

# wave solver with PML

summer 2017

The actual code models the 2D EM wave assuming  $\partial_z = 0$  and quasi-isotropic materials with attenuation for both  $E_z$  and  $\mathbf{H}$ ,

$$\begin{pmatrix} \mu_y & 0 & 0 \\ 0 & \mu_x & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \begin{pmatrix} \dot{H}_y \\ -\dot{H}_x \\ \dot{E}_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & \partial_x \\ 0 & 0 & \partial_y \\ \partial_x & \partial_y & 0 \end{pmatrix} \begin{pmatrix} H_y \\ -H_x \\ E_z \end{pmatrix} - \quad (1)$$

$$\begin{pmatrix} \sigma_y^m & 0 & 0 \\ 0 & \sigma_x^m & 0 \\ 0 & 0 & \sigma_z \end{pmatrix} \begin{pmatrix} H_y \\ -H_x \\ E_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -J_z \end{pmatrix}. \quad (2)$$

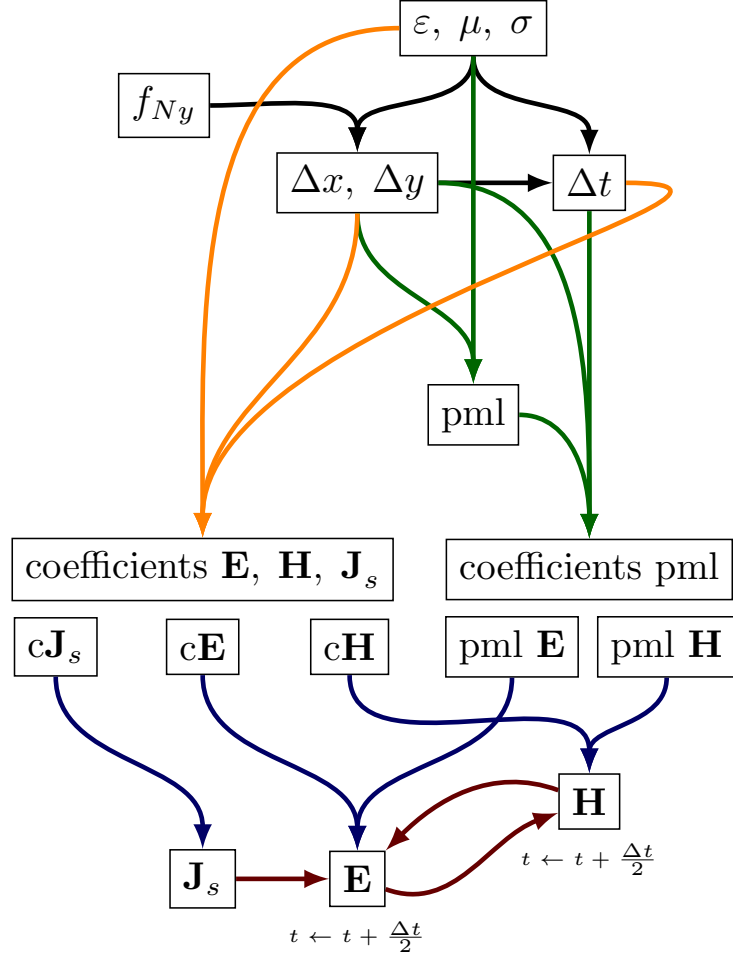


Figure 1: Main code flow.

$$\begin{aligned}
 c_o &= \frac{1}{\sqrt{\varepsilon_o \mu_o}} \\
 c_{min} &= \frac{c_o}{\sqrt{\varepsilon_{max}}} & c_{max} &= \frac{c_o}{\sqrt{\varepsilon_{min}}} \\
 \lambda_{min} &= \frac{c_{min}}{f_{Ny}} \propto \Delta x & \Delta t &= \frac{\text{cfl}}{c_{max} \sqrt{\left(\frac{1}{\Delta x}\right)^2 + \left(\frac{1}{\Delta y}\right)^2}}
 \end{aligned}$$

Figure 2: Calculation of  $\Delta x, \Delta y$  and  $\Delta t$ .

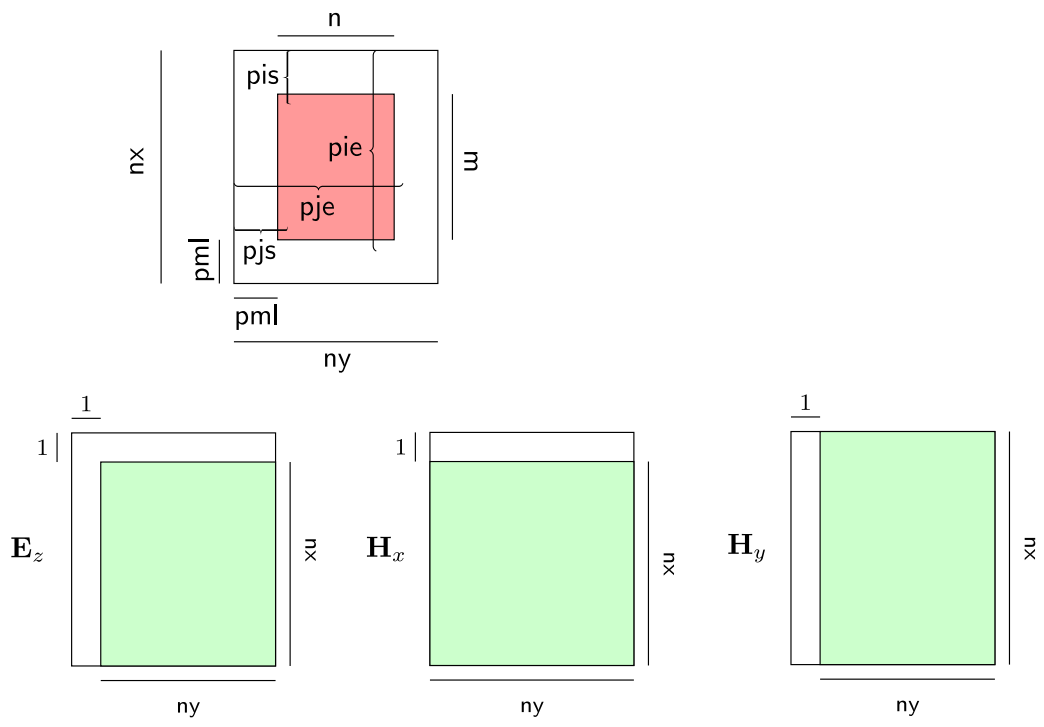


Figure 3: Grid dimensions of  $\mathbf{E}_z$ ,  $\mathbf{H}_x$ ,  $\mathbf{H}_y$ .

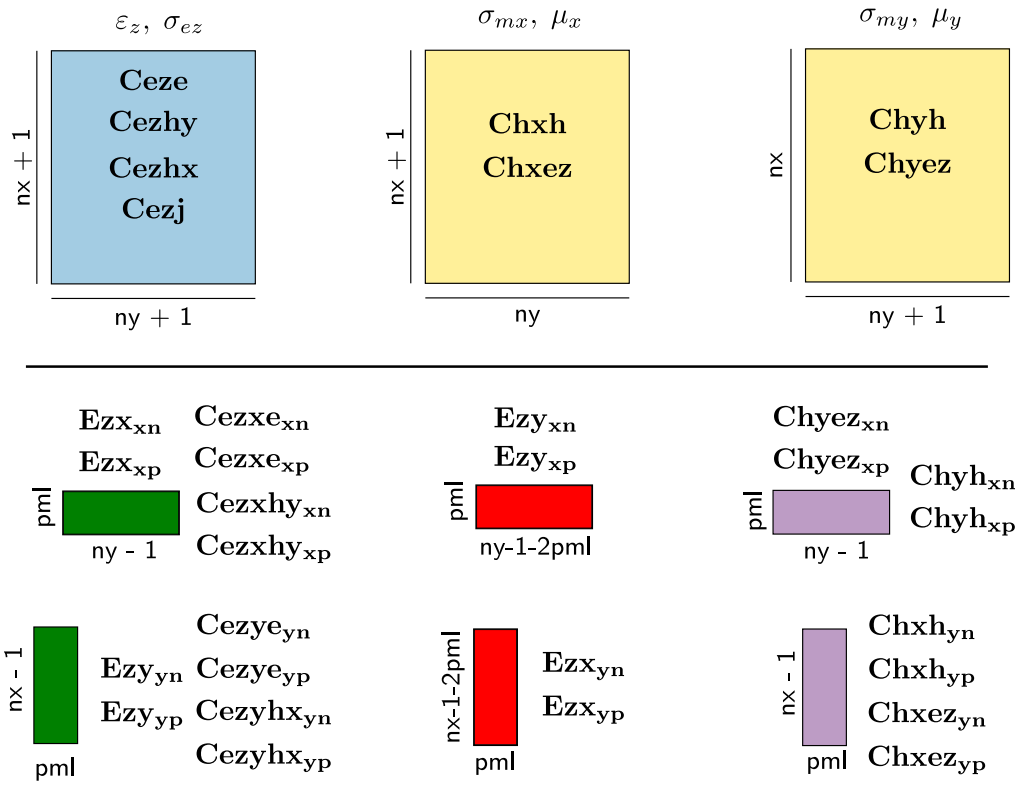


Figure 4: Coefficients of all nodes (**up**), and PML nodes (**down**).

$$\begin{aligned}
\mathbf{H}_x &= \mathbf{Chxh} \odot \mathbf{H}_x \odot \dots \\
&+ \mathbf{Chxez} \odot \left( \mathbf{E}_z - \mathbf{E}_z \right) \\
\mathbf{H}_y &= \mathbf{Chyh} \odot \mathbf{H}_y \odot \dots \\
&+ \mathbf{Chyez} \odot \left( \mathbf{E}_z - \mathbf{E}_z \right)
\end{aligned}$$

Diagram illustrating the update of  $\mathbf{H}_x$  and  $\mathbf{H}_y$  using PML. The diagram shows the update of  $\mathbf{H}_x$  and  $\mathbf{H}_y$  using PML. The update for  $\mathbf{H}_x$  involves a series of operations:  $\mathbf{H}_x$  is updated by  $\mathbf{Chxh}$  and  $\mathbf{Chxez}$  (which is  $\mathbf{E}_z$  multiplied by  $\mathbf{H}_x$ ). The update for  $\mathbf{H}_y$  involves a series of operations:  $\mathbf{H}_y$  is updated by  $\mathbf{Chyh}$  and  $\mathbf{Chyez}$  (which is  $\mathbf{E}_z$  multiplied by  $\mathbf{H}_y$ ). The diagram uses yellow rectangles to represent the matrices and white rectangles to represent the identity matrices. The dimensions of the matrices are indicated by the labels  $pjs$ ,  $pje-1$ ,  $pje$ ,  $pie-1$ , and  $pie$ .

Figure 5: Update of  $\mathbf{H}_x$  and  $\mathbf{H}_y$ . One line of nodes into PML.

$$\begin{aligned}
& \left( \begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \begin{array}{c} \boxed{\phantom{0000}} \\ \hline \end{array} \right) \mathbf{E}_z \parallel \left( \begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \begin{array}{c} \boxed{\phantom{0000}} \\ \hline \end{array} \right) \text{Ceze} \odot \left( \begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \begin{array}{c} \boxed{\phantom{0000}} \\ \hline \end{array} \right) \mathbf{E}_z \dots \\
& \quad \text{pjs} + 1 \quad \text{pje} - 1 \quad \text{pjs} + 1 \quad \text{pje} - 1 \quad \text{pjs} + 1 \quad \text{pje} - 1 \\
& + \left( \begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \begin{array}{c} \boxed{\phantom{0000}} \\ \hline \end{array} \right) \text{Cezhy} \odot \left( \begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \begin{array}{c} \boxed{\phantom{0000}} \\ \hline \end{array} \right) \mathbf{H}_y - \left( \begin{array}{c} \text{pie} - 2 \\ \hline \text{pis} \\ \hline \end{array} \begin{array}{c} \boxed{\phantom{0000}} \\ \hline \end{array} \right) \mathbf{H}_y \\
& \quad \text{pjs} + 1 \quad \text{pje} - 1 \quad \text{pjs} + 1 \quad \text{pje} - 1 \quad \text{pjs} + 1 \quad \text{pje} - 1 \\
& \dots + \left( \begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \begin{array}{c} \boxed{\phantom{0000}} \\ \hline \end{array} \right) \text{Cezhx} \odot \left( \begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \begin{array}{c} \boxed{\phantom{0000}} \\ \hline \end{array} \right) \mathbf{H}_x - \left( \begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \begin{array}{c} \boxed{\phantom{0000}} \\ \hline \end{array} \right) \mathbf{H}_x \\
& \quad \text{pjs} + 1 \quad \text{pje} - 1 \quad \text{pjs} + 1 \quad \text{pje} - 1 \quad \text{pjs} \quad \text{pje} - 2
\end{aligned}$$

Figure 6: Update of  $\mathbf{E}_z$ . One line of nodes into PML.

$$\begin{aligned}
& \left( \begin{array}{c} \text{nx} \\ \hline \end{array} \begin{array}{c} \begin{array}{c} 2 \\ \hline \end{array} \begin{array}{c} \text{H}_x \end{array} \end{array} \right)_{\text{pjs} - 1} = \text{Chxh}_{yn} \odot \left( \begin{array}{c} \text{nx} \\ \hline \end{array} \begin{array}{c} \begin{array}{c} 2 \\ \hline \end{array} \begin{array}{c} \text{H}_x \end{array} \end{array} \right)_{\text{pjs} - 1} \dots \\
& + \text{Chxez}_{yn} \odot \left( \begin{array}{c} \text{nx} \\ \hline \end{array} \begin{array}{c} \begin{array}{c} 2 \\ \hline \end{array} \begin{array}{c} \text{E}_z \end{array} \end{array} \right)_{\text{pjs}} - \left( \begin{array}{c} \text{nx} \\ \hline \end{array} \begin{array}{c} \begin{array}{c} 2 \\ \hline \end{array} \begin{array}{c} \text{E}_z \end{array} \end{array} \right)_{\text{pjs} - 1} \\
& \left( \begin{array}{c} \text{nx} \\ \hline \end{array} \begin{array}{c} \begin{array}{c} 2 \\ \hline \end{array} \begin{array}{c} \text{H}_x \end{array} \end{array} \right)_{\text{pje}}^{\text{ny}} = \text{Chxh}_{yp} \odot \left( \begin{array}{c} \text{nx} \\ \hline \end{array} \begin{array}{c} \begin{array}{c} 2 \\ \hline \end{array} \begin{array}{c} \text{H}_x \end{array} \end{array} \right)_{\text{pje}}^{\text{ny}} \dots \\
& + \text{Chxez}_{yp} \odot \left( \begin{array}{c} \text{nx} \\ \hline \end{array} \begin{array}{c} \begin{array}{c} 2 \\ \hline \end{array} \begin{array}{c} \text{E}_z \end{array} \end{array} \right)_{\text{pje} + 1}^{\text{ny} + 1} - \left( \begin{array}{c} \text{nx} \\ \hline \end{array} \begin{array}{c} \begin{array}{c} 2 \\ \hline \end{array} \begin{array}{c} \text{E}_z \end{array} \end{array} \right)_{\text{pje}}^{\text{ny}}
\end{aligned}$$

Figure 7: PML update for  $\mathbf{H}_x$ . PML proper.

$$\begin{aligned}
& \left( \begin{array}{c} \text{pie} - 1 \\ \hline 2 \end{array} \right) \mathbf{H}_y = \mathbf{Chyh}_{\text{xn}} \odot \left( \begin{array}{c} \text{pie} - 1 \\ \hline 2 \end{array} \right) \mathbf{H}_y \dots \\
& + \mathbf{Chyez}_{\text{xn}} \odot \left( \begin{array}{c} \text{pie} - 1 \\ \hline 2 \end{array} \right) \mathbf{E}_z - \left( \begin{array}{c} \text{pie} - 1 \\ \hline 2 \end{array} \right) \mathbf{E}_z \\
& \left( \begin{array}{c} \text{nx} \\ \hline \text{pie} \\ \hline 2 \end{array} \right) \mathbf{H}_y = \mathbf{Chyh}_{\text{xp}} \odot \left( \begin{array}{c} \text{nx} \\ \hline \text{pie} \\ \hline 2 \end{array} \right) \mathbf{H}_y \dots \\
& + \mathbf{Chyez}_{\text{xp}} \odot \left( \begin{array}{c} \text{nx} + 1 \\ \hline \text{pie} + 1 \\ \hline 2 \end{array} \right) \mathbf{E}_z - \left( \begin{array}{c} \text{nx} + 1 \\ \hline \text{pie} + 1 \\ \hline 2 \end{array} \right) \mathbf{E}_z
\end{aligned}$$

Figure 8: PML update for  $\mathbf{H}_y$ . PML proper.



$$\begin{aligned}
\mathbf{E}_{zx_{xn}} &= \mathbf{C}_{ezx_{e_{xn}}} \odot \mathbf{E}_{zx_{xn}} \dots \\
&+ \mathbf{C}_{ezx_{hy_{xn}}} \odot \left( \frac{\frac{p_{is}}{2}}{ny} \mathbf{H}_y - \frac{\frac{p_{is}-1}{2}}{ny} \mathbf{H}_y \right) \\
\mathbf{E}_{zy_{xn}} &= \mathbf{C}_{ezy_{e_{xn}}} \odot \mathbf{E}_{zy_{xn}} \dots \\
&+ \mathbf{C}_{ezy_{hy_{xn}}} \odot \left( \frac{\frac{p_{js}+1}{p_{je}-1}}{p_{je}-1} \mathbf{H}_x - \frac{\frac{p_{js}}{p_{je}-2}}{p_{je}-2} \mathbf{H}_x \right)
\end{aligned}$$

Figure 9: PML update for  $\mathbf{E}_{zx}$  up. One line of nodes into inner nodes.

$$\begin{aligned}
& \mathbf{E}_{zx_{xp}} = \mathbf{C}_{ezx_{xp}} \odot \mathbf{E}_{zx_{xp}} \dots \\
& + \mathbf{C}_{ezx_{hy_{xp}}} \odot \left( \mathbf{H}_y - \mathbf{H}_y \right) \\
& \mathbf{E}_{zy_{xp}} = \mathbf{C}_{ezy_{xp}} \odot \mathbf{E}_{zy_{xp}} \dots \\
& + \mathbf{C}_{ezy_{hx_{xp}}} \odot \left( \mathbf{H}_x - \mathbf{H}_x \right)
\end{aligned}$$

The diagram illustrates the PML update for  $\mathbf{E}_{zx}$  down. It shows the calculation of the updated electric field components  $\mathbf{E}_{zx_{xp}}$  and  $\mathbf{E}_{zy_{xp}}$  based on the current values and the PML coefficients  $\mathbf{C}_{ezx_{xp}}$ ,  $\mathbf{C}_{ezy_{xp}}$ ,  $\mathbf{C}_{ezx_{hy_{xp}}}$ , and  $\mathbf{C}_{ezy_{hx_{xp}}}$ . The update involves element-wise multiplication ( $\odot$ ) and subtraction ( $-$ ) of the current field values from the PML coefficients. The spatial dimensions are indicated by the subscripts  $x$  and  $y$ , and the PML parameters  $nx$ ,  $ny$ ,  $pie$ ,  $pjs$ , and  $pje$  are shown in the diagrams.

Figure 10: PML update for  $\mathbf{E}_{zx}$  down. One line of nodes into inner nodes.

$$\begin{aligned}
& \mathbf{E}_{zx_{yn}} = \mathbf{C}_{ezx_{e_{yn}}} \odot \mathbf{E}_{zx_{yn}} \dots \\
& + \mathbf{C}_{ezx_{hy_{yn}}} \odot \left( \frac{\text{pie} - 1}{\frac{2}{\text{pjs}}} \left[ \text{box} \right] \mathbf{H}_y - \frac{\text{pie} - 2}{\frac{2}{\text{pjs}}} \left[ \text{box} \right] \mathbf{H}_y \right) \\
& \mathbf{E}_{zy_{yn}} = \mathbf{C}_{ezy_{e_{yn}}} \odot \mathbf{E}_{zy_{yn}} \dots \\
& + \mathbf{C}_{ezy_{hx_{yn}}} \odot \left( \frac{nx}{\frac{2}{\text{pjs}}} \left[ \text{box} \right] \mathbf{H}_x - \frac{nx}{\frac{2}{\text{pjs} - 1}} \left[ \text{box} \right] \mathbf{H}_x \right)
\end{aligned}$$

Figure 11: PML update for  $\mathbf{E}_{zy}$  left. One line of nodes into inner nodes.

$$\begin{aligned}
& \mathbf{Ez}_{xyp} = \mathbf{Cezx}_{yp} \odot \mathbf{Ez}_{xyp} \quad \dots \\
& + \mathbf{Cezxhy}_{yp} \odot \left( \frac{\text{pie} - 1}{\text{pje}} \frac{\text{pis} + 1}{\text{ny}} \mathbf{H}_y - \frac{\text{pie} - 2}{\text{pje}} \frac{\text{pis}}{\text{ny}} \mathbf{H}_y \right) \\
& \mathbf{Ezy}_{yp} = \mathbf{Ceye}_{yp} \odot \mathbf{Ezy}_{yp} \quad \dots \\
& + \mathbf{Cezyh}_{yp} \odot \left( \frac{\text{nx}}{\text{pje}} \frac{2}{\text{ny}} \mathbf{H}_x - \frac{\text{nx}}{\text{pje} - 1} \frac{2}{\text{ny} - 1} \mathbf{H}_x \right)
\end{aligned}$$

Figure 12: PML update for  $\mathbf{E}_{zy}$  right. One line of nodes into inner nodes.

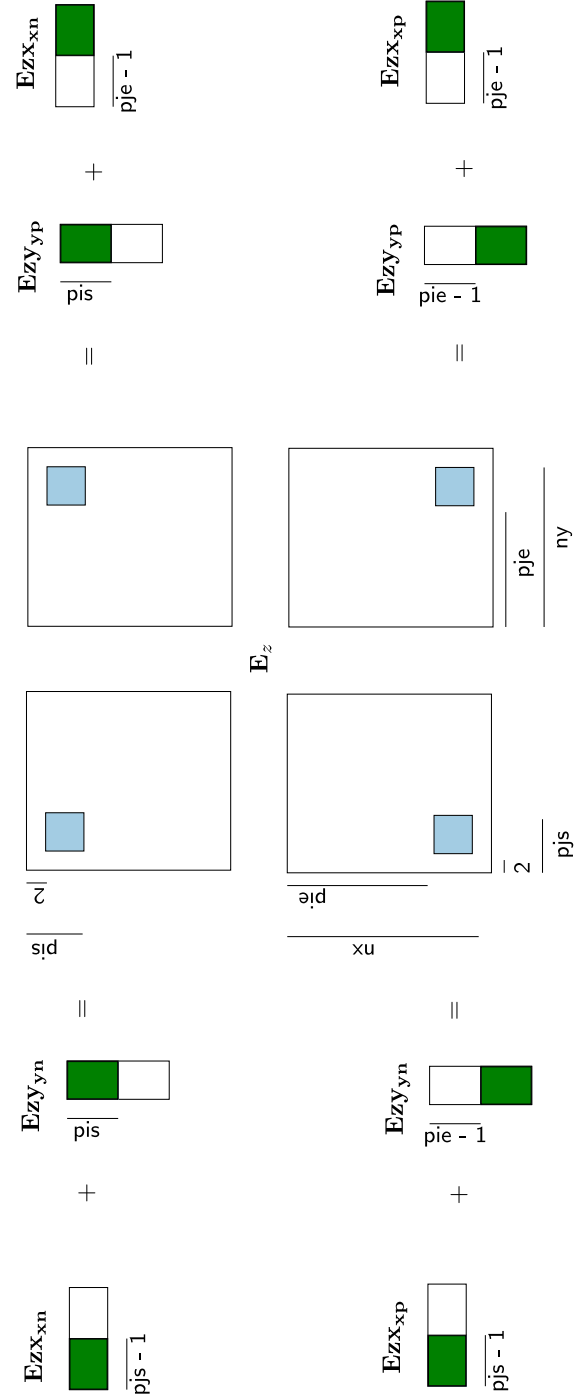


Figure 13: PML update for  $E_z = E_{zx} + E_{zy}$  corners. PML proper.

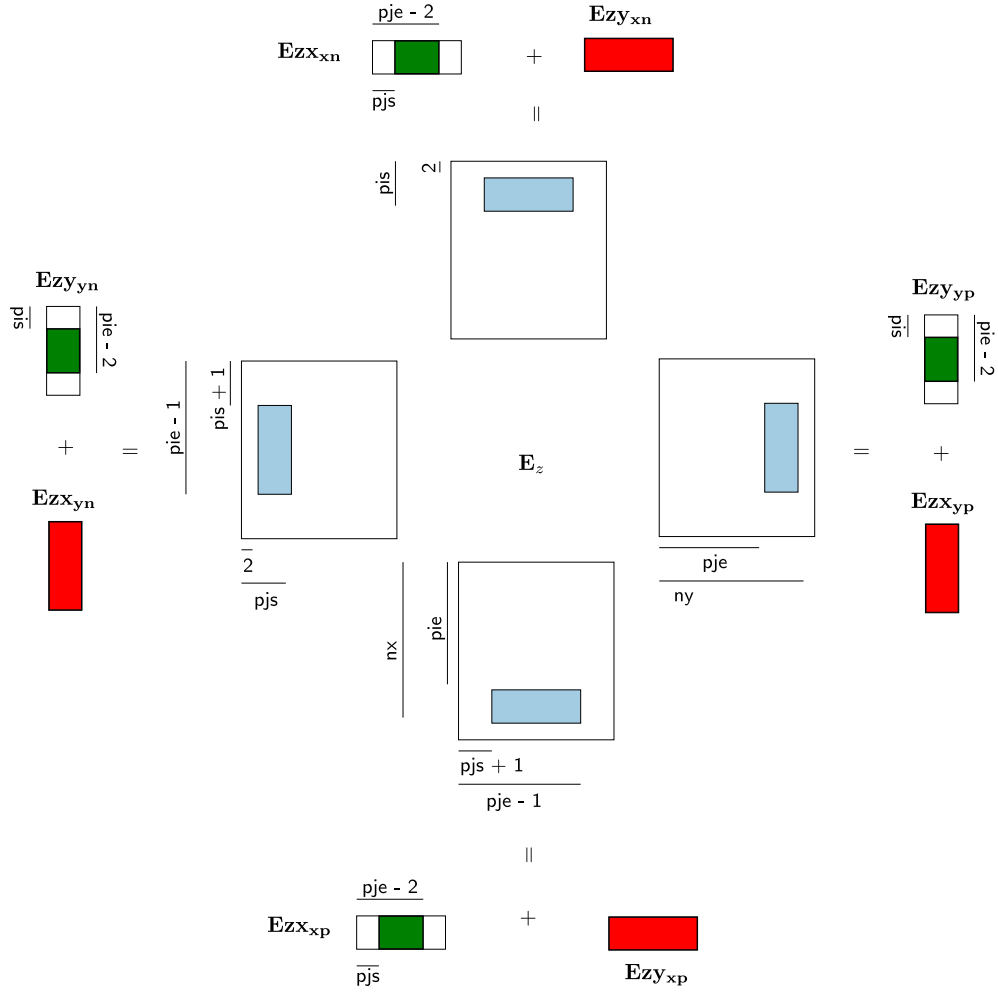


Figure 14: PML update for  $\mathbf{E}_z = \mathbf{E}_{zx} + \mathbf{E}_{zy}$  sides. PML proper.