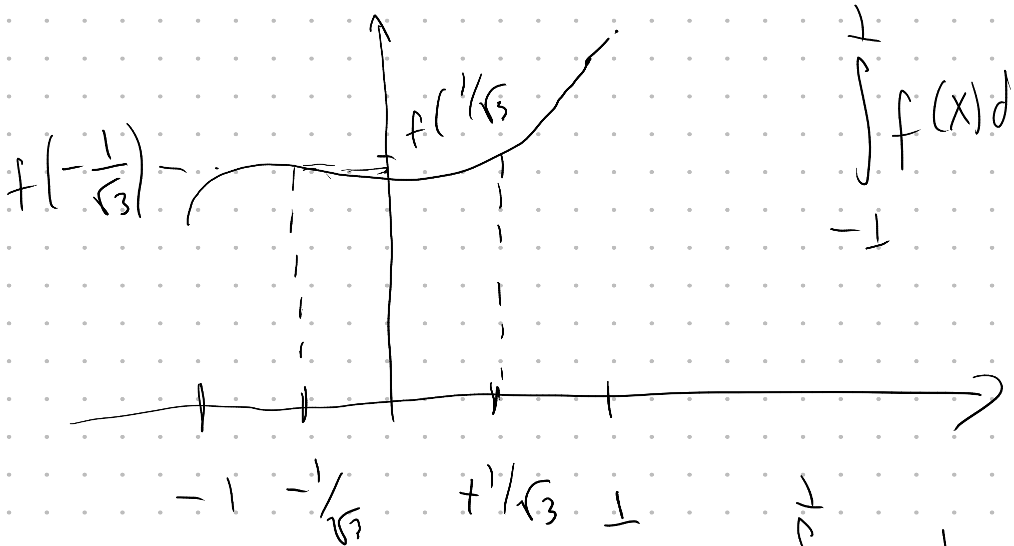
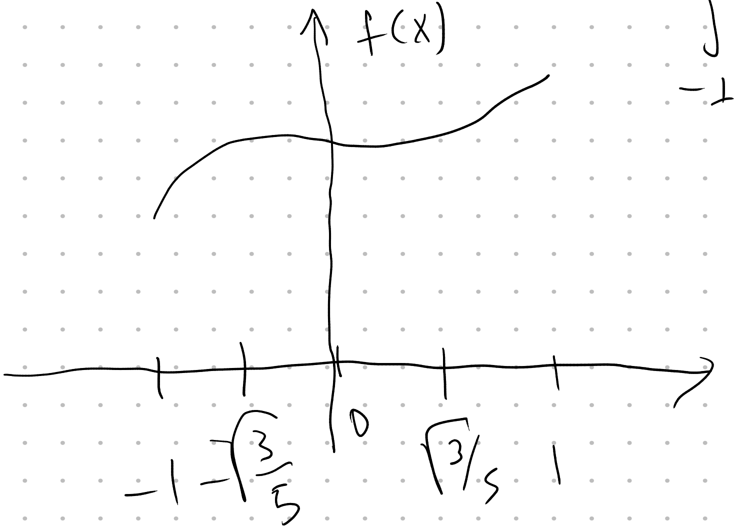
Gaussian Cuadrature



$$\int_{-1}^{1} f(x)\,dx = w_0 f\left(-\frac{1}{\sqrt{3}}\right) + w_1 f\left(\frac{1}{\sqrt{3}}\right)$$

$$= 1\, f\left(-\frac{1}{\sqrt{3}}\right) + 1\, f\left(\frac{1}{\sqrt{3}}\right)$$



$$\int_{-1}^{1} f(x)\,dx = w_0 f\left(-\sqrt{\frac{3}{5}}\right) + w_1 f(0) + w_2 f\left(\sqrt{\frac{3}{5}}\right)$$

$$= \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right)$$

$$\int_{-1}^{1} f(x)\,dx = \sum_{i=1}^{N} w_i f(x_i)$$

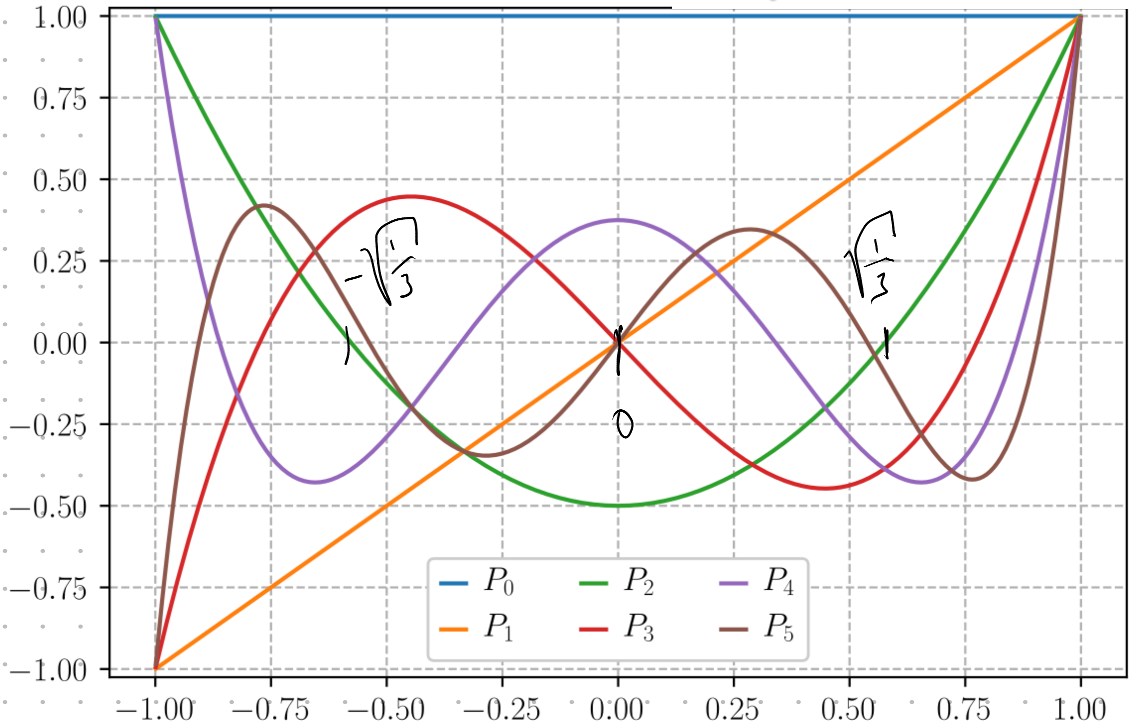
N: Grado de la cuadratura.

Legendre Polynomials

$P_0(x) = 1$ $\qquad$ $P_1(x) = x$

$P_2(x) = \frac{1}{2}(3x^2 - 1)$ $\qquad$ $P_3(x) = \frac{1}{2}(5x^3 - 3x)$

$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$ $\qquad$ $P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$

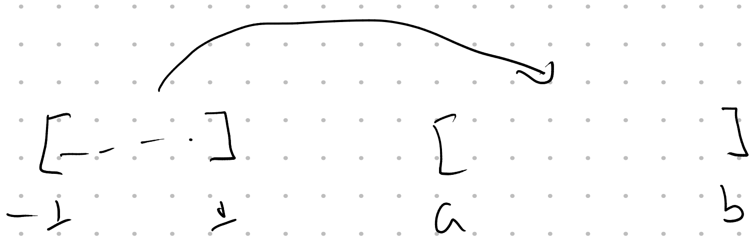


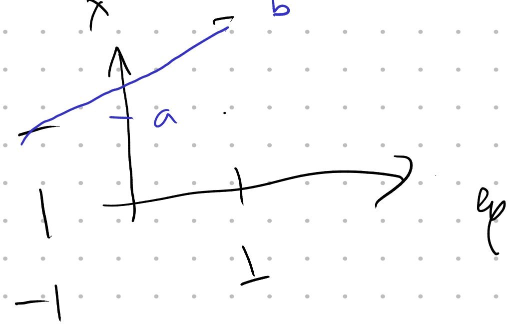
$$\int f(x)\,dx = \sum w_i f(x_i) = \frac{h}{3}\left[f(x_0) + f(x_n) + 4\sum_{imp} f(x_i) + 2\sum_{pare} f(x_i) \right]$$

$$\int_{-1}^{1} f(x)\,dx \approx \sum_{i=1}^{N} w_i f(x_i)$$

$$x = \frac{b-a}{2}\xi + \frac{b+a}{2}$$

$$\begin{cases} \xi = -1 \\ x = \frac{b-a}{2}(-1) + \frac{b+a}{2} = \frac{a-b}{2} + \frac{b+a}{2} = a \\ \xi = 1 \quad x = \frac{b-a}{2} + \frac{b+a}{2} = b \end{cases}$$

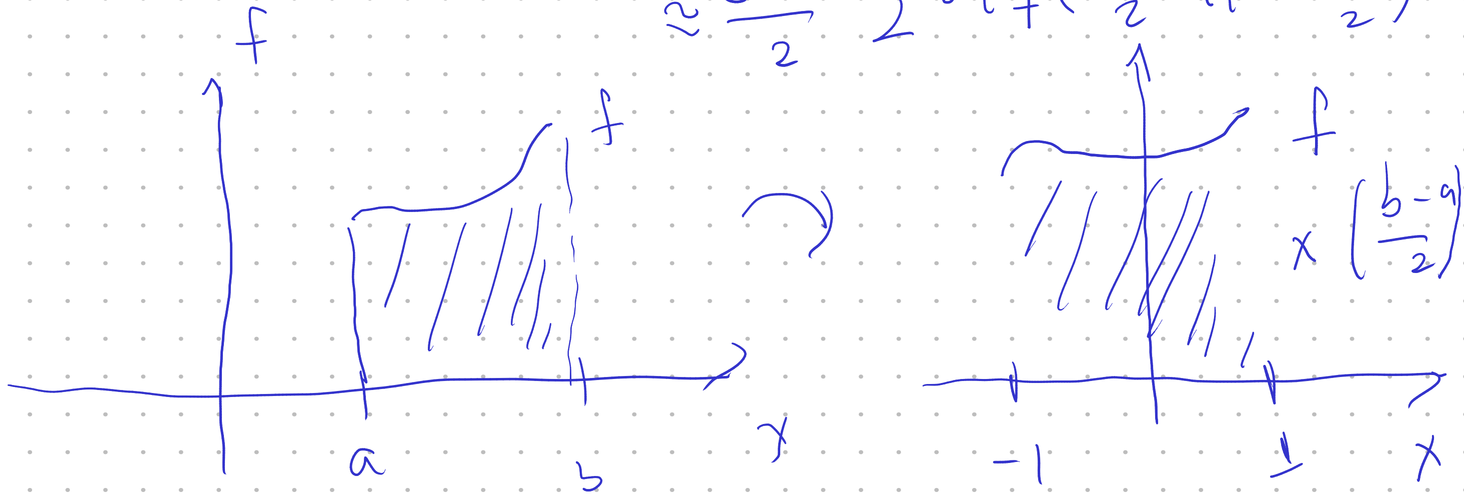




$$\int_a^b f(x)\,dx = \int f\left(\frac{b-a}{2}\xi + \frac{b+a}{2}\right) dx$$

$$dx = \frac{b-a}{2}\, d\psi \qquad \int_a^b f(x)\,dx = \frac{b-a}{2}\int_{-1}^{1} f\left(\frac{b-a}{2}\psi + \frac{b+a}{2}\right) d\psi$$

$$\approx \frac{b-a}{2} \sum w_i\, f\left(\frac{b-a}{2}\psi_i + \frac{b+a}{2}\right)$$



Cuadratura de Gauss Legendre Explicacion

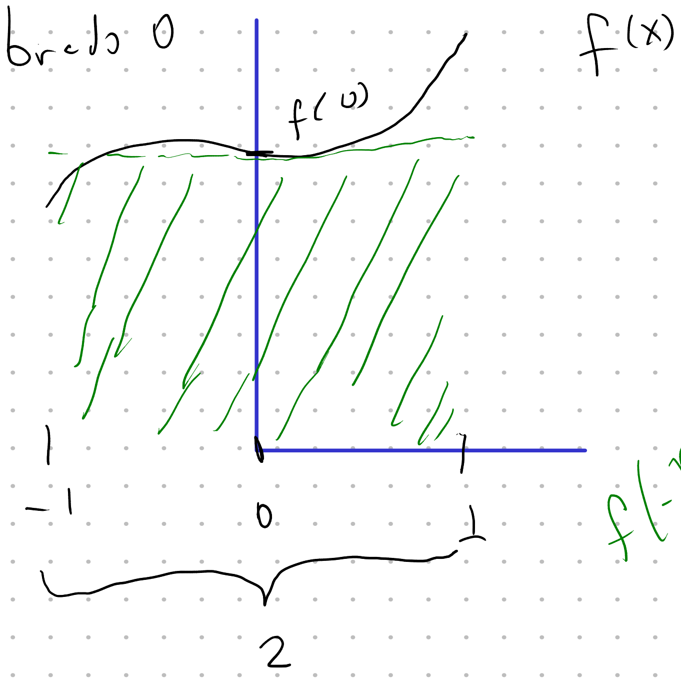
$w_0 = [2]$

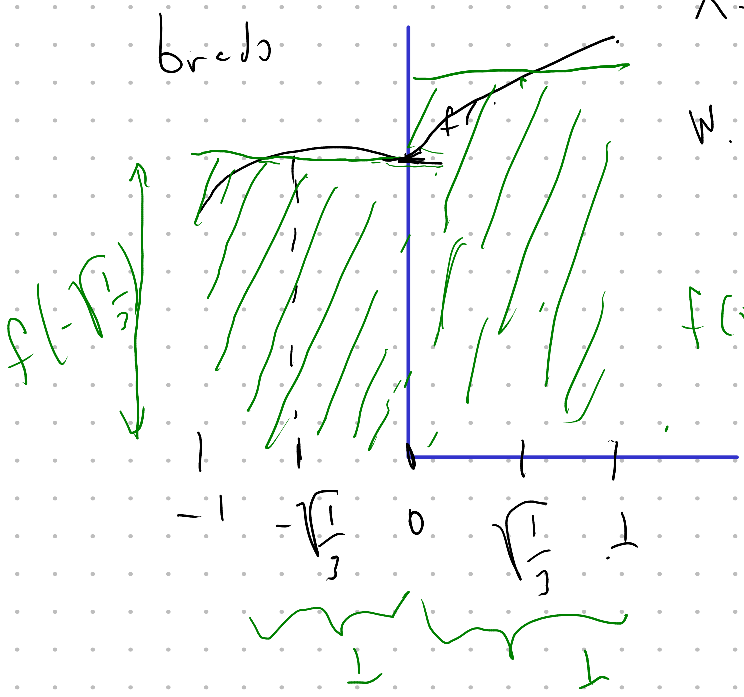
$x_0 = [0]$

$x = \left[-\sqrt{\tfrac{1}{3}}, \sqrt{\tfrac{1}{3}}\right]$

$w = [1, 1]$

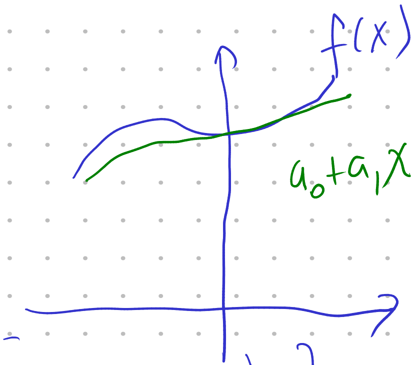
$1 + 1 = 2$





$$\int_{-1}^{1} f(x)\,dx \quad (-1, 1)$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots +$$



$$f(x) = a + bx + cx^2 + \cdots + \cdots$$

$$f(x) \approx a + bx \leftarrow \text{impar}$$

$$\int_{-1}^{1} f(x)\,dx = ax + \frac{bx^2}{2}\Big]_{-1}^{1} = \left[a + \frac{b}{2}\right] - \left[a(-1) + \frac{b}{2}\right]$$

$$= 2a$$

$w = [2] \quad x = [0] \qquad f(0) = a \qquad W = 2$

$$2f(0) = 2a = \int_{-1}^{1} f(x)\,dx \qquad a = f(0)$$

$$W = 2$$

$$f(x) \approx a + bx + cx^2$$

$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} a\,dx + \int_{-1}^{1} bx\,dx + \int_{-1}^{1} cx^2\,dx$$

$$\text{impar } \int = 0$$

$$= \int_{-1}^{1} a\,dx + \int_{-1}^{1} cx^2\,dx = \left[ax + \frac{cx^3}{3}\right]_{-1}^{1}$$

$$= \left[a + \frac{c}{3}\right] - \left[a(-1) + c\frac{(-1)^3}{3}\right]$$

$$\int_{-1}^{1} f(x)\,dx = 2\left[a + \frac{c}{3}\right] *$$

$$X = \left[-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right]$$

$$W = [1, 1]$$

$$f(x) = a + bx + cx^2$$

$$f\left(-\sqrt{\frac{1}{3}}\right) = a - b\sqrt{\frac{1}{3}} + c\frac{1}{3}$$

$$f\left(\sqrt{\frac{1}{3}}\right) = a + b\sqrt{\frac{1}{3}} + c\frac{1}{3}$$

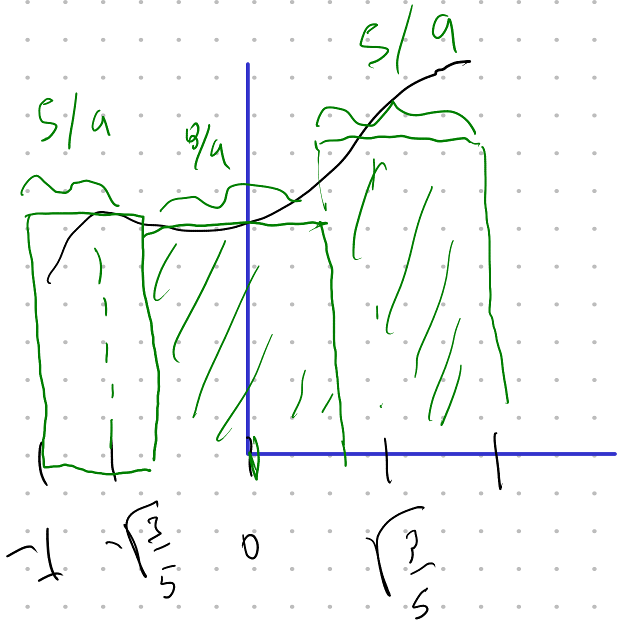
$$w_0 f(x_0) + w_1 f(x_1) = 1\left(a - b\sqrt{\frac{1}{3}} + c\frac{1}{3}\right) + 1\left(a + b\sqrt{\frac{1}{3}} + c\frac{1}{3}\right)$$

$$= 2\left[a + \frac{c}{3}\right] *$$

$$\int a + bx + cx^2 = w_0 f(x_0) + w_1 f(x_1) = 2\left[a + \frac{c}{3}\right]$$

$$\int (a + bx + cx^2 + dx^3)\,dx = \int (a + bx + cx^2)\,dx + \cancel{\int_{-1}^{1} (dx^3)\,dx}^{\,0}$$

$\int x^n$ necesito $\frac{n}{2}$ pesos



$$\frac{5}{9} + \frac{8}{9} + \frac{5}{9} = 2$$

quad