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Lecture 1 — Friday, May 24.

Bambordé Baldé

Important mathematical jargons:

(a) **Axiom:** A fundamental statement that is accepted without proof and serves as a starting point for deducing other truths within a mathematical system.

(b) **Definition:** A precise explanation of the meaning of a mathematical term or concept, used to ensure clarity and consistency proofs.

(c) **Theorem:** A significant mathematical statement that has been rigorously proven based on axioms, definitions, and previously established theorems and propositions. Theorems are central results in mathematics.

(d) **Corollary:** A statement that follows readily from a previously proven theorem, requiring minimal additional proof.

(e) **Lemma:** A preliminary or auxiliary proposition used to help prove a larger theorem. Lemmas are proven statements that are typically less significant on their own but essential for the proof of more important results.

(f) **Proposition:** A statement that is proposed for consideration and is proven to be true based on logical reasoning and previously established principles. Propositions are generally less significant than theorems.

(g) **Conjecture:** An educated hypothesis based on observed patterns or intuition, which has not yet been proven or disproven.

Definition 1.0

A set is a collection of **distinct** objects called elements of the set.

- **Comment 1:**

(a) Let $X = \{a, b, c, d\}$ and $Y = \{a, b, c, d, a\}$. If you give Y to mathematicians they will assume you mean X!

(b) If X is a set and α is an element of X, we write $\alpha \in X$ or else we write $\alpha \notin X$ to indicate that α is not an element of X. So for example, if $X = \{2, 10, 1, 6\}$ then $6 \in X$ but $11 \notin X$.

(c) **Russell's Paradox**: Let S be the set of all sets which are not elements of themselves or more formally $S = \{A \mid A \notin A\}$. Does $S \in S$?

(d) **Popular version**: Consider the barber who shaves all people who don't shave themselves. Who shaves the barber?

Definition 1.1

The empty set denoted $\emptyset$ is a set with no elements.

- **Comment 2:**

(a) The 'ZFC' axiomatic system guarantees the existence $\emptyset$.

(b) It can be proved that $\emptyset$ is unique i.e. there is only one empty set!

(c) The empty set is ubiquitious in modern mathematics built on set theory.

Definition 1.2

Let X be a non-empty set. We say a set A is a subset of X and write $A \subseteq X$ if (only if) whenever an element $\alpha \in A$ then $\alpha \in X$.

- **Comment 3:**

(a) It is clear by the definition that $X \subseteq X$ right?

(b) Is it true that the empty set $\emptyset$ is a subset of X?

(c) If Y is not a subset of X, we write $Y \nsubseteq X$ to indicated that.

Proposition 1.0

Let X, Y, and Z be sets. If $X \subseteq Y$ and $Y \subseteq Z$ then $X \subseteq Z$.

Definition 1.3

Let X and Y be sets. We say $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$ hold.

- **Comment 4:**

(a) We write $X \neq Y$ if the two sets are not equal as per the definition above.

(b) Let $X = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$ and $Y = \{\alpha_6, \alpha_1, \alpha_3, \alpha_4, \alpha_2, \alpha_5\}$. Is $X = Y$?

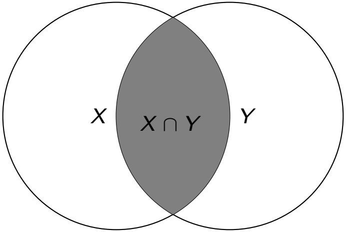
Definition 1.4

If A is a subset of X, then we say A is a proper subset of X if $A \neq X$.

The Intersection of Sets

Definition 1.5

Let X and Y be sets. The *intersection* of X with Y is defined as $X \cap Y = \{\alpha \mid \alpha \in X \text{ and } \alpha \in Y\}$.



Proposition 1.1

Let X, Y and Z be sets. Then the following properties hold:

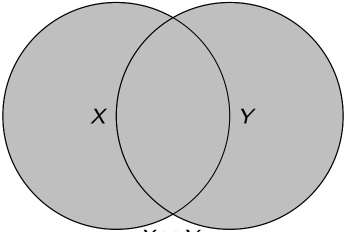
(1) $X \cap \emptyset = \emptyset$.
(2) $X \cap X = X$
(3) $X \cap Y = Y \cap X$ (commutativity).
(4) $(X \cap Y) \cap Z = X \cap (Y \cap Z)$ (associativity).

Definition 1.6

X and Y are said to be disjoint if $X \cap Y = \emptyset$.

Definition 1.7

Let X and Y be sets. The *union* of X with Y is defined as
$X \cup Y = \{\alpha \mid \alpha \in X \text{ or } \alpha \in Y\}$.



$X \cup Y$

- **Comment 5:** $X \cup Y$ may also contain elements that are in both sets!

Proposition 1.2

Let X, Y and Z be sets. Then the following properties hold:

(1) $X \cup \emptyset = X$ and $X \cup X = X$.

(2) $X \cup Y = Y \cup X$ (commutativity).

(3) $(X \cup Y) \cup Z = X \cup (Y \cup Z)$ (associativity).

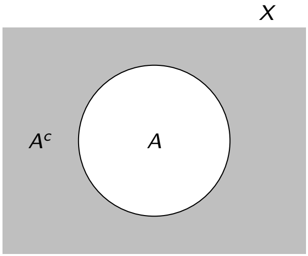
Definition 1.8

Let X and Y be sets. The *relative* complement of Y in X is definited as $X - Y = \{\alpha \in X \mid \alpha \notin Y\}$ i.e. the set of all elements of X that are not in Y. The relative complement is also known as the difference between X and Y!

- **Comment 6:** An alternative notation often used to denote $X - Y$ is $X \setminus Y$.

Definition 1.9

Let A be a subset of X. The *absolute* complement of A in X is $A^c = \{\alpha \in X \mid \alpha \notin A\}$ i.e. the set of all elements of X that are not in A.



Exercise 1.0

Let X, Y and Z be sets. Which of the identities below are true?

(1) $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$.
(2) $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$.
(3) $X \cup (Y \cap Z) = (X \cap Y) \cap (X \cup Z)$.
(4) $X \cup (Y \cap Z) = (X \cap Y) \cup (X \cup Z)$.
(5) $X \cup (Y \cap Z) = (X \cap Y) \cup (X \cap Z)$.

Exercise 1.1

Let A be a subset of X. Which of the identities below are true?

(1) $A \cap A^c = A$.
(2) $A \cap A^c = \emptyset$.
(3) $A \cup A^c = \emptyset$.
(4) $A \cap A^c = X$.
(5) $X \cup (Y \cap Z) = (X \cap Y) \cup (X \cap Z)$.
(6) $X^c = \emptyset$.
(7) $(A^c)^c = A$.
(8) $\emptyset^c = X$.

Exercise 1.2

Let A and B be subsets of X. Which of the identities below are true?

(1) $(A \cup B)^c = A^c \cup B^c$.

(2) $(A \cup B)^c = A^c \cap B^c$.

(3) $(A \cap B)^c = A^c \cap B^c$.

(4) $(A \cap B)^c = A^c \cup B^c$.

Thank you for listening!