Numerical methods for integration

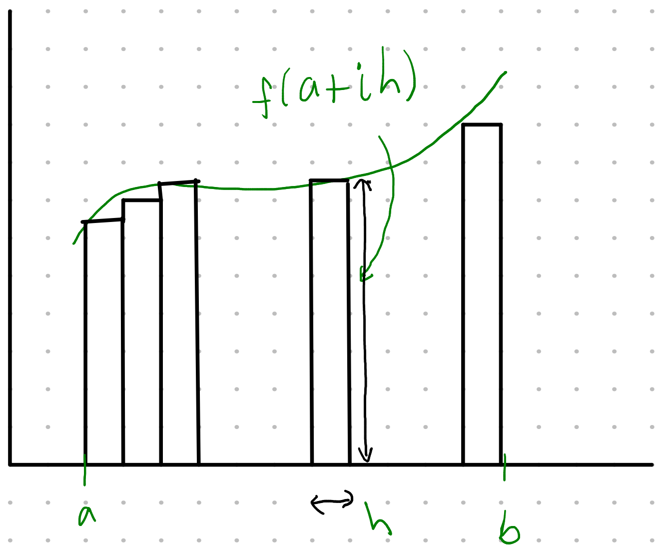
Rectangle Method

Consider a function $f(x)$, and let's estimate $\int_a^b f(x)dx$,
to do that, we may divide the domain $[a,b]$ into small intervals of size h, there are in total $N = \frac{(b-a)}{h}$ such intervals,

the rectangle method would be given by,

$$I(x) = \int_a^b f(x)\,dx \approx \sum_{i=0}^{N-1} f(a+ih)h = \sum_{i=0}^{N-1} f\left(a+i\frac{(b-a)}{N}\right)\frac{b-a}{N}$$

which results from approximating the integral with rectangles,
picture,



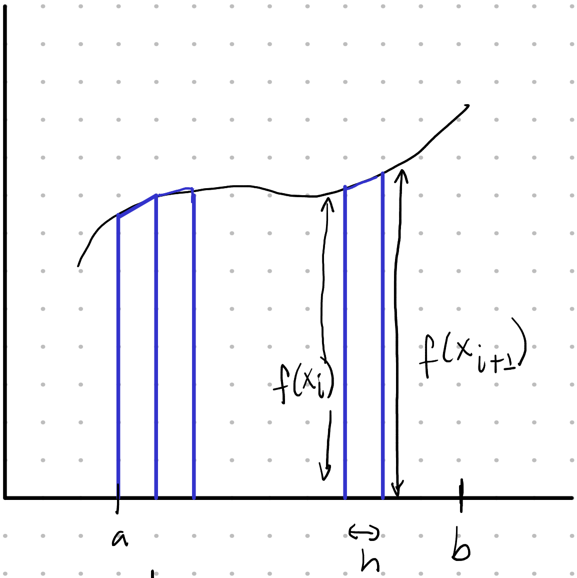
Trapezoid Method

formula →

$$\int_a^b f(x)\,dx \approx \frac{h}{2}(f(x_0)+f(x_n)) + h\sum_{j=1}^{n-1} f(x_j)$$

In the the trapezoid method we divide the integral into trapezoids instead of rectangles,
let's divide the interval $[a,b]$ into smaller intervals separated a distance h, let's say these points are $\{x_0, x_1, \ldots, x_m\}$
so that, $x_{i+1} - x_i = h$, $x_0 = a$, $x_m = b$,

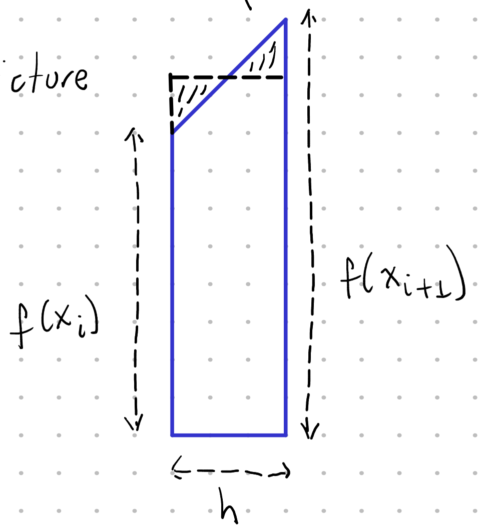
picture



the area of each trapezoid is given by,

$$A_i = h \times \left(\frac{f(x_i) + f(x_{i+1})}{2} \right)$$

picture



the total area is given by,

$$A_T = \sum_{i=0}^{n-1} h * \left(\frac{f(x_i) + f(x_{i+1})}{2} \right)$$

$$= \frac{h}{2} * \left[\sum_{i=0}^{n-1} f(x_i) \right] + \frac{h}{2} \times \left[\sum_{i=0}^{n-1} f(x_{i+1}) \right]$$

$$= \frac{h}{2} * \left[f(x_0) + f(x_n) + 2 \sum_{i=1}^{n-1} f(x_i) \right]$$

$$= \frac{h}{2} \left[f(x_0) + f(x_n) \right] + h \sum_{i=1}^{n-1} f(x_i)$$

In other terms, we have,

$$A_T = h * \left[\frac{\overset{1}{f(x_0)} + f(x_1)}{2} + \frac{f(x_1) + f(x_2)}{2} + \cdots + \frac{f(x_{n-2}) + f(x_{n-1})}{2} + \frac{f(x_{n-1}) + \overset{1}{f(x_n)}}{2} \right]$$

the end points repeat only once, while the points in the middle twice.