Problem Section 2

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Definition Big-O Notation

Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers. We say that $f(x)$ is $O(g(x))$ if there are constants C and k such that

$$\big|f(x)\big| \leq C\big|g(x)\big| \text{ whenever } x > k$$

Also, C and k are referred as the witnesses of the Big-O notation, and are those possible non unique pair of constants that satisfy the definition above. This also applies to Ω and Θ notations.

Definition Big-Ω Notation

Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers. We say that $f(x)$ is $\Omega(g(x))$ if there are constants C and k such that

$$\big|f(x)\big| \geq C\big|g(x)\big| \text{ whenever } x > k$$

Definition Big-Θ Notation

Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers. We say that $f(x)$ is $\Theta(g(x))$ if there are constants C_1, C_2 and k such that

$$C_1\big|g(x)\big| \leq \big|f(x)\big| \leq C_2\big|g(x)\big| \text{ whenever } x > k$$

Problem 1

Describe the shaded area in terms of sets A, B, C.

Problem 2

For sets A, B, and C, is $(A - B) - C - > A - (B - C)$?

Problem 3

in a group of 30 children, 10 like apples, 10 like oranges, 10 like bananas and 6 like all three (nobody likes just two). How many children do not like any of the three fruits?

Apply inclusion-exclusion principle

Problem 4

Give examples of functions from integers to integers that (a) one-to-one but not onto, (b) onto but not one-to-one, (c) one-to-one and onto.

a) $f(x) = 2x$ b) $f(x) = \lfloor x/2 \rfloor$ c) $f(x) = x$

Problem 5

If possible, give exaples of functions from A to A where A

Problem 6

Define "countable set"; show that $\frac{a}{2}, \frac{a}{4}, ...$ where a is in $1, 2, 3, 4, 5, 6, 7, 8, 9$ is countable.

Problem 7

Consider set of all strings over alphabet $0, 1, 2, ..., 9$, prove or disprove claim that set is countable.