Imaging by joint inversion of electromagnetic waves and DC currents

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Abstract

Electromagnetic methods are an important tool for imaging material properties in the subsurface. With the goal of performing a joint inversion using electromagnetic data from different types of energy transport, we developed our **own forward models** for the *electrical resistivity* (ER) and *ground penetrating radar* (GPR) experiments, as well as a **novel inversion algorithm** using an objective function that updates both for GPR and ER parameters in each iteration.

Regarding the transport of electromagnetic energy, we assume the subsurface is made by linear isotropic materials where Ohm's Law holds, there is no free charge and no significant variation along the y axis (horizontal to the ground),

$$\nabla\times\mathbf{E} = -\mu\dot{\mathbf{H}} \qquad \nabla\cdot\mathbf{E} = 0$$
$$\nabla\times\mathbf{H} = \varepsilon\dot{\mathbf{E}} + \sigma\mathbf{E} - \mathbf{J}_s \qquad \nabla\cdot\mathbf{H} = 0.$$

We distinguish two variants of electromagnetic energy transport: steady case (for the ER experiment) and wave propagation (for the GPR experiment). We **model** both experiments in two dimensions and numerically solve for their energy response using a *finite volume method* (FVM) for ER and a *finite difference time domain* (FDTD) for GPR. We also present our **inversion algorithm** making use of the *adjoint method* for calculating the gradient of our proposed objective function.

Steady current (ER) continuous

We denote Ω the halfplane perpendicular to the ground which will be our domain.

By writting $\mathbf{E} = -\nabla\varphi$ we have

$$-\nabla\cdot\sigma\nabla\varphi = \nabla\cdot\mathbf{J}_s.$$

Our forward model equations are

$$\begin{aligned} \nabla\cdot\sigma\nabla\varphi &= \boldsymbol{i}\,(\delta^2_{\mathbf{x}-\mathbf{s}_-} - \delta^2_{\mathbf{x}-\mathbf{s}_+}) && \text{in } \Omega\\ \sigma\nabla\varphi\cdot\hat{\mathbf{n}} &= 0 && \text{on } \Gamma_N\\ \sigma\nabla\varphi\cdot\hat{\mathbf{n}} + \alpha\varphi &= 0 && \text{on } \Gamma_R \end{aligned}$$

where Γ_N is the ground-air interface and Γ_R is the boundary everywhere else.

Wave (GPR) continuous

We do a change of variables

$$\mathbf{v} = (H_z,\ -H_x)^t \qquad u = E_y$$
$$\mathbf{s} = (0,\ 0,\ -J_y)^t \qquad \mathbf{w} = (\mathbf{v},\ u)^t$$

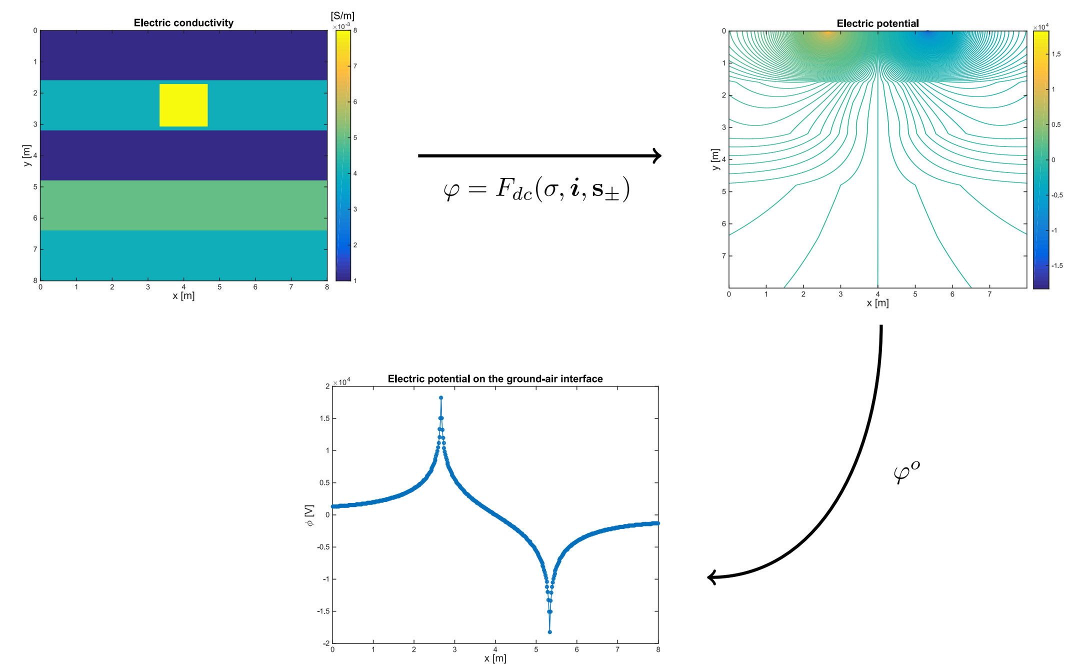
to write our forward model in Ω as an advection transport,

$$\underbrace{\begin{bmatrix}\mu\mathbb{1}_2 & 0\\ 0 & \varepsilon\end{bmatrix}}_{A}\dot{\mathbf{w}} = \underbrace{\begin{bmatrix}0 & \nabla^t\\ \nabla & 0\end{bmatrix}}_{\mathrm{D}}\mathbf{w} - \underbrace{\begin{bmatrix}0 & 0\\ 0 & \sigma\end{bmatrix}}_{B}\mathbf{w} + \mathbf{s}$$

$$\Rightarrow A\dot{\mathbf{w}} + (B - \mathrm{D})\mathbf{w} = \mathbf{s}.$$

On $\partial\Omega$ we impose a *perfectly matched layer* (PML).

Steady current (ER) experiment



We write the differential operator acting on φ as

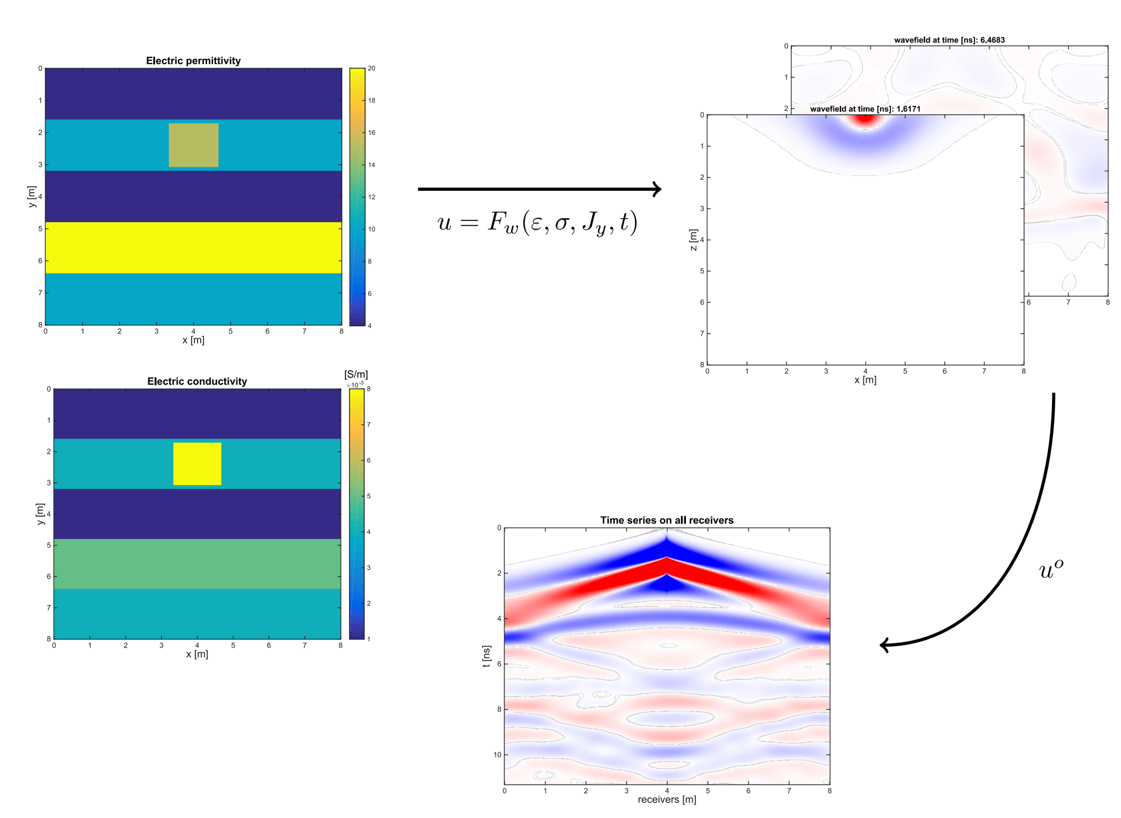
$$L_{dc} = -\nabla\cdot\sigma\nabla$$

so we can write our forward model for the ER experiment as

$$F_{dc}(\sigma, \boldsymbol{i}, \mathbf{s}_\pm) := L_{dc}^{-1}\,\boldsymbol{i}\,(\delta^2(\mathbf{x}-\mathbf{s}_-) - \delta^2(\mathbf{x}-\mathbf{s}_+)) = \varphi.$$

The diagram above shows the work flow of our synthetic ER experiment, restricting our attention on the observed synthetic data φ^o.

Wave (GPR) experiment



We write the differential operator acting on $\mathbf{w}$ as

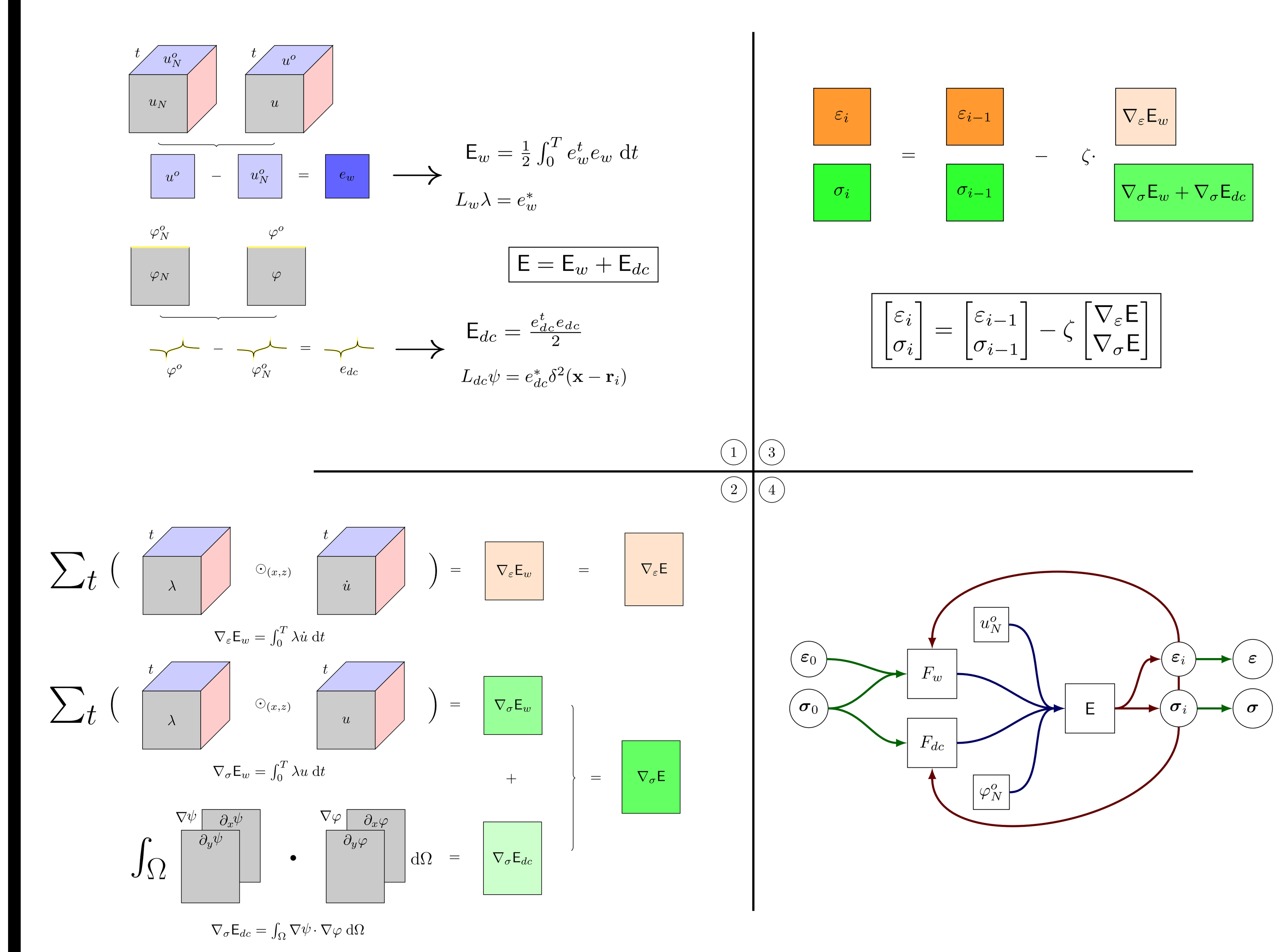
$$L_w = A\partial_t + B - \mathrm{D}$$

so we can write our forward model for the GPR experiment as

$$F_w(\sigma, \varepsilon, J_y, t) := L_w^{-1}\mathbf{s} = \mathbf{w}.$$

The diagram above shows the work flow of our synthetic GPR experiment restricting our attention on the electric $E_y = u$ component, and finally on the synthetically observed u^o data.

Joint inversion



Diagrams represent discretization schemes, and rectangles should be thought of as matrices. Typeset formulas represent the continuous case.

Panels 1-3. We present the gradient calculation of E with respect to the parameters ε and σ of our proposed joint inversion algorithm using the *adjoint method* for both the wavefield u and the potential φ, in order to perform a Newton algorithm for updating ε and σ. Notice that each time panel 1 is called, our forward models are also called to calculate the adjoint fields λ and ψ. Subscript N represents the natural processes that actually take place when we perform the experiments.

Panel 4. We present the work flow of the inversion process with ε_o and σ_o as initial guesses. The box holding E performs steps in panels 1-3.

Conclusions and future work

In order to perform a joint inversion of ER and GPR data, we have implemented FVM and FDTD methods to numerically solve for the electric potential (φ) and electric wavefield (u) respectively. We perform the discretization on a single grid (both F_{dc} and F_w are solved in the same grid) that is allowed to vary the size of its cells. Both methods are accurate and relatively fast to run on a common laptop for moderate sized problems. We have also presented an algorithm for performing a 2 dimensional joint inversion using an *adjoint method* approach on both u and φ (in the case of u this is a *full wavefield inversion* (FWI)) that has permitted us to formulate an objective function E that naturally joins both e_w and e_{dc} and updates simultaneously on u and φ *and* on ε and σ without the need of external regularization. In future work we will write computer code to perform this joint inversion algorithm on synthetically generated data, proceed to test our algorithm on real laboratory data, and ultimately on field data.

References

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Code: github.com/diegozain