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Lecture 4A — Friday, July 5.

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Recap of the basic notions covered in lecture 3:

(a) **The notion of cardinality of a set**

(b) **Countable and uncountable sets**

(c) **Cartesian product of two sets**

(d) **Alternative definition of maps**

Questions:

(a) **Did anyone find it hard to grasp these basic concepts?**

(b) **Did anyone find it hard to solve the assignments?**

If the answer to the above is yes, then you should not proceed with the content on lecture 4 until you feel comfortable with the basics of lecture 3!

Definition 1.0

We say a set X is indexed by a set I if there is surjective map $index : I \longrightarrow X$. I is then called an index set for X.

(a) By convention, for $i \in I$ and $index(i) = x \in X$, we just write x_i.

(b) If I is made of natural numbers, we can think of x_i as the ith element of X.

Concrete examples:

(a) Let $I = \{1, 2, 3\}$. Then, we can say $X = \{x_1, x_2, x_3\}$ is indexed by I.

(b) Let X be a set and $I = X$. We can define the indexing of X by I as $x_i = i$ for all $i \in I$. This is too trivial to be useful, but still valid!☺

(c) Let $X = \mathbb{N}$ and $I = \mathbb{Z}$. We can define the indexing of X by I as $x_i = |i| + 1$ for all $i \in I$.

Definition 1.1

A collection of sets $\mathcal{C}$ is an indexed family of sets with indexing set I if there is surjective map $index : I \longrightarrow \mathcal{C}$.

(a) By convention, for $i \in I$ and $index(i) = A \in \mathcal{C}$, we just write A_i.

(b) If I is made of natural numbers, we can think of A_i as the ith element of the collection $\mathcal{C}$.

(c) We write $\mathcal{C} = \{A_i\}_{i \in I}$ or just $\{A_i\}_{i \in I}$ to denote the indexed family of sets by I.

Definition 1.2

Let $\{A_i\}_{i \in I}$ be an indexed family of sets. Their union is defined as
$\bigcup\limits_{i \in I} A_i = \{x \mid \text{there exists at least one } i \in I \text{ such that } x \in A_i\}$ or alternatively
$\bigcup\limits_{i \in I} A_i = \{x \mid x \in A_i \text{for at least one } i \in I\}$.

(a) If $I = \mathbb{N}$, the we normally write $\bigcup\limits_{i=1}^{\infty} A_i$.

(b) If $I = \{1, \ldots, k\}$ for some $k \in \mathbb{N}$, we write $\bigcup\limits_{i=1}^{k} A_i$.

Definition 1.3

Let $\{A_i\}_{i \in I}$ be an indexed family of sets. Their interesection is defined as
$\bigcap\limits_{i \in I} A_i = \{x \mid x \in A_i \text{for all} i \in I\}$.

(a) If $I = \mathbb{N}$, the we normally write $\bigcap\limits_{i=1}^{\infty} A_i$.

(b) If $I = \{1, \ldots, k\}$ for some $k \in \mathbb{N}$, we write $\bigcap\limits_{i=1}^{k} A_i$.

Exercise 1.0

$\{A_i\}_{i\in I}$ be an indexed family of sets. Prove or disprove the following:

(a) $(\bigcup_{i\in I} A_i)^c = \bigcup_{i\in I} A_i^c$.

(b) $(\bigcup_{i\in I} A_i)^c = \bigcap_{i\in I} A_i^c$.

(c) $(\bigcap_{i\in I} A_i)^c = \bigcap_{i\in I} A_i^c$.

(d) $(\bigcap_{i\in I} A_i)^c = \bigcup_{i\in I} A_i^c$.

Exercise 1.1

$\{A_i\}_{i\in I}$ be an indexed family of sets and X be a set. Prove or disprove the following:

(a) $X \cup (\bigcup_{i\in I} A_i) = \bigcup_{i\in I}(X \cup A_i)$.

(b) $X \cap (\bigcup_{i\in I} A_i) = \bigcup_{i\in I}(X \cap A_i)$.

(c) $X \cap (\bigcap_{i\in I} A_i) = \bigcap_{i\in I}(X \cap A_i)$.

(d) $X \cup (\bigcap_{i\in I} A_i) = \bigcap_{i\in I}(X \cup A_i)$.

Thank you for listening!