Variables Aleatorias

Lanzo una moneda tres veces

Espacio muestral:

$$\Omega = \{ (a_1, a_2, a_3) ; a_i \in \{C, S\} \}$$

$$\Omega = \{ (C, C, C), (C, C, S), (C, S, S) \dots, (S, S, S) \}$$

$$len(\Omega) = 8$$

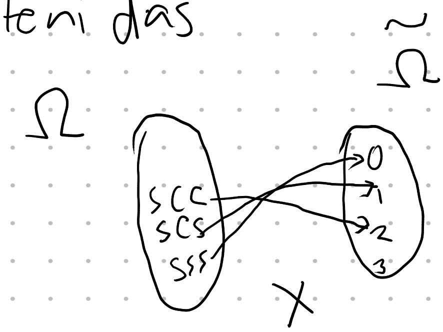
Evento: $A :=$ "La primera es cara"

$$P(A) = \frac{1}{2}$$

Variable aleatoria

$X :=$ "Numero de caras obtenidas"

$$X : \Omega \to \tilde{\Omega}$$



$$\tilde{\Omega} = \{ 0, 1, 2, 3 \}$$

$$P(0) = \frac{1}{8} \left\{ \frac{(S, S, S)}{\Omega} \right\}$$

$$P(1) = \frac{(C, S, S), (S, C, S), (S, S, C)}{\Omega}$$

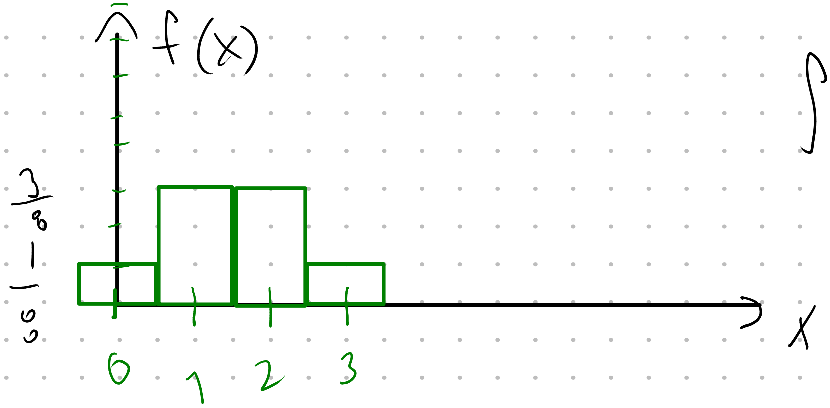
$$P(1) = \frac{3}{8}$$

$$P(2) = \frac{(S, C, C), (C, S, C), (C, C, S)}{\Omega}$$

$$P(2) = \frac{3}{8}$$

$$P(3) = \frac{1}{8}$$

$$\sum P(A_i) = 1$$



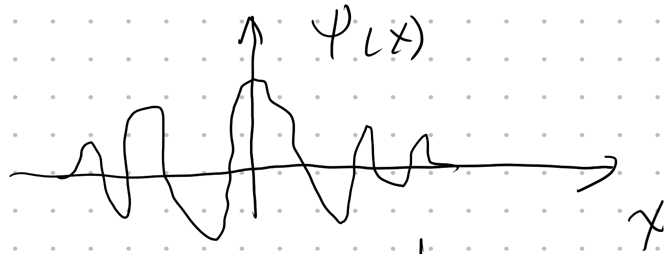
$$\int f(x)\,dx = 1$$

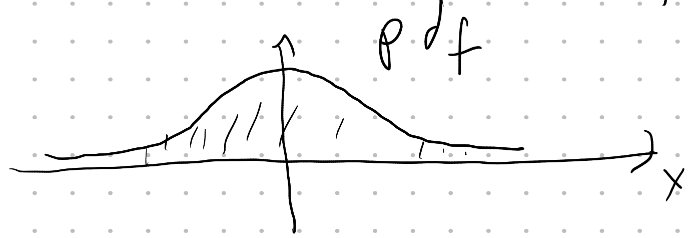
$f(x) \geq 0$ para todo X

función de densidad de probabilidad

probability density function.

$$P(x) = \Psi(x,t)\,\Psi^*(x,t)$$





Función de distribución cumulativa.

$P(C)$ $\quad C :=$ "obtener al menos 0 caras"

$$P(C) = P(0) = \frac{1}{8}$$

$P(D)$ $\quad D :=$ "obtener al menos 1 cara"

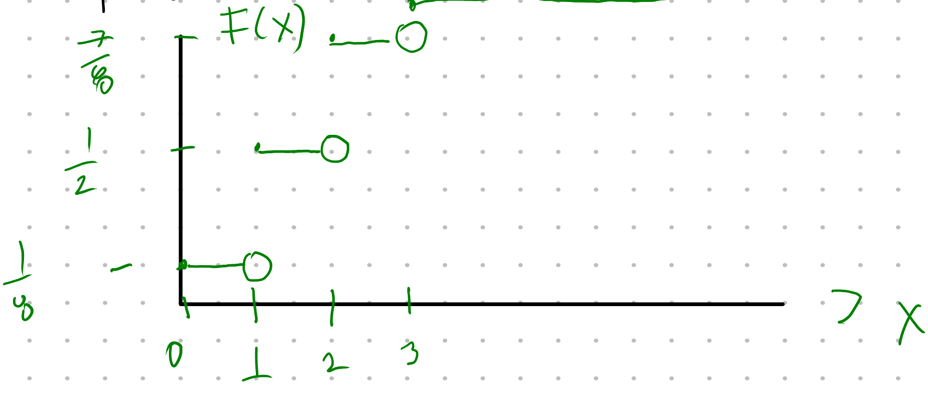
$$P(D) = P(0) + P(1) = \frac{1}{3} + \frac{3}{8} = \frac{1}{2}$$

$$P(X \leq 1) = \frac{1}{2}$$

$$P(X \leq 2) = P(0) + P(1) + P(2) = \frac{1}{3} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$$

$$P(X \leq 3) = P(\Omega) = 1$$

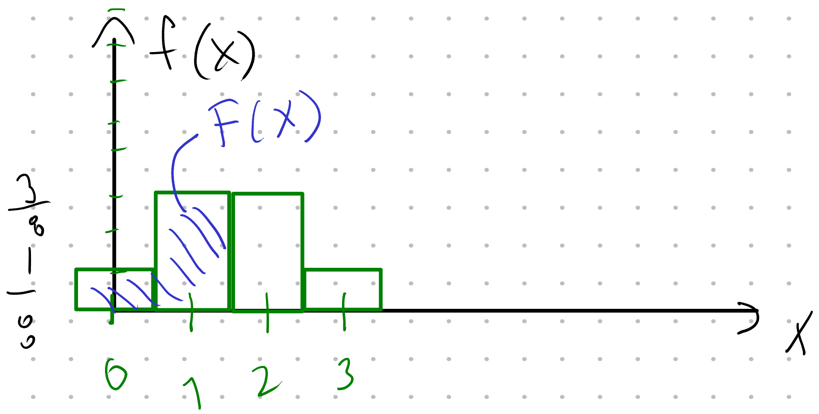
$$F(x) = P(X \le x)$$



$$\lim_{x \to \infty} F(x) = 1$$

$$\lim_{x \to -\infty} F(x) = 0$$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$



Función de distribución acumulativa: Considere una variable aleatoria X. La función de distribución acumulativa se define por:

$$F_X(x) := P_X((-\infty, x]) = P(X \leq x)$$

Función de densidad de probabilidad: Una función f_X se dice que es una función de densidad de probabilidad si es positiva, si integra a uno, y si su integral da una función acumulativa, es decir:

$$f_X(t) \geq 0 \quad \text{para todo} \quad t$$

$$\int_{-\infty}^{\infty} f_X(t)dt = 1$$

$$F_x(X) = \int_{-\infty}^{x} f_X(t)dt$$

Ej: $f(x) = \begin{cases} \alpha x(2-x) & \text{si } 0 \leq x \leq 2 \\ 0 & \text{en otro lado.} \end{cases}$

1) Calcular α,

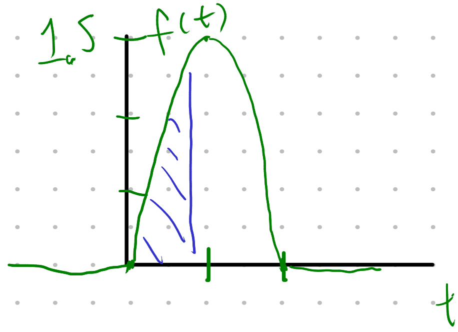
$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \rightarrow \int_0^2 \alpha(2x - x^2)\,dx = 1$$

$$\frac{\alpha 2x^2}{2}\Big|_0^2 - \frac{\alpha x^3}{3}\Big|_0^2 = 1$$

$$4\alpha - \frac{\alpha 8}{3} = 1 \qquad 12\alpha - 8\alpha = 3$$

$$4\alpha = 3 \qquad \alpha = \frac{3}{4}$$

$$f(x) = \begin{cases} \frac{3}{4} x(2-x) & 0 \le x \le 2 \\ 0 & \end{cases}$$



Cual es la función cumulativa

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

$$= \int_0^x \frac{3}{4} t(2-t)\,dt = \frac{3}{2}\frac{t^2}{2} - \frac{3}{4}\frac{t^3}{3}\Big|_0^x$$

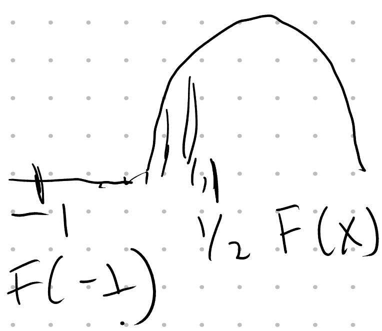
$$= \frac{3}{4}x^2 - \frac{x^3}{4} = \frac{1}{4}x^2(3-x)$$

$$F(x) = \begin{cases} 0 & \text{si } x < 0 \\ \frac{1}{4}x^2(3-x) & \text{si } 0 \le x \le 2 \\ 1 & \text{si } x > 2 \end{cases}$$

c) $$P\left(-1 \le X \le \frac{1}{2}\right) = \int_{-1}^{1/2} f(x)\,dx = \int_{-\infty}^{1/2} f(x)\,dx - \int_{-\infty}^{-1} f(x)\,dx$$

$$= F\left(\frac{1}{2}\right) - F(-1)$$

$$= \frac{5}{64}$$



Valor esperado: El valor esperado EX de una variable aleatoria X está definido por:

Caso discreto:

$$EX = \sum_{i=1}^{\infty} x_i P(X = x_i)$$

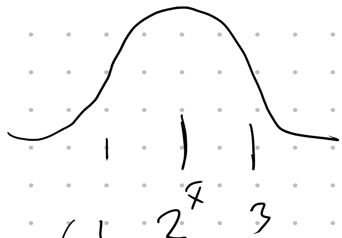
Caso continuo:

$$EX = \int_{-\infty}^{\infty} x f_X(x) dx$$

Varianza: La varianza se define como la dispersión de los datos alrededor de la media:

$$Var(X) = E[(X - EX)^2]$$

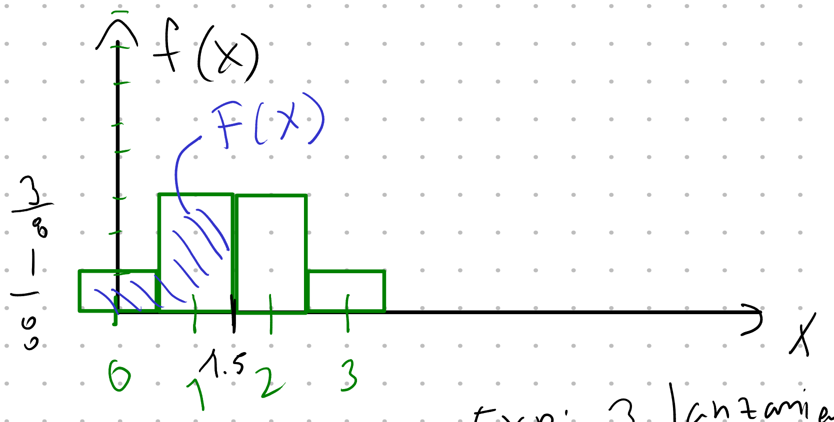
$\frac{1}{N} \sum (x - \bar{x})^2$ MSE



$\frac{1}{N} \sum |x - \bar{x}|$ MAE

$x - \bar{x} = 1$

$x - \bar{x} = 1 - 2 = -1$



Exp: 3 lanzamientos

8 Exps:

$$X = \{0, 1, 1, 1, 2, 2, 2, 3\}$$

$$\bar{X} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{3+6+3}{8} = \frac{12}{8} = \frac{3}{2} \leftarrow$$

Promedio

Valor esperado

$$\bar{X} = E[X] = 0 \times \left(\frac{1}{8}\right) + 1 \times \left(\frac{3}{8}\right) + 2 \times \left(\frac{3}{8}\right) + 3 \times \left(\frac{1}{8}\right)$$

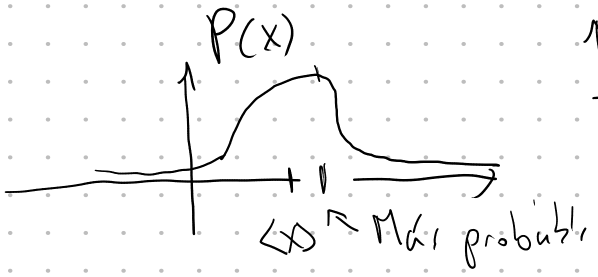
$$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$$

Valor esperado

$$E[X] = \sum_{i=1}^{\infty} P(X = x_i)\, x_i \leftarrow \text{Promedio.}$$

P $\psi^* \psi$

$$\langle x \rangle = \int \psi^* \psi \, x \, dx$$



$$CM = \frac{\int \lambda(x)\, x\, dx}{M}$$

Varianza: Mide la dispersión de mis datos.

$$V(x) = \frac{\sum (x_i - \bar{x})^2}{N}$$

↑ Varianza

$$\sigma(x) = \sqrt{V(x)}$$

↑ Desviación estandar.

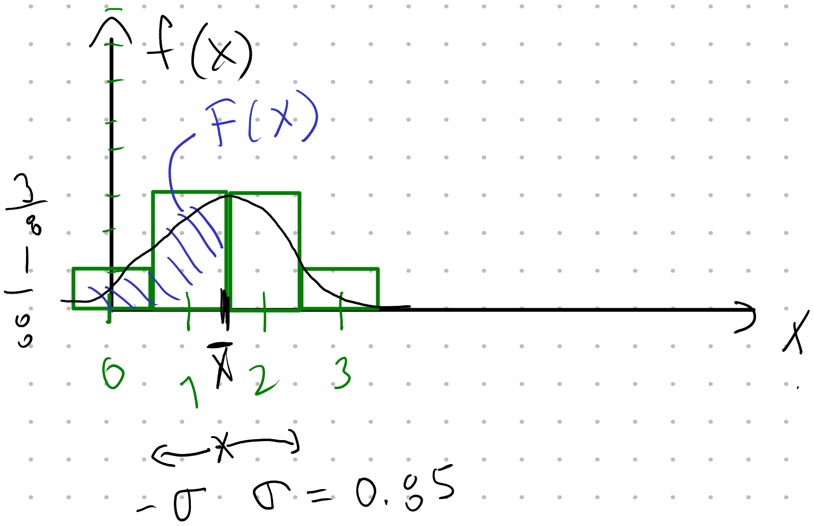
$$X = \{0, 1, 1, 1, 2, 2, 2, 3\}$$

$$V(x) = \frac{\sum_{i=1}^{8} (x_i - \bar{x})^2}{8} = \frac{1}{8}\Big[1(0-1.5)^2 + 3\cdot(1-1.5)^2 + 3(2-1.5)^2 + 1(3-1.5)^2\Big]$$

$$= \frac{1}{8}\left[\frac{9}{4} + \frac{3}{4} + \frac{3}{4} + \frac{9}{4}\right]$$

$$V(x) = \frac{1}{8}\left(\frac{24}{4}\right) = \frac{3}{4}$$

$$\sigma(x) = \frac{\sqrt{3}}{2} \simeq 0.85$$



$$X = \{0, 1, 1, 1, 2, 2, 2, 3\}$$

$$X = 1.5 \pm 0.85 = \bar{x} \pm \sigma(x)$$