AC to DC Converters

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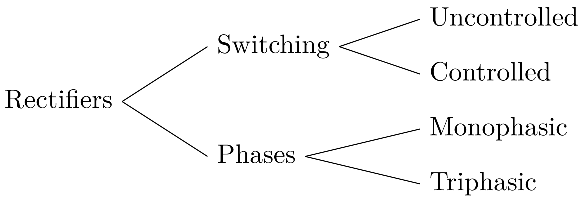
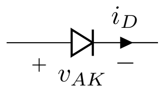
1 AC to DC Converters, Rectifiers

A AC/DC Converter, or rectifier, is an electrical device that converts alternating current (AC), which periodically reverses direction, to direct current (DC), which flows in only one direction. Physically, rectifiers take a number of forms, including vacuum tube diodes, mercury-arc valves, copper and selenium oxide rectifiers, semiconductor diodes, silicon-controlled rectifiers and other silicon-based semiconductor switches. Historically, even synchronous electromechanical switches and motors have been used. #

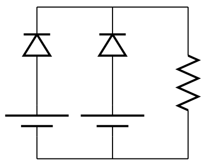


- Uncontrolled rectifiers use diodes as switching devices. They don't need a control circuit.

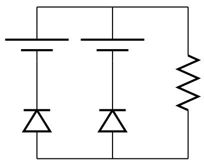
Diodes

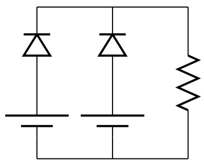
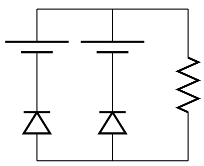


Common Cathode Diodes If two or more diodes are connected with a common cathode, the closed diode is the one with the most positive anode voltage. The rest of the diodes are open.



Common Anode Diodes If two or more diodes are connected with a common anode, the closed diode is the one with the most negative anode voltage. The rest of the diodes are open.



Configuration	Circuit diagram	Conducting Diode
Common Cathode		The diode with the most positive anode voltage
Common Anode		The diode with the most negative cathode voltage

General rule: The most polarized diode is the one conducting. The one with the biggest v_{AK}

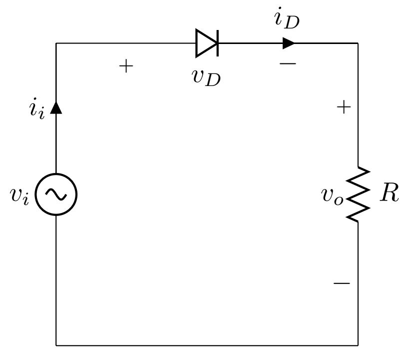
Thyristors

(Ideal)

- Closing conditions
  - $v_A K > 0$, the thyristor is forward polarized
  - i_G, current in the gate (thyristors are current-controlled)
- Opening conditions
  - $i_A K < 0$, current is reversed

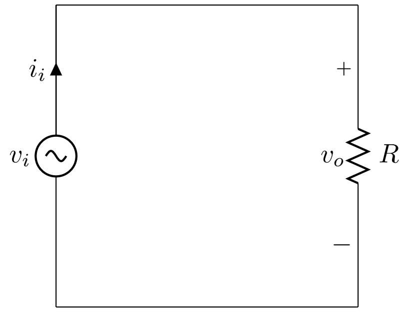
2 Monophasic Rectifiers

2.1 Half Wave Rectifier

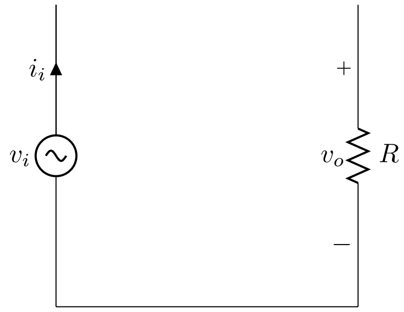


- $v_i > 0$: D ON, $i > 0$
- $v_i < 0$: D OFF, $i = 0$

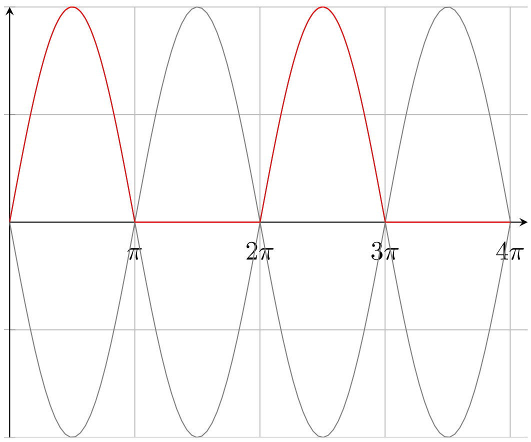
$v_i > 0$ (D ON) equivalent circuit



$v_i < 0$ (D OFF) equivalent circuit

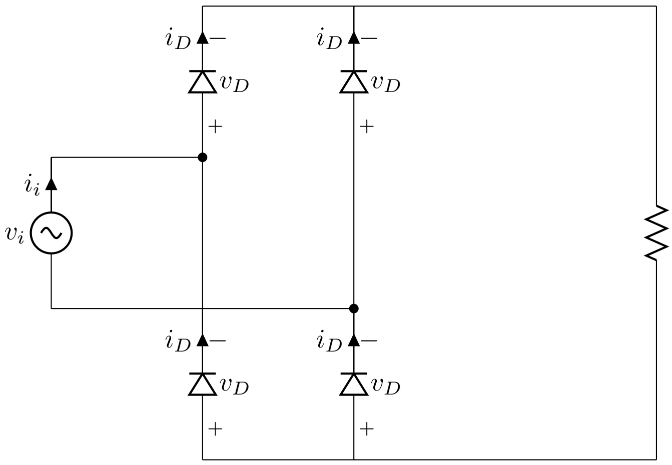


Output voltage, v_o



The voltage in the resistive load, v_R, and the current demanded to the source depends on the load type, and can be different for a same kind of rectifier depending on the load that is connected to it.

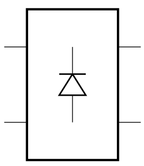
2.2 Full Wave Rectifier



where

- v_i is a pure sinusoidal wave $v_i = V_i\sqrt{2}\sin(2\pi f)$
  - RMS value is V_i
  - Peak value, or V_{ip} is $V_i\sqrt{2}$
  - Frecuency is f
  - $\varphi = 0$ beacause it is the reference for phase shifts
- R is resitance that models the load

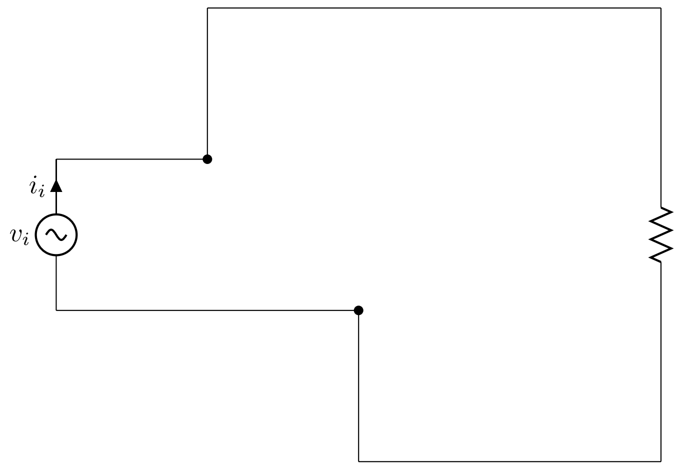
Full Wave Rectifier Symbol



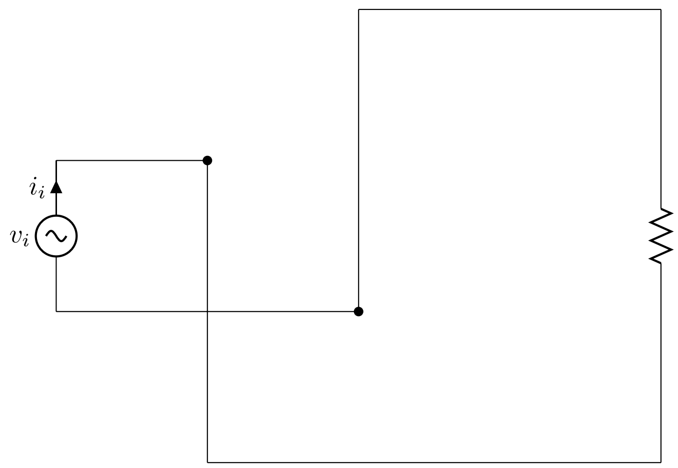
Diode table

	D1	D2	D3	D4
$v_i > 0$	ON	OFF	OFF	ON
$v_i < 0$	OFF	ON	ON	OFF

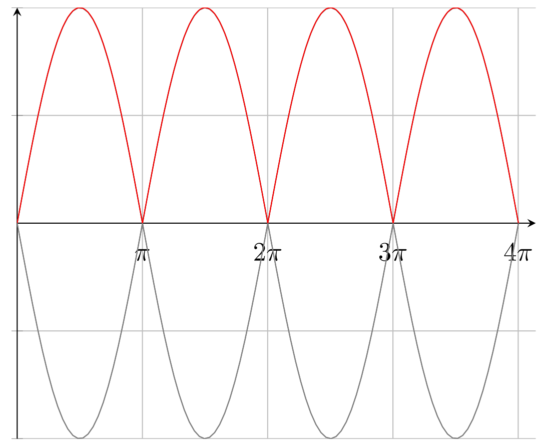
$v_i > 0$ **equivalent circuit:** D1, D4 are ON



$v_i < 0$ **equivalent circuit:** D2, D3 are ON



Output voltage, v_o

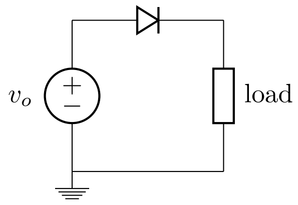


The voltage in the resistive load, v_R, and the current demanded to the source depends on the load type, and can be different for a same kind of rectifier depending on the load that is connected to it.

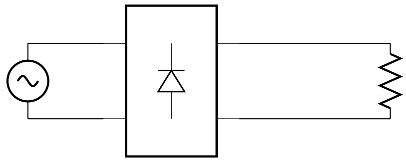
Note: Although this v_o waveform can always be used with the reduced model circuit, where it is behind a diode, in some cases, like the RC load, the v_o node in the complete circuit coincides with the v_R node and the output voltage directly after the rectifier has not this typical rectified waveform, but a filtered version of it.

Reduced model of the rectifier If v_o is already known, a simplified model of the rectifier can be represented to study the loaded rectifier. The left hand side of the circuit is substituted by

- a v_o voltage source, that represents the bridge output voltage
- a diode that represents that current cannot enter in the rectifier

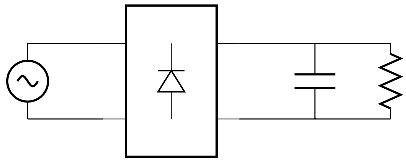


2.2.1 R Load



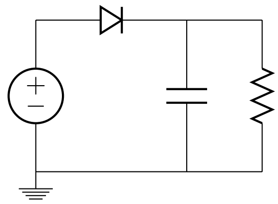
2.2.2 RC Load

A condenser can be added in parallel with the R load to smooth the voltage across it. In this case, the circuit is



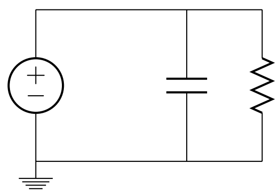
Output voltage, v_o; Load voltage, v_R

- Mean value



Not exactly like this, it's more complex.

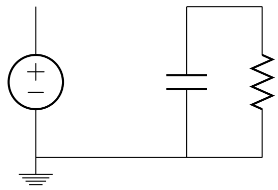
- When the diode conducts, $v_o = v_R$ and the capacitor is charged with the current from the source.



The charge is sinusoidal:

$$i_C(t) = \frac{dv(t)}{dt} = sinusoidal$$

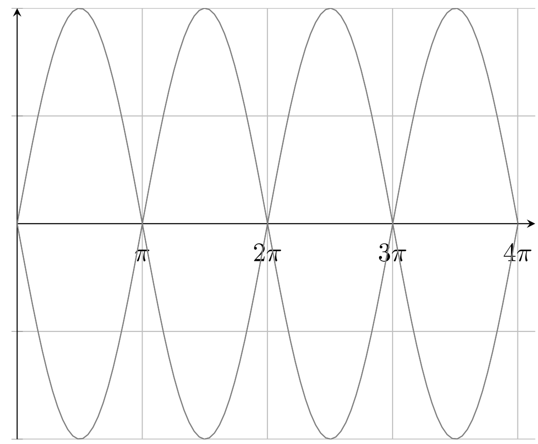
- When the diode is not conducting, the capacitor provides the current to the load. It would continue to discharge until $v_o = v_i$.



The discharge is exponential:

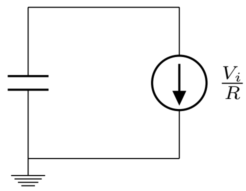
$$formulita$$

The voltage in the load is:



Triangular approximation

- The charge is considered instantaneous: $t_{\text{charge}} = 0 \rightarrow t_{\text{discharge}} = \frac{T}{2}$
- The discharge is linear, with constant current: $I_R = V_m/R$
- The diode opens in the maximum of v_o: $I_R = V_i/R$ (in the slides it says "if ripple is small enough").



The ripple of the wave using the triangular approximation is

$$\Delta v_o = \frac{V \Delta T}{C}$$

$$\Delta v_o = \frac{V_i T}{2RC}$$

Mean value Can be calculated from the triangular approximation

$$\bar{v_o} = V_{ip} - \frac{1}{2} \Delta v_o$$

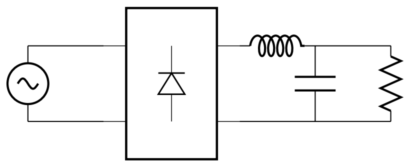
Advantages of a smoothing condenser

- Small size
- Cheap
- Robust

Disadvantages of a smoothing condenser

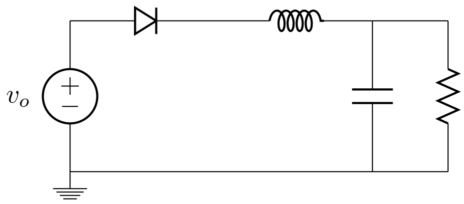
- High peaks of current through the diodes.
- Harmonics in the source current.
- Need of a big capacitor if the required ripple is small.

2.2.3 RLC Load

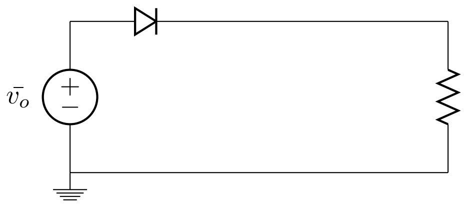


v_R mean value

The mean value of v_o can be calculated by using superposition with the circuit:



For the mean values, the resulting DC circuit is:



The resulting $\bar{v_R}$ is

$$\bar{v_R} = \bar{v_o} = \frac{2V_{ip}}{\pi}$$

RLC circuit transfer function

$$\mathbf{H} = \frac{\mathbf{V_o}}{\mathbf{V_i}} = \frac{1}{1 + \frac{L}{R}j\omega - LC\omega^2}$$

$$H = |\frac{\mathbf{V_o}}{\mathbf{V_i}}| = \frac{1}{\sqrt{(1 - LC\omega^2)^2 + (\frac{L}{R}\omega)^2}}$$

v_R ripple

Ripple is due to the first harmonic:

$$\Delta v_R = H_{filter} \cdot |V_{o1}|$$

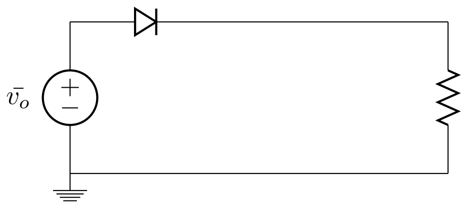
$$H = \frac{1}{\sqrt{(1 - LC\omega^2)^2 + (\frac{L}{R}\omega)^2}}$$

$$|V_{o1}| = \frac{4}{\pi}\frac{1}{3}V_{ip}$$

Inductor current, i_L

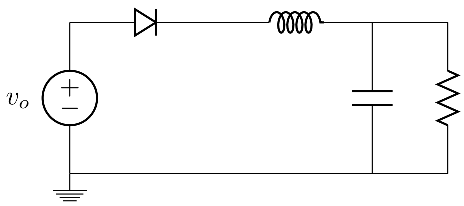
Inductor current can be approximated with its mean value and ripple.

- **Mean value**. Using superposition



$$i_L = \frac{\bar{v_o}}{R}$$

- **Ripple**. As in the v_R ripple, maximum variation of the current is due to the 1st harmonic of v_o. Thus, ripple can be calculated:



$$I_{1p} = \frac{V_{o1}}{Z_1}$$

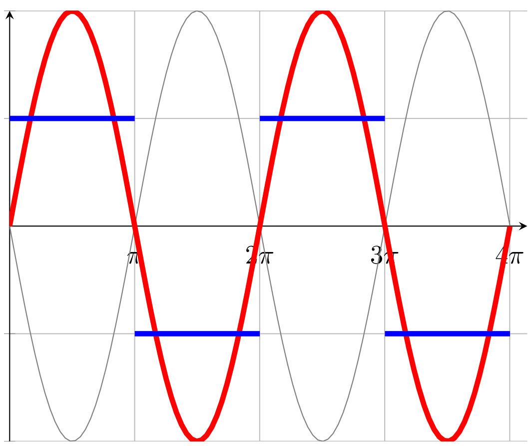
If Δv_R is already calculated, it can be used to compute the ripple:

$$\Delta i_L = \frac{\Delta v_R}{Z_{RC}}$$

The rectifier is in CCM if the inductor is always conducting some current:

$$\frac{1}{2}\Delta i_l < \bar{i_L}$$

Power factor in the source



$$\text{PF} = \frac{P}{S}$$

- Only first harmonic delivers power:

$$P = V_1 I_1 \cos(\varphi)$$
$$P = \frac{V_{ip}}{\sqrt{2}} \frac{4}{\pi} \frac{I_o}{\sqrt{2}} \cos(\alpha)$$

- Apparent power

$$S = V_{\mathrm{RMS}} I_{\mathrm{RMS}}$$
$$S = \frac{V_{ip}}{\sqrt{2}} I_o$$

$$\mathrm{PF} = \frac{P}{S} = \frac{\frac{V_{ip}}{\sqrt{2}} \frac{4}{\pi} \frac{I}{\sqrt{2}} \cos(\alpha)}{\frac{V_{ip}}{\sqrt{2}} I_o}$$
$$\mathrm{PF} = \frac{4}{\pi} \frac{1}{\sqrt{2}} = \frac{2\sqrt{2}}{\pi} \cos(\alpha)$$

Advantages of LC filtering

- No current peaks in the diodes.
- Less condenser ripple.
- Less capacity and current capacitor required.

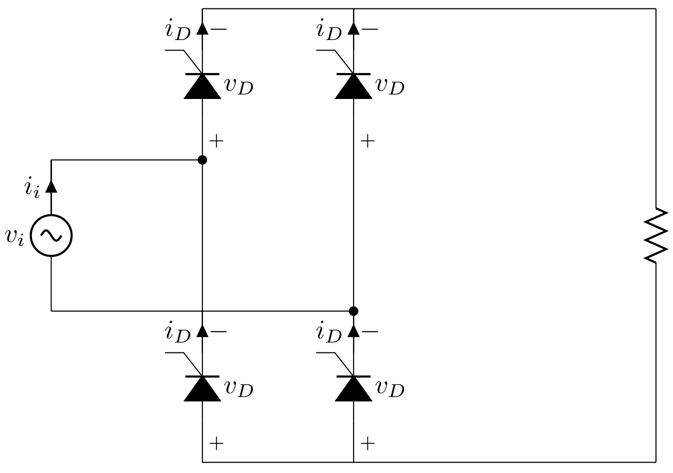
Disadvantages of LC filtering

- Size and weight of the inductor

2.2.4 RL load

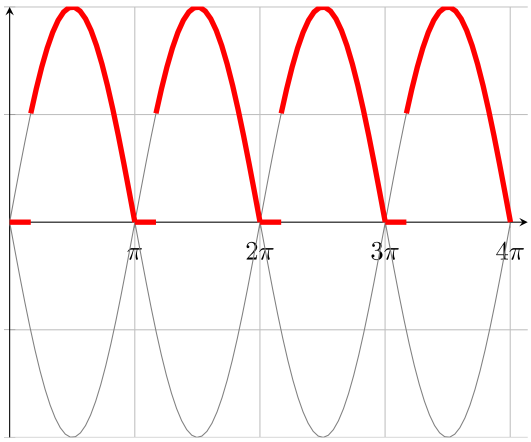
Flyback Diode

2.2.5 Controlled Full Wave Rectifier R



where

- v_i is a pure sinusoidal wave $v_i = V_i\sqrt{2}\sin(2\pi f)$
  - RMS value is V_i
  - Peak value, or V_{ip} is $V_i\sqrt{2}$
  - Frecuency is f
  - $\varphi = 0$ beacause it is the reference for phase shifts
- R is resitance that models the load
- α is the delay angle for thyristors

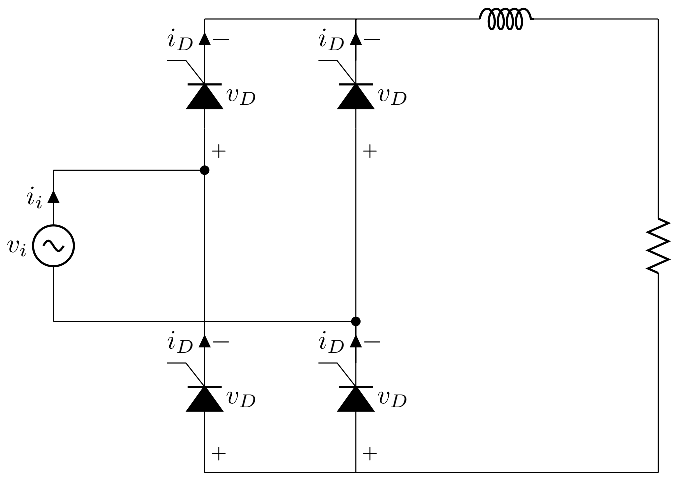


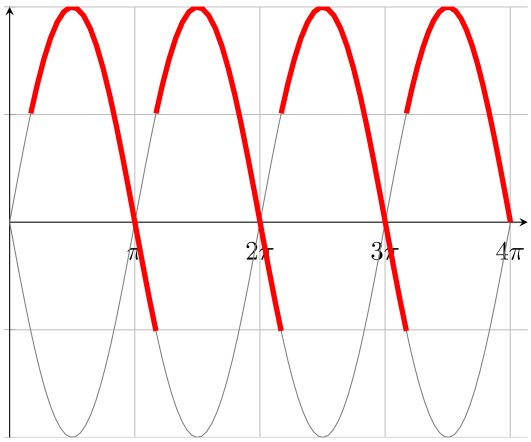
v_o mean value, $\bar{v_o}$

$$\begin{aligned}
\bar{v_o} &= \frac{1}{T}\int_{\theta_0}^{\theta_0+T} v_o(\theta)d\theta \\
&= \frac{1}{\pi}\int_{0}^{\pi} v_o(\theta)d\theta \\
&= \cancelto{0}{\frac{1}{\pi}\int_{0}^{\alpha} 0d\theta} + \frac{1}{\pi}\int_{\alpha}^{\pi} V_{ip}\sin\left(\theta\right)d\theta \\
&= \frac{V_{ip}}{\pi}[-\cos\left(\theta\right)]_{\alpha}^{\pi} \\
&= \frac{V_{ip}}{\pi}[-\cos\left(\pi\right)+\cos\left(\alpha\right)] \\
&= \frac{V_{ip}}{\pi}(\cos\left(\alpha\right)+1)
\end{aligned}$$

2.2.6 Controlled Full Wave Rectifier RL

When it is loaded with a RL load, current cannot be suddenly cut and thyristos continue to be closed even when they're polarized backwards, until the other bramch is triggered. The





v_o mean value, $\bar{v_o}$

$$
\begin{aligned}
\bar{v_o} &= \frac{1}{T}\int_{\theta_0}^{\theta_0+T} v_o(\theta)d\theta \\
&= \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_{ip}\sin(\theta)\,d\theta \\
&= \frac{V_{ip}}{\pi}[-\cos(\theta)]_{\alpha}^{\alpha+\pi} \\
&= \frac{V_{ip}}{\pi}[-\cos(\alpha+\pi)+\cos(\alpha)] \\
&= \frac{2V_{ip}}{\pi}\cos(\alpha)
\end{aligned}
$$

$\bar{v_o}$ with flyback diode Same as R loaded converter

$$
\begin{aligned}
\bar{v_o} &= \frac{1}{T}\int_{\theta_0}^{\theta_0+T} v_o(\theta)d\theta \\
&= \frac{1}{\pi}\int_{0}^{\pi} v_o(\theta)d\theta \\
&= \cancelto{0}{\frac{1}{\pi}\int_{0}^{\alpha} 0d\theta} + \frac{1}{\pi}\int_{\alpha}^{\pi} V_{ip}\sin(\theta)\,d\theta \\
&= \frac{V_{ip}}{\pi}[-\cos(\theta)]_{\alpha}^{\pi} \\
&= \frac{V_{ip}}{\pi}[-\cos(\pi)+\cos(\alpha)] \\
&= \frac{V_{ip}}{\pi}(\cos(\alpha)+1)
\end{aligned}
$$

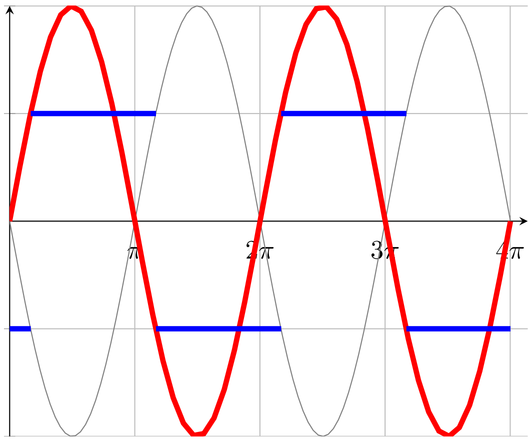
Phases *hacer mejor, poner ciruiticos, explicar por que se extingue la corriente a traves de los tiristores*

1. $v_i > 0$, T1 and T4 triggered
   - T2 and T3 are backwards polarized
   - T1 and T4 have been triggered and are conducting
   - $i_i > 0$
2. $v_i < 0$, T2 and T3 haven't been triggered yet
   - T2 and T3 are forward polarized but haven't been triggered yet
   - T1 and T4 are still conduction because current through the inductor cannot be discontinued
   - $i_i > 0$
3. $v_i < 0$, T2 and T3 triggered

- T1 and T4 are triggered and start conducting.
- Current extingueshes through T2 and T3, that cur and become backwards polarized
- $i_i < 0$

4. $v_i > 0$, T2 and T3 haven't been triggered yet
   - $i_i < 0$

Source current, i_i



Tip: For drawing the current, when the thyristors are triggered during the positive semicycle of the v_i current, current flows in the positive direction. Or, α delays the current square wave.

Power

Calculated in the source:

Only first harmonic produces power:

$$P = V_{1RMS} I_{1RMS} \cos(\varphi_1)$$
$$P = \frac{V_{ip}}{\sqrt{2}} \frac{4}{\pi} \frac{I}{\sqrt{2}} \cos(\alpha)$$
$$P = \frac{2V_{ip}I}{\pi} \cos(\alpha)$$

Power factor in the source

$$\text{PF} = \frac{P}{S}$$

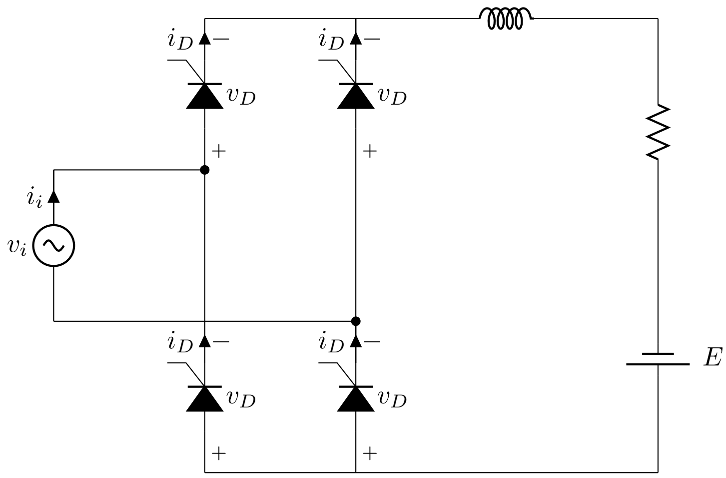
$$P = \frac{2V_{ip}I}{\pi} \cos(\alpha)$$

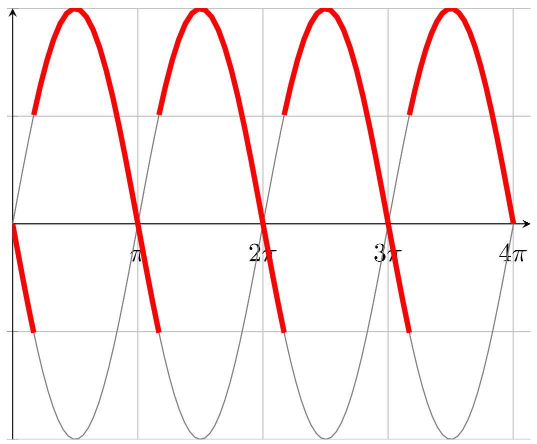
$$S = V_{RMS} I_{RMS}$$
$$S = \frac{V_{1p}}{\sqrt{2}} I$$

$$\text{PF} = \frac{P}{S} = \frac{2\sqrt{2}}{\pi} \cos(\alpha)$$

2.2.7 Controlled Full Wave Rectifier RLE

RLE loads can be used to transfer power to the net, for example in solar powers. Using a rectifier to inject power in the net is cheaper than using an inverter because it uses thyristors instead of MOSFETs/IGBTs, which are more expensive.

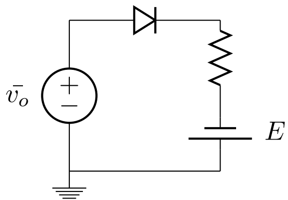




v_o mean value, $\bar{v_o}$

$$
\begin{aligned}
\bar{v_o} &= \frac{1}{T}\int_{\theta_0}^{\theta_0+T} v_o(\theta)d\theta \\
&= \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_{ip}\sin(\theta)\,d\theta \\
&= \frac{V_{ip}}{\pi}[-\cos(\theta)]_{\alpha}^{\alpha+\pi} \\
&= \frac{V_{ip}}{\pi}[-\cos(\alpha+\pi)+\cos(\alpha)] \\
&= \frac{2V_{ip}}{\pi}\cos(\alpha)
\end{aligned}
$$

Power flow Three operanting regions, depending of the value of $\bar{v_o}$:



$$i = \frac{\bar{v_o} - (-E)}{R}$$

- if $i > 0$
  - if $\bar{v_o} > 0$
    * The rectifier produces power
    * The E source produces power
    * R consumes all the power
    * Bad operating regime
  - if $\bar{v_o} < 0$
    * The rectifier consumes power
    * The E source produces power
    * R consumes some power
    * Good operating regime if you want to inject power in the net
- if $i = 0$ the diode blocks the current and no power is consumed or generated

So, if you want to inject power from a DC source into the net with a rectifier, the output voltage of the rectifier should be negative, but current must be flowing from it.

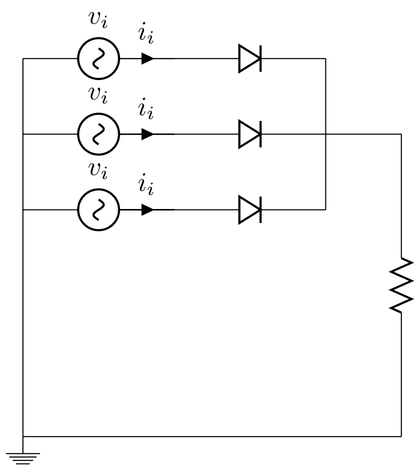
The limit angle is calculated from the condition $i = 0$:

$$0 = \frac{\bar{v_o} - (-E)}{R}$$

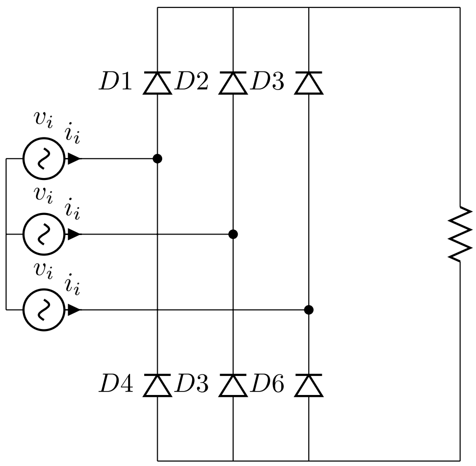
$$\bar{v_o} = -E$$

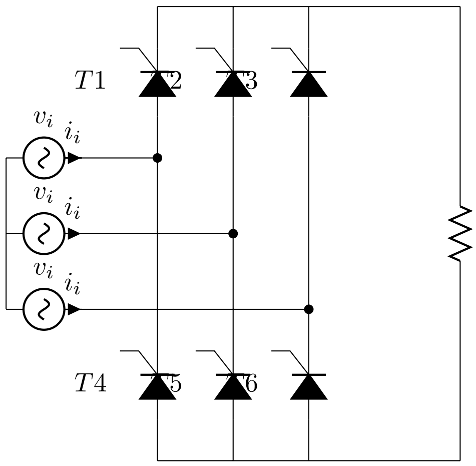
3 Triphasic Rectifiers

3.1 Half Wave Triphasic Rectifier



3.2 Full Wave Triphasic Rectifier





4 Reference

- https://en.wikipedia.org/wiki/Rectifier

$$\alpha_{max} = 180 \text{ para cualquier convertidor}$$