Maximum Likelihood Estimation

(Estimación de Máxima Verosimilitud).

$$f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)}{2\sigma^2}\right\}$$

μ, σ : Parametros que vamos a estimar,

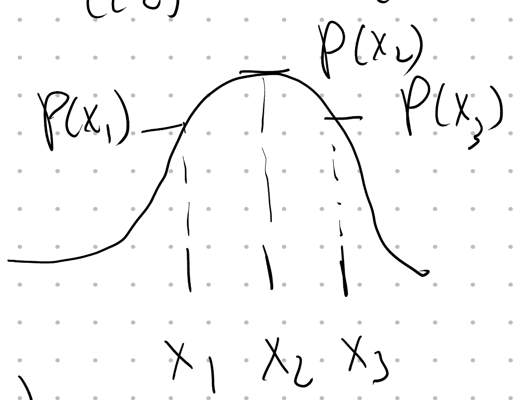
Ejemplo: Probalidad de pasar un examen con dos personas.

$P_1 : 0.05$ $\quad P_2 : 0.01$ } Caso 1

$P_1 : 0.66$ $\quad P_2 : 0.48$ } Caso 2.

Caso 2 es más probable por $P_1 P_2 (c2) > P_1 P_2 (caso\ 1)$

Maximizar la verosimilitud



$$l = P(x_1) P(x_2) P(x_3)$$

$x_1, \dots, x_N$, con prob: $P(x_1) \cdots P(x_N)$

Maximizar

$$P(x_1) P(x_2) \cdots P(x_N) = \prod_{i=1}^{N} P(x_i)$$

$$P(\mu,\sigma|\{x_i\}) = \prod \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{\frac{(x_i-\mu)^2}{2\sigma^2}\right\}$$

$$\frac{\partial P}{\partial \mu} = 0 \quad , \quad \frac{\partial P}{\partial \sigma} = 0$$

$$P(\mu,\sigma|\{x_i\}) = \prod \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{\frac{(x_i-\mu)^2}{2\sigma^2}\right\}$$

$$l(\mu,\sigma) = \log(P(\mu,\sigma)) = \log\left(\prod \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{\frac{(x_i-\mu)^2}{2\sigma^2}\right\}\right)$$

$$= \sum_{i=1}^{N} \log\left(\frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{\frac{-(x_i-\mu)^2}{2\sigma^2}\right\}\right)$$

$$\log(ab) = \log(a) + \log(b) \qquad \log(\prod y_i) = \sum \log(y_i)$$

$$l(\mu,\sigma) = N\log\left(\frac{1}{\sqrt{2\pi}}\right) - N\log\sigma + \sum \log\left[\exp\left\{\frac{-(x_i-\mu)^2}{2\sigma^2}\right\}\right]$$

$$\sum_{i=1}^{N} \log\left(\frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma}\right) = \sum_{i=1}^{N} \log\left(\frac{1}{\sqrt{2\pi}}\right) + \sum_{i}^{N} \log\left(\frac{1}{\sigma}\right)$$

$$= N\log\left(\frac{1}{\sqrt{2\pi}}\right) + N\log(\sigma^{-1})$$

$$= N\log\left(\frac{1}{\sqrt{2\pi}}\right) - N\log(\sigma)$$

$$l(\mu,\sigma) = N\log\left(\frac{1}{\sqrt{2\pi}}\right) - N\log\sigma + \sum \log\left\{\exp\left(\frac{-(x_i-\mu)^2}{2\sigma^2}\right)\right\}$$

$$l(\mu,\sigma) = N\log\left(\frac{1}{\sqrt{2\pi}}\right) - N\log\sigma + \sum\left(\frac{-(x_i-\mu)^2}{2\sigma^2}\right)$$

$$\frac{\partial l}{\partial \mu} = 0 + 0 + \sum\left(\frac{-(x_i-\mu)}{2\sigma^2}\right)(-1) = 0$$

$$\frac{(-1)(-1)}{2\sigma^2} \sum (x_i - \mu) = 0$$

$$\sum (x_i - \mu) = 0 \Rightarrow \sum x_i - \sum_{i=1}^{N} \mu = 0$$

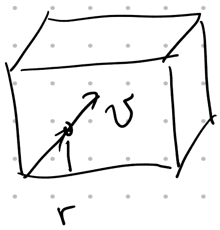
$$\sum x_i - N\mu = 0$$

$$\mu = \frac{\sum x_i}{N} \text{ : promedio.}$$

$$\frac{\partial l(\mu, \sigma)}{\partial \sigma} \qquad \sigma^2 = \frac{\sum (x_i - \mu)^2}{N} \text{ : Varianza.}$$

Ejemplos de máxima verosimolitud.

Física Estadística

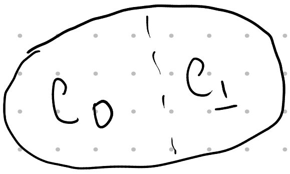


→ Espacio de Fase

El estado de equilibrio es el estado que máximiza la verosimilitud. A partir de esto se encuentra la temperatura T.

Redes Neuronales:

Datos

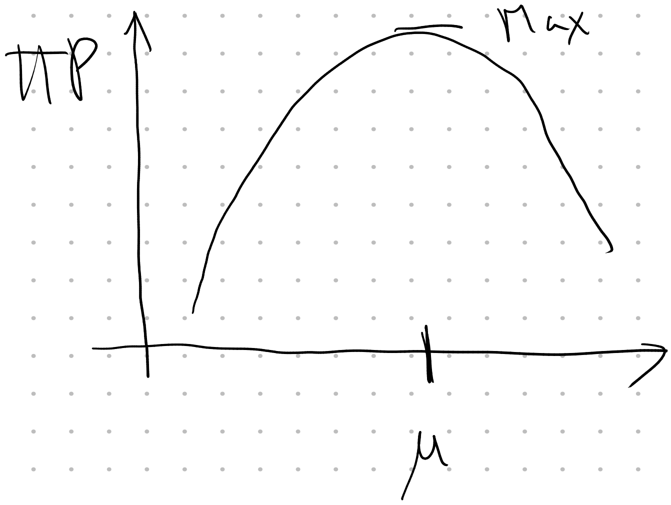


Etiquetas 0, 1

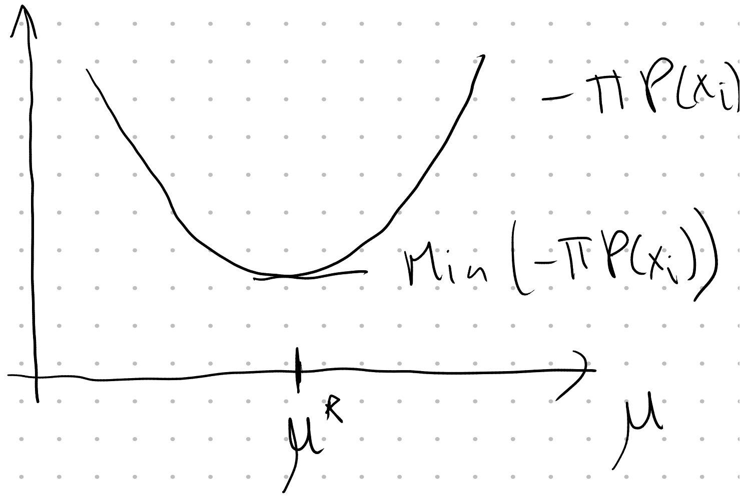
$$\prod P(C_0|X) \prod P(C_1|X)$$

Categorical cross entropy

Se entrena con gradiante descente.



ΠP
Max
μ



$-\Pi P(x_i)$
$\text{Min}\left(-\Pi P(x_i)\right)$
μ^*
μ