcopy shall include any relevant derivations and emphasize the final results (i.e. boxed, highlighted, etc.). INCLUDE ALL CODING RESULTS (including plots, final values) IN THE HARDCOPY.*
  - *Shall be submitted as a single file according to the provided template with the following naming scheme:* "LastnameHW#.pdf"
- *Coding Submission:*
  - *Due on CANVAS at 11:59 PM on the day of the deadline.*
  - *Shall be submitted as a single file according to the provided template with the following naming scheme:* "LastnameHW#.py"
  - *The script shall print out all outputs asked for in the problem.*
- *Late submissions will be accepted with a 10 point deduction per day late.*

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1. **Integration: Trapezoid and Midpoint (15 pts)**
   Consider the integral $I = \displaystyle\int_0^{\pi} e^{-2x} \sin(3x)\, \mathrm{d}x$, whose exact value is, $I = \dfrac{3}{13}\left(1 + e^{-2\pi}\right)$.
   (**a**) **Code.** For a range of partitions, $n = [5, 20]$, compute an approximation of the integral using the Midpoint method. Plot the number of partitions versus the value of the integral, and the absolute error with respect to the true value on a semi-log scale.
   (**b**) **Code.** Do the same using the Trapezoid method.

2. **Integration: Richardson Extrapolation (20 pts)**
   Code. Using the values of the integral computed by the Trapezoid method in Problem 1 (**b**) for $n = 7$ and $n = 8$ partitions, estimate the integral using Richardson extrapolation. (Use the error bound for trapezoid given in the slides.)

3. **Integration: Simpson (10 pts)**
Consider $f(x) = \dfrac{1}{2+x^2}$.

(a) **(5 pts) By-hand.** Compute $\displaystyle\int_{-1}^{+1} f(x)\,\mathrm{d}x$ using two Simpson partitions (five points). Show all the calculations in *exact arithmetic* (i.e. use fractions throughout).

(b) **(5 pts) Code.** Compute the integral with Simpson partitions ranging from 3 to 12. Plot the number of partitions versus the value of the integral, and the absolute error with respect to the true value ($I_{\text{true}} = \sqrt{2}\tan^{-1}(1/\sqrt{2})$ on a semi-log scale.

4. **Integration: Gauss-Legendre (20 pts)**

(a) **(10 pts) By-hand.** Use a 3-point Gauss-Legendre quadrature to approximate the distance covered by a rocket from $t = 2$ sec to $t = 20$ sec, as given by the integral

$$I = \int_{2}^{20} \left[2{,}000\,\ln\left(\frac{110{,}000}{110{,}000 - 1{,}600\,t}\right) - 9.8\,t \right] \mathrm{d}t.$$

Report the approximate distance in the hard copy.

(b) **(10 pts) Code.** Compute the integral with Gauss-Legendre using 3, 4, and 5 points. *Report the values of the integral in a table.*

5. **ODE: Improved Euler (15 pts)**
Solve the following initial value problems:

1. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3y}{x}$, subject to: $y(1) = 2 \;\rightarrow\; y_{\text{true}}(x) = 2x^3$

2. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x - 2y$, subject to: $y(0) = 1 \;\rightarrow\; y_{\text{true}}(x) = \frac{1}{4}(2x + 5e^{-2x} - 1)$

over the interval $x = [1, 4]$ (for the first equation) and $x = [0, 3]$ (for the second equation) using 30 and 300 intervals. You will use the given true solution to determine the absolute error of the methods. For each equation, provide a plot with all 3 absolute errors on a semi-log scale and a plot with all 3 method solutions and the actual solution.

(a) **(5 pts) Code.** Using Euler's method.

(b) **(5 pts) Code.** Using Improved Euler's method using average derivative.

(c) **(5 pts) Code.** Using Improved Euler's method using derivative at the midpoint.

6. ODE: Runge-Kutta (20 pts)

Solve the two differential equations given in Problem 5 over the same time ranges and intervals. For each equation, provide a plot with both absolute errors on a semi-log scale and a plot with both method's solutions and the actual solution.

(a) **(10 pts) Code.** Using 4th-order Runge-Kutta method.

(b) **(10 pts) Code.** Using `scipy.integrate.solve_ivp`.