AC Cheatsheet

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1 Sinusoidal signal

A sinusoidal signal in AC takes the form

$$s(t) = \sqrt{2}A \cdot \sin(\omega t + \varphi)$$

where

- $A \equiv A_{RMS}$ is the RMS value of the signal
- ω is the angular frecuency, $\omega = 2\pi f$ $[rad]$
- Frecuency, $f = \omega/2\pi$
- Period, $T = 1/f$
- Amplitude of the signal is $A/\sqrt{2}$

Note: In AC the sinusoidal function amplitude is taken as $A/\sqrt{2}$, so A is the RMS value of the signal. This way, we can directly use the RMS values of the signals to make calculations with power.

Average and RMS values

- Average value: $S_{AVG} = 0$
- RMS value: $S_{RMS} = A$

1 Phasors and complex numbers

1 Phasor

$$s(t) = \sqrt{2}A \cdot \sin(\omega t + \varphi) \rightarrow \mathbf{S} = A\angle\varphi$$

In physics and engineering, a phase vector, or phasor, is a representation of a sinusoidal function whose amplitude, frequency, and phase are time-invariant

Euler's formula indicates that sinusoids can be represented mathematically by the sum of two complex-valued functions:

$$A \cdot \cos(\omega t + \theta) = A \cdot \frac{e^{i(\omega t+\theta)} + e^{-i(\omega t+\theta)}}{2}$$

or by the real part of one of the functions:

$$A \cdot \cos(\omega t + \theta) = \operatorname{Re}\left\{A \cdot e^{i(\omega t+\theta)}\right\} = \operatorname{Re}\left\{Ae^{i\theta} \cdot e^{i\omega t}\right\}.$$

The term phasor can refer to either $Ae^{i\theta}e^{i\omega t}$ or just the complex constant, $Ae^{i\theta}$. In the latter case, it is understood to be a shorthand notation, encoding the amplitude and phase of an underlying sinusoid.

An even more compact shorthand is angle notation: $A\angle\theta$.

1 Generalized Ohm Law, complex impedance

The **Generalized Ohm Law**:

$$\mathbf{V} = \mathbf{ZI}$$

where $\mathbf{Z}$ is

- $\mathbf{Z} = R$ for resistors
- $\mathbf{Z} = \frac{1}{j\omega C} = \frac{-j}{\omega C}$ for capacitors
- $\mathbf{Z} = j\omega L$ for inductors

1 AC Power

Power in a two-terminal component

- Voltage wave with RMS value V: $v(t) = \sqrt{2}V \cdot \cos(\omega t)$ ($\varphi = 0$ because it's the reference)
- Current wave with RMS value I: $v(t) = \sqrt{2}I \cdot \cos(\omega t + \varphi)$

$$p(t) = v(t)i(t) = 2V \cdot \sin(\omega t)\, I \cdot \cos(\omega t + \varphi)$$
$$p(t) = VI\cos(\varphi) + VI\cos(2\omega t - \varphi)$$

Active and reactive power

- Active power: $P = VI\cos(\varphi)$
- Reactive power: $Q = VI\sin(\varphi)$

Complex and apparent power

- Complex power is the complex number $\mathbf{S} = \mathbf{V}\mathbf{I}^* = P + Qj$
- Apparent power is the module of the complex power, $S = VI$

Power factor

$$PF = \frac{P}{S}$$

$$PF = \cos(\varphi)$$

AC power in a resistor

$$P = VI$$
$$Q = 0$$

Power in a inductor

Power in a capacitor