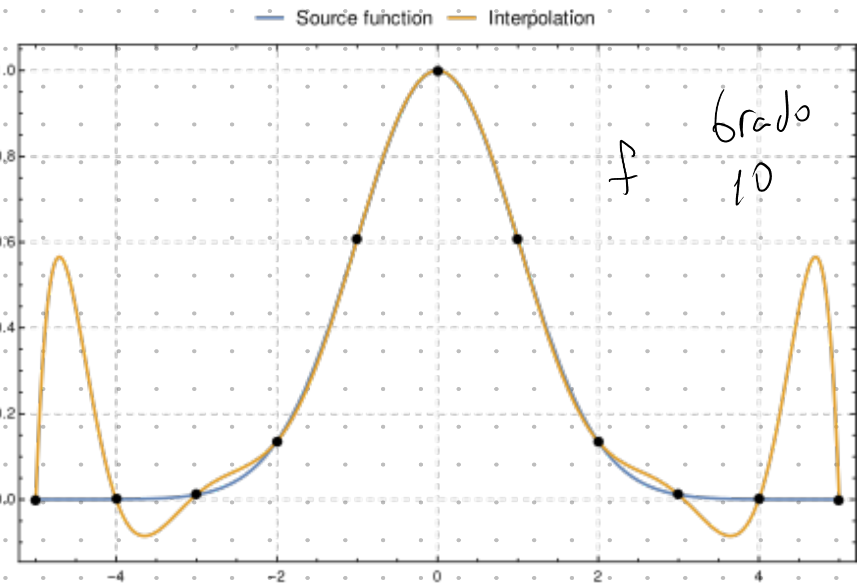
Lagrange Interpolation

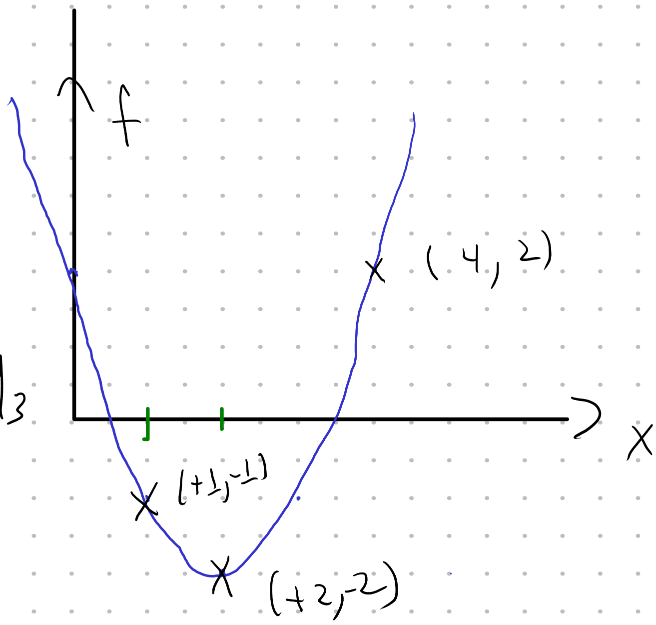
Dados n puntos en el plano x, y encontrar la función de grado $n-1$ que pasa por todos los puntos



$$f(x) = x^2$$

$$f(x) = (x-2)^2 - 2$$

$$(x_1, y_1), (x_2, y_2), (x_3, y_3)$$



$$f(x) = l_1(x) y_1 + l_2(x) y_2 + l_3(x) y_3$$

$$l_1(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}$$

$$l_1(x_2) = \frac{(x_2-x_2)\cdots}{\cdots} = 0 \qquad l_1(x_3) = 0 \qquad l_1(x_1) = 1$$

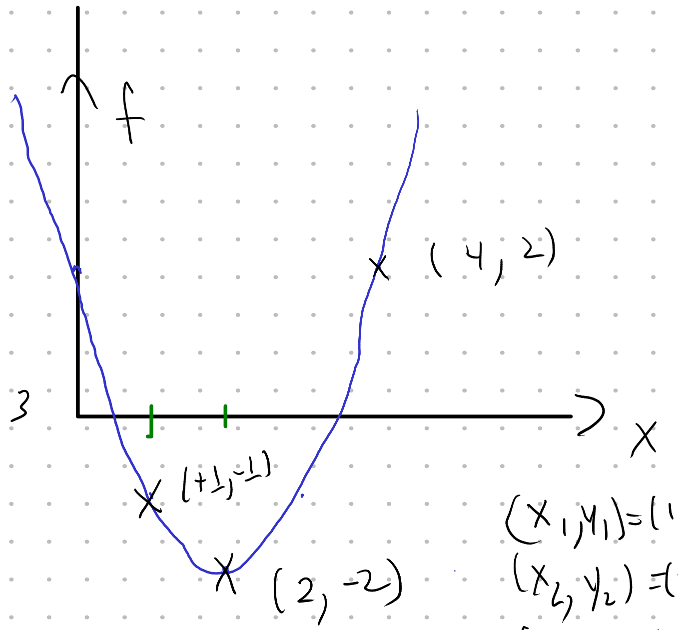
$$l_1(x_1) = \frac{(x_1-x_2)(x_1-x_3)}{(x_1-x_2)(x_1-x_3)} = 1$$

$$f(x_1) = 1 \cdot y_1 + 0 \cdot y_2 + 0 \cdot y_3 = y_1$$

$$f(x) = (x-2)^2 - 2$$

$$(x_1, y_1),\ (x_2, y_2),\ (x_3, y_3)$$

$$f(x) = l_1(x)\,y_1 + l_2(x)\,y_2 + l_3(x)\,y_3$$



$(x_1, y_1) = (1, -1)$
$(x_2, y_2) = (2, -2)$
$(x_3, y_3) = (4, 2)$

$$l_1(x) = \frac{(x-2)(x-4)}{(1-2)(1-4)}$$

$$l_1(x_1) = l_1(1) = 1 \qquad l_1(2) = 0 \qquad l_1(4) = 0$$

$$f(x) = \frac{(x-2)(x-4)}{(1-2)(1-4)}(-1) + \frac{(x-1)(x-4)}{(2-1)(2-4)}(-2) + \frac{(x-1)(x-2)}{(4-1)(4-2)}(2)$$

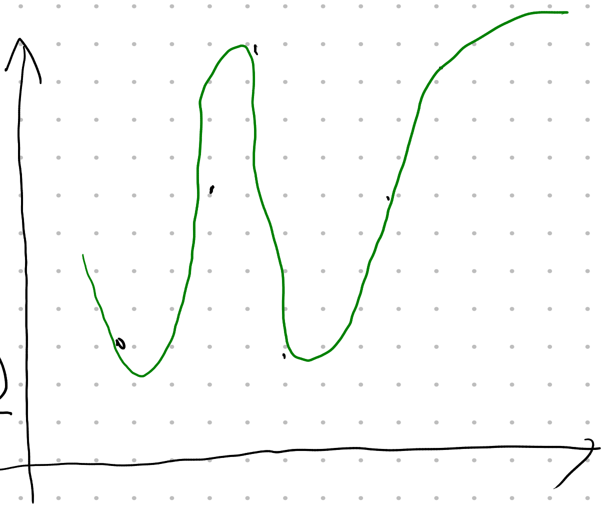
$$f(x_1) = f(1) = -1 \qquad f(x_2) = f(2) = -2 \qquad f(x_3) = f(4) = 2$$

$$f(x) = x^2 - 2x + 2$$

$$l_i(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_{i-1})(x-x_{i+1})\cdots\overset{+2,}{(x-x_n)}}{(x_i-x_1)(x_i-x_2)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x-x_n)}$$

$, (x_1, y_1), \cdots, (x_n, y_n)$

Pregunta: $f(x)$



$$f(x) = l_1(x)y_1 + \cdots + l_n(x)y_n$$

$$l_i(x) = \frac{(x-x_1)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_n)}{(x_i-x_1)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_n)}$$

$$l_i(x_j) = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

$$l_i(x_j) = \delta_{ij}$$

$$f(x_i) = 0 + 0 + l_i(x_i)y_i + \cdots + 0$$

$$f(x_i) = 1 \cdot y_i = y_i$$

when you use Lagrange interpolation to find the function given by points (0,0), (1,2), and (2,4)

Theorem: $2^{1/n}$ is irrational for $n \geq 3$.

Proof: Suppose that $2^{1/n}$ is rational. Then

$$\begin{aligned} 2^{1/n} &= a/b \\ &\Rightarrow 2 = a^n/b^n \\ &\Rightarrow 2b^n = a^n. \end{aligned}$$

But since a and b are both non-zero this implies that $b^n + b^n = a^n$ has a non-trivial solution, which contradicts Fermat's last theorem.$\square$