

# Filters

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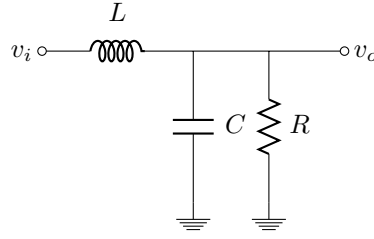
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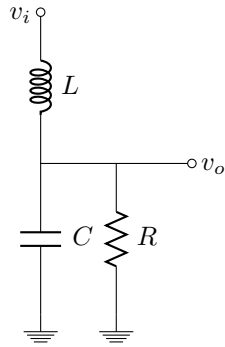
# 1 Filters

## 1.1 LC filter

LC filter loaded with R transfer function



In the voltage-divider form:



$$V_o = V_i \frac{Z_{R//C}}{Z_L + Z_{R//C}}$$

**Butterworth filter.** Maximum flatness filter. The normalized Butterworth polynomials can be used to determine the transfer function for any low-pass filter cut-off frequency  $\omega_c$ , as follows

$$H(s) = \frac{G_0}{B_n(a)}$$

where  $a = \frac{s}{\omega_c}$ .

To four decimal places, Butterworth polynomials are (expressed in quadratic factors)

| Order | Polynomial $B_n(s)$  |
|-------|--|
| 1     | $(s + 1)$  |
| 2     | $s^2 + 1.4142s + 1$  |
| 3     | $(s + 1)(s^2 + s + 1)$   |
| 4     | $(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)$                                       |
| 5     | $(s + 1)(s^2 + 0.6180s + 1)(s^2 + 1.6180s + 1)$                                |
| 6     | $(s^2 + 0.5176s + 1)(s^2 + 1.4142s + 1)(s^2 + 1.9319s + 1)$                    |
| 7     | $(s + 1)(s^2 + 0.4450s + 1)(s^2 + 1.2470s + 1)(s^2 + 1.8019s + 1)$             |
| 8     | $(s^2 + 0.3902s + 1)(s^2 + 1.1111s + 1)(s^2 + 1.6629s + 1)(s^2 + 1.9616s + 1)$ |

From [Butterworth filter, Wikipedia](#)

- two frequencies which ratio is 1:10 are separate by a decade
- two frequencies which ratio is 1:2 are separate by a octave
- 1 decade is 3.33 octaves

$$\frac{6dB}{oct} = \frac{20dB}{dec}$$