Metropolis-Hasting Algorithm

Dada una función $f(x)$, Obtener una distribución de probabilidad $P(x)$

$$\int P(x)\,dx = 1$$

Q < Uniforme / Gausiana

X_1

$$a = \frac{f(X_{new})}{f(X_{old})} \frac{Q(X_{old} | X_{new})}{Q(X_{new} | X_{old})}$$

Algoritmo:

X_1

$X_2 = X_1 + \delta(X_1) = X_1 + \text{random.normal}(0, 1)$

$$a = \frac{f(X_2)}{f(X_1)} \frac{Q(X_2 \mid X_1)}{Q(X_1 \mid X_2)}$$

if $a > 1$:

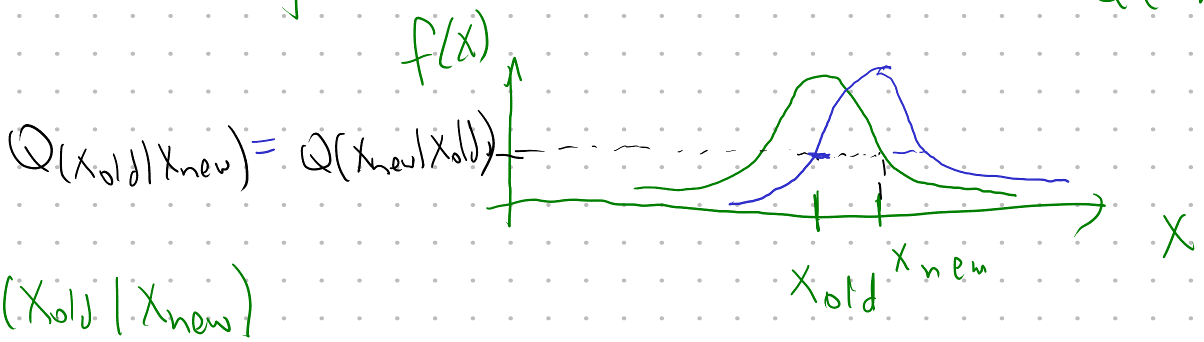
$X_1 = X_2$

if $a < 1$

Acepto $X_1 = X_2$ con probabilidad a.

$$a = \frac{f(X_{new})}{f(X_{old})} \frac{Q(X_{old} \mid X_{new})}{Q(X_{new} \mid X_{old})}$$

Q es una función Gausiana $\quad Q(X_{new} \mid X_{old})$

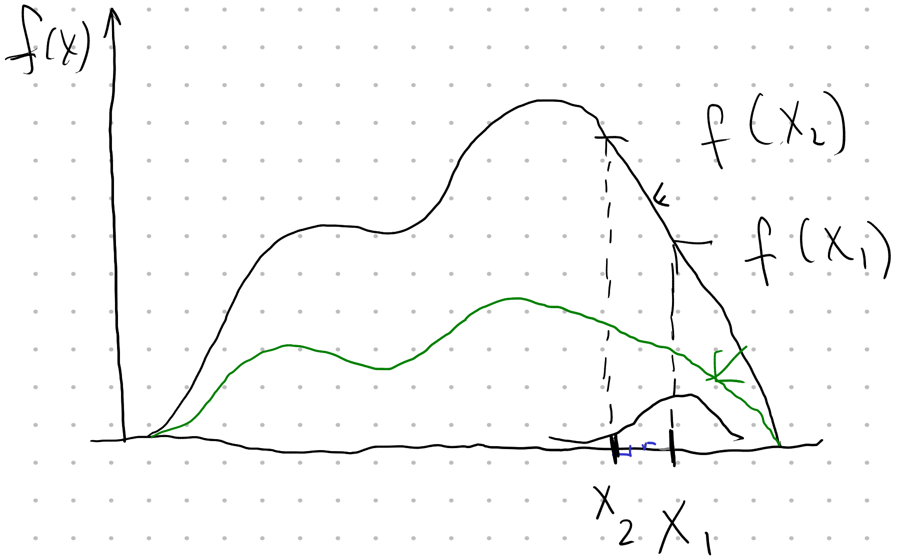


$Q(X_{old} \mid X_{new})$

$$a = \frac{f(X_{new})}{f(X_{old})}$$

if $a > 1$
acepto el punto

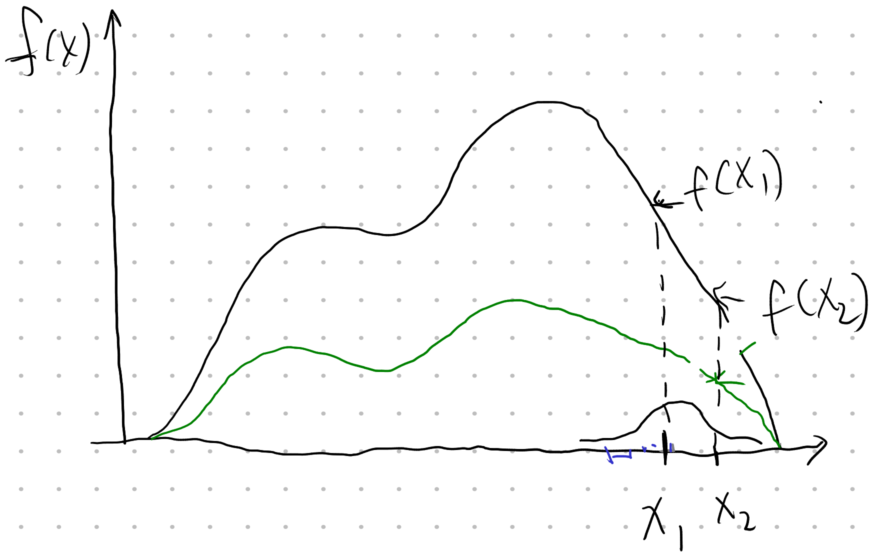
if $a < 1$
acepto el punto con probabilidad a



$$a = \frac{f(x_2)}{f(x_1)}$$

$$a > 1$$

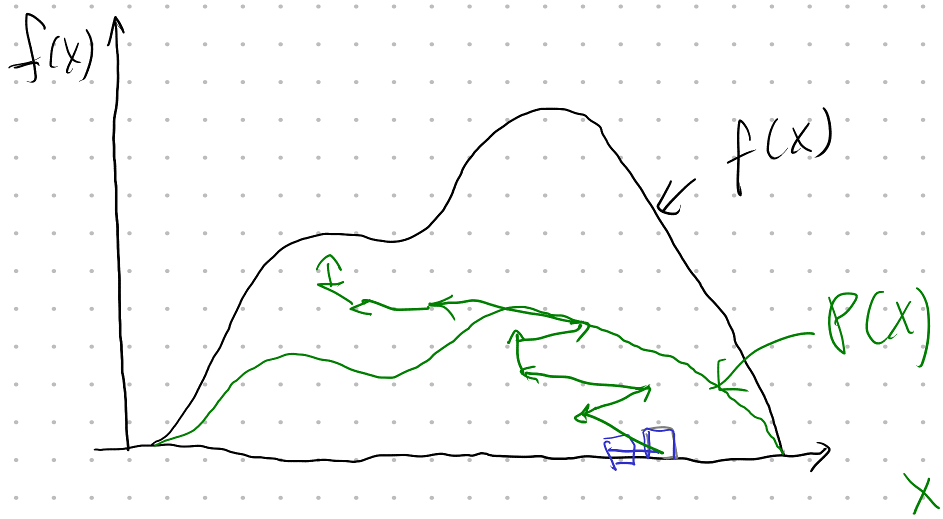
lo acepto



$$a = \frac{f(x_2)}{f(x_1)}$$

$$a < 1$$

Lo acepto
on probabilidad
a



Integración con Metropolis-Hastings

$h(x) = f(x)g(x)$ $\qquad f(x) \rightarrow P(x)$

$$\int_{-\infty}^{\infty} f(x)g(x)\,dx \qquad \int f(x)\,dx < M \qquad f(x) > 0$$

$\int f(x)\,dx = C$: Factor de normalización

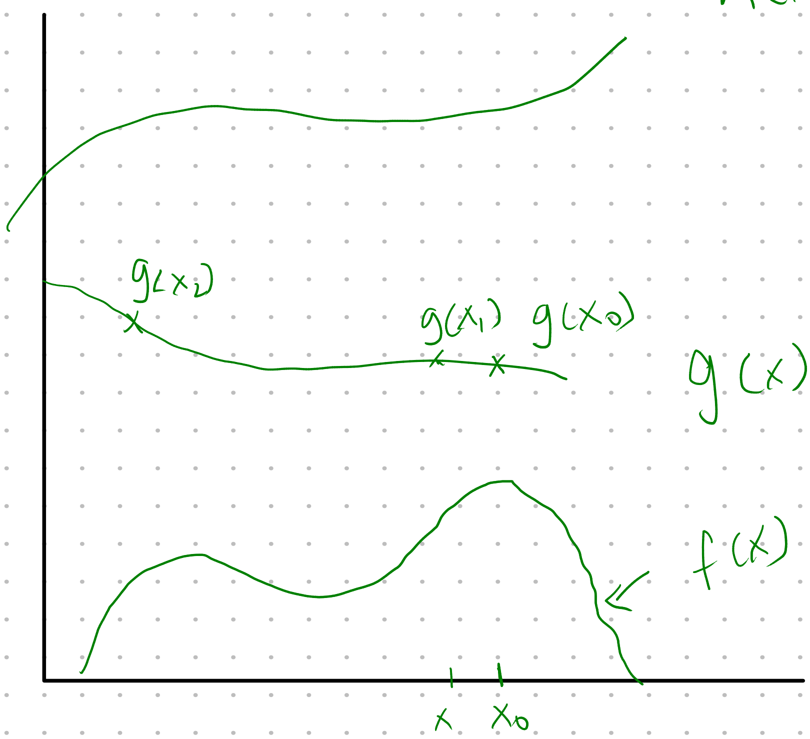
$$\frac{\int f(x)g(x)\,dx}{\int f(x)\,dx} = \frac{1}{N}\sum_{i=1}^{N} g(x_i)$$

: Donde estos puntos x_i son muestreados con M-H a partir de la función $f(x)$

$x_1, x_2, \dots, x_N$ muestreados de la función $f(x)$ con M-H

$$\int f(x)g(x)\,dx = C\frac{1}{N}\sum g(x_i) = C\langle g(x)\rangle_{f(x)}$$

$h(x) = g(x)f(x)$



$$\frac{1}{N}\sum g(x_i) = \frac{\int g(x)f(x)\,dx}{\int f(x)\,dx}$$

$$\frac{\int g(x)\, f(x)\, dx}{\int f(x)\, dx}$$

$$CM = \frac{m_1 x_1 + m_2 x_2 + \cdots m_n x_n}{m_1 + \cdots + m_n}$$

$$CM = \frac{\sum m_i x_i}{\sum m_i} = \frac{\int x\, dm}{\int dm}$$

$$dm = \rho\, dV$$

$$CM = \frac{\int x \rho\, dV}{\int \rho\, dV} \qquad \frac{\int f(x)\, g(x)\, dx}{\int f(x)\, dx}$$

$$g(x) = x \qquad f(x) = \rho(x)$$

$$CM = \frac{\int \vec{r}\, \rho(\vec{r})\, dV}{\int \rho(\vec{r})\, dV} = \frac{1}{N} \sum \vec{r}_i \leftarrow$$ r_i son muestreadas a partir de la densidad del objeto