AERO 422 Homework #1

Instructor: Vedang Deshpande

Due: September 15, 2021 at 12:40p.m.

Fall 2021

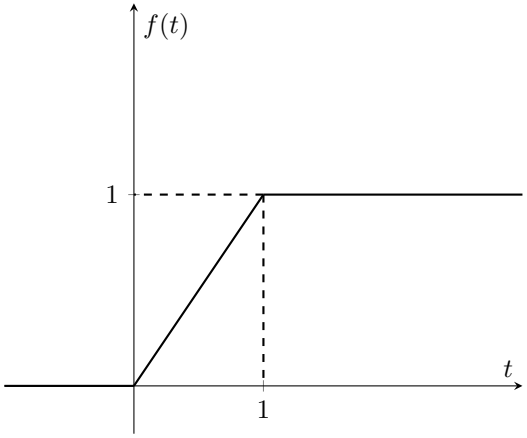
(**20 Points**)

1. (**5 points**) For the following rational function, i) determine the poles and zeros, ii) plot the pole-zero map using MATLAB (or another appropriate alternative), and iii) find the magnitude and phase at $s = -1 + j$.

$$F(s) = \frac{6(s^2 + 2)}{s(s^2 + 2s + 5)}$$

2. (a) (**2 points**) Using the *definition* of the Laplace transform, find $F(s) = \mathcal{L}\{f(t)\}$ for $f(t) = e^{-at} \cdot 1(t)$.

   (b) (**3 points**) Using the Laplace table and its properties, find the Laplace transform of $f(t)$ shown in the figure.



Hint: Express $f(t)$ in terms of simple components like ramp the function and unit step function.

3. (**6 points**) For each of the following differential equations with specified initial conditions, use the properties of the Laplace transform to solve for $X(s)$, where $X(s) = \mathcal{L}\{x(t)\}$.

   (a) $\ddot{x}(t) + 2\dot{x}(t) + 5x(t) = 3 \cdot 1(t), \;\; x(0^+) = 0, \; \dot{x}(0^+) = 0$

   (b) $\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = 0, \;\; x(0^+) = a, \; \dot{x}(0^+) = b$

   (c) $\dot{x}(t) + ax(t) = A \sin \omega t, \;\; x(0^+) = b$

4. (**4 points**) Assuming that $\{f(t), F(s)\}$ are a Laplace transform pair, where

$$F(s) = \frac{3s + 1}{s^2 + s + 1},$$

   determine the values of $f(0^+)$, $\dot{f}(0^+)$, and $f(\infty)$.

   Hint: Apply the initial value theorem approach to $\ddot{f}(t)$.