Monte-Carlo Methods, and Markov Chain

1. Randomized Methods History
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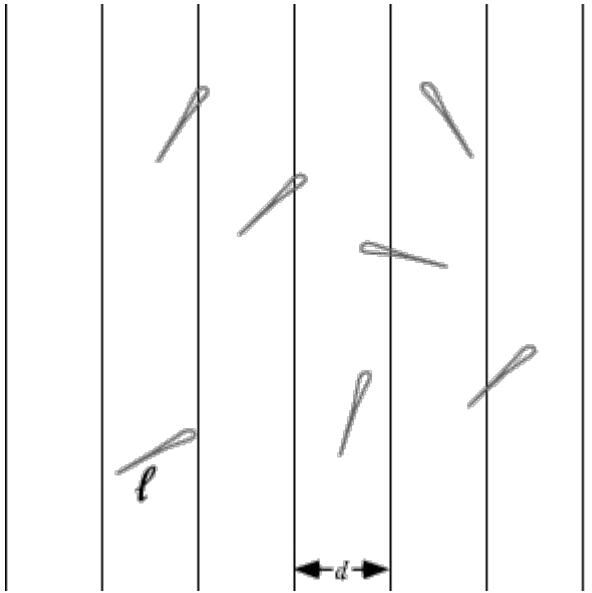
Randomized Methods History

Georges Louis Leclerc (1777)
calculating π using needles

$$p = \frac{2 * l}{\pi * d}$$

$$p \approx \frac{P}{N}$$

$$\pi = \frac{N * 2 * l}{P * d}$$



Randomized Methods History

Stanislaw Ulam played hundreds of games to find the probability of a perfect hand while he was in a hospital.

In collaboration with Nicholas Metropolis and John von Neumann (1949) published the first paper on Monte Carlo methods

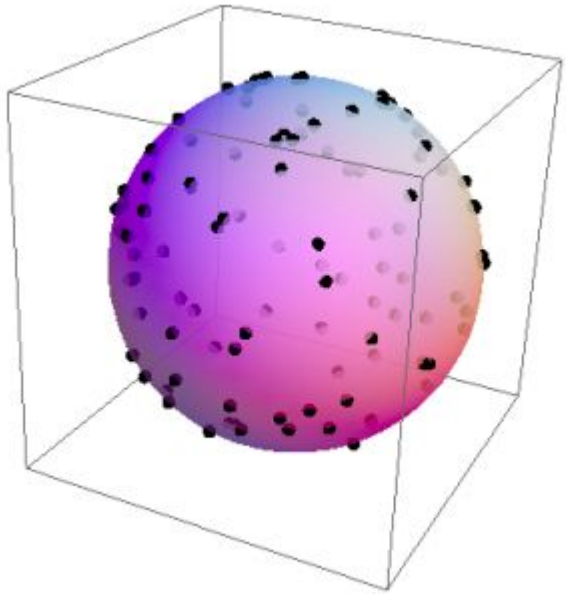
Monte Carlo

A set of randomized algorithms and resource constraint, that may or may not be correct within a certain margin of error. In other words, it gambles on the correct answer with some probability (usually high).

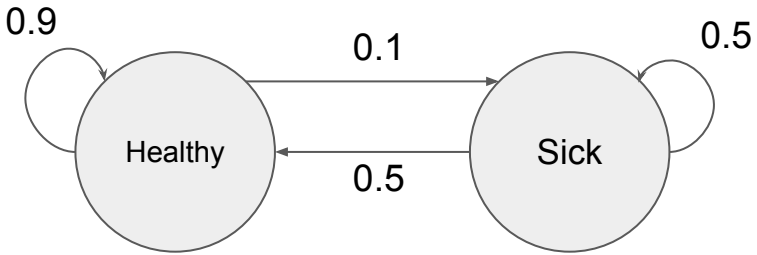
Monte Carlo

Used:

- Approximate integrals
- Optimization
- Statistical Distributions



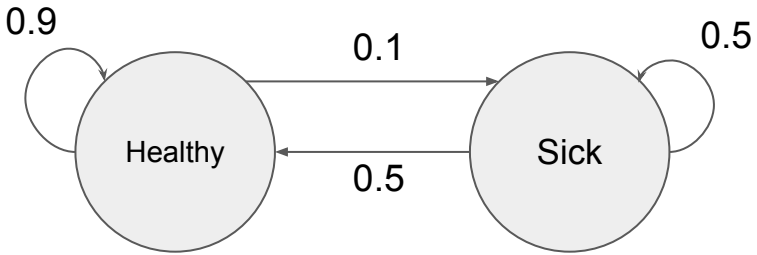
Markov Chain



It can start in any state

T	Healthy	Sick
0	1	0
1	0.9	0.1
2	(0.9*0.9) + (0.1*0.5) = 0.86	(0.9*0.1) + (0.1*0.5) = 0.14
3	(0.86*0.9) + (0.14*0.5) = 0.844	(0.86*0.1) + (0.14*0.5) = 0.156
	...	...
t →∞	0.833	0.167

Stationary Distribution



T	Healthy	Sick
$t \to \infty$	0.833	0.167

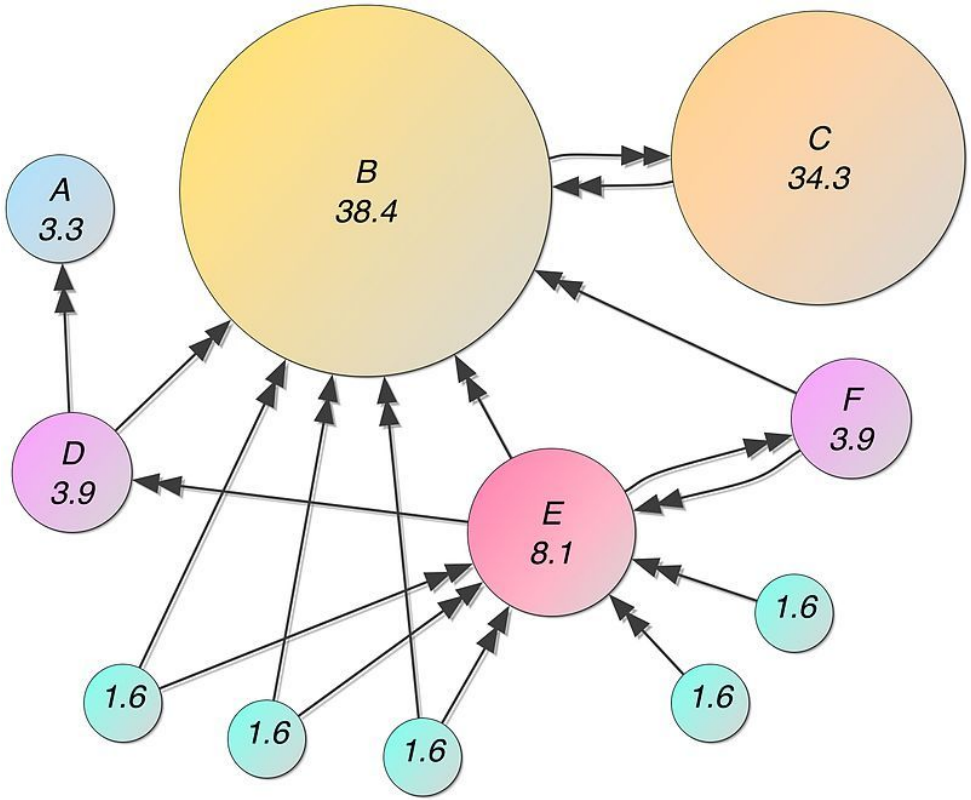
Knowing the stationary distribution, it's easier to sample from.

$\rho \rightsquigarrow U(x)$

$$\text{Sample} = \begin{cases} \rho \le 0.833 & \text{Healthy} \\ \rho > 0.833 & \text{Sick} \end{cases}$$

Page Rank

- Similar to a Markov Chain to determine the connected web pages.
- Built as product of a PhD research by Sergei Brin and Larry Page.
- Used to improve web searching.
- The origin of google.



Markov Chain Monte Carlo method for sampling from any probability distribution (possibly intractable).

References

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