The Amplituhedron: algebra, combinatorics & physics

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Steven N. Karp's Talk

POSITIVE GEOMETRIES [ABL] {CAM's talk}

SPACE	CAN FORM	PHYSICAL OBS
(X, X_+)	$\Omega(X)$	ϑ
combinatorial geometric properties	analytical properties	physical properties
* BOUNDARIES ∂X_i	* $\text{Res}_{\partial X_i} \Omega(X) = \Omega(\partial X_i)$	* locality, unitarity, ...
* TRIANGULATIONS $X_+ = \bigcup_a X_a$	* $\Omega(X) = \sum_a \Omega(X_a)$	* Different Repr. * hidden properties * Dualities

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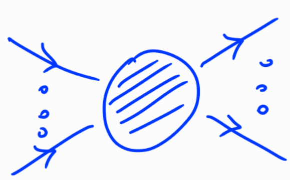
$\vartheta \equiv$ Scattering Amplitudes

-  encodes probability interaction of elementary particles in a certain THEORY

- Computed perturbatively [strength interaction] in QFT
  - → leading order "TREE-LEVEL" ↔ RATIONAL functions [POLES, RESIDUES]
  - → L-th subleading order "L LOOPS" ↔ COMPLICATED zoo of functions

$$\int \mathcal{I} = \text{[BRANCH PTS, MONODROMIES]}$$

INTEGRAND
RATIONAL function

↓ BEAUTIFUL story too!

CLUSTER ALGEBRAS $Gr_{4,n}$.

Amplituhedron (2013) [AT]

"m=4" * $\mathcal{A}_{n,k,4}$ – tree-level $\mathcal{N}=4$ SYM [momentum twistors]

"loop" * $\mathcal{A}^{(L)}_{n,k}$ – L loops " "

"m=2" * $\mathcal{A}_{n,k,2}$ – 1 Loop ["MHV & NMHV" sector] " "

{LAUREN's talk}

Momentum Amplituhedron (2019) [DFLP, LPW]

* $\mathcal{M}_{n,k,4}$ – tree-level $\mathcal{N}=4$ SYM [momentum space]

→ T-dual of $\mathcal{A}_{n,k,4}$ → Dual Formulation

Associahedron (2018) [ABHY] – tree-level ϕ^3 biadjoint scalar

[AHST] ... – loop-level " "

{HUGH's talk}

⋮

Questions. (Amplituhedron)

* ODD/EVEN m

Recall $\mathcal{A}_{n,k,\underline{\underline{m}}} \subset Gr_{k,k+m}$

→ properties expected for any m.

→ nice/bad properties for even/odd m.

* KWZ conjecture

$\#$ top-cells in a triangulation of $\mathcal{A}_{n,k,m} = M(k, n-k-m, \frac{m}{2})$

- Does it hold for ANY triangulation? [e.g. cyclic polytope]
- What can we say about odd m?
- What happens if we sum M over k?

* REGIONS & SIGN VARIATION

$\Big\{$ (Bounded) regions R_a characterised by sign-variation

Images I_a positroid cells $\subseteq Gr^{\geq 0}_{k,n}$ in $\mathcal{A}_{n,k,1}$ (injective)

- $R \overset{?}{=} \bigcap_a I_a$, $I \overset{?}{=} \bigcup_a R_a$.
- combinatorics of triangulations? (how many? $\#$ pos. cells in secondary geometry etc.).