3 A history of factor model

3.1 A time-line of important studies

Year	*Authors*	*Descriptions*
1972	Black, Jensen, Scholes	Support Black-version CAPM
1973	Fama and Macbeth	Support CAPM
1973	Merton	Intertemporal or ICAPM
1976	Ross	Arbitrage Pricing Theory or APT
1986	Chen, Roll, Ross	Industrial Production, Default spread, and Term Spread
1991	Cochrane	Use aggregate investment's return and relate it expected stock returns
1992	Fama and French	Reject CAPM, support size and BE/ME
1993	Fama and French	3 factor model for equity and 2 factor for bond (Term and Default)
1993	Jegadeesh and Titman	Short term price momentum
1994	Lakonishok, Shleifer, Vishny	Value premium
1996	Fama and French	3 factor model solves size, B/M anomalies, but fail to solve the Momentum
1996	Chan, Jegadeesh, Lakonishok	Earning momentum
1996	Jagannathan and Wang	Conditional CAPM
1997	Carhart	A 4-factor model: 3 factor of FF3 and Momentum
2004	Titman, Wei, Xie	Capex and negative returns
2004	Vassalou and Xing	Default risk as FF3, but not reject FF3
2005	Zhang	Value premium
2006	Petkova	ICAPM idea, reject FF3
2008	Cooper et al.	Asset Growth and negative returns
2010	Wu, Zhang, Zhang	Add investment factor to traditional model to explain accruals
2011	Chen, Novy-Marx, Zhang	Alternative 3-factor q-factor model: Market, I/A, and ROE
2013	Belo et al.	A supply model to compare between actual Tobin's q and predicted q
2015	Hou, Xue, Zhang	A 4-factor q-factor model: Market, Size, I/A, ROE
2015	Fama and French	5-factor model: add RMW and CMA

#Red ink means investment-related models.

A short review of CAPM and factor model

Classical CAPM

Markowitz (1958) laid a very first groundwork for CAPM. He proposes that investors would optimally hold a mean-variance frontier portfolio: a portfolio gives the highest expected return for a given level of variance. Sharpe (1964) and Lintner (1965) develope this idea that if investors are homogeneous expectations and optimally hold mean-variance frontier portfolios, then the market portfolio will be itself a mean-variance portfolio. The Sharpe-Lintner CAPM model is:

$$
\begin{aligned}
E[R_i] &= R_f + \beta_{im}\left(E[R_m] - R_f\right) \\
\beta_{im} &= cov\left(R_i, R_m\right)/var(R_m)
\end{aligned}
$$

where the R_m is return on market portfolio, R_f is risk-free rate. The model could be written in excess return too:

$$E[R_i - R_f] = \beta_{im}\left(E[R_m] - R_f\right)$$

Black (1972) propose a more general Black-version CAPM:

$$E[R_i] = R_{0m} + \beta_{im}\left(E[R_m] - E[R_{0m}]\right)$$

where R_{0m} is return on zero-beta portfolio associated with m.

Early empirical evidence

Early evidence such as Black, Jensen, Scholes (1972) and Fama and Macbeth (1973) support the CAPM. Using time-series regression approach for 10 beta-sort portfolios, Black, Jensen, Scholes (1972) support the CAPM. Fama and Macbeth (1973) use a cross-section method to test the CAPM. Their method includes two main steps:

1. For each t in T periods, assuming beta is known, run a cross-section regression: $E[R_i - R_f] = \gamma_{0t} + \gamma_{1t}\beta_{im}$

2. Given the time series of estimates γ_{0t} and γ_{1t}, use an usual t-test that $\gamma_0 = 0$ and $\gamma_1 > 0$

$$t - statistic = \frac{\bar{\hat{\gamma}}_j}{s(\hat{\gamma}_j)}$$

Again, their evidence supports the CAPM.

Anomalies

However, CAPM fails to explain stock returns of some portfolio formed from specific factors. First, Fama and French (1992) find that beta tends to be related to size, small stocks earn higher returns than large stocks. Thus, they control for size when forming the beta. Then, applying the Fama-Macbeth procedure, they find that beta cound not explain the cross-section returns, even when it is the only explanatory variable. However, size and ratio of book equity to market equity (B/M) have significant power to exlain stock returns. Other factors (such as leverage, earning E/P) fail to explain stock returns, after controlling size and B/M.

Debondt and Thaler (1985) find a long-term reversal in stock returns, while Jegadeesh and Titman (1993) find a short-term momentum. Momentum effect refers to a portfolio formed by buying past winners and selling past losers has a higher average return than the CAPM predicts. Lakonishok, Shleifer, and Vishny (1994) find the value premium that value stocks outperform the growth stocks. Value stocks refer to stock with low price relative to fundamental value such as high B/M, E/P, C/P (cash), and low SG (sale growth). Thus, a contrarian strategy that sells growth stocks and buys value stocks could earn positive returns.

Factor models

Fama and French (1993) show 5 common risk factors to explain equity return and bond return. For stock returns, they propose three factors: market excess return, size, and B/M. For government and corporate bond return, they propose two additional factors for maturity spread (TERM) and default spread (DEF). Using a time-series approach as in Black, Jensen, Scholes (1972), they find that these mimicking factors formed based on these factors can capture stock return and bond return.

Fama and French (1996) again use the 3 factor model (hereafter FF3) includes market excess return, size, and B/M to explain anomalies in literature.

$$R_i - R_f = a_i + b_i(R_m - R_f) + s_i SMB + h_i HML$$

where SMB is small-minus-big mimicking portfolio return, HML is high-minus-low.

The idea is if the FF3 can explain the portfolio return of anomalies, the intercept will be close to zero. The FF3 successfully explain size, B/M, and long-term reversal anomalies, but fail to explain the momentum. Thus, Carhart (1997) add momentum and construct a 4-factor Carhart model:

$$R_i - R_f = a_i + b_i(R_m - R_f) + s_i SMB + h_i HML + u_i UMD$$

where UMD is winner-minus-loser.

Conditional CAPM

While traditional CAPM assumes a static beta, Jagannathan and Wang (1996) relax two assumptions: (i) allow beta and market premium depend on a set of information at specific time point and vary over time and (ii) add an additional human capital return together with market return. Their model is as follows:

$$E[R_i] = c_0 + c_{size}ln(ME) + c_{vw}\beta^{vw} + c_{prem}\beta^{prem} + c_{labor}\beta^{labor}$$

where $\beta^{vw} = cov(R_i, R_{vw})/var(R_{vw})$ and R_{vw} is value-weighted market return;

$\beta^{prem} = cov(R_i, R_{prem})/var(R_{prem})$ and R_{prem} is defaul spread between BAA and AAA rated bonds;

$\beta^{labor} = cov(R_i, R_{labor})/var(R_{labor})$ and R_{labor} is growth rate in per capital labor income.

Next question is which factors that FF3 proxies for? This question is quite interesting because in Fama and French (1993), they propose that SMB and HML proxy for state variables that describe time variation in the investment opportunity set (quite similar to ICAPM). Fama and French (1996) argue that SMB and HML might proxy for financial distress.

APT and ICAPM

Ross (1976) propose the APT model that its factors f account for the common variation in asset returns so that the disturbance term e for large well-diversified portfolios vanishes. Thus, there is a SDF $m = a + b'f$ which linear in f and prices the returns. Chen, Roll, and Ross (1986) use financial theory to add some factors to model: maturity spread, inflation, industrial production, default spread. The results show that some notable factors affect stock returns include: market return, industrial production growth, unanticipated inflation, change in risk premium, yield curve.

Vassalou and Xing (2004) use default risk to check whether FF3's factors proxy for default risk. They find that size effect and B/M effect exist only in subsample of stocks with high default risk. In addition, the stock with high default risk earn higher stock return than those with low default risk only in subsample of small stock or high B/M ratio. They conclude that FF3 contain some default-related information but they don't reject the FF3. They argue that FF3 contain other important priced information that unrelated to default risk.

Merton (1973) proposes the ICAPM model that is a multifactor model that includes current wealth (market return) and other factors explain the future investment opportunities. Petkova (2006) applies this idea to extract the innovations u^K from state variables that describible the yield curve (risk free and TERM) and conditional distribution of asset returns (interest rate, aggregate dividend yield DIV, and default spread DEF). Then she propose an ICAPM:

$$R_{it} = a_i + \beta_{im} R_{mt} + \sum(\beta_{i,u^k}) u_t^k + e_{it}$$

and use this model to compare with FF3. She proves that her model perform better than the conventional FF3 model.

A recent 5-factor model

As an attempt to fix their FF3 that fails to explain a bunch of anomalies, Fama and French (2015) finally add two additional factor profitability and investment to their old FF3.

$$R_i - R_f = a_i + b_i(R_m - R_f) + s_i SMB + h_i HML + r_i RMW + c_i CMA$$

where RMW is robust-minus-weak (profitability) and CMA is conservative-minus-aggressive (investment) factors.