Power Electronics

Diego Trapero

Table of contents

1 Power Electronics

Power electronics is . . .

2 Signals

Circuit signal A circuit signal, $s(t)$, is a function of time that represents the variation of an electrical magnitude in a circuit.

Periodic signal A periodic signal is a signal $s(t)$ that satisfies $s(t) = s(t+T)$ for all t

- *Note:* Constant signals are periodic signals because they fulfill the condition.

Periodic signal parameters

- The period is the smallest value of T satisfying $u(t+T) = u(t)$ for all t
- Frecuency, $f = 1/T\ [Hz]\ [s^{-1}]$
- Angular frecuency, $\omega = 2\pi f\ [rad]$
- Amplitudes
  - Peak-to-peak amplitude
  - Peak amplitude, commonly referred as just amplitude

2 Average value

Average value of a periodic signal The average value S_{avg} or $\bar{s(t)}$ of a periodic signal $S(t)$ is

$$S_{avg} = \bar{s(t)} = \frac{1}{T}\int_{t}^{t+T} s(t)dt$$

2 Root Mean Square Value

Root Mean Square of a periodic signal The Root Mean Square S_{RMS} of a periodic signal $s(t)$ is

$$S_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} s(t)^2 dt}$$

Motivation for RMS value RMS value of a voltage or current signal (constant or non-constant) has the property that is the value of an equivalent DC voltage or current signal that would dissipate the same power through a resistance.

2 Average and RMS values table

Signal	Average value	Root Mean Square Value
Constant	A	A
Square Wave		

Sinusoidal Wave	0	$\frac{A}{\sqrt{2}}$

2 Periodic signal decomposition in Fourier Series

Motivation The phasor method can be used to solve circuits with sinusoidal signals (AC circuits), avoiding the necessity of solving the time-domain differential equations. A circuit with other type of waveforms cannot be analysed with this method, unless it is decomposed into its sinusoidal components. The Fourier series allows us to decompose any waveform to a series of sine and cosine functions. Then, we can solve each circuit (an AC circuit for each sinusoidal component of the wave) separatedly and, apllying superposition, add up all the solutions to get the solution of the original circuit.

Fourier decomposition Any periodic signal can be decomposed in a sum of a constant value plus a (possibly infinite) series of sine and cosine functions.

$$s(x) = S_{avg} + \sum_{n=1}^{\infty} A_n \cos(n\omega t) + \sum_{n=1}^{\infty} B_n \sin(n\omega t)$$

Superposition theorem The superposition theorem for electrical circuits states that for a linear system the response (voltage or current) in any branch of a bilateral linear circuit having more than one independent source equals the algebraic sum of the responses caused by each independent source acting alone, while all other independent sources are cancelled.

2 Parseval Formula

Parseval formula The Parseval formula expresses the RMS value of a signal in terms of the RMS values of the harmonic components of the signal

$$S^2_{RMS} = S^2_{avg} + S^2_{1RMS} + S^2_{2RMS} + S^2_{3RMS} + ...$$

- *The squared RMS value of a signal is the sum of all the squared RMS values its sinusoidal components*
- *The RMS value of a signal is the geometric mean of the RMS values of its sinusoidal components*

Ripple RMS value In constant signals, ripple are the harmonics other than the signal average value. According to Parseval formula, the RMS value of the ripple is the geometric mean of the RMS values of its sinusoidal components:

$$S^2_{RMSripple} = S^2_{1RMS} + S^2_{2RMS} + S^2_{3RMS} + ...$$

We can derive the ripple RMS value from the DC signal's Parseval formula:

$$S^2_{RMS} = S^2_{avg} + S^2_{1RMS} + S^2_{2RMS} + S^2_{3RMS} + ... = S^2_{avg} + S^2_{RMSripple}$$

$$S^2_{RMSripple} = S^2_{RMS} - S^2_{avg}$$
$$S_{RMSripple} = \sqrt{S^2_{RMS} - S^2_{avg}}$$

3 Electric power

Electric power Electric power, $p(t)$ is

$$p(t) = v(t)i(t)$$

Average power Average power or active power, P, is the average of the $p(t)$ signal in a component

$$P = \bar{p(t)} = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)dt$$

Aparent power Apparent power, S, in a component is the product

$$S = V_{RMS}I_{RMS}$$

Power factor Power factor is the quotient

$$PF = \frac{P}{S}$$

3 Power in AC

Power in AC refers to the analysis of power in circuits in which all signals are sinusoidal waves. Although power electronics circuits are not usually pure AC circuits, this theory is useful when solving circuits with superposition of the Fourier components.

Active and reactive power

- Active power: $P = VI\cos(\varphi)$
- Reactive power: $Q = VI\sin(\varphi)$

Complex and apparent power

- Complex power is the complex number $\mathbf{S} = \mathbf{V}\mathbf{I}^* = P + Qj$
- Apparent power is the module of the complex power, $S = VI$

Power factor

$$PF = \frac{P}{S}$$

$$PF = \cos(\varphi)$$

3 Power with superposition

3 Power from harmonics

Average power from harmonics

$$P = V_{avg}I_{avg} + V_{1RMS}I_{1RMS}\cos(\varphi_1) + V_{2RMS}I_{2RMS}\cos(\varphi_2) + ...$$

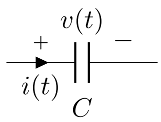
- Only same frecuency harmonics produce power.

4 Circuit components

4 Resistor

4 Capacitor

4 Capacitor equation



The differential equation of the capacitor is

$$i(t) = C\frac{dv(t)}{dt} \tag{CAPEQ}$$

- **Continuity of the voltage $v(t)$ and existance of a solution for the equation** For the equation to have a solution in any time t, $\frac{dv(t)}{dt}$, must exist in al the real domain. If $v(t)$ has a discontinuity or is not differentiable in a point of its domain, then $\frac{dv(t)}{dt}$ is undetermined and so is $i(t)$. This doesn't means *per se* that $i(t)$ is infinite, it's just indetermined.

- **Sudden changes in voltage** If $v(t)$ changes abruptly, then its derivative $\frac{dv(t)}{dt}$ is very big, and so is the current throught the capacitor, $i(t)$.

- **Voltage throught the capacitor should be continuous** In a real circuit, the voltage function $v(t)$ of a capacitor may not be discontinuous, but vary from one value to another in a very short period of time. This situation is often modelled by a discontinuity in the $v(t)$ function, althought in reality is a very step, almost vertical, part of the function (it is not vertical because voltage cannot take two different values in the same time, and in the real world, it can neither go from one value to another without passing through all the values inbetween [how this last thing be probed, if it can be proved?]). In this case,

$\frac{dv(t)}{dt}$ is very big, what leads to a very large $i(t)$ current. A real capacitor can't withstand the very large current that such a sudden change drives through it, and the theoretical model has no solution if there is a discontinuity. This is the reason why, in this model, the voltage throught the capacitor is required to be continuous in time.

- **Capacitor in DC steady state is an open circuit** In DC steady state, $\frac{dv(t)}{dt} = 0$, so, using the capacitor equation, $i(t) = \frac{dv(t)}{dt} = 0$. In DC steady state, the no current flows through the capacitor, it acts as an open circuit.

4 Capacitor voltage equation

Solving the capacitor differential equation using separation of variables, we obtain the capacitor voltage equation:

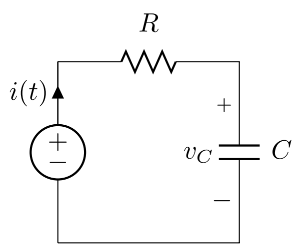
$$i(t) = C\frac{dv(t)}{dt}$$

$$\frac{1}{C}i(t) = dv(t)$$

$$\int_{v(t=-\infty)}^{v(t)} dv(t) = \frac{1}{C}\int_{t=-\infty}^{t} i(t)dt$$

$$v(t) = \frac{1}{C}\int_{t=t_0}^{t} i(t)dt + v(t=t_0)$$

4 Charging a capacitor with a constant voltage source



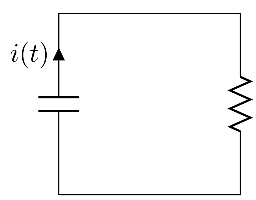
- $v_s(t)$ is a step function
- $v_c(t=0) = 0$, the capacitor starts discharged

$$i(t) = \frac{v_c(t) - v_s(t)}{R}$$

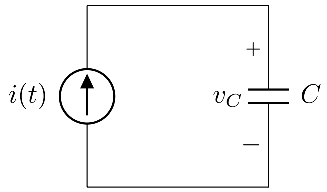
$$\frac{v_c(t) - v_s(t)}{R} = C\frac{dv(t)}{dt}$$

Solution is exponential

4 Discharging a capacitor through a resistor



4 Charging a capacitor with a constant current source



Charging a capacitor with a switched current source problem enuntiation
Given the circuit represented above, with a switched current source connected in series to a capacitor of capacitance C...

- The current throught the capacitor $i(t)$ is a step function:

$$i(t) = blah$$

- Initial condition: $v_C(t = 0) = v_0$

Calculate the voltage across the capacitor v_C

1. for $t > 0$, $i(t) = I$, constant, and the capacitor equation becomes

$$I = \frac{dv(t)}{dt}$$

Solve the separated variables equation:

$$Idt = dv(t)$$

$$\int_{t=0}^{t} Idt = \int_{v(t=0)}^{v(t)} dv(t)$$

$$It - I \cdot 0 = v(t) - v(t = 0)$$

$$v(t) = It + v_0$$

In this condition, a real capacitor would eventually break by “dielectric rupture”

2. for $t < 0$, using the continuity of the voltage in a capacitor principle, the voltage is $v(t) = v(t = 0) = v_0$
3. if the value of the current source becomes 0 at time t_OFF, the differential equation is

$$0 = \frac{dv(t)}{dt}$$

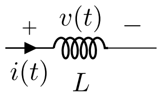
$$v(t) = K, Kinrealnumbers$$

This means that the voltage remains constant. Due to the continuity capacitor voltage, the voltage after t_off is

$$v(t > t_{OFF}) = v(t = t_{OFF})$$

4 Inductor

4 Inductor equation



The differential equation of the inductor is

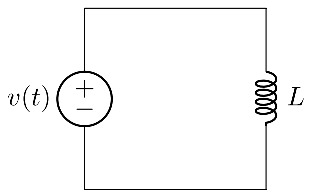
$$v(t) = L\frac{di(t)}{dt} \tag{INDEQ}$$

Continuity of current in the inductor In an inductor, $i(t)$ is a continuous function, $i(t_0^+) = i(t_0^-) foranytinR$

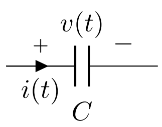
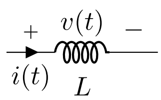
- For inductors, the $i - v$ characteristics equation $v(t) = \frac{di(t)}{dt}$ implies that if there were a dis- continuity in the current waveform, an infinite amount of voltage should appear across the inductor. Therefore, the current waveform in an inductor should be continuous. This means that if a switch is thrown at time t_0, we should have $i(t_0^+) = i(t_0^-)$. Note the inductor voltage waveform can have a discontinuity. #
- Proof/Justification? if $v(t)$ is not continuous in $t = t_0$, then $dv(t)/dt$ doesn't exists, and neither $i(t)$ through the capacitor

Inductor in DC steady state In steady state, $\frac{di(t)}{dt} = 0$, so, using the inductor equation, $v(t) = \frac{di(t)}{dt} = 0$. In DC steady state, there is no voltage drop across the inductor, it acts as a shortcircuit.

4 Charging an inductor with a constant voltage source (switched)



4 Inductor vs Capacitor table

What	Capacitor	Inductor
Symbol		
Differential equation	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = L\frac{di(t)}{dt}$
Hydraulic analogy		

Voltage	$v(t) = \frac{1}{C}\int_{t=t_0}^{t} i(t)dt + v(t=t_0)$	$v(t) = L\frac{di(t)}{dt}$
Current	$i(t) = C\frac{dv(t)}{dt}$	$i(t) = \frac{1}{L}\int_{t=t_0}^{t} v(t)dt + i(t=t_0)$

4 Transformers

4 Semiconductor devices

5 Analysis hypothesis

The analysis hypothesis that are often used in a first analysis of a power electronics circuit are:

1. Periodic Steady State condition. The converter is in a periodic steady state regime, so all the waveforms are periodic. If the circuit is in PSS, the following is also true:
   i. At the inductors, the average voltage is zero, $v_L = 0$.
   ii. At the capacitors, the average current is zero, $i_L = 0$
2. No power loss, unitary efficiency: $\eta = \frac{P_o}{P_i} = 1$

3. The voltage in the capacitors is considered to be approximately constant, $v_c \approx$ constant
4. Ideal components: diodes, switches, capacitors...

Proof of 1.ii:

Small ripple approximation: In a well-designed converter, the output voltage ripple is small. Hence, the waveforms can be easily determined by ignoring the ripple: v_o(t) approx V (same as 3.)

6 Reference