

wave solver with PML

summer 2017

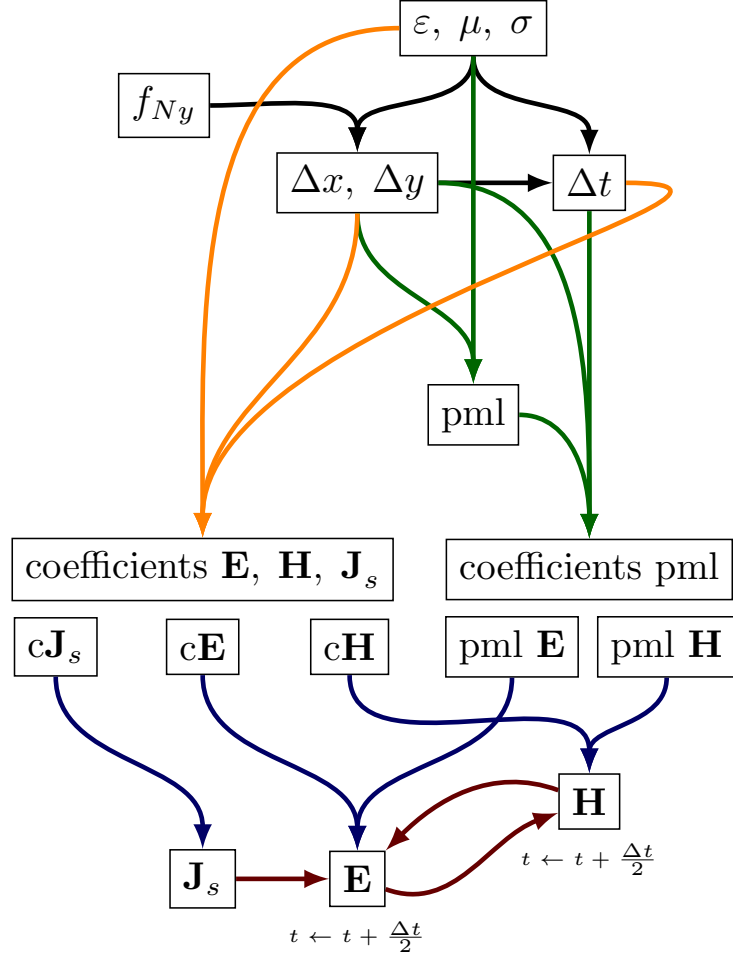


Figure 1: Main code flow.

$$\begin{aligned}
 c_o &= \frac{1}{\sqrt{\epsilon_o \mu_o}} \\
 c_{min} &= \frac{c_o}{\sqrt{\epsilon_{max}}} & c_{max} &= \frac{c_o}{\sqrt{\epsilon_{min}}} \\
 \lambda_{min} &= \frac{c_{min}}{f_{Ny}} \propto \Delta x & \Delta t &= \frac{cfl}{c_{max} \sqrt{\left(\frac{1}{\Delta x}\right)^2 + \left(\frac{1}{\Delta y}\right)^2}}
 \end{aligned}$$

Figure 2: Calculation of $\Delta x, \Delta y$ and Δt .

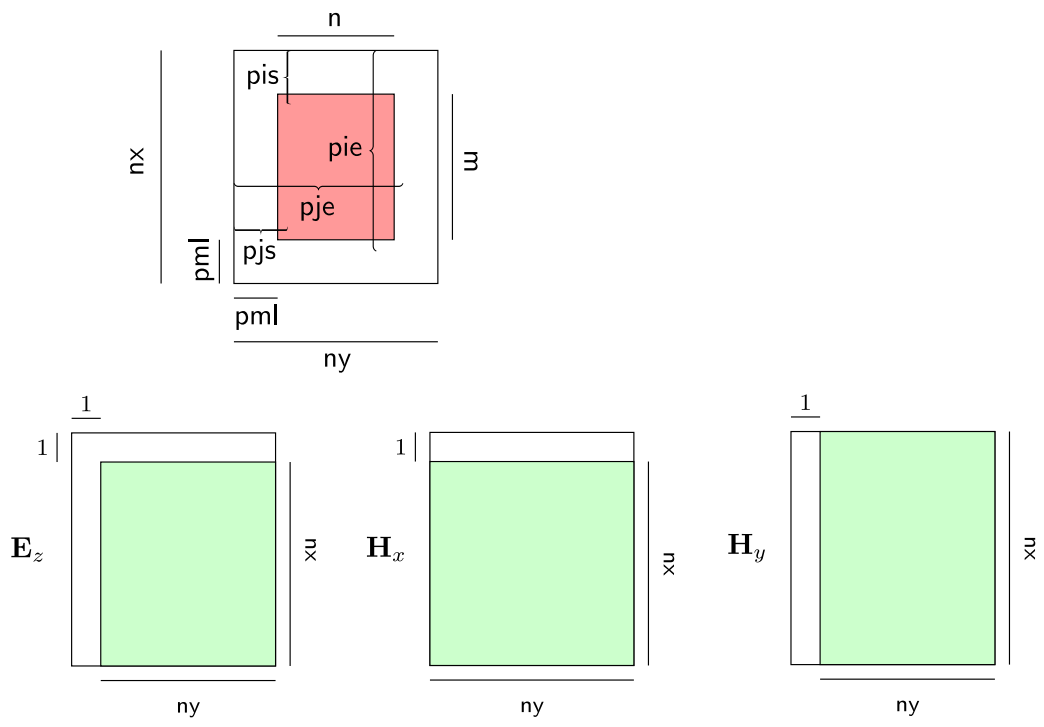


Figure 3: Grid dimensions of \mathbf{E}_z , \mathbf{H}_x , \mathbf{H}_y .

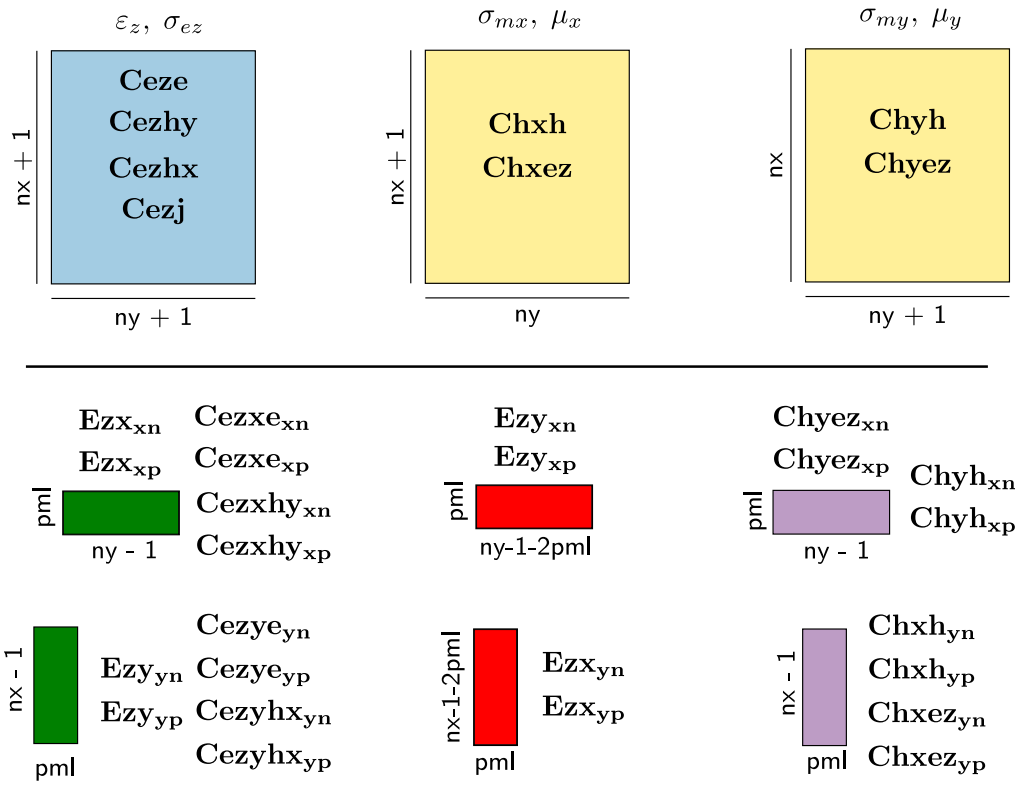


Figure 4: Coefficients of all nodes (**up**), and PML nodes (**down**).

$$\begin{aligned}
\mathbf{H}_x &= \mathbf{Chxh} \odot \mathbf{H}_x \odot \dots \\
&+ \mathbf{Chxez} \odot \left(\mathbf{E}_z - \mathbf{E}_z \right)
\end{aligned}$$

The diagram illustrates the update of \mathbf{H}_x and \mathbf{H}_y using a PML (Perfectly Matched Layer) approach. The matrices are represented as yellow rectangles with white borders. The dimensions are indicated by labels: pjs (top), $pje-1$ (bottom), pje (bottom), and $pje-1$ (bottom). The update involves a series of operations: \mathbf{H}_x is updated by \mathbf{Chxh} and \mathbf{Chxez} , and \mathbf{H}_y is updated by \mathbf{Chyh} and \mathbf{Chyez} . The operations are performed element-wise (\odot) and the results are summed ($+$). The \mathbf{E}_z matrices are also updated.

$$\begin{aligned}
\mathbf{H}_y &= \mathbf{Chyh} \odot \mathbf{H}_y \odot \dots \\
&+ \mathbf{Chyez} \odot \left(\mathbf{E}_z - \mathbf{E}_z \right)
\end{aligned}$$

The diagram illustrates the update of \mathbf{H}_y using a PML (Perfectly Matched Layer) approach. The matrices are represented as yellow rectangles with white borders. The dimensions are indicated by labels: pjs (top), $pje-1$ (bottom), pje (bottom), and $pje-1$ (bottom). The update involves a series of operations: \mathbf{H}_y is updated by \mathbf{Chyh} and \mathbf{Chyez} , and \mathbf{H}_z is updated by \mathbf{E}_z . The operations are performed element-wise (\odot) and the results are summed ($+$).

Figure 5: Update of \mathbf{H}_x and \mathbf{H}_y . One line of nodes into PML.

$$\begin{aligned}
& \left[\begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \right] \left[\begin{array}{c} \text{pjs} + 1 \\ \hline \text{pje} - 1 \\ \hline \end{array} \right] \mathbf{E}_z \parallel \left[\begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \right] \left[\begin{array}{c} \text{pjs} + 1 \\ \hline \text{pje} - 1 \\ \hline \end{array} \right] \text{Ceze} \odot \left[\begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \right] \left[\begin{array}{c} \text{pjs} + 1 \\ \hline \text{pje} - 1 \\ \hline \end{array} \right] \mathbf{E}_z \dots \\
& + \left[\begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \right] \left[\begin{array}{c} \text{pjs} + 1 \\ \hline \text{pje} - 1 \\ \hline \end{array} \right] \text{Cezhy} \odot \left(\left[\begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \right] \left[\begin{array}{c} \text{pjs} + 1 \\ \hline \text{pje} - 1 \\ \hline \end{array} \right] \mathbf{H}_y - \left[\begin{array}{c} \text{pie} - 2 \\ \hline \text{pis} \\ \hline \end{array} \right] \left[\begin{array}{c} \text{pjs} + 1 \\ \hline \text{pje} - 1 \\ \hline \end{array} \right] \mathbf{H}_y \right) \\
& \dots + \left[\begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \right] \left[\begin{array}{c} \text{pjs} + 1 \\ \hline \text{pje} - 1 \\ \hline \end{array} \right] \text{Cezhx} \odot \left(\left[\begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \right] \left[\begin{array}{c} \text{pjs} + 1 \\ \hline \text{pje} - 1 \\ \hline \end{array} \right] \mathbf{H}_x - \left[\begin{array}{c} \text{pie} - 1 \\ \hline \text{pis} + 1 \\ \hline \end{array} \right] \left[\begin{array}{c} \text{pjs} \\ \hline \text{pje} - 2 \\ \hline \end{array} \right] \mathbf{H}_x \right)
\end{aligned}$$

Figure 6: Update of \mathbf{E}_z . One line of nodes into PML.

$$\begin{aligned}
& \left(\begin{array}{c} \text{nx} \\ \hline \begin{array}{c} \boxed{\text{H}_x} \\ \text{pjs} - 1 \end{array} \end{array} \right) = \text{Chxh}_{yn} \odot \left(\begin{array}{c} \text{nx} \\ \hline \begin{array}{c} \boxed{\text{H}_x} \\ \text{pjs} - 1 \end{array} \end{array} \right) \dots \\
& + \text{Chxez}_{yn} \odot \left(\begin{array}{c} \text{nx} \\ \hline \begin{array}{c} \boxed{\text{E}_z} \\ \text{pjs} \end{array} \end{array} - \begin{array}{c} \text{nx} \\ \hline \begin{array}{c} \boxed{\text{E}_z} \\ \text{pjs} - 1 \end{array} \end{array} \right) \\
& \left(\begin{array}{c} \text{nx} \\ \hline \begin{array}{c} \boxed{\text{H}_x} \\ \text{pje} \\ \text{ny} \end{array} \end{array} \right) = \text{Chxh}_{yp} \odot \left(\begin{array}{c} \text{nx} \\ \hline \begin{array}{c} \boxed{\text{H}_x} \\ \text{pje} \\ \text{ny} \end{array} \end{array} \right) \dots \\
& + \text{Chxez}_{yp} \odot \left(\begin{array}{c} \text{nx} \\ \hline \begin{array}{c} \boxed{\text{E}_z} \\ \text{pje} + 1 \\ \text{ny} + 1 \end{array} \end{array} - \begin{array}{c} \text{nx} \\ \hline \begin{array}{c} \boxed{\text{E}_z} \\ \text{pje} \\ \text{ny} \end{array} \end{array} \right)
\end{aligned}$$

Figure 7: PML update for \mathbf{H}_x . PML proper.

$$\begin{aligned}
& \begin{array}{c} \text{pis} - 1 \\ \hline \end{array} \begin{array}{|c|} \hline \text{[Diagram: Box with purple rectangle at top-left]} \\ \hline \end{array} \mathbf{H}_y = \begin{array}{c} \text{Chyh}_{\text{xn}} \\ \text{[Diagram: Purple rectangle]} \end{array} \odot \begin{array}{c} \text{pis} - 1 \\ \hline \end{array} \begin{array}{|c|} \hline \text{[Diagram: Box with purple rectangle at top-left]} \\ \hline \end{array} \mathbf{H}_y \dots \\
& \quad \quad \quad \begin{array}{c} \text{ny} \\ \hline \end{array} \quad \quad \quad \begin{array}{c} \text{ny} \\ \hline \end{array} \\
& + \begin{array}{c} \text{Chyez}_{\text{xn}} \\ \text{[Diagram: Purple rectangle]} \end{array} \odot \left(\begin{array}{c} \text{pis} \\ \hline \end{array} \begin{array}{|c|} \hline \text{[Diagram: Box with green rectangle at top-left]} \\ \hline \end{array} \mathbf{E}_z - \begin{array}{c} \text{pis} - 1 \\ \hline \end{array} \begin{array}{|c|} \hline \text{[Diagram: Box with green rectangle at top-left]} \\ \hline \end{array} \mathbf{E}_z \right) \\
& \quad \quad \quad \begin{array}{c} \text{ny} \\ \hline \end{array} \quad \quad \quad \begin{array}{c} \text{ny} \\ \hline \end{array} \\
& \begin{array}{c} \text{nx} \\ \hline \end{array} \begin{array}{|c|} \hline \begin{array}{c} \text{pie} \\ \hline \end{array} \begin{array}{|c|} \hline \text{[Diagram: Box with purple rectangle at bottom-left]} \\ \hline \end{array} \mathbf{H}_y = \begin{array}{c} \text{Chyh}_{\text{xp}} \\ \text{[Diagram: Purple rectangle]} \end{array} \odot \begin{array}{c} \text{nx} \\ \hline \end{array} \begin{array}{|c|} \hline \begin{array}{c} \text{pie} \\ \hline \end{array} \begin{array}{|c|} \hline \text{[Diagram: Box with purple rectangle at bottom-left]} \\ \hline \end{array} \mathbf{H}_y \dots \\
& \quad \quad \quad \begin{array}{c} \text{ny} \\ \hline \end{array} \quad \quad \quad \begin{array}{c} \text{ny} \\ \hline \end{array} \\
& + \begin{array}{c} \text{Chyez}_{\text{xp}} \\ \text{[Diagram: Purple rectangle]} \end{array} \odot \left(\begin{array}{c} \text{nx} + 1 \\ \hline \end{array} \begin{array}{|c|} \hline \begin{array}{c} \text{pie} + 1 \\ \hline \end{array} \begin{array}{|c|} \hline \text{[Diagram: Box with green rectangle at bottom-left]} \\ \hline \end{array} \mathbf{E}_z - \begin{array}{c} \text{nx} \\ \hline \end{array} \begin{array}{|c|} \hline \begin{array}{c} \text{pie} \\ \hline \end{array} \begin{array}{|c|} \hline \text{[Diagram: Box with green rectangle at bottom-left]} \\ \hline \end{array} \mathbf{E}_z \right) \\
& \quad \quad \quad \begin{array}{c} \text{ny} \\ \hline \end{array} \quad \quad \quad \begin{array}{c} \text{ny} \\ \hline \end{array}
\end{aligned}$$

Figure 8: PML update for \mathbf{H}_y . PML proper.

$$\begin{aligned}
\mathbf{E}_{zx_{xn}} &= \mathbf{C}_{ezx_{e_{xn}}} \odot \mathbf{E}_{zx_{xn}} \dots \\
&+ \mathbf{C}_{ezx_{hy_{xn}}} \odot \left(\frac{\frac{p_{is}}{2}}{ny} \mathbf{H}_y - \frac{\frac{p_{is}-1}{2}}{ny} \mathbf{H}_y \right) \\
\mathbf{E}_{zy_{xn}} &= \mathbf{C}_{ezy_{e_{xn}}} \odot \mathbf{E}_{zy_{xn}} \dots \\
&+ \mathbf{C}_{ezy_{hy_{xn}}} \odot \left(\frac{\frac{p_{js}+1}{p_{je}-1}}{p_{je}-1} \mathbf{H}_x - \frac{\frac{p_{js}}{p_{je}-2}}{p_{je}-2} \mathbf{H}_x \right)
\end{aligned}$$

Figure 9: PML update for \mathbf{E}_{zx} up. One line of nodes into inner nodes.

$$\begin{aligned}
& \mathbf{E}_{zx_{xp}} = \mathbf{C}_{ezx_{xp}} \odot \mathbf{E}_{zx_{xp}} \dots \\
& + \mathbf{C}_{ezx_{hy_{xp}}} \odot \left(\mathbf{H}_y - \mathbf{H}_y \right) \\
& \mathbf{E}_{zy_{xp}} = \mathbf{C}_{ezy_{xp}} \odot \mathbf{E}_{zy_{xp}} \dots \\
& + \mathbf{C}_{ezy_{hx_{xp}}} \odot \left(\mathbf{H}_x - \mathbf{H}_x \right)
\end{aligned}$$

The diagram illustrates the PML update for \mathbf{E}_{zx} down. It shows the calculation of the updated electric field components $\mathbf{E}_{zx_{xp}}$ and $\mathbf{E}_{zy_{xp}}$ based on the current field values and the PML coefficients $\mathbf{C}_{ezx_{xp}}$, $\mathbf{C}_{ezy_{xp}}$, $\mathbf{C}_{ezx_{hy_{xp}}}$, and $\mathbf{C}_{ezy_{hx_{xp}}}$. The update involves element-wise multiplication (\odot) and subtraction ($-$) of the current field values from the PML coefficients. The spatial dimensions are indicated by the subscripts x , y , and z , and the PML parameters pjs and pje are shown in the diagram.

Figure 10: PML update for \mathbf{E}_{zx} down. One line of nodes into inner nodes.

$$\begin{aligned}
& \mathbf{E}_{zx_{yn}} = \mathbf{C}_{ezx_{e_{yn}}} \odot \mathbf{E}_{zx_{yn}} \dots \\
& + \mathbf{C}_{ezx_{hy_{yn}}} \odot \left(\frac{\text{pie} - 1}{\frac{2}{\text{pjs}}} \left[\text{box} \right] \mathbf{H}_y - \frac{\text{pie} - 2}{\frac{2}{\text{pjs}}} \left[\text{box} \right] \mathbf{H}_y \right) \\
& \mathbf{E}_{zy_{yn}} = \mathbf{C}_{ezy_{e_{yn}}} \odot \mathbf{E}_{zy_{yn}} \dots \\
& + \mathbf{C}_{ezy_{hx_{yn}}} \odot \left(\frac{nx}{\frac{2}{\text{pjs}}} \left[\text{box} \right] \mathbf{H}_x - \frac{nx}{\frac{2}{\text{pjs} - 1}} \left[\text{box} \right] \mathbf{H}_x \right)
\end{aligned}$$

Figure 11: PML update for \mathbf{E}_{zy} left. One line of nodes into inner nodes.

$$\begin{aligned}
& \mathbf{Ez}_{xyp} = \mathbf{Cezx}_{yp} \odot \mathbf{Ez}_{xyp} \quad \dots \\
& + \mathbf{Cezx}_{hyyp} \odot \left(\left(\frac{\text{pie} - 1}{\text{pje}} \right)^{\frac{\text{pis} + 1}{\text{ny}}} \mathbf{H}_y - \left(\frac{\text{pie} - 2}{\text{pje}} \right)^{\frac{\text{pis}}{\text{ny}}} \mathbf{H}_y \right) \\
& \mathbf{Ezy}_{yp} = \mathbf{Ceye}_{yp} \odot \mathbf{Ezy}_{yp} \quad \dots \\
& + \mathbf{Ceyh}_{xyp} \odot \left(\left(\frac{\text{nx}}{\text{pje}} \right)^{\frac{2}{\text{ny}}} \mathbf{H}_x - \left(\frac{\text{nx}}{\text{pje} - 1} \right)^{\frac{2}{\text{ny} - 1}} \mathbf{H}_x \right)
\end{aligned}$$

Figure 12: PML update for \mathbf{E}_{zy} right. One line of nodes into inner nodes.

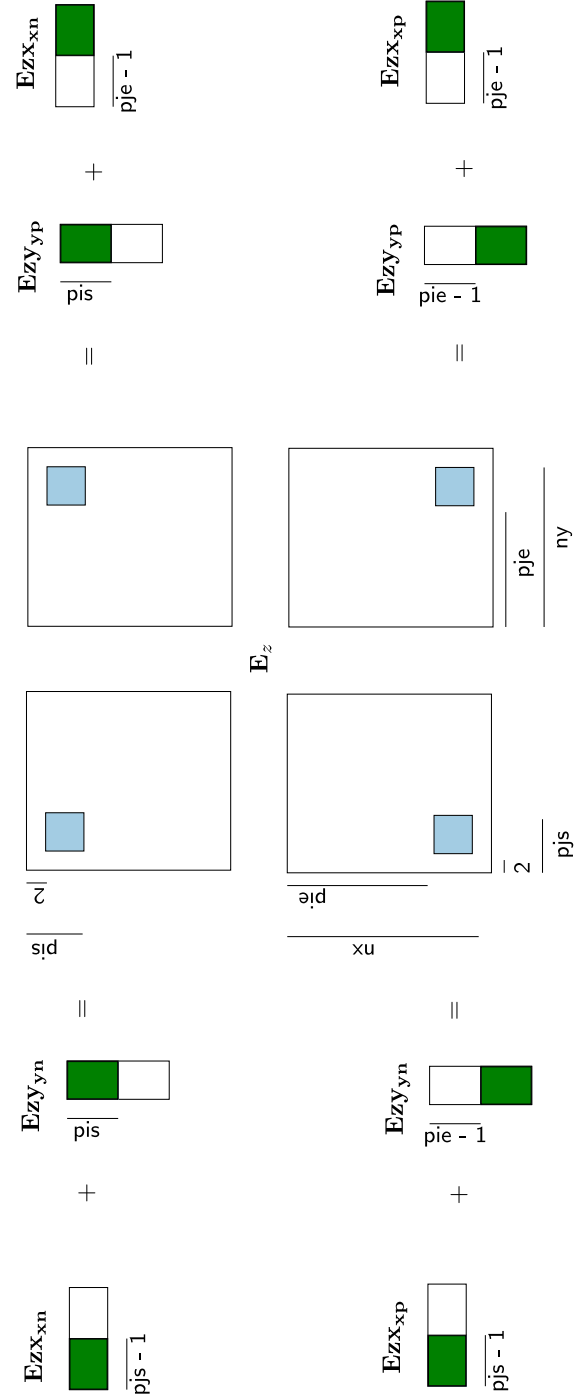


Figure 13: PML update for $E_z = E_{zx} + E_{zy}$ corners. PML proper.

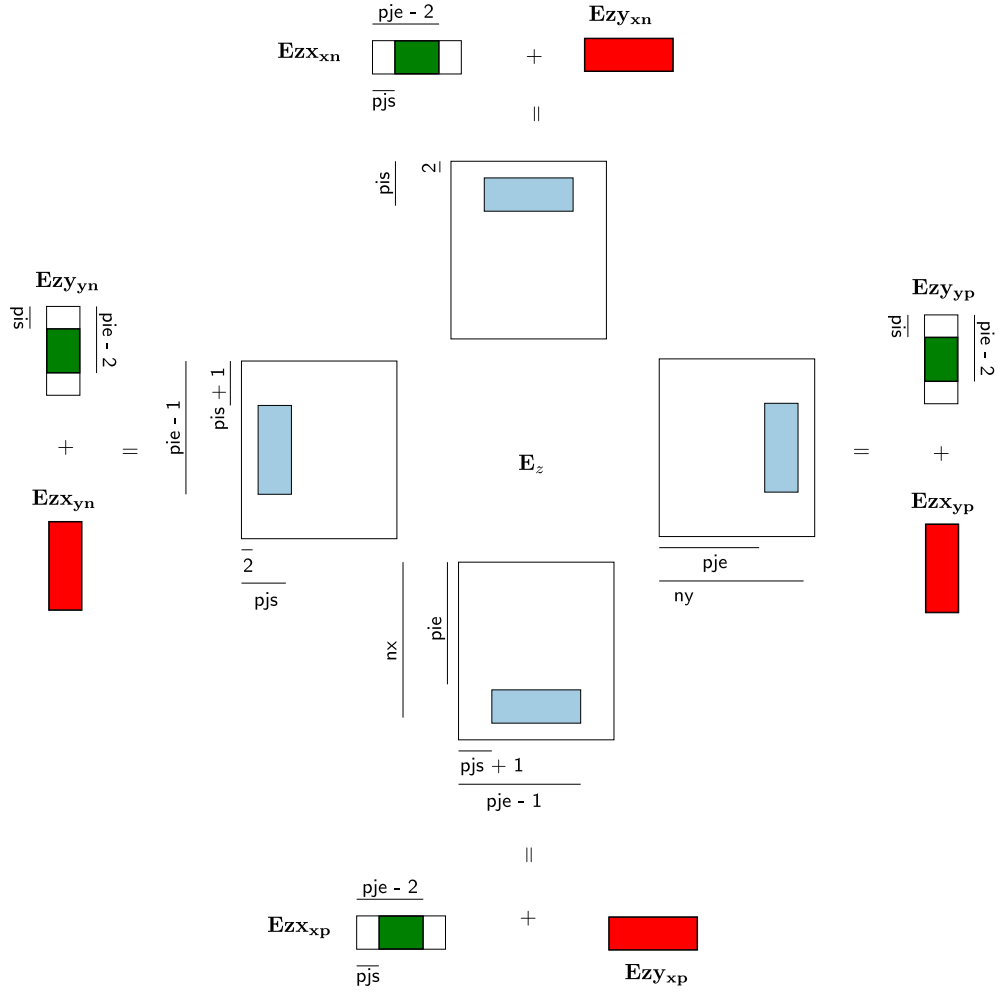


Figure 14: PML update for $\mathbf{E}_z = \mathbf{E}_{zx} + \mathbf{E}_{zy}$ sides. PML proper.