2d inversion recipes

ER inversion

This process is repeated for each experiment (i.e. each source s and all its receivers),

- 1. compute synthetic data d and error $e=d-d^{o}$,
- 2. correct error amplitude by its variance,

$$e \leftarrow \frac{e}{1 + \operatorname{std}(d^o)}$$

- 3. compute $g=J^{\top}e$, filter out high spatial-frequencies and normalize by largest amplitude,
- 4. compute $\sigma_{err}=\sigma-\sigma_{ref}$, filter out high spatial-frequencies and normalize by largest amplitude,
- 5. $g \leftarrow g + \beta \sigma_{err}$ and normalize g by largest amplitude,
- 6. compute step-size α_s using Pica and set $\Delta\sigma_s=-\alpha_s g$
- 7. store $\Delta \sigma_s$ and α_s .

After all sources have their update direction,

$$\alpha = \frac{1}{n_s} \sum \alpha_s$$

$$\Delta \sigma = \sum \Delta \sigma_s$$

$$\Delta \sigma_{dc} = \alpha \frac{\Delta \sigma}{\max |\Delta \sigma|}.$$

The conductivity is updated as,

$$\sigma \leftarrow \sigma \odot \exp{\{\sigma \odot \Delta \sigma_{dc}\}}.$$

GPR inversion ε

This process is repeated for each experiment (i.e. each source s and all its receivers),

- 1. compute synthetic data d and error $e = d d^o$,
- 2. how to treat noise in e?
- 3. compute g_{ε} and apply Kurzmann preconditioner,
- 4. on $g_{w,\varepsilon}$ filter out high spatial-frequencies and normalize by largest amplitude,
- 5. compute step-size α using Pica and set $\Delta \varepsilon_s = -\alpha g_{\varepsilon}$
- 6. store $\Delta \varepsilon_s$,
- 7. invert for source wavelet with a Wiener filter,

$$\tilde{a}(\omega_o) = \frac{d_{w_o} \cdot (d_{w_o}^o)^*}{d_{w_o} \cdot d_{w_o}^*} \qquad \text{for all } \omega_o \in \omega,$$
$$\tilde{s} \leftarrow \tilde{a} \odot \tilde{s}.$$

After all sources have their update direction,

$$\Delta \varepsilon = \frac{1}{n_s} \sum \Delta \varepsilon_s.$$

The permittivity is updated as,

$$\varepsilon \leftarrow \varepsilon \odot \exp\{\varepsilon \odot \Delta \varepsilon\}.$$

GPR inversion σ

This process is repeated for each experiment (i.e. each source s and all its receivers),

- 1. compute synthetic data d and error $e = d d^o$,
- 2. how to treat noise in e?
- 3. compute $g_{w,\sigma}$ and apply Kurzmann preconditioner,

- 4. on $g_{w,\sigma}$ filter out high spatial-frequencies and normalize by largest amplitude.
- 5. compute step-size α using Pica and set $\Delta\sigma_s=-\alpha g_{w,\sigma}$
- 6. store $\Delta \sigma_s$,
- 7. invert for source wavelet with Wiener filter,

$$\tilde{a}(\omega_o) = \frac{d_{w_o} \cdot (d_{w_o}^o)^*}{d_{w_o} \cdot d_{w_o}^*}$$
 for all $\omega_o \in \omega$,
$$\tilde{s} \leftarrow \tilde{a} \odot \tilde{s}.$$

After all sources have their update direction,

$$\Delta \sigma_w = \frac{1}{n_s} \sum \Delta \sigma_s.$$

The conductivity is updated as,

$$\sigma \leftarrow \sigma \odot \exp{\{\sigma \odot \Delta \sigma_w\}}.$$

$\mathbf{GPR}\mathbf{+ER}\ \mathbf{inversion}\ \sigma$

The update direction is

$$\Delta\sigma \leftarrow \frac{1}{2}(\Delta\sigma_w + \Delta\sigma_{dc})$$

and the update for both the GPR and ER is

$$\sigma \leftarrow \sigma \odot \exp{\{\sigma \odot \Delta\sigma\}}.$$