Circuit Signals

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1 Circuit Signals

Circuit signal A circuit signal, $s(t)$, is a function of time that represents the values that an electrical magnitude in a circuit takes over time.

1 Periodic signals

Periodic signal A periodic signal is a signal $s(t)$ that satisfies $s(t) = s(t+T)$ for all t

- *Note* Constant signals are periodic signals because they fulfill the condition.

Periodic signal parameters

- The period is the smallest value of T satisfying $u(t+T) = u(t)$ for all t
- Frecuency, $f = 1/T\ [Hz]\ [s^{-1}]$
- Angular frecuency, $\omega = 2\pi f\ [rad]$
- Amplitudes
  - Peak-to-peak amplitude
  - Peak amplitude, commonly referred as just amplitude

1 Average value of a periodic signal

The average value S_{avg} or $\bar{s(t)}$ of a periodic signal $S(t)$ is

$$S_{avg} = \bar{s(t)} = \frac{1}{T}\int_{t}^{t+T} s(t)dt$$

- S_{avg} can be negative, positive or zero

Average power Average power in a component is a case of the average value of a periodic signal. The average power P or $\bar{p(t)}$ dissipated in a component is

$$P = \bar{p(t)} = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)dt$$

1 Root Mean Square of a periodic signal

The Root Mean Square S_{RMS} of a periodic signal $s(t)$ is

$$S_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} s(t)^2 dt}$$

Motivation for RMS value RMS value of a voltage or current signal (constant or non-constant) has the property that is the value of an equivalent DC voltage or current signal that would dissipate the same power through a resistance.

Average power has the expression $P = \bar{p(t)} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)dt$ and using the RMS value of the signals we want to express it in a similar way to DC, where power is the product of two constants: $P = VI$.

a) **RMS value of current**

a.1) Using Ohm Law derivate the power expression in DC: $P = VI = RI^2$.

a.2) Calculate the expression for I_{RMS}, that satisfies $P = RI_{RMS}^2$.

$$P = RI_{RMS}^2 = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt$$

$$RI_{RMS}^2 = \frac{1}{T}\int_{t_0}^{t_0+T} Ri^2(t)\,dt$$

$$\cancel{R}I_{RMS}^2 = \cancel{R}\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt$$

$$I_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt}$$

b) **RMS value of voltage**

b.1) Using Ohm Law derivate the power expression in DC: $P = VI = \frac{V^2}{R}$.

b.2) Calculate the expression for I_{RMS}, that satisfies $P = VI = \frac{V_{RMS}^2}{R}$.

$$P = \frac{V_{RMS}^2}{R} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt$$

$$\frac{V_{RMS}^2}{R} = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{1}{R}v^2(t)\,dt$$

$$\cancel{\frac{1}{R}}V_{RMS}^2 = \cancel{\frac{1}{R}}\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\,dt$$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\,dt}$$

c) Check results with the $P = VI$ formula. *I don't know how to prove it yet, or even if it is true*

$$P = VI = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\,dt}\sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt}$$

$$P = \frac{1}{T}\sqrt{\int_{t_0}^{t_0+T} v^2(t)\,dt \cdot \int_{t_0}^{t_0+T} i^2(t)\,dt}$$

Properties of RMS value

- The RMS value of a function is the same as the RMS value of the absolute value of the function

$$RMS[f(t)] = RMS[|f(t)|]$$

1 Signal Examples

1.1.3.1 Sinusoidal signal

$$s(t) = A \cdot \sin(\omega t + \varphi)$$

Average value of the sinusoidal signal

$$S_{AVG} = \frac{1}{T}\int_{t}^{t+T} A\cos(t)\,dt$$

$$S_{AVG} = \frac{A}{T}\left[\sin(t)\right]_t^{t+T} dt = 0$$

RMS value of the sinusoidal signal

$$
\begin{aligned}
S_{RMS} &= \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T}[A\cos{(t)}]^2 dt} \\
&= \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T}\left[A\cos{(t)}^2\, dt\right.} \\
&= \sqrt{\frac{A^2}{T}\int_{t_0}^{t_0+T}\frac{1+\cos{(2t)}}{2}dt} \\
&= \sqrt{\frac{A^2}{T}[1/2(t+1/2\sin{(2t)})]_t^{t+T}} \\
&= \sqrt{\frac{A^2}{2T}T} = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}
\end{aligned}
$$

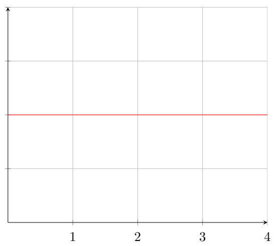
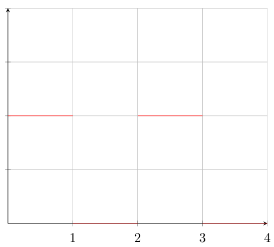
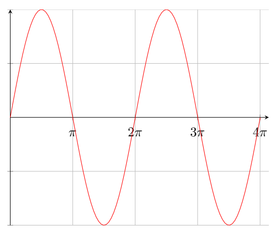
1 Normalized Average and RMS values table

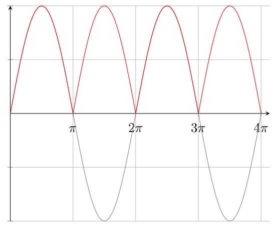
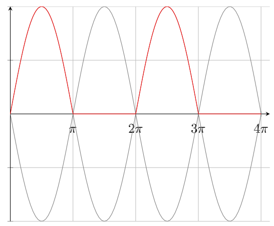
- Amplitudes are normalized with the peak value of the signal

$$s_N = \frac{s_R}{S_{MAX}}$$

- Frecuency of sinusoidal waves is normalized for the first period to be in the interval $[0, 2\pi]$

$$\theta = \omega t$$
$$t = \frac{\theta}{\omega}$$

Signal	Average value	Root Mean Square Value
Constant 	1	1
Square Wave 	d	$\sqrt{d}$
Sinusoidal Wave 	0	$\frac{1}{\sqrt{2}}$

Fully Rectified Sinusoidal Wave 	$\frac{2}{\pi}$	$\frac{1}{\sqrt{2}}$
Half Rectified Sinusoidal Wave 	$\frac{1}{\pi}$	$\frac{1}{2}$

1 Periodic signal decomposition in Fourier Series

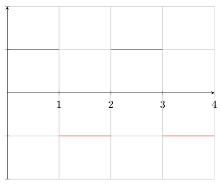
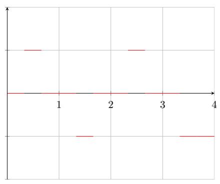
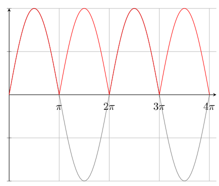
Motivation The phasor method can be used to solve circuits with sinusoidal signals (AC circuits), avoiding the necessity of solving the time-domain differential equations. A circuit with other type of waveforms cannot be analysed with this method, unless it is decomposed into its sinusoidal components. The Fourier series allows us to decompose any waveform to a series of sine and cosine functions. Then, we can solve each circuit (an AC circuit for each sinusoidal component of the wave) separatedly and, apllying superposition, add up all the solutions to get the solution of the original circuit.

Fourier decomposition Any periodic signal can be decomposed in a sum of a constant value plus a (possibly infinite) series of sine and cosine functions.

$$s(x) = S_{avg} + \sum_{n=1}^{\infty} A_n \cos(n\omega t) + \sum_{n=1}^{\infty} B_n \sin(n\omega t)$$

Fourier series table

Function	Fourier Series

	$$\frac{4}{\pi}\left(\frac{\sin(t)}{1}+\frac{\sin(3t)}{3}+\frac{\sin(5t)}{5}+\ldots\right)$$
	$$\frac{4}{\pi}\left(\frac{\sin(t)\cos(\beta)}{1}+\frac{\sin(3t)\cos(3\beta)}{3}+\frac{\sin(5t)\cos(5\beta)}{5}+\ldots\right)$$
	$$\frac{2}{\pi}-\frac{4}{\pi}\left(\frac{\sin(t)}{1\cdot 3}+\frac{\sin(2t)}{3\cdot 5}+\frac{\sin(3t)}{5\cdot 7}+\ldots\right)$$

Superposition theorem The superposition theorem for electrical circuits states that for a linear system the response (voltage or current) in any branch of a bilateral linear circuit having more than one independent source equals the algebraic sum of the responses caused by each independent source acting alone, while all other independent sources are cancelled.

Solving a circuit with a Fourier decomposed periodic signal From the superposition theorem, we know that the reponse of the circuit can be derived from the contribution of many different signals . . .

Parseval formula The Parseval formula expresses the RMS value of a signal in terms of the RMS values of the harmonic components of the signal

$$S_{RMS}^2 = S_{avg}^2 + S_{1RMS}^2 + S_{2RMS}^2 + S_{3RMS}^2 + \ldots$$

- *The squared RMS value of a signal is the sum of all the squared RMS values its sinusoidal components*
- *The RMS value of a signal is the geometric mean of the RMS values of its sinusoidal components*

Ripple RMS value In constant signals, ripple are the harmonics other than the signal average value. According to Parseval formula, the RMS value of the ripple is the geometric mean of the RMS values of its sinusoidal components:

$$S_{RMSripple}^2 = S_{1RMS}^2 + S_{2RMS}^2 + S_{3RMS}^2 + \ldots$$

We can derive the ripple RMS value from the DC signal's Parseval formula:

$$S_{RMS}^2 = S_{avg}^2 + S_{1RMS}^2 + S_{2RMS}^2 + S_{3RMS}^2 + \ldots = S_{avg}^2 + S_{RMSripple}^2$$

$$S_{RMSripple}^2 = S_{RMS}^2 - S_{avg}^2$$
$$S_{RMSripple} = \sqrt{S_{RMS}^2 - S_{avg}^2}$$

Average power from harmonics

$$P = V_{avg}I_{avg} + V_{1RMS}I_{1RMS}\cos(\varphi_1) + V_{2RMS}I_{2RMS}\cos(\varphi_2) + \ldots$$

- Only same frecuency harmonics produce power.