Homework 5

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1 Problem 1

a. How many 32-bit integers can be stored in a 16-byte cache block? Each 32-bit integer occupies 4 bytes:

$$\frac{16 \text{ bytes}}{4 \text{ bytes/integer}} = 4 \text{ integers.}$$

- b. References to which variables exhibit temporal locality? Temporal locality is exhibited by the loop variables I, J and addresses of A and B since they are accessed repeatedly in every iteration of the loop. Also B(I, 0) is accessed multiple times in the inner loop and therefore exhibits temporal locality, only in the inner loop as I changes in the outer loop and breaks temporal locality.
- c. References to which variables exhibit spatial locality? Spatial locality is exhibited by the variables A(J, I) it is accessed contiguously in memory because arrays are stored in row-major order. Notice that A(J, I) is accessed in column-major which doesnt exhibit spatial locality in C. And B(I, 0) is not accessed contiguously as it is jumping row by row in the outer loop.
- d. How many 16-byte cache blocks are needed to store all 32-bit matrix elements being referenced? Given that the matrix has 8 rows and 8000 columns, and each element is a 32-bit integer (4 bytes), the total number of cache blocks required is:

$$\frac{8\times8000\times4~\mathrm{bytes}}{16~\mathrm{bytes/block}} = 16000~\mathrm{blocks}.$$

e. References to which variables exhibit temporal locality? Temporal locality is exhibited by the variables I, J and addresses of A and B. Also B(I, 0) is accessed multiple times in the inner loop and therefore exhibits temporal locality, only in the inner loop as I changes in the outer loop and breaks temporal locality.

f. References to which variables exhibit spatial locality? This time A(J, I) and B(I, 0) exhibits spatial locality, as it is accessed contiguously through the outer loop. In this case A(I, J) is not accessed contiguously as it will jump in memory because of the column-major order.

2 Problem 2

| Binary Address | Tag | Index | Hit/Miss |
|----------------|------|-------|--------------|
| 00000011 | 0000 | 0011 | miss |
| 10110100 | 1011 | 0100 | $_{ m miss}$ |
| 00101011 | 0010 | 1011 | $_{ m miss}$ |
| 00000010 | 0000 | 0010 | $_{ m miss}$ |
| 10111111 | 1011 | 1111 | $_{ m miss}$ |
| 01011000 | 0101 | 1000 | miss |
| 10111110 | 1011 | 1110 | miss |
| 00001110 | 0000 | 1110 | miss |
| 10110101 | 1011 | 0101 | miss |
| 10110101 | 1011 | 0101 | miss |
| 00101100 | 0010 | 1100 | miss |
| 10111010 | 1011 | 1010 | miss |
| 11111101 | 1111 | 1101 | miss |

Table 1: Cache A: Direct-mapped cache with 16 one-word blocks

a.

| Binary Address | Tag | Index | Hit/Miss |
|----------------|-----|-------|----------|
| 00000011 | - | 0001 | miss |
| 10110100 | 101 | 1010 | miss |
| 00101011 | 1 | 0101 | miss |
| 00000010 | - | 0001 | hit |
| 10111111 | 101 | 1111 | miss |
| 01011000 | 10 | 1100 | miss |
| 10001110 | 100 | 0111 | miss |
| 00001110 | _ | 0111 | miss |
| 00001110 | _ | 0111 | hit |
| 10110101 | 101 | 1010 | hit |
| 00101100 | 1 | 0110 | miss |
| 10111010 | 101 | 1101 | miss |
| 11111101 | 111 | 1110 | miss |

Table 2: Cache B: Direct-mapped cache with two-word blocks and a total of 8 blocks

b.

3 Problem 3

- a. The cache block size is 4 words.
- b. The cache has **32 entries**.
- c. The ratio between total bits required for such a cache implementation over the data storage bits is approximately ${\bf 1.1796875}$.
- 1. d.

| Hex Address | Tag | Index | Offset | Hit/Miss | Replaced Tag |
|-------------|---|-------|--------|----------|---|
| 00 | 000000000000000000000000000000000000000 | 00000 | 0000 | miss | - |
| 04 | 00000000000000000000000 | 00000 | 0100 | hit | - |
| 10 | 000000000000000000000000000000000000000 | 00000 | 0000 | hit | - |
| 84 | 000000000000000000000000000000000000000 | 00100 | 0100 | miss | - |
| E8 | 000000000000000000000000000000000000000 | 00111 | 1000 | miss | - |
| A0 | 000000000000000000000000000000000000000 | 00101 | 0000 | miss | - |
| 400 | 000000000000000000000000000000000000000 | 00000 | 0000 | miss | 00000000000000000000000 |
| 1E | 000000000000000000000000000000000000000 | 00000 | 1110 | miss | 00000000000000000000000000001 |
| 8C | 000000000000000000000000000000000000000 | 00100 | 1100 | hit | - |
| C1C | 00000000000000000000011 | 00000 | 1100 | miss | 000000000000000000000000 |
| B4 | 000000000000000000000000000000000000000 | 00101 | 0100 | hit | - |
| 884 | 000000000000000000000000000000000000000 | 00100 | 0100 | miss | 000000000000000000000000000000000000000 |

Table 3: Cache actions for each address reference.

- e. The hit ratio for the given cache references is 0.3333 (or 33.33%).
- f. <00000, 0000000000000000000011>
 - <00100, 00000000000000000000010>

Question 4 Answers

| | Word Address | Hit/Miss |
|----|--------------|----------|
| | 2 | miss |
| | 3 | miss |
| | 11 | miss |
| | 16 | miss |
| | 21 | miss |
| | 13 | miss |
| | 64 | miss |
| a. | 48 | miss |
| | 19 | miss |
| | 11 | hit |
| | 3 | miss |
| | 22 | miss |
| | 4 | miss |
| | 27 | miss |
| | 6 | miss |
| | 11 | miss |
| | | |

Final cache contents:

| Set Index | Words |
|-----------|--------|
| 0 | 64, 48 |
| 1 | - |
| 2 | 2 |
| 3 | 27, 11 |
| 4 | 4 |
| 5 | 21, 13 |
| 6 | 22, 6 |
| 7 | _ |

Hit/Miss Word Address 2 miss 3 ${\rm miss}$ 11 ${\rm miss}$ 16 ${\rm miss}$ 21 ${\rm miss}$ 13 $_{
m miss}$ 64 miss b. 48 miss 19 miss 11 hit 3 hit 22 miss 4 ${\rm miss}$ 27 miss 6 ${\rm miss}$ 11 hit