FTAN image

Let d_r^o be the observed waveform at receiver r for all times. FTAN first computes an image I in the (t, f) domain and then computes group velocity using the source-receiver distance to achieve an image in the (f, v_g) domain. Let I be the matrix in the (t, f) plane, then for a fixed frequency f_o the row $I(f_o, :)$ is,

$$I(f_o, :) = \mathcal{F}^{-1}\{\ \mathcal{F}\{\mathcal{H}(d_r^o)\} \odot g(\alpha, f_o)\ \}$$

where $\mathcal{F}$ is the Fourier transform, $\mathcal{H}$ is the hilbert transform and g is a gaussian filter with width parameter α.

MASW image

Let $\tilde{d^o_{f_o}}$ be the observed waveform for a fixed frequency f_o on all (linearly arranged) receivers r_x and s_p be the phase slowness. MASW first computes an image I on the (f, s_p) plane and then computes an image on (f, v_p). Let I be the matrix in the (f, s_p) plane, then for a fixed frequency f_o the column $I(:, f_o)$ is,

$$I(:, f_o) = L^*_{f_o}\, \tilde{d^o_{f_o}}, \qquad\qquad L_{f_o} = \exp(2\pi i f_o \cdot r_x \otimes s_p).$$

Phase and group velocities

Let v_g, v_p, ω and k denote group velocity, phase velocity, angular frequency and wavenumber respectively. We have,

$$v_g = \partial_k\, \omega,$$
$$v_p = \frac{\omega}{k}.$$

Let s denote slowness,

$$v_g = \partial_k(k\, v_p) \qquad \text{or} \qquad s_g = \partial_\omega(\omega\, s_p)$$

and so

$$v_g = k\partial_k\, v_p + v_p \qquad \text{or} \qquad s_g = \omega\partial_\omega\, s_p + s_p.$$

Solving these differential equations we have,

$$v_p(\omega) = \frac{1}{k}\left(k_o\, v_p^o + \int_{k_o}^{k} v_g(k)\ \mathrm{d}k\right) \qquad \text{or} \qquad s_p(\omega) = \frac{1}{\omega}\left(\omega_o\, s_p^o + \int_{\omega_o}^{\omega} s_g(\omega)\ \mathrm{d}\omega\right).$$