AERO 422 Homework #1

Instructor: Vedang Deshpande

Due: September 15, 2021 at 12:40p.m.

Fall 2021

(**20 Points**)

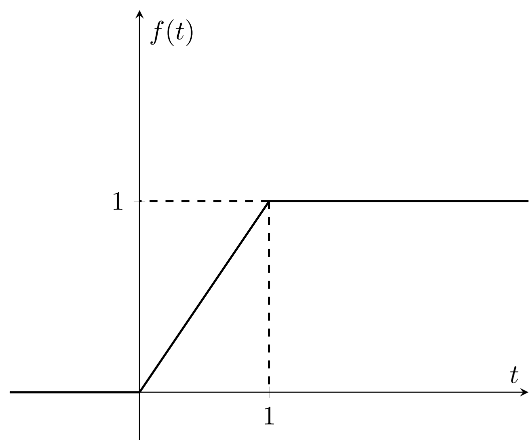
1. (**5 points**) For the following rational function, i) determine the poles and zeros, ii) plot the pole-zero map using MATLAB (or another appropriate alternative), and iii) find the magnitude and phase at $s = -1 + j$.

$$F(s) = \frac{6(s^2+2)}{s(s^2+2s+5)}$$

Solution in other document.

2. (a) (**2 points**) Using the *definition* of the Laplace transform, find $F(s) = \mathcal{L}\{f(t)\}$ for $f(t) = e^{-at} \cdot 1(t)$.
Solution in other document.

(b) (**3 points**) Using the Laplace table and its properties, find the Laplace transform of $f(t)$ shown in the figure.



Hint: Express $f(t)$ in terms of simple components like ramp the function and unit step function.
Solution:

Express the piecewise function in terms of unit step function and ramp function. Simplify and then apply the Laplace transform to both sides.

$$\begin{aligned} f(t) &= t \cdot 1(t) - t \cdot 1(t-1) + 1(t-1) \\ &= t \cdot 1(t) + (1-t) \cdot 1(t-1) \\ &= t \cdot 1(t) - (t-1) \cdot 1(t-1) \end{aligned}$$

For the second term, apply this rule from the Laplace table: $\mathcal{L}\{f(t-a) \cdot 1(t-a)\} = e^{-as}F(s)$.

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t \cdot 1(t)\} - \mathcal{L}\{(t-1) \cdot 1(t-1)\}$$

For the first term, recall that $1(t) = 1$ when $t \geq 0$, which is also the domain of the Laplace transform.

$$\begin{aligned} F(s) &= \mathcal{L}\{t\} - \mathcal{L}\{(t-1) \cdot 1(t-1)\} \\ &= \frac{1}{s^2} - e^{-s} \cdot \mathcal{L}\{t\} \\ &= \frac{1}{s^2} - \frac{e^{-s}}{s^2} \end{aligned}$$

3. (**6 points**) For each of the following differential equations with specified initial conditions, use the properties of the Laplace transform to solve for $X(s)$, where $X(s) = \mathcal{L}\{x(t)\}$.

   (a) $\ddot{x}(t) + 2\dot{x}(t) + 5x(t) = 3 \cdot 1(t), \quad x(0^+) = 0, \ \dot{x}(0^+) = 0$

   (b) $\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = 0, \quad x(0^+) = a, \ \dot{x}(0^+) = b$

   (c) $\dot{x}(t) + ax(t) = A\sin\omega t, \quad x(0^+) = b$

   Solutions in other document.

4. (**4 points**) Assuming that $\{f(t), F(s)\}$ are a Laplace transform pair, where

$$F(s) = \frac{3s+1}{s^2+s+1},$$

   determine the values of $f(0^+)$, $\dot{f}(0^+)$, and $f(\infty)$.

   Hint: Apply the initial value theorem approach to $\ddot{f}(t)$.

   Solutions to $f(0^+)$ and $\dot{f}(0^+)$ in other document.

   Solution to $f(\infty)$:

   Apply Initial Value Theorem.

$$\begin{aligned}
\lim_{t\to\infty} f(t) &= \lim_{s\to 0} s \cdot F(s) \\
&= \lim_{s\to 0} s \cdot \frac{3s+1}{s^2+s+1} \\
&= \lim_{s\to 0} \frac{3s^2+s}{s^2+s+1} \\
&= \frac{0}{1} \\
&= 0
\end{aligned}$$