Step sizes for w

A tale of units. Theory

The gradients of the objective function with respect to the parameters computed using the *fwi* scheme for electromagnetic waves are,

$$g_\varepsilon = \int_0^T \dot{u}(-t) \cdot a_u(t)\ \mathrm{d}t,$$
$$g_{\sigma,w} = \int_0^T u(-t) \cdot a_u(t)\ \mathrm{d}t.$$

The updates for the parameters will be of the form,

$$\Delta\varepsilon = -\alpha_\varepsilon\, g_\varepsilon,$$
$$\Delta\sigma = -\alpha_{\sigma,w}\, g_{\sigma,w},$$

and so $\Delta\varepsilon$ and $\Delta\sigma$ must have units of ε and σ respectively ($[F/m]$, $[S/m]$). In units, the gradients take the form

$$[g_\varepsilon] = \left[\frac{V}{s\,m}\right] \cdot \left[\frac{V}{m}\right] \cdot [s] = \frac{m^2\,kg^2}{s^4\,A^2},$$
$$[g_{\sigma,w}] = \left[\frac{V}{m}\right] \cdot \left[\frac{V}{m}\right] \cdot [s] = \frac{m^2\,kg^2}{s^3\,A^2}.$$

By taking the step sizes,

$$[\alpha_\varepsilon] = \frac{1}{A^2} \cdot \frac{s^6\,A^6}{m^3\,kg^3} \cdot \frac{s^2}{m^2},$$
$$[\alpha_{\sigma,w}] = \frac{1}{A^2} \cdot \frac{s^6\,A^6}{m^3\,kg^3} \cdot \frac{1}{m^2},$$

we ensure the gradients have the right units.

$$\left[\frac{F}{m}\right] = \frac{s^4\,A^2}{m^3\,kg}, \qquad \left[\frac{S}{m}\right] = \frac{s^3\,A^2}{m^3\,kg}, \qquad \left[\frac{H}{m}\right] = \frac{m\,kg}{s^2\,A^2}, \qquad \left[\frac{V}{m}\right] = \frac{m\,kg}{s^2\,A}.$$

$$\varepsilon_o = 8.8541 \times 10^{-12} \qquad [F/m],$$
$$\mu_o = 4\pi \times 10^{-7} \qquad [H/m].$$

A tale of units. Practice

In practice we compute,

$$\begin{aligned} g_\varepsilon &= \sum_t \dot{u}(-t) \odot a_u(t)\ \Delta t, \\ g_{\sigma,w} &= \sum_t u(-t) \odot a_u(t)\ \Delta t, \end{aligned}$$

and correct for the right amplitudes by,

$$\begin{aligned} g_\varepsilon &\leftarrow \frac{1}{i^2} \cdot \frac{\Delta t^2}{\varepsilon_o\, \mu_o^3\ \Delta x^2} \cdot g_\varepsilon, \\ g_{\sigma,w} &\leftarrow \frac{1}{i^2} \cdot \frac{1}{\mu_o^3\ \Delta x^2} \cdot g_{\sigma,w}. \end{aligned}$$

The actual step sizes are computed using Pica,

$$\begin{aligned} \varepsilon_\bullet &= \varepsilon - k_\varepsilon g_\varepsilon, \\ d_\bullet &= d(\varepsilon,\sigma) - d(\varepsilon_\bullet,\sigma), \\ \alpha_\varepsilon &= k_\varepsilon \frac{d_\bullet(:)^\top \cdot e(:)}{d_\bullet(:)^\top \cdot d_\bullet(:)}, \end{aligned}$$

with a similar equation for α_σ.

Finding k_ε

If k_ε is too large, then $\varepsilon_\bullet$ falls off the stability region for the wave solver and α_ε becomes a NaN. If too small, α_ε is also too small and no relevant update is made on ε.

To find the right k_ε, a heuristic search is performed scanning from large to small values of k_ε and checking if $\varepsilon_\bullet$ falls the stability region or not.

The implementation of this search is two while loops, the first loop searches in decreasing order on a coarse discretization of possible values of k_ε. The second loop sweeps a finer discretization of possible values of k_ε in increasing order.