Two-dimensional diffusion

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2D EM wave equation

$$\boldsymbol{\mu}\dot{\mathbf{H}} = -\nabla \times \mathbf{E} + \mathbf{M}_s \tag{1}$$

$$\boldsymbol{\varepsilon}\dot{\mathbf{E}} = \nabla \times \mathbf{H} - \boldsymbol{\sigma}\mathbf{E} - \mathbf{J}_s \tag{2}$$

Assuming quasi-isotropic materials and $\partial_y = 0$, in expanded-matrix form we have the TE mode,

$$\begin{pmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \varepsilon \end{pmatrix} \begin{pmatrix} -\dot{H}_z \\ \dot{H}_x \\ \dot{E}_y \end{pmatrix} = \begin{pmatrix} 0 & 0 & \partial_x \\ 0 & 0 & \partial_z \\ \partial_x & \partial_z & 0 \end{pmatrix} \begin{pmatrix} -H_z \\ H_x \\ E_y \end{pmatrix} - \tag{3}$$

$$\sigma \begin{pmatrix} 0 \\ 0 \\ E_y \end{pmatrix} + \begin{pmatrix} M_z \\ M_x \\ -J_y \end{pmatrix}. \tag{4}$$

The terms M_z, M_x, and J_y are the source terms for H_z, H_x, and E_y respectively. The velocity of propagation is given by $v = 1/\sqrt{\mu\varepsilon}$.

For example, when modeling an electric dipole antenna parallel to the y-axis, then $M_z = M_x = 0$, and only J_y varies as a function of time. When modeling a magnetic dipole antenna parallel to the z-axis, then $J_y = M_x = 0$, and only M_z varies as a function of time.

An EM field exhibits wave propagation when the media is a *poor conductor*,

$$\frac{\sigma}{\omega\varepsilon} << 1, \tag{5}$$

where $\omega = 2\pi f_o$ and f_o is the fundamental frequency of the source term.

2D EM diffusion

An EM field exhibits diffusion propagation when the media is a *good conductor*,

$$1 << \frac{\sigma}{\omega\varepsilon}. \tag{6}$$

We re-write the *good/bad conductor* conditions as follows,

$$\varepsilon >> \frac{\sigma}{\omega} \qquad \text{poor conductor,} \tag{7}$$

$$\varepsilon << \frac{\sigma}{\omega} \qquad \text{good conductor.} \tag{8}$$

Now, by looking at Greens solutions for wave and diffusion propagation it can be shown that they agree asymptotically when [Oristaglio and Hohmann, 1984],

$$\frac{2\varepsilon}{\sigma} << \sqrt{t^2 - \frac{r^2}{v^2}}, \tag{9}$$

where r is the distance from source to receiver and t is the time it takes for the field to arrive to the receiver.

We can re-write this equation in order to get a criteria on when propagation is diffusive,

$$r^* = \sqrt{v^2 \cdot \left(t^2 - \frac{4\varepsilon^2}{\sigma^2}\right)}. \tag{10}$$

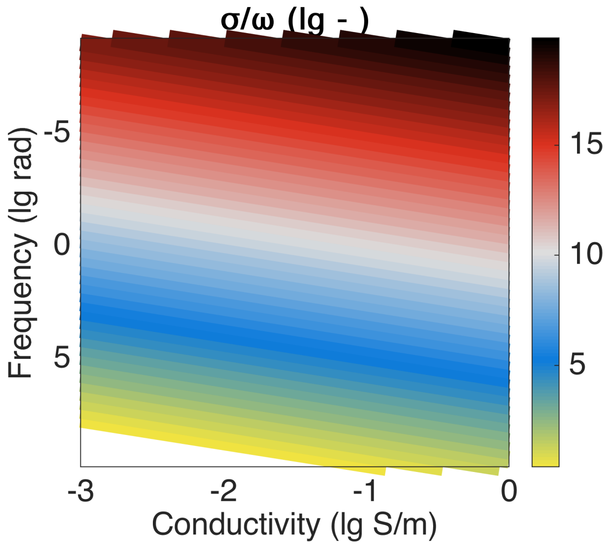
The difference between r and r^* is that r is the real distance between source and receiver, and r^* is a function of v, t, ε, and σ.

Is it EM diffusion or wave propagation?

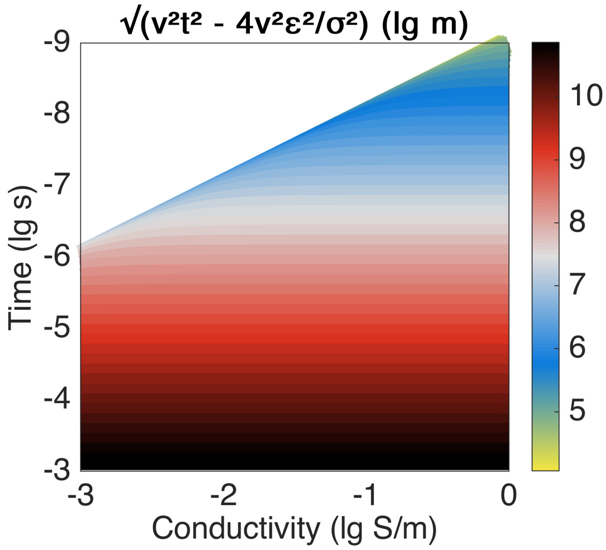
First, we can plot σ/ω. If the target values for ε are below those of the plot, then it is diffusion. See Figure 1a.

Then, for a fixed value of ε we can plot r^* (Figure 1b). Then we compare with our survey acquisition,

- if $r << r^*$ it is diffusion,
- if $0 < r^* << r$ it is on the edge of being a wave or diffusion,
- if $r^* \in \mathbb{C}$ it is a wave.



(a) For crazy large would-be values of relative permittivity (colors), the media is diffusive.



(b) If your receiver is placed at distances much shorter than r^* (colors) then there is no difference between wave and diffusion. In this case $\varepsilon_r = 50$ just because.

Figure 1: **Can I use my wave code to solve for diffusion?**

Comments on a magnetic dipole source

The EM system HGG uses is a magnetic source driven by a *square* wired coil. I think the modeling team got confused and modeled a square coil as two electric dipoles. That would explain the in-out motion of the electric field in the gif.

If the coil is actually a coil and not two electric dipoles, then the induced source is a magnetic dipole and not two infinitely long electric dipoles. So the modeling team got that wrong.

References

Michael L Oristaglio and Gerald W Hohmann. Diffusion of electromagnetic fields into a two-dimensional earth: A finite-difference approach. *Geophysics*, 49(7): 870–894, 1984.