AC to DC Converters

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Table of contents

1 AC to DC Converters, Rectifiers

A AC/DC Converter, or rectifier, is an electrical device that converts alternating current (AC), which periodically reverses direction, to direct current (DC), which flows in only one direction. Physically, rectifiers take a number of forms, including vacuum tube diodes, mercury-arc valves, copper and selenium oxide rectifiers, semiconductor diodes, silicon-controlled rectifiers and other silicon-based semiconductor switches. Historically, even synchronous electromechanical switches and motors have been used. #
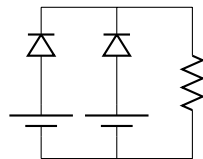
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                                        ┌─── Uncontrolled
                        Switching ──────┤
                       ╱                └─── Controlled
            Rectifiers ┤
                       ╲                ┌─── Monophasic
                        Phases ─────────┤
                                        └─── Triphasic
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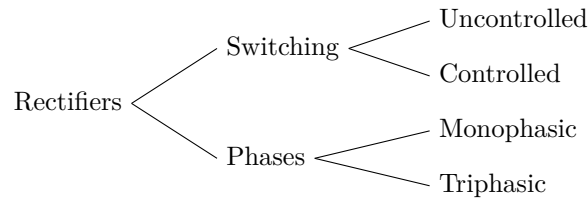
- Uncontrolled rectifiers use diodes as switching devices. They don't need a control circuit.

Diodes

$$ \xrightarrow{\hspace{2cm}} \;\; \triangleright\!\!|\; \xrightarrow{i_D} $$
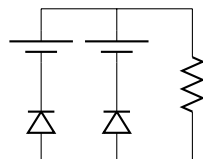$$ \underset{+ \quad v_{AK} \quad -}{} $$
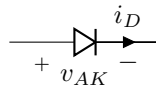
Common Cathode Diodes If two or more diodes are connected with a common cathode, the closed diode is the one with the most positive anode voltage. The rest of the diodes are open.

Common Anode Diodes If two or more diodes are connected with a common anode, the closed diode is the one with the most negative anode voltage. The rest of the diodes are open.

Configuration	Circuit diagram	Conducting Diode
Common Cathode		The diode with the most positive anode voltage
Common Anode		The diode with the most negative cathode voltage

General rule: The most polarized diode is the one conducting. The one with the biggest v_{AK}
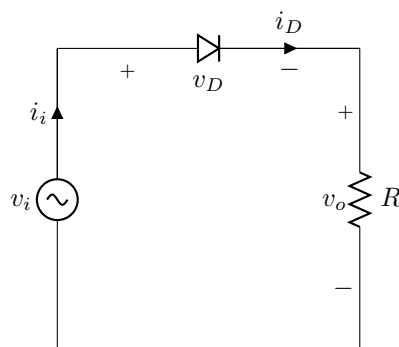
Thyristors

(Ideal)

- Closing conditions
  - $v_A K > 0$, the thyristor is forward polarized
  - i_G, current in the gate (thyristors are current-controlled)
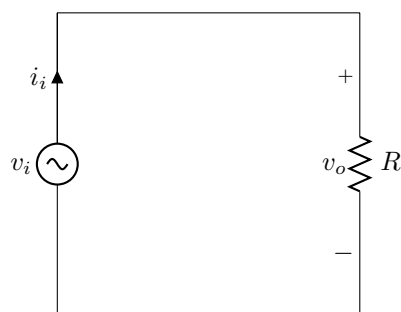- Opening conditions
  - $i_A K < 0$, current is reversed

2 Monophasic Rectifiers

2.1 Half Wave Rectifier

- $v_i > 0$: D ON, $i > 0$
- $v_i < 0$: D OFF, $i = 0$

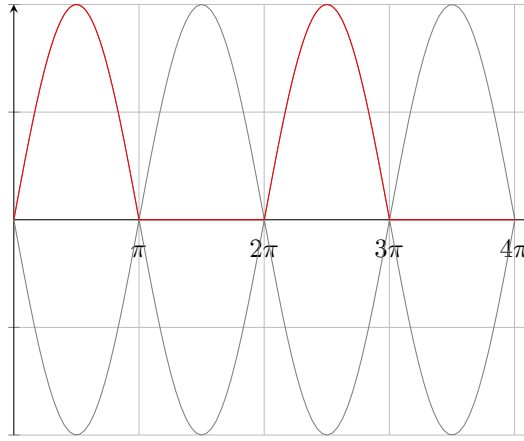$v_i > 0$ (**D ON**) equivalent circuit
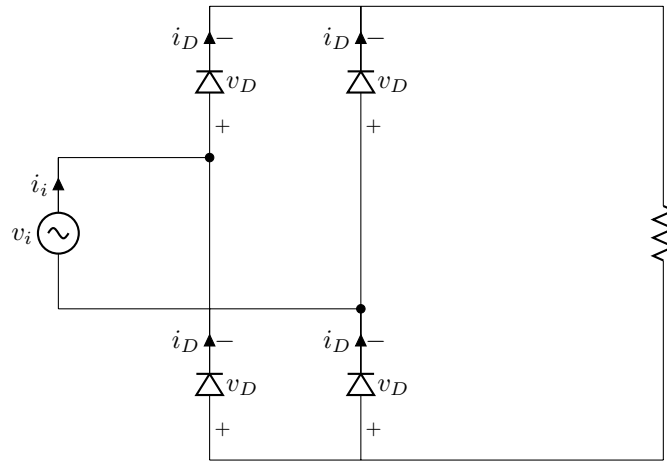

$v_i < 0$ (**D OFF**) equivalent circuit
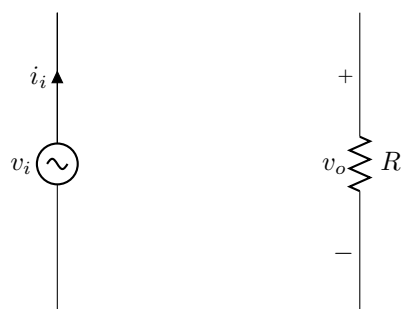
Output voltage, v_o

The voltage in the resistive load, v_R, and the current demanded to the source depends on the load type, and can be different for a same kind of rectifier depending on the load that is connected to it.
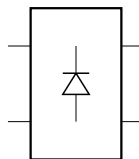
2.2 Full Wave Rectifier

where

- v_i is a pure sinusoidal wave $v_i = V_i\sqrt{2}\sin\left(2\pi f\right)$
  - RMS value is V_i
  - Peak value, or V_{ip} is $V_i\sqrt{2}$
  - Frecuency is f
  - $\varphi = 0$ beacause it is the reference for phase shifts
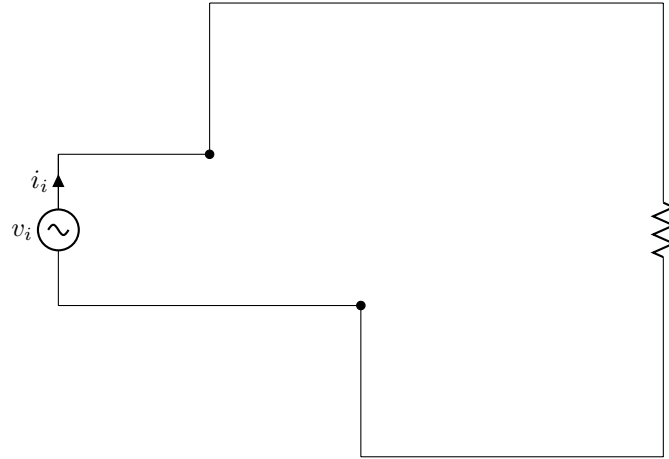- R is resitance that models the load
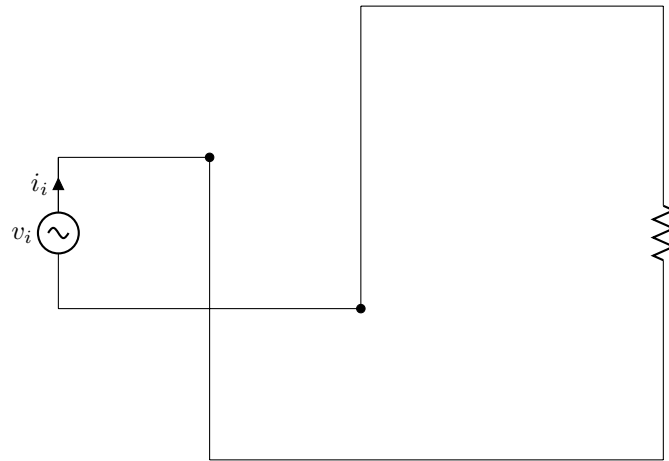
Full Wave Rectifier Symbol

Diode table

	D1	D2	D3	D4
$v_i > 0$	ON	OFF	OFF	ON
$v_i < 0$	OFF	ON	ON	OFF

4

$v_i > 0$ **equivalent circuit:** D1, D4 are ON

$v_i < 0$ **equivalent circuit:** D2, D3 are ON
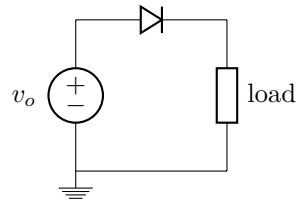
Output voltage, v_o
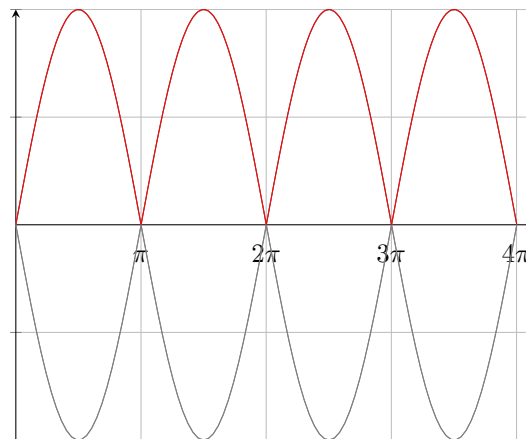
The voltage in the resistive load, v_R, and the current demanded to the source depends on the load type, and can be different for a same kind of rectifier depending on the load that is connected to it.

Note: Although this v_o waveform can always be used with the reduced model circuit, where it is behind a diode, in some cases, like the RC load, the v_o node in the complete circuit coincides with the v_R node and the output voltage directly after the rectifier has not this typical rectified waveform, but a filtered version of it.

Reduced model of the rectifier If v_o is already known, a simplified model of the rectifier can be represented to study the loaded rectifier. The left hand side of the circuit is substituted by

- a v_o voltage source, that represents the bridge output voltage
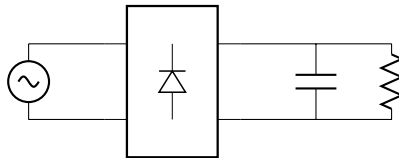- a diode that represents that current cannot enter in the rectifier
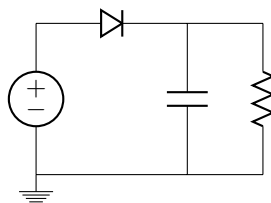

2.2.1   R Load
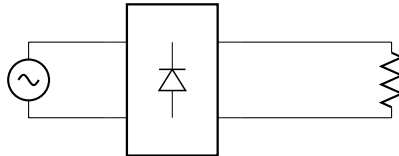
2.2.2   RC Load

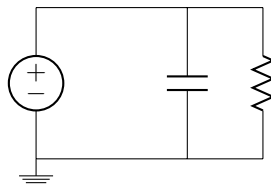A condenser can be added in parallel with the R load to smooth the voltage across it. In this case, the circuit is

Output voltage, v_o; Load voltage, v_R

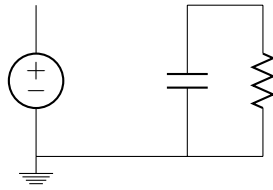- Mean value

Not exactly like this, it's more complex.

- When the diode conducts, $v_o = v_R$ and the capacitor is charged with the current from the source.

The charge is sinusoidal:
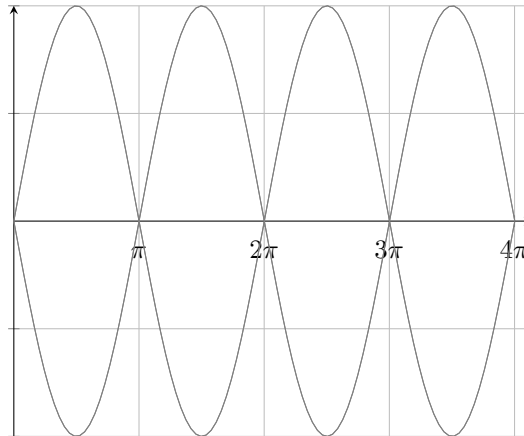
$$i_C(t) = \frac{dv(t)}{dt} = sinusoidal$$

- When the diode is not conducting, the capacitor provides the current to the load. It would continue to discharge until $v_o = v_i$.
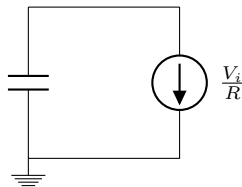
6

The discharge is exponential:

$$formulita$$

The voltage in the load is:

Triangular approximation

- The charge is considered instantaneous: $t_{\text{charge}} = 0 \rightarrow t_{\text{discharge}} = \frac{T}{2}$
- The discharge is linear, with constant current: $I_R = V_m/R$
- The diode opens in the maximum of v_o: $I_R = V_i/R$ (in the slides it says "if ripple is small enough").

The ripple of the wave using the triangular approximation is

$$\Delta v_o = \frac{V\Delta T}{C}$$

$$\Delta v_o = \frac{V_i T}{2RC}$$

Mean value Can be calculated from the triangular approximation
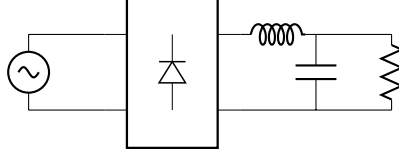
$$\bar{v}_o = V_{ip} - \frac{1}{2}\Delta v_o$$

Advantages of a smoothing condenser

- Small size
- Cheap
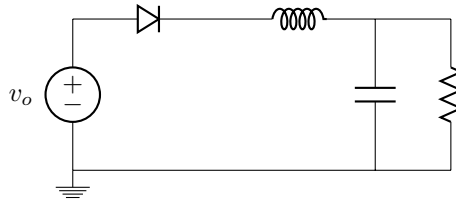- Robust

Disadvantages of a smoothing condenser

- High peaks of current through the diodes.
- Harmonics in the source current.
- Need of a big capacitor if the required ripple is small.
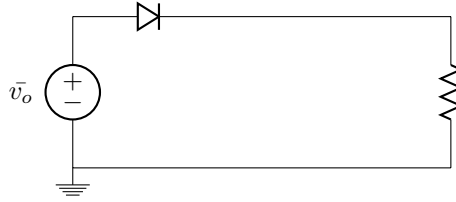
2.2.3 RLC Load

v_R **mean value**

The mean value of v_o can be calculated by using superposition with the circuit:

For the mean values, the resulting DC circuit is:

The resulting $\bar{v}_R$ is

$$\bar{v}_R = \bar{v}_o = \frac{2V_{ip}}{\pi}$$

RLC circuit transfer function

$$\mathbf{H} = \frac{\mathbf{V_o}}{\mathbf{V_i}} = \frac{1}{1 + \frac{L}{R}j\omega - LC\omega^2}$$

$$H = |\frac{\mathbf{V_o}}{\mathbf{V_i}}| = \frac{1}{\sqrt{(1 - LC\omega^2)^2 + (\frac{L}{R}\omega)^2}}$$

v_R **ripple**

Ripple is due to the first harmonic:

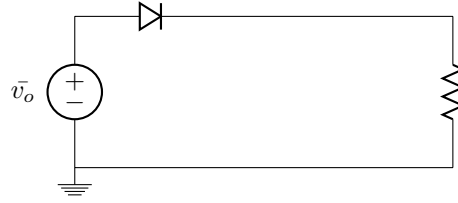$$\Delta v_R = H_{filter} \cdot |V_{o1}|$$

$$H = \frac{1}{\sqrt{(1 - LC\omega^2)^2 + (\frac{L}{R}\omega)^2}}$$

$$|V_{o1}| = \frac{4}{\pi}\frac{1}{3}V_{ip}$$

Inductor current, i_L

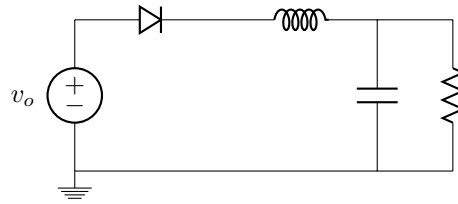Inductor current can be approximated with its mean value and ripple.

- **Mean value**. Using superposition

$$i_L = \frac{\bar{v}_o}{R}$$

- **Ripple**. As in the v_R ripple, maximum variation of the current is due to the 1st harmonic of v_o. Thus, ripple can be calculated:
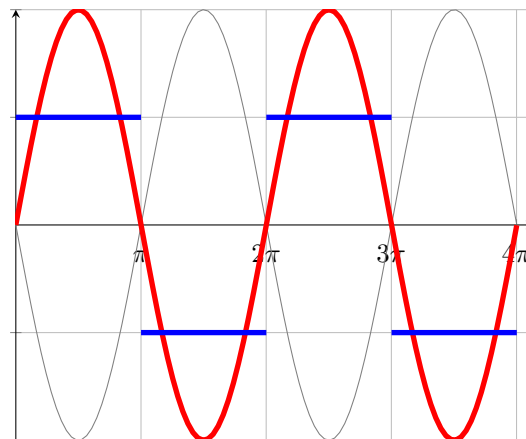

$$I_{1p} = \frac{V_{o1}}{Z_1}$$

If Δv_R is already calculated, it can be used to compute the ripple:

$$\Delta i_L = \frac{\Delta v_R}{Z_{RC}}$$

The rectifier is in CCM if the inductor is always conducting some current:

$$\frac{1}{2}\Delta i_l < \bar{i}_L$$

Power factor in the source

$$\text{PF} = \frac{P}{S}$$

- Only first harmonic delivers power:
$$P = V_1 I_1 \cos(\varphi)$$
$$P = \frac{V_{ip}}{\sqrt{2}} \frac{4}{\pi} \frac{I}{\sqrt{2}} \cos(0)$$

- Apparent power
$$S = V_{\text{RMS}} I_{\text{RMS}}$$
$$S = \frac{V_{ip}}{\sqrt{2}} I$$

$$\text{PF} = \frac{P}{S} = \frac{\frac{V_{ip}}{\sqrt{2}} \frac{4}{\pi} \frac{I}{\sqrt{2}} \cos(0)}{\frac{V_{ip}}{\sqrt{2}} I}$$

$$\text{PF} = \frac{4}{\pi} \frac{1}{\sqrt{2}} \approx 0.9003...$$

Advantages of LC filtering

- No current peaks in the diodes.
- Less condenser ripple.
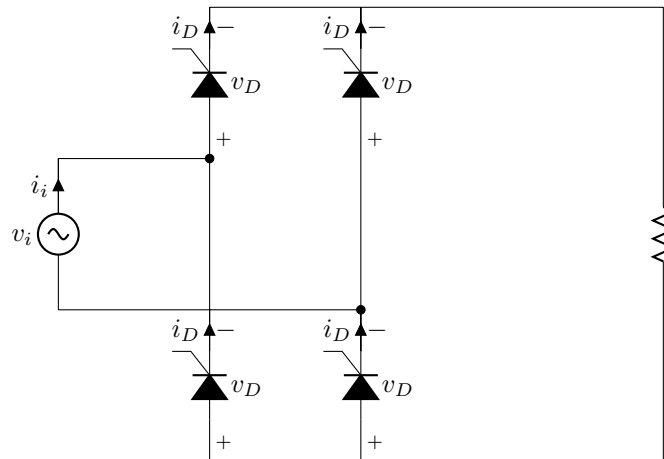- Less capacity and current capacitor required.

Disadvantages of LC filtering
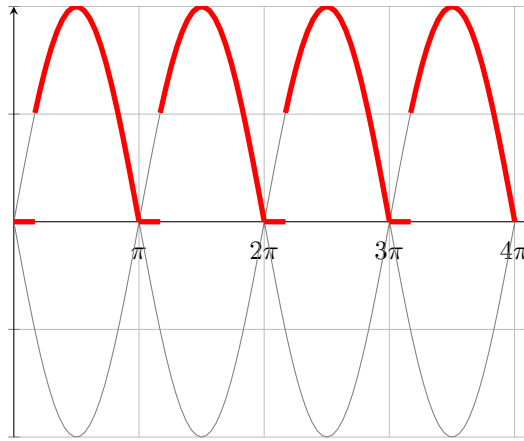
- Size and weight of the inductor

2.2.4 RL load

Flyback Diode

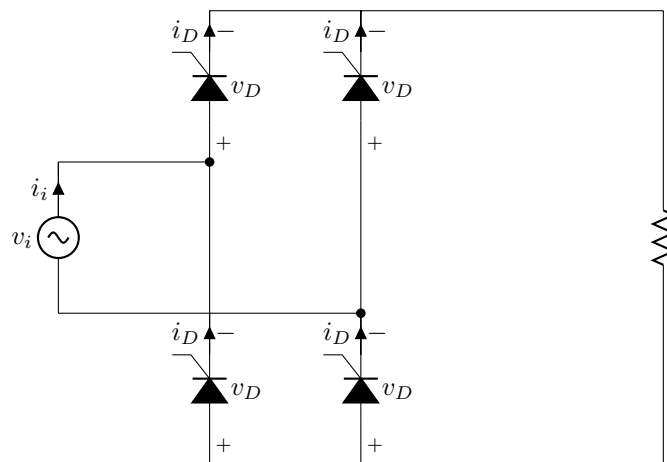### 2.2.5 Controlled Full Wave Rectifier R

where

- v_i is a pure sinusoidal wave $v_i = V_i\sqrt{2}\sin(2\pi f)$
  - RMS value is V_i
  - Peak value, or V_{ip} is $V_i\sqrt{2}$
  - Frecuency is f
  - $\varphi = 0$ beacause it is the reference for phase shifts
- R is resitance that models the load
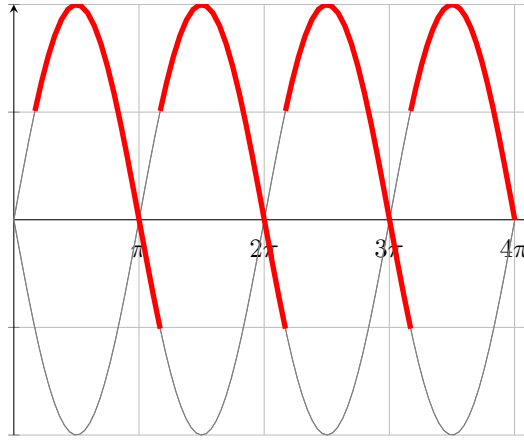- α is the delay angle for thyristors

v_o **mean value,** $\bar{v}_o$

$$
\begin{aligned}
\bar{v}_o &= \frac{1}{T} \int_{\theta_0}^{\theta_0 + T} v_o(\theta) d\theta \\
&= \frac{1}{\pi} \int_0^\pi v_o(\theta) d\theta \\
&= \frac{1}{\pi} \int_0^\alpha 0 d\theta + \frac{1}{\pi} \int_\alpha^\pi V_{ip} \sin(\theta) d\theta \\
&= \frac{V_{ip}}{\pi} [- \cos(\theta)]_\alpha^\pi \\
&= \frac{V_{ip}}{\pi} [- \cos(\pi) + \cos(\alpha)] \\
&= \frac{V_{ip}}{\pi} (\cos(\alpha) + 1)
\end{aligned}
$$

2.2.6 Controlled Full Wave Rectifier RL

v_o **mean value,** $\bar{v_o}$

$$\bar{v_o} = \frac{1}{T} \int_{\theta_0}^{\theta_0 + T} v_o(\theta) d\theta$$

$$= \frac{1}{\pi} \int_{\alpha}^{\pi + \alpha} \sin(\theta) \, d\theta$$

$$= \frac{1}{\pi} \int_{0}^{\alpha} 0 \, d\theta + \frac{1}{\pi} \int_{\alpha}^{\pi} V_{ip} \sin(\theta) \, d\theta$$

$$= \frac{V_{ip}}{\pi} [-\cos(\theta)]_{\alpha}^{\pi}$$

$$= \frac{V_{ip}}{\pi} [-\cos(\pi) + \cos(\alpha)]$$

$$= \frac{2V_{ip}}{\pi} \cos(\alpha)$$

$\bar{v_o}$ **with flyback diode** Same as R loaded converter

$$\bar{v_o} = \frac{1}{T} \int_{\theta_0}^{\theta_0 + T} v_o(\theta) d\theta$$

$$= \frac{1}{\pi} \int_{0}^{\pi} v_o(\theta) d\theta$$

$$= \frac{1}{\pi} \int_{0}^{\alpha} 0 \, d\theta + \frac{1}{\pi} \int_{\alpha}^{\pi} V_{ip} \sin(\theta) \, d\theta$$

$$= \frac{V_{ip}}{\pi} [-\cos(\theta)]_{\alpha}^{\pi}$$

$$= \frac{V_{ip}}{\pi} [-\cos(\pi) + \cos(\alpha)]$$

$$= \frac{V_{ip}}{\pi} (\cos(\alpha) + 1)$$

3 Triphasic Rectifiers

3.1 Half Wave Triphasic Rectifier

3.2 Full Wave Triphasic Rectifier

4 Reference

- https://en.wikipedia.org/wiki/Rectifier