Diferencias Finitas

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$f''(x) = \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}$$

$$f(x_0+h) = f(x_0) + f'(x_0)h + \frac{1}{2}f''(x_0)h^2$$

$$+ \quad f(x_0-h) = f(x_0) - f'(x_0)h + \frac{1}{2}f''(x_0)h^2$$

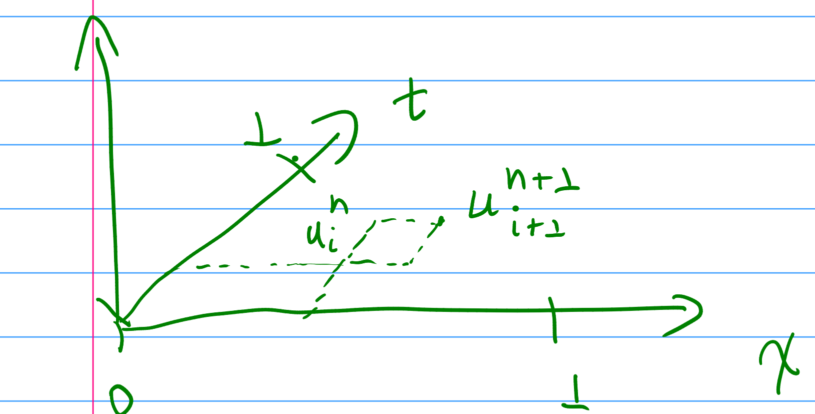
$$\frac{f(x_0+h) + f(x_0-h) - 2f(x_0)}{h^2} = f''(x_0)$$

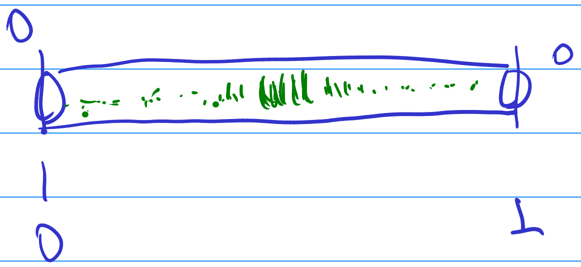
$$\frac{dy}{dt} \to y_{n+1}$$

$$\frac{\partial u}{\partial t} = \beta \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \quad u_{\boxed{i}}^{\boxed{n}}$$

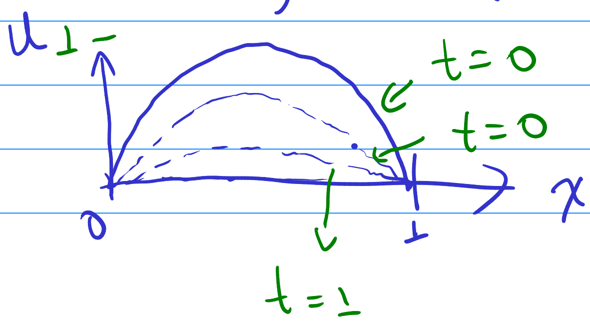
tiempo (n), espacio (i)





$$u(0,x) = \sin(\pi x)$$

$$u(t,x) = \sin(\pi x)\, e^{-t/c}$$



$$\frac{\partial u}{\partial t} = \beta \frac{\partial^2 u}{\partial x^2}$$

u_i^n ← tiempo (n), ← espacio (i)

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \beta \left(\frac{u_{i+1}^n + u_{i-1}^n - 2u_i^n}{(\Delta x)^2} \right)$$

$$u_i^{n+1} = u_i^n + \frac{\beta \Delta t}{(\Delta x)^2} u_{i+1}^n + \frac{\beta \Delta t}{\Delta x^2} u_{i-1}^n - \frac{\beta \Delta t}{(\Delta x)^2} u_i^n$$

$$u_i^{n+1} = \left(1 - \frac{2\beta \Delta t}{(\Delta x)^2}\right) u_i^n + \frac{\beta \Delta t}{(\Delta x)^2}\left(u_{i+1}^n + u_{i-1}^n\right)$$

$$a = \left(1 - \frac{2\beta \Delta t}{(\Delta x)^2}\right) \qquad r = \frac{\beta \Delta t}{(\Delta x)^2}$$

$$u_i^{n+1} = a u_i^n + r\left(u_{i+1}^n + u_{i-1}^n\right)$$

$$a > 0 \qquad 1 - \frac{2\beta \Delta t}{\Delta x^2} > 0$$

$$\Delta t < \frac{(\Delta x)^2}{2\beta} \qquad 0.01 \qquad \Delta t \; \frac{(\Delta x)^2}{\beta}$$

$$\Delta t < (\quad)$$