2.5d inversion recipes

ER inversion

This process is repeated for each experiment (i.e. each source s and all its receivers). Let the 2.5d weights for source s be $\{k, \omega\}$ and stored in disk,

1. compute synthetic electric potentials $\{\tilde{u}_i\}$, data d and error $e = d - d^o$,
   (a) retrieve from memory k and ω,
   (b) choose $k_i \in k$ and build L_i with the right boundary conditions for that k_i,
   $$L^i \approx -\nabla \cdot \sigma \nabla,$$
   $$L_i = L^i + k_i^2\, \sigma,$$
   (c) solve $L_i \tilde{u}_i = \frac{s}{2}$ for $\tilde{u}_i$ and store,
   (d) after all k has been used,
   $$u = \sum_i \omega_i\, \tilde{u}_i \rightarrow d = Mu \rightarrow e = d - d^o.$$
2. correct error amplitude by its variance,
   $$e \leftarrow \frac{e}{1 + \mathrm{std}\,(d^o)}$$
3. compute g,
   (a) choose $k_i \in k$,
   (b) build L_i and $S_i = -((\nabla_\sigma L^i)\tilde{u}_i)^\top - k_i^2\, \mathrm{diag}(\tilde{u}_i)^\top$
   (c) solve $L_i^\top a_i = M^\top e$ for a_i
   (d) compute and store $g_i = S_i a_i$,
   (e) after all k has been used,
   $$g = \frac{2}{\pi} \sum \omega_i g_i.$$
4. On g filter out high spatial-frequencies and normalize by largest amplitude,

5. compute $\sigma_{err} = \sigma - \sigma_{ref}$, filter out high spatial-frequencies and normalize by largest amplitude,
6. $g \leftarrow g + \beta\sigma_{err}$ and normalize g by largest amplitude,
7. compute step-size α_s using Pica and set $\Delta\sigma_s = -\alpha_s g$
8. store $\Delta\sigma_s$ and α_s.

After all sources have their update direction,

$$\alpha = \frac{1}{n_s}\sum \alpha_s$$
$$\Delta\sigma = \sum \Delta\sigma_s$$
$$\Delta\sigma_{dc} = \alpha\frac{\Delta\sigma}{\max|\Delta\sigma|}.$$

The conductivity is updated as,

$$\sigma \leftarrow \sigma \odot \exp\{\sigma \odot \Delta\sigma_{dc}\}.$$

GPR inversion ε

This process is repeated for each experiment (i.e. each source s and all its receivers),

1. compute synthetic data d and error $e = d - d^o$,
2. how to treat noise in e?
3. compute g_ε and apply Kurzmann preconditioner,
4. on $g_{w,\varepsilon}$ filter out high spatial-frequencies and normalize by largest amplitude,
5. compute step-size α using Pica and set $\Delta\varepsilon_s = -\alpha g_\varepsilon$
6. store $\Delta\varepsilon_s$,
7. invert for source wavelet with a Wiener filter.

After all sources have their update direction,

$$\Delta\varepsilon = \frac{1}{n_s}\sum \Delta\varepsilon_s.$$

The permittivity is updated as,

$$\varepsilon \leftarrow \varepsilon \odot \exp\{\varepsilon \odot \Delta\varepsilon\}.$$

GPR inversion σ

This process is repeated for each experiment (i.e. each source s and all its receivers),

1. compute synthetic data d and error $e = d - d^o$,
2. how to treat noise in e?
3. compute $g_{w,\sigma}$ and apply Kurzmann preconditioner,
4. on $g_{w,\sigma}$ filter out high spatial-frequencies and normalize by largest amplitude,
5. compute step-size α using Pica and set $\Delta\sigma_s = -\alpha g_{w,\sigma}$
6. store $\Delta\sigma_s$,
7. invert for source wavelet with Wiener filter.

After all sources have their update direction,

$$\Delta\sigma_w = \frac{1}{n_s}\sum \Delta\sigma_s.$$

The conductivity is updated as,

$$\sigma \leftarrow \sigma \odot \exp\{\sigma \odot \Delta\sigma_w\}.$$

GPR+ER inversion σ

The update direction is

$$\Delta\sigma \leftarrow \frac{1}{2}(\Delta\sigma_w + \Delta\sigma_{dc})$$

and the update for *both* the GPR and ER is

$$\sigma \leftarrow \sigma \odot \exp\{\sigma \odot \Delta\sigma\}.$$