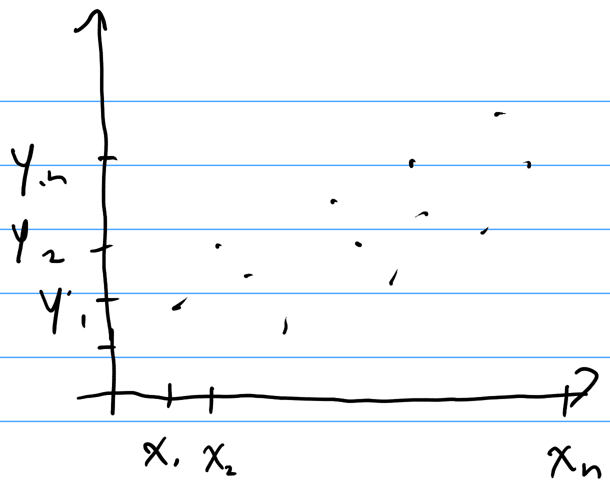
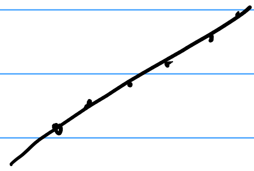
Linear Regression (Matrix Approach)



pts = n

m, b

$$x_1 m + b = y_1$$
$$x_2 m + b = y_2$$
$$x_n m + b = y_n$$



$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

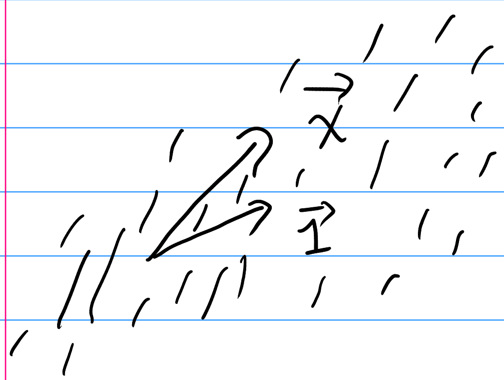
$$\begin{bmatrix} | & | \\ \vec{1} & \vec{x} \\ | & | \end{bmatrix} \vec{v}^* = \begin{bmatrix} | \\ \vec{y} \\ | \end{bmatrix}$$

$\vec{1}, \vec{x}, \vec{y} \in \mathbb{R}^n$

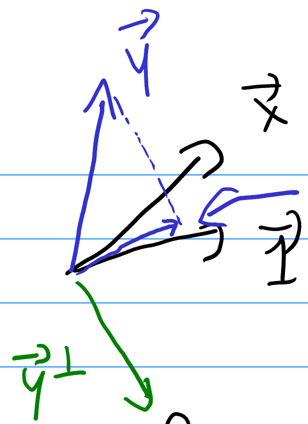
$\vec{v}^* \in \mathbb{R}^2$

$$P\vec{v} = \vec{y}$$

$$C(P) = \{ \vec{z} : \vec{z} = a_1 \vec{1} + a_2 \vec{x} \}$$



$$\vec{y} = m\vec{x} + b\vec{1}$$



$\vec{y}' = \text{Proj}_{C(P)}\vec{y}$

$$\text{Proj}_{C(P)}\vec{y} = \vec{y}' = m'\vec{x} + b\vec{1}$$

$$= P\begin{bmatrix} m' \\ b' \end{bmatrix} = P\vec{v}^*$$

Objetivo: m', b'

$$P\vec{v}' = \vec{y}'$$

$$P\vec{v}' - \vec{y} = \vec{y}' - y = \vec{y}^\perp$$

$$P\vec{v}' - \vec{y} = \vec{y}^\perp \qquad \begin{aligned} \vec{x}\cdot\vec{y}^\perp &= 0 \\ \vec{1}\cdot\vec{y}^\perp &= 0 \end{aligned}$$

$$\begin{bmatrix} — & \vec{1} & — \\ — & \vec{x} & — \end{bmatrix} \begin{bmatrix} | \\ \vec{y}^\perp \\ | \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

$$P^T\vec{y}^\perp = \vec{0} \Rightarrow P^T(P\vec{v}' - \vec{y}) = \vec{0}$$

$$P^TP\vec{v}^* - P^T\vec{y} = 0$$

$$P^TP\vec{v}^* = P^T\vec{y}$$

$$\vec{v}^* = (P^TP)^{-1}P^T\vec{y} \qquad \vec{v}^* = \begin{bmatrix} b^* \\ m^* \end{bmatrix}$$