Joint inversion of GPR and ER Data using the Adjoint Method

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INTRODUCTION

Ground penetrating radar (GPR) and electrical resistivity (ER) data are sensitive in complementary ways to electrical conductivity: GPR is sensitive to conductivity through reflection and attenuation in high frequencies while ER is directly sensitive to conductivity in the DC frequency. We have developed a joint inversion algorithm that takes into account the GPR and ER sensitivities of the conductivity at each step of a non-linear adjoint method inversion. We formulate our imaging algorithm by finding parameters ε_*, σ_* that satisfy,

$$\begin{aligned}\{\varepsilon_*, \sigma_*\} &= \arg\min E(\varepsilon, \sigma)\\ E &= E_w(\varepsilon, \sigma;\, d_w^o) + E_{dc}(\sigma;\, d_{dc}^o).\end{aligned} \qquad (1)$$

Both E_w and E_{dc} are the sum squared errors of the synthetic and observed data. We optimize (1) using gradient descent.

GPR INVERSION

The physics of the GPR experiment are given by the time dependent Maxwell's equations,

$$\begin{pmatrix}\mu_o & 0 & 0\\ 0 & \mu_o & 0\\ 0 & 0 & \varepsilon\end{pmatrix}\begin{pmatrix}\dot H_z\\ -\dot H_x\\ \dot E_y\end{pmatrix} = \begin{pmatrix}0 & 0 & \partial_x\\ 0 & 0 & \partial_z\\ \partial_x & \partial_z & 0\end{pmatrix}\begin{pmatrix}H_z\\ -H_x\\ E_y\end{pmatrix} - \sigma\begin{pmatrix}0\\ 0\\ E_y\end{pmatrix} + \begin{pmatrix}0\\ 0\\ -J_y\end{pmatrix}. \qquad (2)$$

The discretized version of equation (2),

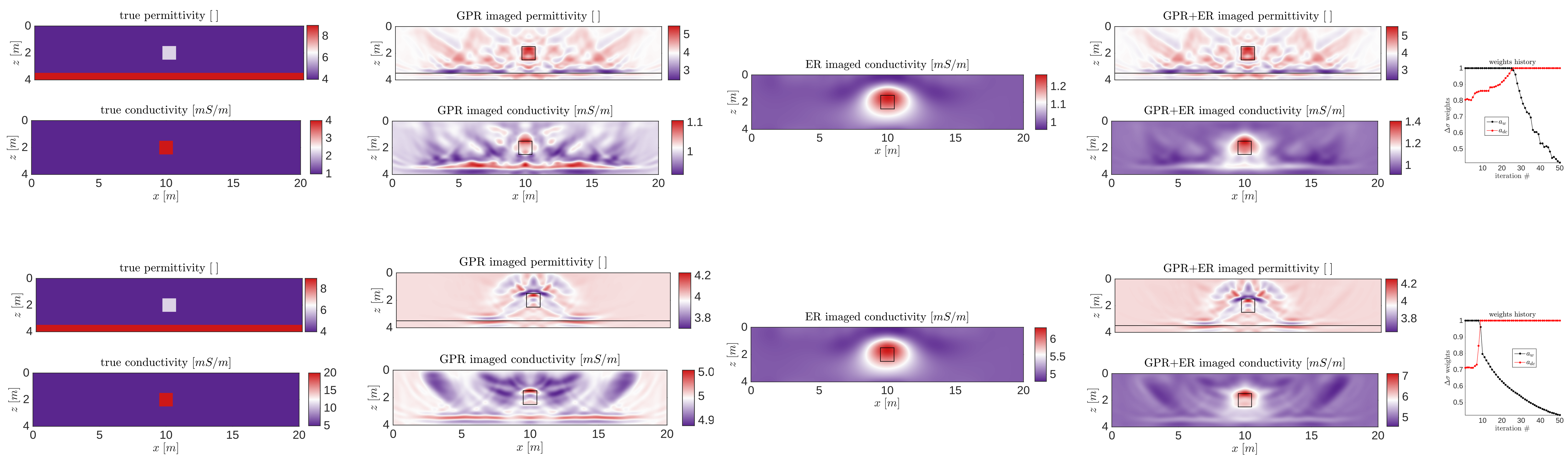
$$\begin{aligned}u &= L_w\, s_w,\\ d_w &= M_w\, u\end{aligned} \qquad (3)$$

where L_w is the discretized differential operator of (2), u is the electric field y component, s_w is the source term, M_w is the measuring operator and $d_w = M_w\, u$ is the data. We compute the gradients of E_w with respect to σ and ε by using the *adjoint wave field* a_w,

$$\begin{aligned}a_w &= L_w\, e_w(-t),\\ g_{w,\sigma} &= -\sum_t u(-t) \odot a_w(t)\cdot \Delta t,\\ g_{w,\varepsilon} &= -\sum_t \dot u(-t) \odot a_w(t)\cdot \Delta t,\end{aligned} \qquad (4)$$

where $e_w = d_w - d_w^o$ is the error of the synthetic vs the observed data.

EXAMPLES



GPR experiments: 5 sources equally spaced with receivers all along the transect spaced $\lambda_o/4$ between each other and λ_o away from the source. The source was a Ricker wavelet with f_o =250MHz. **ER experiments**: electrodes placed 1m apart with all possible dipole-dipole and Wenner arrays. **Inversion**: All initial models were taken with a homogeneous background. Aside from removing high amplitudes near the sources and smoothing the gradients, no further regularization was made.

ER INVERSION

The physics of the ER experiment are given by the steady state Maxwell's equations where Ohm's law holds,

$$-\nabla\cdot\sigma\nabla\varphi = \mathbf{i}(\delta(x-s_+) - \delta(x-s_-)), \qquad (5)$$

where φ is the electric potential, $\mathbf{i}$ is the current intensity and $s_\pm$ is the source-sink position. We write the discretized version of (5) as,

$$\begin{aligned}L_{dc}\varphi &= s_{dc},\\ d_{dc} &= M_{dc}\varphi,\end{aligned} \qquad (6)$$

where L_{dc} is the discretized differential operator of (5), φ is the electric potential, s_{dc} is the source term, M_{dc} is the measuring operator that computes observed voltages and d_{dc} is the data. We compute the gradient of E_{dc} with respect to σ by using the *adjoint potential field* a_{dc},

$$\begin{aligned}L_{dc}^\top a_{dc} &= M_{dc}^\top e_{dc},\\ g_{dc} &= S_{dc} a_{dc},\end{aligned} \qquad (7)$$

where $S_{dc} = -((\nabla_\sigma L_{dc})\varphi)^\top$ and $e_{dc} = d_{dc} - d_{dc}^o$ is the error of the synthetic vs the observed data.

JOINT INVERSION

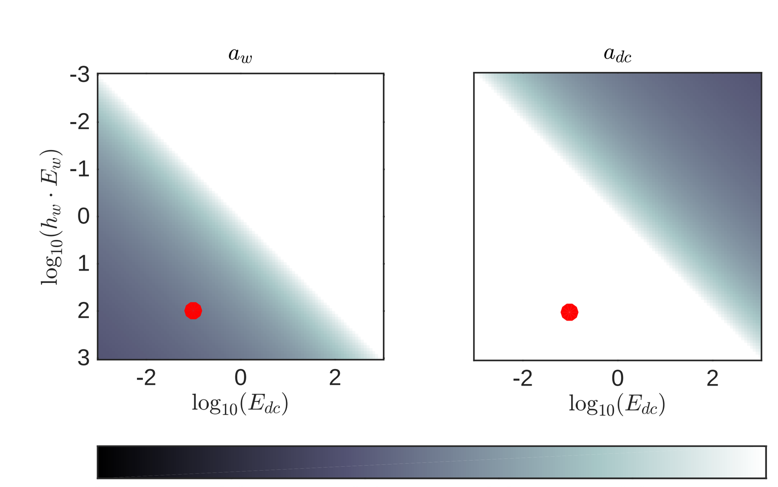
Let the updates for σ be,

$$\begin{aligned}\Delta\sigma_w &= -\frac{1}{n_w}\sum_{i=1}^{n_w}\alpha_\sigma\, g_{w,\sigma},\\ \Delta\sigma_{dc} &= -\frac{1}{n_{dc}}\sum_{i=1}^{n_{dc}}\alpha_{dc}\, g_{dc},\end{aligned} \qquad (8)$$

where n_w and n_{dc} denote the number of GPR and ER experiments respectively. We define the joint update,

$$\Delta\sigma = a_w\,\Delta\sigma_w + a_{dc}\,\Delta\sigma_{dc}, \qquad (9)$$

where a_w and a_{dc} are scalar weights. The choice for weights a_w and a_{dc} is made with the paradigm of letting both updates $\Delta\sigma_w$ and $\Delta\sigma_{dc}$ always contribute to $\Delta\sigma$ in proportion to their objective function value at a given iteration.



CONCLUSIONS

- Our joint inversion algorithm of GPR and ER data allows for both data sets to work cooperatively and **regularize each other while honoring the physics**.
- Our algorithm performs well on **one sided acquisition** and makes **no assumption of the subsurface geometry**.
- Both when the media is strongly and weakly conductive, our algorithm **enhances the spatial and magnitude resolution of** σ.
- Possible enhancements to the algorithm are a **stepped-frequency GPR inversion** as in Adam Mangel's poster NS31B-0753.

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References

[1] D. Domenzain, J. Bradford & J. Mead, *Joint inversion of GPR and ER data*, SEG Technical Program Expanded Abstracts 2018, 4763-4767.

[2] https://github.com/diegozain/gerjoii