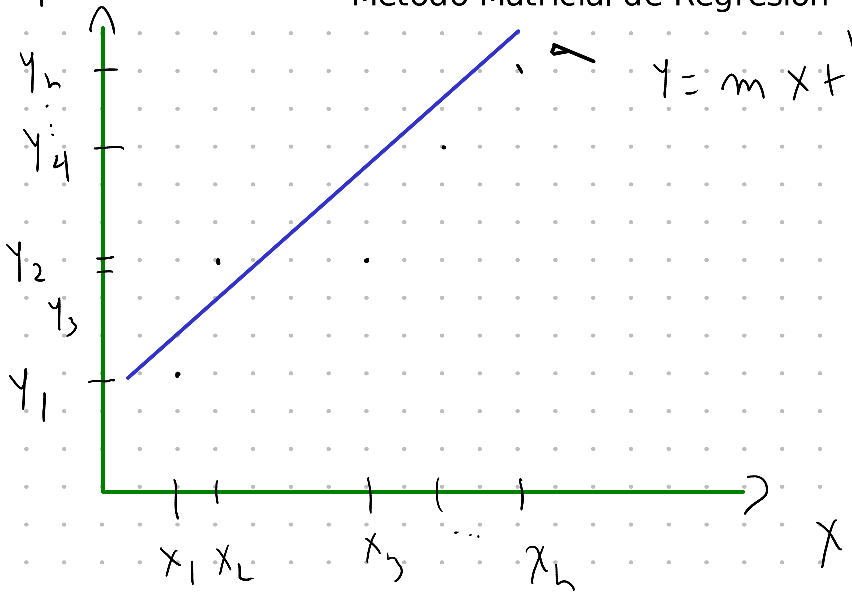
Metodo Matricial de Regresión



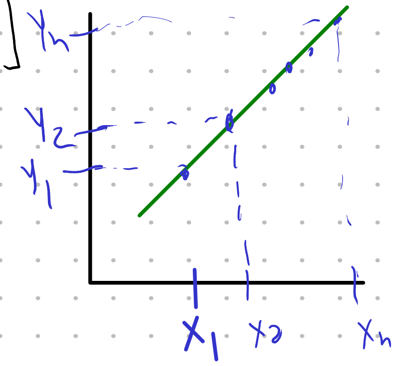
$$y = mx + b$$

$$y_1 = m x_1 + b$$
$$y_2 = m x_2 + b$$
$$\vdots$$
$$y_n = m x_n + b$$

$$\left[\begin{array}{l} y_1 = \begin{bmatrix} 1 & x_1 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} \\ \vdots \\ y_n = \begin{bmatrix} 1 & x_n \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} \end{array}\right.$$

$$y_n = 1 * b + m x_n$$
$$y_n = m x_n + b$$

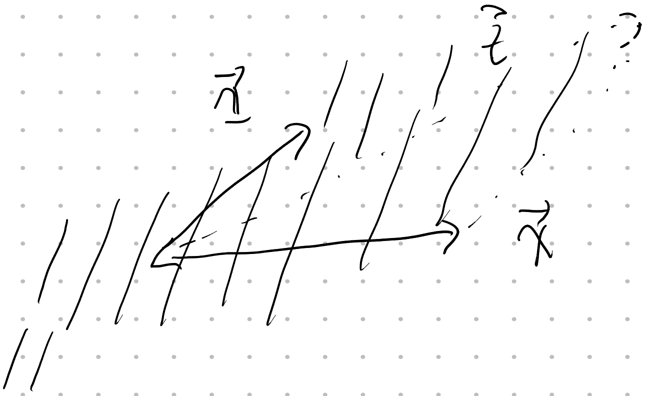
$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix}$$



$$\vec{y}' = P \vec{v}^* \qquad \vec{v}^* = \begin{bmatrix} b^* \\ m^* \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & x_1 \\ \vdots & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} = \begin{bmatrix} \vec{1} & \vec{x} \end{bmatrix}, \quad \vec{1}, \vec{x} \in \mathbb{R}^n$$

$$C(P) = \{ \vec{z} \in \mathbb{R}^n : \vec{z} = c_1 \vec{1} + c_2 \vec{x}, \ c_1, c_2 \text{ constantes} \}$$



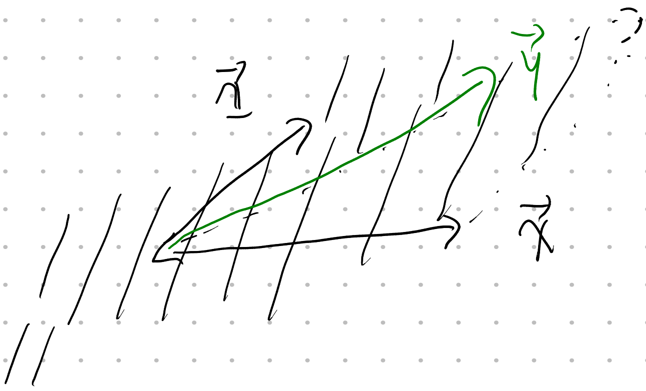
Ej: $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\vec{x}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \hat{i}$, $\vec{x}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \hat{j}$

$C(I_2) = \{ c_1 \hat{i} + c_2 \hat{j} \}$

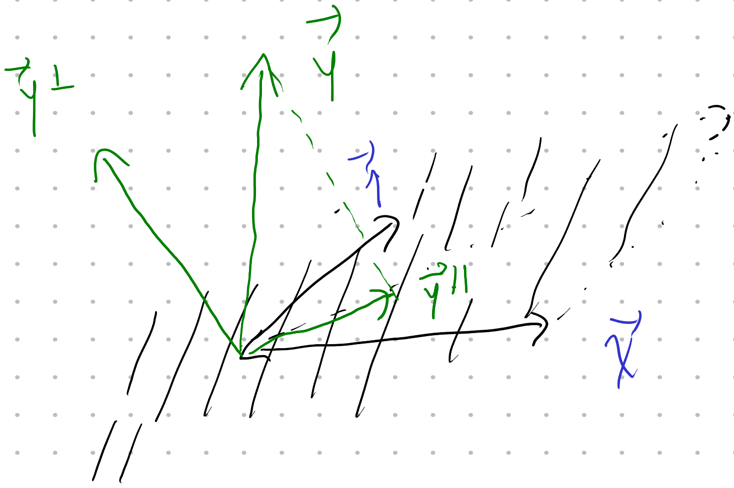
$\vec{y} = P\vec{v}$

$$\vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \begin{bmatrix} \vec{1} & \vec{x} \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix} = \underline{b\vec{1} + m\vec{x} = \vec{y}}$$



$\vec{y}^{\parallel} = P\vec{v}^*$

$$\vec{y}^{\parallel} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} b^* \\ m^* \end{bmatrix}$$

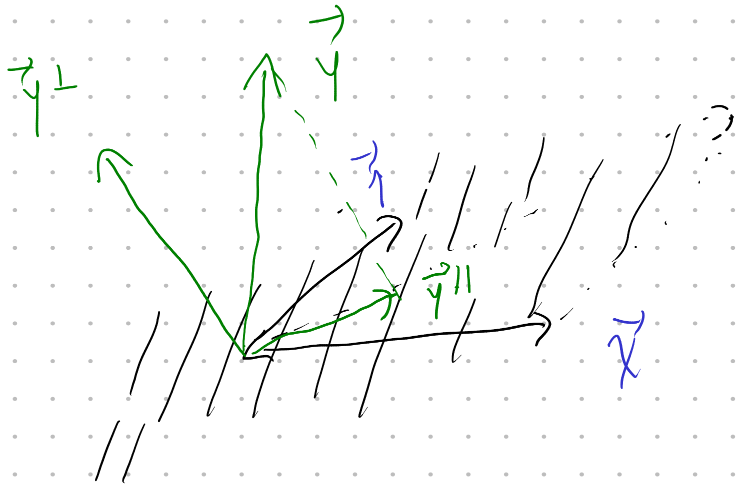


$\vec{y}^{\parallel} = \mathrm{Proj}_{C(P)} \vec{y}$

$\vec{y}^{\perp} + \vec{y}^{\parallel} = \vec{y}$

$\vec{y}^{\parallel} = \vec{y} - \vec{y}^{\perp}$

$\vec{y}^{\parallel} = P\vec{v}^*$ $\qquad$ $\vec{y} - \vec{y}^{\perp} = P\vec{v}^*$



$\vec{y}^{\perp} \cdot \vec{1} = 0$

$\vec{y}^{\perp} \cdot \vec{x} = 0$

$$\begin{bmatrix} \vec{1} \\ \vec{x} \end{bmatrix} \begin{bmatrix} \vec{y}^{\perp} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} y_1^{\perp} \\ y_2 \\ \vdots \\ y_n^{\perp} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

P^T

$\vec{y} - \vec{y}^{\perp} = P\vec{v}^*$

$[\vec{y} - P\vec{v} = \vec{y}^{\perp}]$

$P^T[\vec{y} - P\vec{v}^*] = P^T\vec{y}^{\perp} = \vec{0}$

$P^T \vec{y}^{\perp} = \vec{0}$

$P^T\vec{y} - P^T P\vec{v}^* = 0$

$P^T P \vec{v}^* = P^T \vec{y}$

$\vec{v}^* = (P^T P)^{-1} P^T \vec{y}$

$(x_1, y_1), (x_2, y_2)$

$$\begin{bmatrix} y_1^{\parallel} \\ y_2^{\parallel} \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix}}^{P} \begin{bmatrix} b^* \\ m^* \end{bmatrix}$$

$$\begin{bmatrix} b^* \\ m^* \end{bmatrix} = \left[\begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$C \begin{cases} t \\ n \end{cases}$$

$$100 \quad [$$

$$C = c_1 t + c_2 n + c_3$$

$$C = \begin{bmatrix} 1 & t_1 & n_1 \\ 1 & t_2 & n_2 \\ \\ 1 & t_N & n_N \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix} \leftarrow \text{modelo}$$

$$P$$