AERO 422 Homework #2

Instructor: Vedang Deshpande

Due: September 22, 2021 at 12:40p.m.

Fall 2021

(**25 Points**)

1. Consider the function $f(t) = te^{2t} \sin 3t$

   (a) (**2 points**) Find the Laplace transform using the table. Mention which entries from the table are being used.

   Use the following properties from the Laplace table

   $$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n F(s)}{ds^n} \quad (7), \qquad \mathcal{L}\{e^{at} \sin kt\} = \frac{k}{(s-a)^2 + k^2} \quad (22)$$

   Let $f(t) = e^{2t} \sin 3t$. Apply (7) to get

   $$\mathcal{L}\{te^{2t} \sin 3t\} = \mathcal{L}\{t f(t)\}$$
   $$= (-1)\frac{dF(s)}{ds}$$
   $$= (-1)\frac{d\left[\mathcal{L}\{e^{2t} \sin 3t\}\right]}{ds}$$

   Apply (22) and simplify using the quotiet rule.

   $$\mathcal{L}\{te^{2t} \sin 3t\} = (-1)\frac{d}{ds}\left\{\frac{3}{(s-2)^2 + 3^2}\right\}$$
   $$= (-1)\frac{-3 \cdot 2(s-2)}{((s-2)^2 + 9)^2}$$
   $$= \frac{6s - 12}{((s-2)^2 + 9)^2}$$

   (b) (**1 point**) Can we use the F.V.T. to determine $f(\infty)$? Why or why not?

2. Find the inverse Laplace transform using the table and partial fraction expansion. Show your work.

   (a) (**2 points**)
   $$F(s) = \frac{s + 10}{s^2 + 2s + 10}$$

   (b) (**3 points**)
 $$F(s) = \frac{s^2 + 1}{s(s-1)^3}$$

3. A given system is found to have a transfer function that is

   $$\frac{Y(s)}{R(s)} = \frac{10(s+2)}{s^2 + 8s + 15}$$

   (a) (**3 points**) Using partial fractions, determine $y(t)$ when $r(t)$ is a unit step input. Show your work.

(b) (**1 point**) Can we use F.V.T. to find $y(\infty)$? If the answer is yes, apply F.V.T. If not, explain why.

4. (a) (**3 points**) Using the convolution integral, find the step response of the system whose impulse response is given below

$$h(t) = \begin{cases} te^{-t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

(b) (**2 points**) Now use the Laplace transform table and partial fraction expansion to find $y(t)$.

(c) (**2 points**) Apply I.V.T. and F.V.T. (if applicable) to find $y(0)$ and $y(\infty)$.

5. For each of the following block diagrams, reduce the block diagram to find $T(s)$, where $T(s)$ is defined by $Y(s) = T(s)R(s)$.
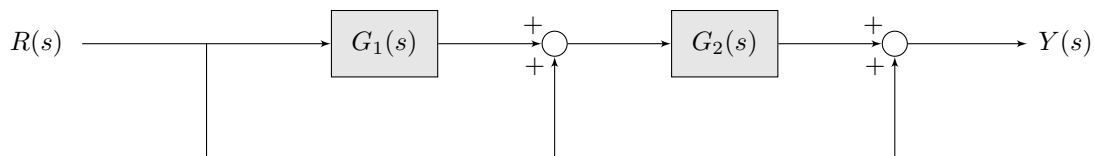
(a) (**1.5 points**)
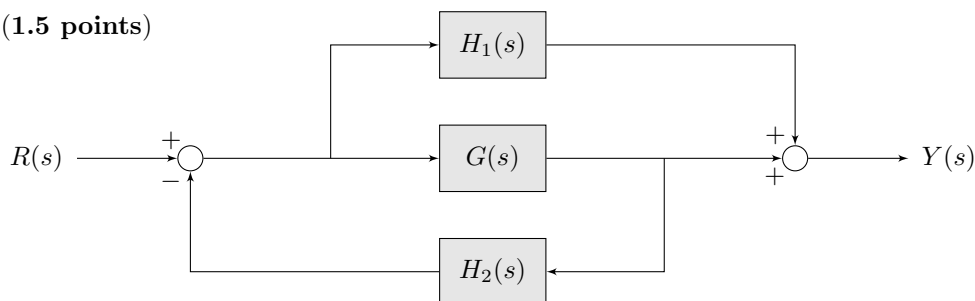
(b) (**3 points**)
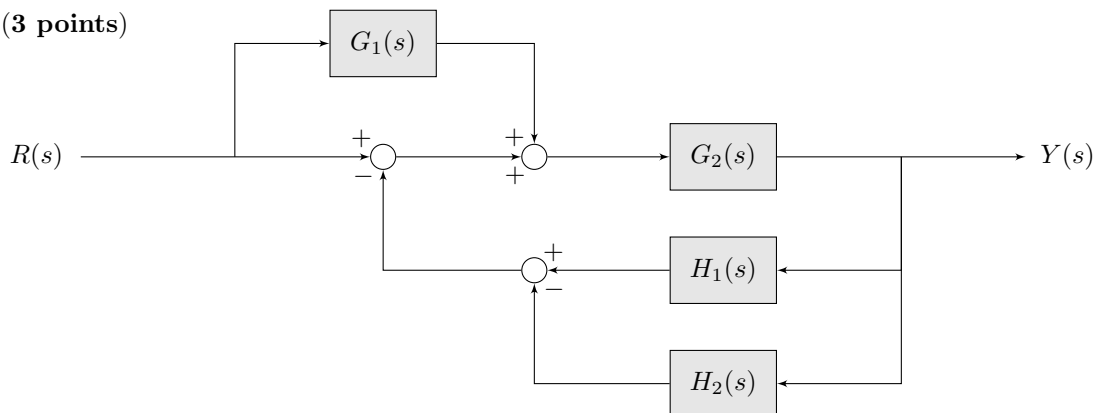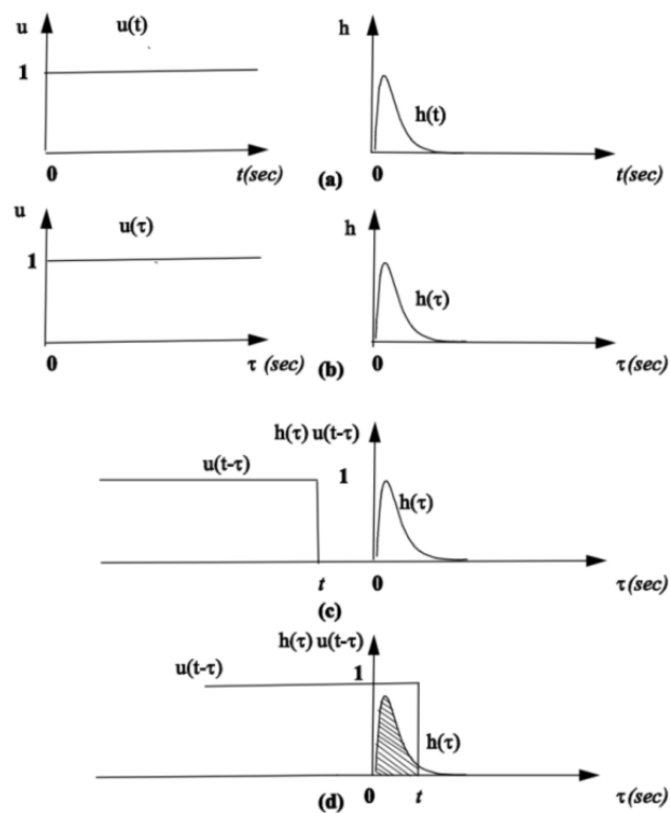

(c) (**1.5 points**)

Illustration of convolution.

Figure 1: Convolution integral (reference for problem 4)