AC Cheatsheet

Diego Trapero

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1 AC Cheatsheet

1 Sinusoidal signal

A sinusoidal signal in AC takes the form

$$s(t) = \sqrt{2}A \cdot \cos(\omega t + \varphi)$$

where

- $A \equiv A_{RMS}$ is the RMS value of the signal
- ω is the angular frecuency, $\omega = 2\pi f$ $[rad]$
- Frecuency, $f = \omega/2\pi$
- Period, $T = 1/f$
- Amplitude of the signal is $A/\sqrt{2}$

Note: In AC the sinusoidal function amplitude is taken as $A/\sqrt{2}$, so A is the RMS value of the signal. This way, we can directly use the RMS values of the signals to make calculations with power.

Average and RMS values

- Average value: $S_{AVG} = 0$
- RMS value: $S_{RMS} = A$

1 Phasors and complex numbers

1 Phasor

$$s(t) = \sqrt{2}A \cdot \cos(\omega t + \varphi) \to \mathbf{S} = A\angle\varphi$$

In physics and engineering, a phase vector, or phasor, is a representation of a sinusoidal function whose amplitude, frequency, and phase are time-invariant

Euler's formula indicates that sinusoids can be represented mathematically by the sum of two complex-valued functions:

$$A \cdot \cos(\omega t + \theta) = A \cdot \frac{e^{i(\omega t + \theta)} + e^{-i(\omega t + \theta)}}{2}$$

or by the real part of one of the functions:

$$A \cdot \cos(\omega t + \theta) = \operatorname{Re}\left\{A \cdot e^{i(\omega t + \theta)}\right\} = \operatorname{Re}\left\{Ae^{i\theta} \cdot e^{i\omega t}\right\}.$$

The term phasor can refer to either $Ae^{i\theta}e^{i\omega t}$ or just the complex constant, $Ae^{i\theta}$. In the latter case, it is understood to be a shorthand notation, encoding the amplitude and phase of an underlying sinusoid.

An even more compact shorthand is angle notation: $A\angle\theta$.

1 Generalized Ohm Law, complex impedance

The **Generalized Ohm Law**:

$$\mathbf{V} = \mathbf{Z}\mathbf{I}$$

where $\mathbf{Z}$ is

- $\mathbf{Z} = R$ for resistors
- $\mathbf{Z} = \frac{1}{j\omega C} = \frac{-j}{\omega C}$ for capacitors
- $\mathbf{Z} = j\omega L$ for inductors

1 AC Power

Power in a two-terminal component

- Voltage wave with RMS value V: $v(t) = \sqrt{2}V \cdot \cos(\omega t)$ ($\varphi = 0$ because it's the reference)
- Current wave with RMS value I: $v(t) = \sqrt{2}I \cdot \cos(\omega t - \varphi)$

Instantaneous power

1. Instantaneous power definition

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= \sqrt{2}V\cos(\omega t) \cdot \sqrt{2}I\cos(\omega t - \varphi) \\ &= 2VI\cos(\omega t)\cos(\omega t + \varphi) \end{aligned}$$

2. Transform into sum of cosines using the identity $\cos(a \cdot b) = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$

$$\begin{aligned} p(t) &= \cancel{2}VI \cdot \frac{\cancel{1}}{\cancel{2}}[\cos(\cancel{\omega t} - \cancel{\omega t} + \varphi) + \cos(\omega t + \omega t - \varphi)] \\ &= VI[\cos(\varphi) + \cos(2\omega t - \varphi)] \\ &= VI\cos(\varphi) + VI\cos(2\omega t - \varphi)] \end{aligned}$$

The instantaneous power in an AC component is:

$$p(t) = VI\cos(\varphi) + VI\cos(2\omega t - \varphi)$$

We observe

- Power in the component has a constant term $VI\cos(\varphi)$ and a fluctuating term that varies with time $VI\cos(2\omega t - \varphi)$.

Average power

The instantaneous power is $p(t) = VI\cos(\varphi) + VI\cos(2\omega t - \varphi)$, so we can calculate the average power using the definition:

$p(\bar{t}) = integral VI\cos(\varphi) + VI\cos(2\omega t - \varphi)$

Active and reactive power

- Active power: $P = VI\cos(\varphi)$
- Reactive power: $Q = VI\sin(\varphi)$

Complex and apparent power

- Complex power is the complex number $\mathbf{S} = \mathbf{VI}^* = P + Qj$
- Apparent power is the module of the complex power, $S = VI$

Power factor

$$PF = \frac{P}{S}$$

$$PF = \cos(\varphi)$$

AC power in a resistor

$$P = VI$$

$$Q = 0$$

Power in a inductor

Power in a capacitor