DC to DC Converters

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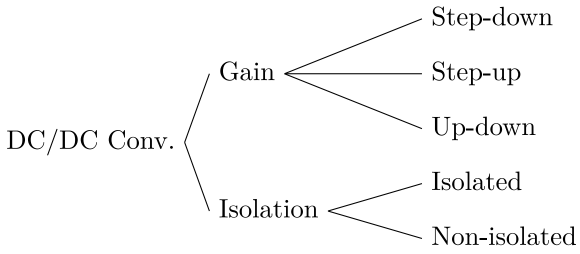
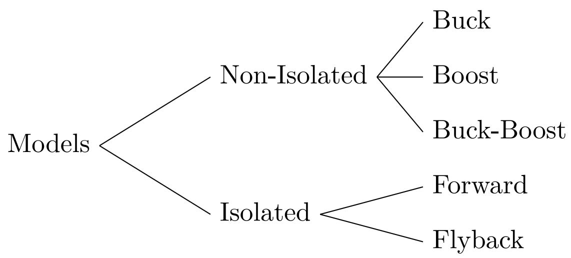
Table of contents

1 DC to DC Converters

Converter types:



- **Step-down**: A converter where output voltage is lower than the input voltage (like a Buck converter).
- **Step-up**: A converter that outputs a voltage higher than the input voltage (like a Boost converter).



Converter operation regimes, conduction modes:

- **Continuous Current Mode**: Current and thus the magnetic field in the inductive energy storage never reach zero. In CCM
$$\frac{1}{2}\Delta i_l < \bar{i_L}$$
- **Discontinuous Current Mode**: Current and thus the magnetic field in the inductive energy storage may reach or cross zero. In DCM
$$\frac{1}{2}\Delta i_l > \bar{i_L}$$

Control signal The control signal of a DC converter is a square signal, which applied to the MOSFET or IGBT gate is responsible of switching of the current.

The control signal is a square function

$$f(t) =$$

Some of the parameters of the control signal are:

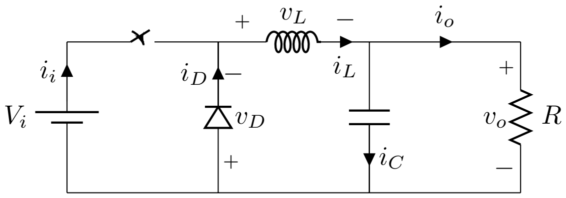
- T, period of the signal
- t_{ON}, time of high value
- t_{OFF}, time of low value (zero)
- D, duty cycle

$$D = \frac{t_{ON}}{T}$$

Switching component

2 Non-isolated converters

2.1 Step-down Converter (Buck)



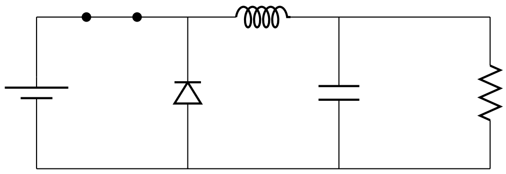
Known variables:

- Input voltage, V_i (constant, positive voltage)
- Switch square control signal, D and T
- Circuit components, L, C, R.

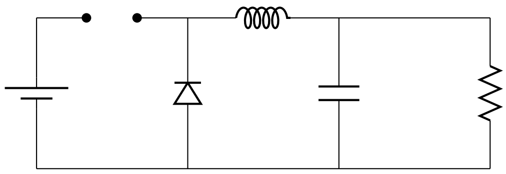
State of the diode

- **ON Circuit**: During the ON state of the circuit, the diode is backwards polarized by the DC source (OFF, not conducting).

$$v_D = -V_i$$



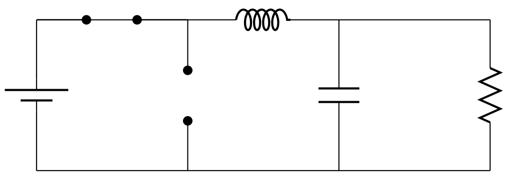
- **OFF circuit**. We can't know the state of the diode until the v_o and v_L voltages are known. Since in the ON circuit the diode is not conducting, the diode should be conducting in the OFF circuit or it won't have a purpouse in the circuit. We suposse the diode to be forward polarized (ON or conducting state), although at the end of the circuit analysis it will be necessary to check if the hypothesis was correct.



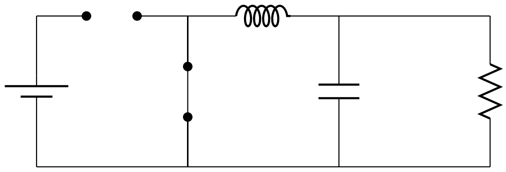
Diode table

Switch	D
ON	OFF
OFF	ON

ON equivalent circuit

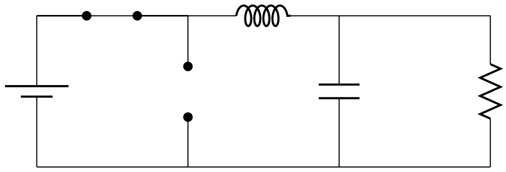


OFF equivalent circuit

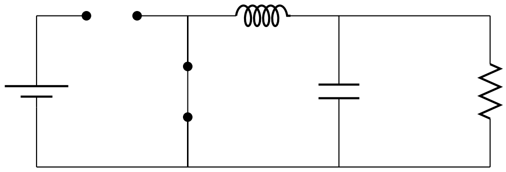


Inductor voltage

1. **ON**. KVL @ Inductor: $v_L = V_i - v_o$



2. **OFF**. KVL @ Inductor: $v_L = -v_o$



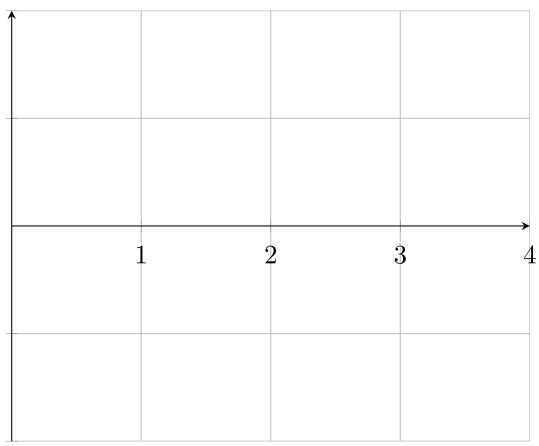
The voltage in the inductor is

$$v_L(t) = \begin{cases} V_i - v_o & \text{ON} \\ -v_o & \text{OFF} \end{cases}$$

If we

- apply the small ripple approximation (consider the $v_o(t)$ constant, V_o),
- suppose that $V_i - V_o > 0$ and $-V_o > 0$, for the inductor to be in periodic steady state (we will check in the output voltage calculation). If we asume that the output voltage sign is placed correctly and $V_i > V_o$, as it would be normal in a step-up converter, then the assumptions are correct.

we can represent the inductor voltage waveform:



Output voltage

In order to calculate the output voltage of the converter, we have to apply the condition $\bar{v_L} = 0$ in the inductor.

1. In periodic steady state, the average voltage of an inductor is zero, $\bar{v_L} = 0$

   1.1 Mean value definition

$$\bar{v_L}(t) = \frac{1}{T}\int_{t_0}^{t_0+T} v_L(t)dt = 0$$

   1.2 Substitute the definition of $v_L(t)$ and split the integral

$$\cancel{\frac{1}{T}}\left[\int_0^{DT}(V_i - v_o)dt + \int_{DT}^{T}(-v_o(t))dt\right] = 0$$

2. In order to obtain a value for $v_o(t)$ from the integral we have to suppose it is contant. We apply the small-ripple approximation: $v_o(t)$ can be considered constant: $v_o(t) \approx V_o$.

$$(V_i - V_o)D\cancel{T} + (-V_o)(1-D)\cancel{T} = 0$$

$$(V_i D - \cancel{V_o D} - V_o + \cancel{V_o D} = 0$$

$$V_o = V_i D$$

Since $V_i > 0$ and $D < 1$, we can say

- $V_o > 0$, so the sign criteria chosen for the diagram shows the correct polarization.
- $V_o < V_i$, so it is indeed a step-down converter.

From now in advance, all the calculations will be made with the small ripple approximation, so

- $v_o(t)$ ripple is negligible, so $v_o(t) \approx \bar{v_o}(t) = V_o$
- If v_o is constant, then also $i_o(t)$ is constant: $i_o(t) = \bar{i_o}(t) = I_o$

Duty Cycle

$$D = \frac{V_o}{V_i}$$

Output current

Since we have applied the Small-Ripple approximation, we have already stated that $i_o(t) = \bar{i_o}(t) = I_o$. Output current can be calculated from the problem data:

$$I_o = \frac{V_o}{R}$$

Other ways of calculating output current are

$$I_o = \frac{P_o}{V_o}$$

$$I_o = \sqrt{\frac{P_o}{R}}$$

Output power

We can calculate the power the load is consuming from the problem data

$$P_o = \frac{V_o^2}{R}$$

Other ways of calculating output power are

$$P_o = I_o^2 R$$

Inductor current

Since the inductor voltage is a square wave with positive and negative voltages, the inductor current is a triangular wave.

The inductor current triangular wave can be described with its mean value and the ripple:

- **Mean value,** $\bar{i_L}$

  1. KCL @ v_o node: $i_L(t) = i_c(t) + i_o(t)$
  2. Superposition: $\bar{i_L}(t) = \bar{i_c}(t) + \bar{i_o}(t)$
  3. $\bar{i_o}(t) = I_o = \frac{V_o}{R}$: $\bar{i_L}(t) = \bar{i_c}(t) + \frac{V_o}{R}$
  4. PSS: $\bar{i_c}(t) = 0$

$$\bar{i_L}(t) = I_o = \frac{V_o}{R}$$

  From power:

$$\bar{i_L}(t) = I_o = \frac{P_o}{V_o}$$

- **Ripple,** Δi_L

$$\Delta i_L = \frac{V \Delta t}{L}$$

$$\Delta i_L = \frac{(V_i - V_o)DT}{L}$$

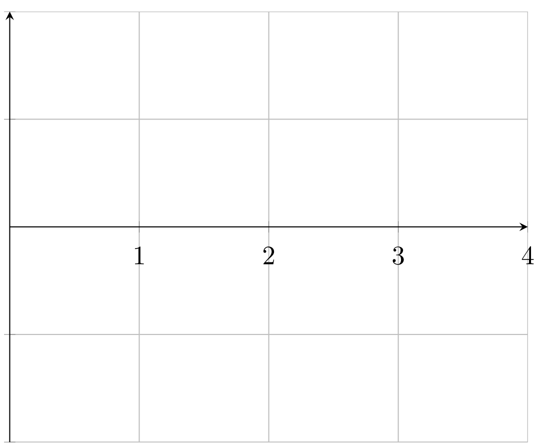
$$\Delta i_L = \frac{V_o(1-D)T}{L}$$

The maximum and minimum value of the current waveform are:

$$I_1 = \bar{i_L} + \frac{1}{2}\Delta i_L$$

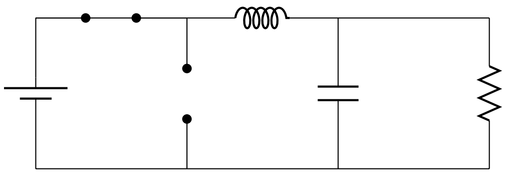
$$I_2 = \bar{i_L} - \frac{1}{2}\Delta i_L$$

The representation is:

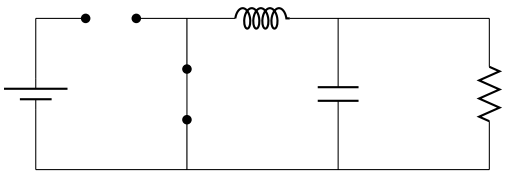


Switch tension

- When the switch is conducting (in the ON phase of the circuit), the voltage drop across the switch is 0, according to the ideal switch model: $v_S = 0$

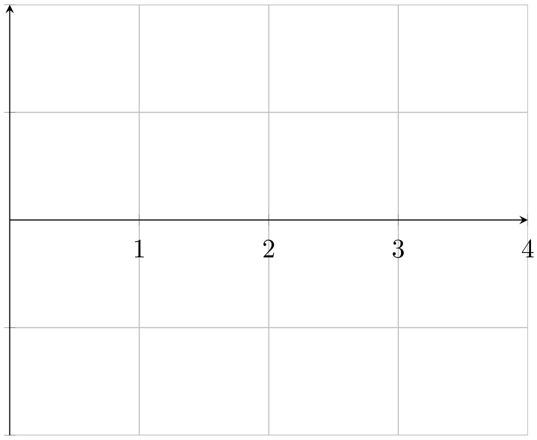


- When the switch is not conducting (in the OFF phase of the circuit), voltage drop is $v_S = V_i - 0 = V_i$ (KVL @ Switch).



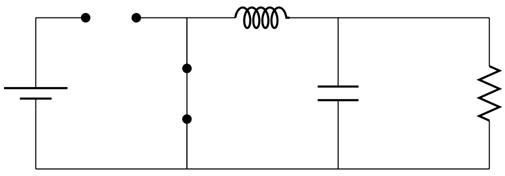
The switch voltage is

$$v_S(t) = \begin{cases} 0 & \text{ON} \\ V_i & \text{OFF} \end{cases}$$

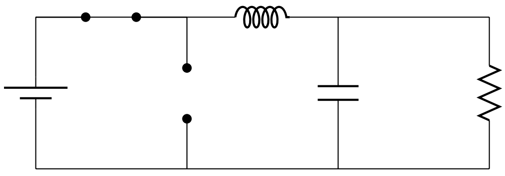


Switch current

- When the switch is not conducting (in the OFF phase of the circuit), the current through the switch is 0, according to the ideal switch model: $i_S = 0$



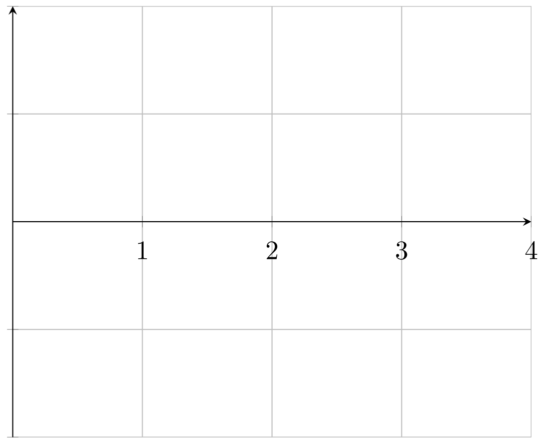
- When the switch is conducting (in the ON phase of the circuit), voltage drop is $i_S = i_L$ (KCL @ v_D node).



The switch current is

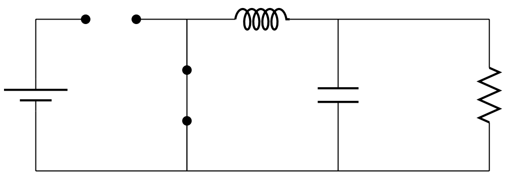
$$i_S(t) = \begin{cases} i_L & \text{ON} \\ 0 & \text{OFF} \end{cases}$$

This means that the switch conducts the inductor current in the ON phase of the circuit.

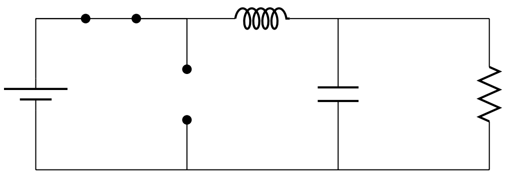


Diode tension

- When the diode is conducting (in the OFF phase of the circuit), the voltage drop across the diode is 0, according to the ideal diode model: $v_D = 0$

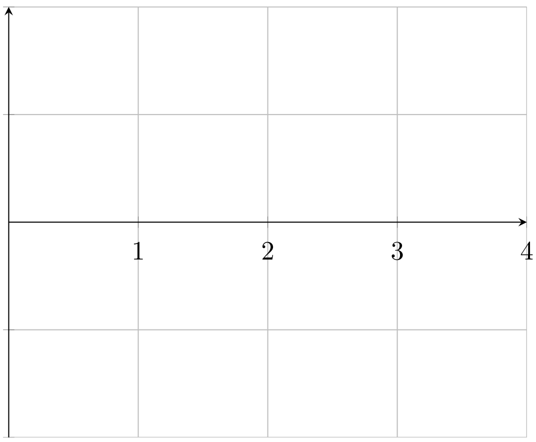


- When the diode is not conducting (in the ON phase of the circuit), current is $v_D = 0 - V_i = -V_i$ (KVL @ Diode).



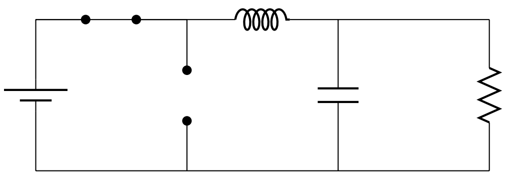
The diode voltage is

$$v_D(t) = \begin{cases} -V_i & \text{ON} \\ 0 & \text{OFF} \end{cases}$$

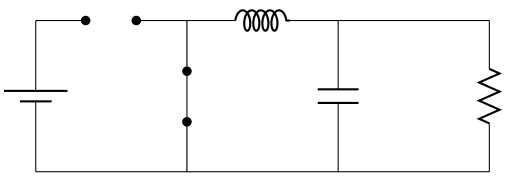


Diode current

- When the diode is not conducting (in the ON phase of the circuit), the current through the diode is 0, according to the ideal diode model: $i_D = 0$



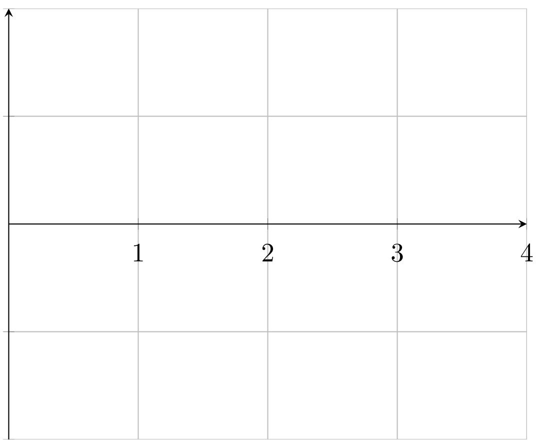
- When the diode is conducting (in the ON phase of the circuit), current is $i_S = i_L$ (KCL @ v_D node).



The switch current is

$$i_D(t) = \begin{cases} 0 & \text{ON} \\ i_L & \text{OFF} \end{cases}$$

This means that the diode conducts the inductor current in the OFF phase of the circuit.



Capacitor current

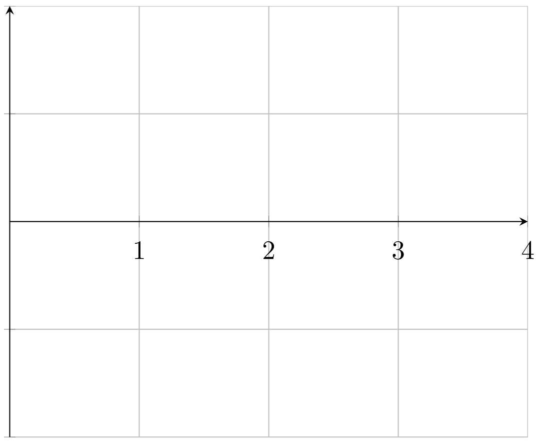
- We already know that the capacitor current mean value is 0, because it is the steady state condition: $\bar{i_c} = 0$
- If we apply KCL @ v_o node, we obtain the equation $i_L(t) = i_C(t) + i_o(t)$. We isolate $i_C(t)$:

$$i_C(t) = i_L(t) - i_o(t)$$
$$i_C(t) = i_L(t) - I_o$$

So $i_C(t)$ is the same wave as $i_L(t)$, but displaced vertically by a constant I_o. If we substitute in the maximum and minimum points of $i_L(t)$, we obtain the extreme values of $i_C(t)$:

$$I_1 = \bar{i_L} + \frac{1}{2}\Delta i_L - I_o =$$
$$I_2 = \bar{i_L} - \frac{1}{2}\Delta i_L - I_o =$$

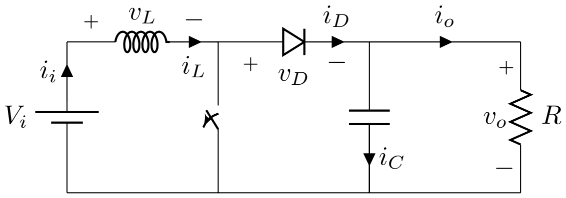
$$I_1 = +\frac{1}{2}\Delta i_L$$
$$I_2 = -\frac{1}{2}\Delta i_L$$



Voltage ripple

Source waveforms

2.2 Step-up Converter (Boost)

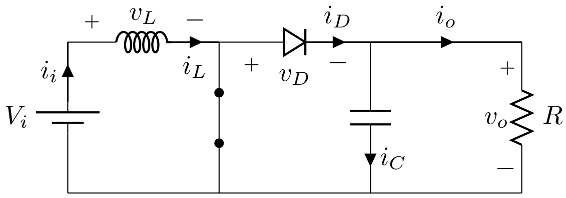


Known variables:

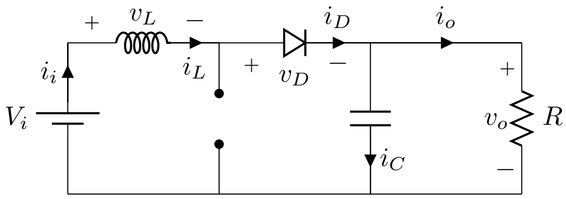
- Input voltage, V_i (constant, positive voltage)
- Switch square control signal, D and T
- Circuit components, L, C, R.

State of the diode

- **ON Circuit**: Since $v_D = 0 - v_o = -v_o$, the diode is reverse polarized (OFF), if $v_o > 0$. We can make that asumprion and check later.



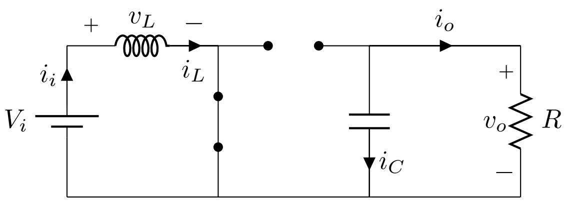
- **OFF circuit**: $v_D = V_i - v_L - V_o$. We assume the diode is ON for the current in the inductor to be continuous.



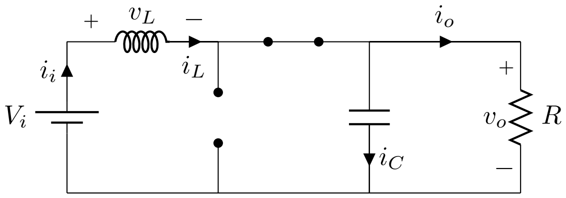
Diode table

Switch	D
ON	OFF
OFF	ON

ON equivalent circuit

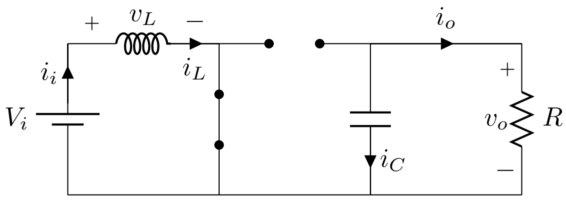


OFF equivalent circuit

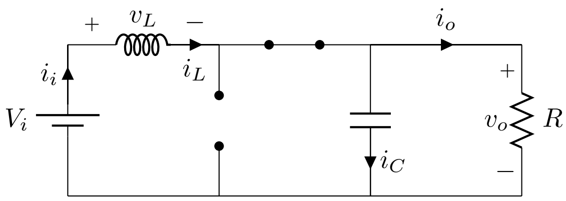


Inductor voltage

1. **ON**. KVL @ Inductor: $v_L = V_i$



2. **OFF**. KVL @ Inductor: $v_L = V_i - v_o$



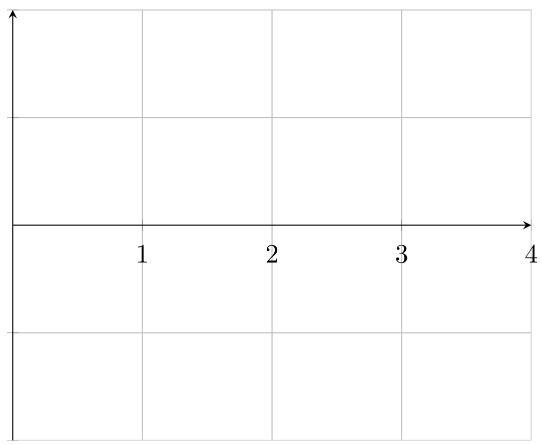
The voltage in the inductor is

$$v_L(t) = \begin{cases} V_i & \text{ON} \\ V_i - v_o & \text{OFF} \end{cases}$$

If we

- apply the small ripple approximation (consider the $v_o(t)$ constant, V_o),
- suppose that $V_i - V_o < 0$, for the inductor to be in periodic steady state (we will check in the output voltage calculation). If we asume that the output voltage sign is placed correctly and $V_o > V_i$, as it would be normal in a step-up converter, then $V_i - V_o < 0$ is true.

we can represent the inductor voltage waveform:



Output voltage

In order to calculate the output voltage of the converter, we have to apply the condition $\bar{v_L} = 0$ in the inductor.

1. In periodic steady state, the average voltage of an inductor is zero, $\bar{v_L} = 0$

   1.1 Mean value definition

$$\bar{v_L}(t) = \frac{1}{T}\int_{t_0}^{t_0+T} v_L(t)dt = 0$$

   1.2 Substitute the definition of $V_L(t)$ and split the integral

$$\cancel{\frac{1}{T}}\left[\int_0^{DT}(V_i)dt + \int_{DT}^{T}(V_i - v_o(t))dt\right] = 0$$

2. In order to obtain a value for $v_o(t)$ from the integral we have to suppose it is contant. We apply the small-ripple approximation: $v_o(t)$ can be considered constant: $v_o(t) \approx V_o$.

$$V_iD\cancel{T} + (V_i - V_o)(1-D)\cancel{T} = 0$$

$$\cancel{V_iD} + V_i - \cancel{V_iD} - V_o + V_oD = 0$$

$$V_i = V_o(1-D)$$

$$V_o = V_i\frac{1}{1-D}$$

Since $V_i > 0$ and $D < 1$, we can say

- $V_o > 0$, so the sign criteria chosen for the diagram shows the correct polarization.
- $V_o > V_i$, so it is indeed a step-up converter.

From now in advance, all the calculations will be made with the small ripple approximation, so

- $v_o(t)$ ripple is negligible, so $v_o(t) \approx \bar{v_o}(t) = V_o$
- If v_o is constant, then also $i_o(t)$ is constant: $i_o(t) = \bar{i_o}(t) = I_o$

Duty Cycle

$$D = 1 - \frac{V_i}{V_o}$$

Output power

$$P_o = \frac{V_o^2}{R}$$

Inductor current

Switch tension

Switch current

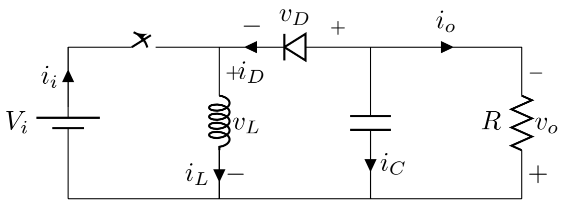
Diode tension

Diode current

Capacitor current

Output current

2.3 Buck-Boost Converter



Known variables:

- Input voltage, V_i (constant, positive voltage)
- Switch square control signal, D and T
- Circuit components, L, C, R.

State of the diode

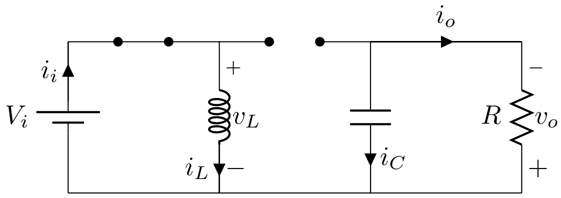
- **ON Circuit**:

- **OFF circuit**

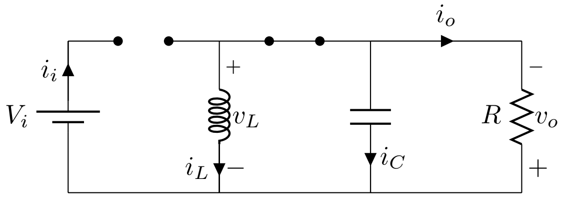
Diode table

Switch	D
ON	OFF
OFF	ON

ON equivalent circuit



OFF equivalent circuit



Inductor voltage

Output voltage

$$V_o = \frac{D}{1-D}V_i$$

Inductor current

Switch tension

Switch current

Diode tension

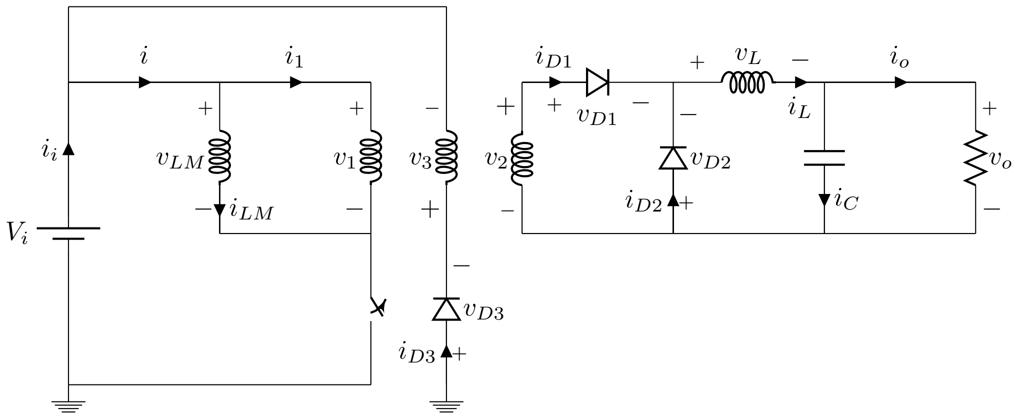
Diode current

Capacitor current

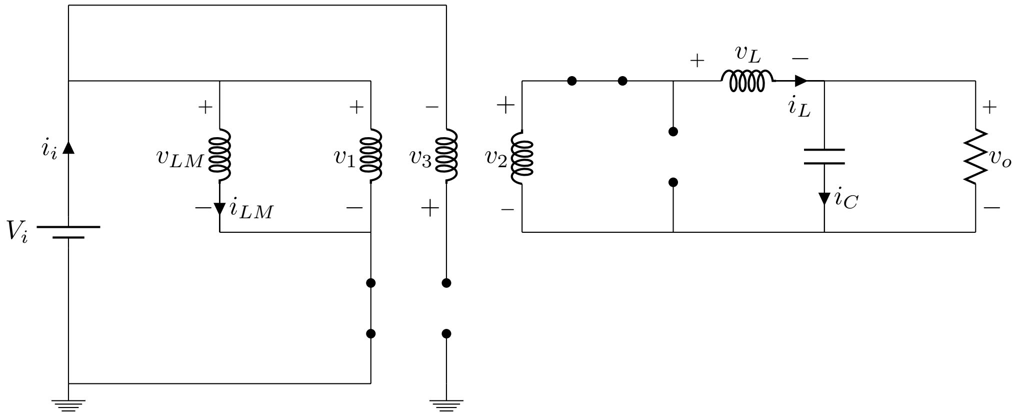
Output current

3 Isolated Converters

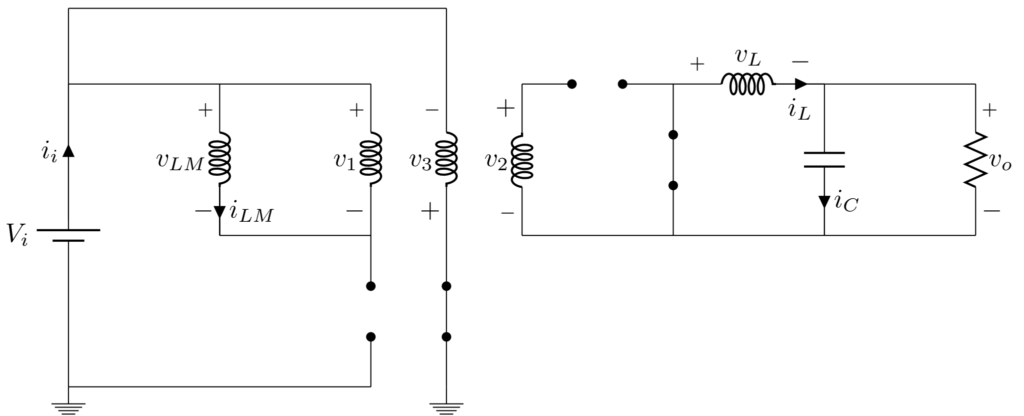
3.1 Forward Converter



ON circuit



OFF circuit



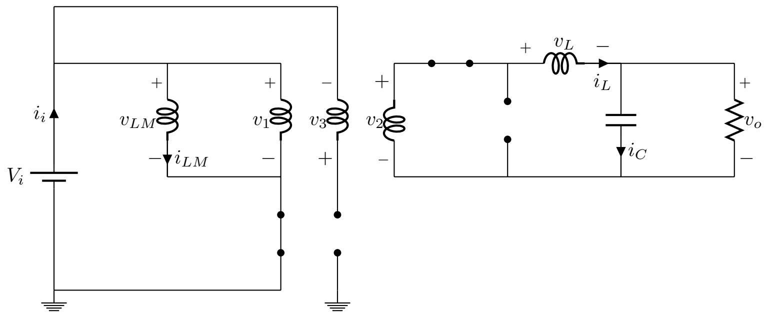
Inductor voltage

- **ON**

$$v_L = v_2 - v_o$$

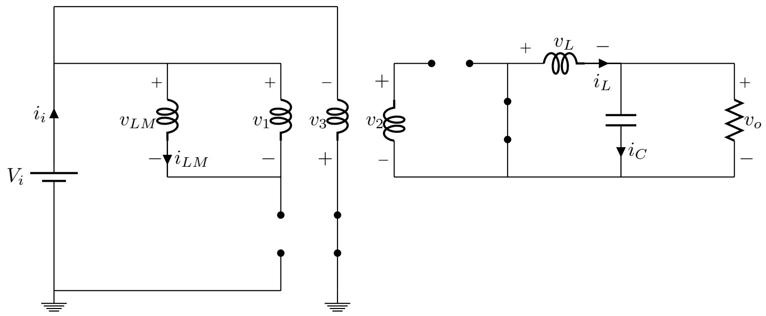
$$v_L = \frac{N_2}{N_1} v_1 - v_o$$

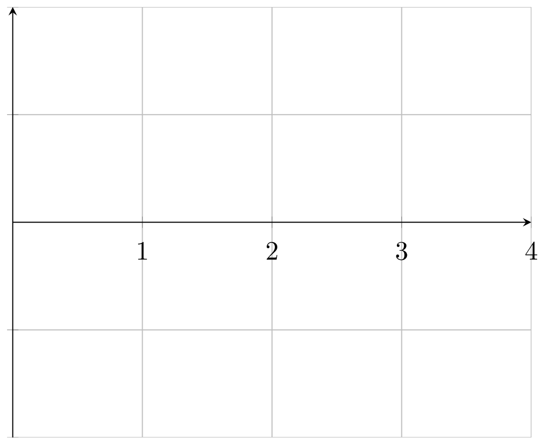
$$v_L = \frac{N_2}{N_1} V_i - v_o$$



- **OFF**

$$v_L = 0 - v_o = -v_o$$





Output voltage

$$\left(\frac{N_2}{N_1}V_i - V_o\right) D\cancel{T} + (-V_o)(1-D)\cancel{T} = 0$$

$$\frac{N_2}{N_1}DV_i - \cancel{V_o D} + -V_o + \cancel{V_o D} = 0$$

$$V_o = \frac{N_2}{N_1}DV_i$$

Magnetizing inductor voltage

- ON

$$v_{LM} = V_i$$

- OFF

$$v_{LM} = v_2$$

$$v_2 = \frac{N_1}{N_3} v_3$$

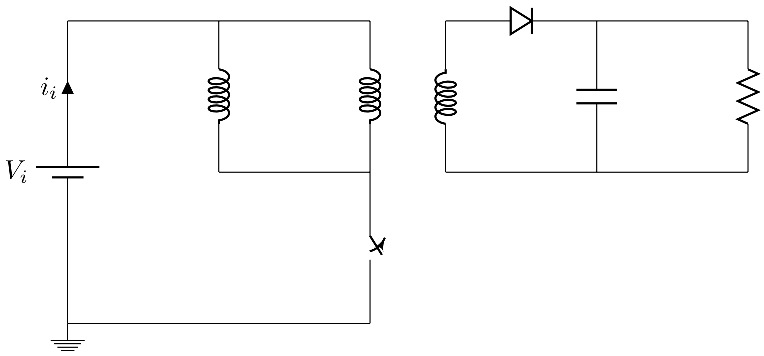
$$v_3 = -v_1$$

$$v_{LM} = \frac{N_1}{N_3} V_i$$

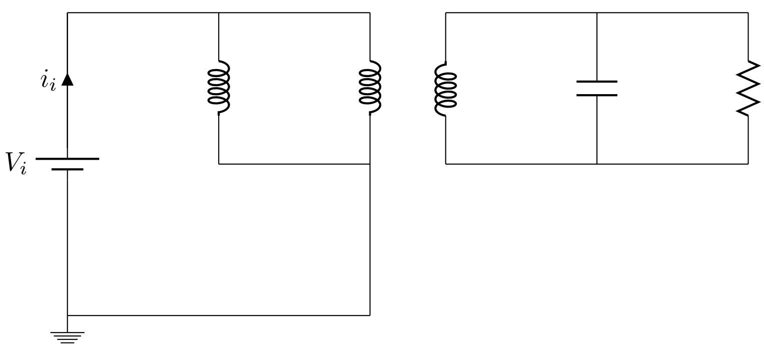
Maximum Duty Cycle

$$V_{up} DT + V_{down}(1 - D)T = 0$$

3.2 Flyback Converter



ON circuit



OFF circuit