

AERO 422 Homework #1

Instructor: Vedang Deshpande

Due: September 15, 2021 at 12:40p.m.

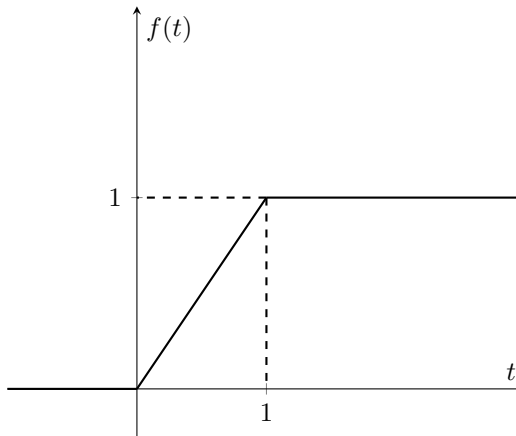
Fall 2021

(20 Points)

1. (5 points) For the following rational function, i) determine the poles and zeros, ii) plot the pole-zero map using MATLAB (or another appropriate alternative), and iii) find the magnitude and phase at $s = -1 + j$.

$$F(s) = \frac{6(s^2 + 2)}{s(s^2 + 2s + 5)}$$

2. (a) (2 points) Using the *definition* of the Laplace transform, find $F(s) = \mathcal{L}\{f(t)\}$ for $f(t) = e^{-at} \cdot 1(t)$.
(b) (3 points) Using the Laplace table and its properties, find the Laplace transform of $f(t)$ shown in the figure.



Hint: Express $f(t)$ in terms of simple components like ramp the function and unit step function.

3. (6 points) For each of the following differential equations with specified initial conditions, use the properties of the Laplace transform to solve for $X(s)$, where $X(s) = \mathcal{L}\{x(t)\}$.
- (a) $\ddot{x}(t) + 2\dot{x}(t) + 5x(t) = 3 \cdot 1(t)$, $x(0^+) = 0$, $\dot{x}(0^+) = 0$
 - (b) $\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2x(t) = 0$, $x(0^+) = a$, $\dot{x}(0^+) = b$
 - (c) $\dot{x}(t) + ax(t) = A \sin \omega t$, $x(0^+) = b$

4. (4 points) Assuming that $\{f(t), F(s)\}$ are a Laplace transform pair, where

$$F(s) = \frac{3s + 1}{s^2 + s + 1},$$

determine the values of $f(0^+)$, $\dot{f}(0^+)$, and $f(\infty)$.

Hint: Apply the initial value theorem approach to $\ddot{f}(t)$.