Some notes on empirical asset pricing method

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1 Fama Macbeth and Fama French

The cross-section of expected stock returns Fama and French (1992) check the validity of CAPM. They reject *beta* but support *size* and *BM*. Their method follows a Fama-Macbeth procedure as follows:

1. Sort *size*: In June of year t, sort stocks into 10 *Size* portfolios using NYSE decile breakpoints[1]
2. Sort *beta*: To allow *beta* unrelated to *size*: independently sort stocks into another 10 pre-ranking *beta* portfolios (from time-series regression of each stock), using only NYSE deciles, so we have 100 $size - beta$ portfolios
3. Estimate portolio *beta*:
   (a) Calculate post-ranking *equal-weighted monthly returns* R_i^p of portfolios for next 12 months (July year t to June year $t+1$). The sample time period is 330 months from July 1963 to December 1990 on 100 portfolios.
   (b) Model to estimate portfolio *beta*, run time-series regression: $R_{it}^p = \alpha_i^p + \beta_{i1}^p R_t^M + \beta_{i2}^p R_{t-1}^M + e_p$ where i is portfolio i, R^M is market return from value-weighted of NYSE, AMEX, and NASDAQ (after 1972) stocks.
   (c) Totally, we run 100 regressions for 100 portfolios, each has 330 observation. We get estimated $\hat{\beta}_i^p = \hat{\beta}_{i1}^p + \hat{\beta}_{i2}^p$ for each portfolio (sum of slopes of current and prior month's market return).
4. Cross-section regression: For next one year (testing period), run cross-section regression for each month:
   (a) Model: $R_{pi} = \gamma_0 + \gamma_1 \hat{\beta}_i^p + \gamma_2 ln(ME)_i^p + \epsilon_i$, where *size* of portfolio is average size of a portfolio (time series average of monthly averages of $ln(ME)$ for *stocks in portfolio at the end of June* each year; hints: only take stocks at June than average all firms in one year).
   (b) Other models add other characteristics such as B/M or E/P: $R_{pi} = \gamma_0 + \gamma_1 \hat{\beta}_i^p + \gamma_2 ln(ME)_i^p + \gamma_3 ln(BE/ME) + \epsilon_i$
   (c) From that, we get a time series of estimated $\hat{\gamma}_1$, $\hat{\gamma}_2$, $\hat{\gamma}_3$...
5. t-test: $t\left(\bar{\hat{\gamma}}_j\right) = \frac{\bar{\hat{\gamma}}_j}{s(\hat{\gamma}_j)/\sqrt{n}}$ then test whether estimated of $\hat{\gamma}_1$, $\hat{\gamma}_2$, $\hat{\gamma}_3$ equals zero.

The results of Fama-Macbeth regression show that:

- When measured it unrelated to size, β is not related to stock returns, even when it's the only explanatory variable.

[1] Reason why use NYSE stocks to sort: to avoid dominance of small stocks from Nasdaq are added into the market after 1973 (and Amex stocks since 1963).

- E/P effect weakens after we control *size* and B/M
- Size and B/M capture the cross-sectional variation of stock returns, and they reflect the multidimensional risks in asset pricing
- What factors they proxy for? Distress risk or investment?

Multifactor explanations of asset pricing anomalies The main finding of Fama and French (1996) is they propose 3-factor model ($FF3$) to solve most of anomalies *except for momentum.* The existing anomalies are: long-term reversal (DeBondt and Thaler, 1985), short-term momentum (Jegadeesh and Titman, 1993), size, B/M (Lakonishok, Shleifer, and Vishny, 1994), which cannot be explained by CAPM.

Form factors:

- At the end of June in year t, independently sort median ME of NYSE stocks and 3 groups of BE/ME (30-40-30%) of NYSE stocks. We got 6 portfolios.
- Value-weighted monthly returns on portfolios are calculated from July year t to June year $t+1$
- Two factors: $SMB = \frac{SL+SM+SH}{3} - \frac{BL+BM+BH}{3}$; $HML = \frac{SH+BH}{2} - \frac{SL+BL}{2}$

Three-factor models

$$R_i - R_f = a_i + b_i(R_m - R_f) + s_i SMB + h_i HML + e$$

If the three factors can proxy for all risk exposure to unobservable factors, we should see the intercepts are close to zero.

Fama and French often analyze by using two methods:

- Two-way sort
- Run factor model regression of 10 deciles (sort based on the anomalies) then test GRS that all intercepts are zero. If we accept the null, meaning that FF3 could explain the anomaly because all intercepts are zero. For example, if they test B/M, they first sort sample to 10 deciles of B/M then run factor models for 10 portfolios. Then, they test all 10 intercepts are zero by GRS.

GRS test is the F-statistic of Gibbons, Ross, Ahanken (1989) The null hypothesis is all intercepts of regressions are all zero. If the 3-factor model could explain the anomaly, all intercepts seem to be zero, so if we see GRS test accepts the null hypothesis, it means that the model could explain the anomaly.

The evidence shows that GRS test could not reject null in long-term reversal an B/M anomalies, but reject the momentum effect, meaning that the model could not explain the momentum anomaly.

Fama and French (1996) show that the model fails to explain momentum because the short-term losers' returns load more on SMB and HML (behave more like small distressed stocks) than winners. Thus, the model predicts a reversal for the short-term losers (or the model can explain a mean-reversal) but fails to explain the observed continuation. This explanation seems to be a dumb one, I think.

2 Value-premium

Classic explanation: extrapolation (overreaction) and risk Lakonishok et al. (1994) propose two contrasting hypotheses to explain why value stocks earn abnormal returns:

1. Overreaction/extrapolation: investors seem to overreact to good news and believe that past successful perfromance will continue
2. Risk: value stocks have higher fundamental risks, so they have higher returns

The evidence supports the behavioral view. Contrarian strategy (buy value, sell glamour

Their method

- Proxies of value stocks: BE/ME; C/P (cash/price), E/P (earning/price), and GS (growth in sale). For GS, remember that low GS is value stocks and high GS is glamour stocks.
- One-way sort: sort stocks to 10 groups of value measures (e.g., B/M) then take value-weighted returns for each groups. Then they compare between V-G (value-glamour) between high B/M (value) and low B/M (glamour). The results show that value stocks outperform glamour stocks.
- Two-way sort: sort B/M and GS. A high B/M and low GS is most value stocks, while a low B/M and high GS is the most glamour stocks. Results show that the most value stocks earn highest returns, while the most glamour stocks earn lowest returns.
- Extrapolation test:
  - They want to test whether the past successful performance will persist in the future
  - They compare the past- and future- performance between glamour stocks (BM1, CP1, or GS3) and value stocks (BM10, CP3, GS1)
  - In the past (-5;0): glamour stocks have higher fundamental ratio (E/P, C/P, B/M) than value stocks
  - In the future (0;5): all tend to reverse, glamour stocks underperform than value stocks
  - It means that the past succeed will not continue in the future
- Whether contrarian strategy is riskier?
  - Whether the contrarian strategy perform worse during recession? No. $Value - Glamour$ return seem to positive in most of the time. In all recession year (NBER) and declined year (equal-weighted CRSP return declines), V-G always positive.
  - When the split sample based on market performance and re-apply two-way sorting of B/M (or C/P, E/P) and GS, the V-G is again positive in all sub-sample
  - They also first one-way sort and two-way sort to define the value and glamour stocks, then they compare risks such as *beta* and standard deviation (rather than portfolio returns as in main test). The contrarian strategy is not more risks than the naive strategy (or extrapolation strategy).

3 Spanning test

The idea of spanning test is we want to test whether adding a new set K of risky assets to an existing set N could enlarge the mean-variance frontier.

Consider K basis assets with returns R_{1t} and N test assets with returns R_{2t}.

$$
\begin{aligned}
R_t &= \left[R_{1t}^{'}, R_{2t}^{'}\right] \, and \, \mu = E[R_t] = \left[\begin{array}{c} \mu_1 \\ \mu_2 \end{array}\right] \\
V &= Var[R_t] = \left[\begin{array}{cc} V_{11} & V_{12} \\ V_{21} & V_{22} \end{array}\right]
\end{aligned}
$$

Project R_{2t} on R_{1t} we have to run regression: $R_{2t} = \alpha + \beta R_{1t} + e_t$
then $\alpha = \mu_2 - \beta\mu_1$ and $\beta = V_{21}V_{11}^{-1}\Sigma = V_{22} - V_{21}V_{11}^{-1}V_{12}$
The null hypothesis:

$$H_0 : \alpha = 0_N \, and \, \delta = 1_N - \beta 1_K = 0_N$$

Intuitively, if the null holds, the basis assets K have same mean as every test assets N, but with smaller variance.

The mean-variance spanning test is actually the test of whether N test assets have zero weight in these two portfolios that lie on the minimum-variance frontier. By *two fund separation*, if the above holds, every portfolio on the minimum-variance frontier will have zero weight on N test assets, i.e., no expansion by adding the N test assets into K basis assets.

The test statistics:

$$
\begin{aligned}
W &= T(\lambda_1 + \lambda_2) \sim \chi^2_{2N} \\
LM &= T\left(\frac{\lambda_1}{1+\lambda_1} + \frac{\lambda_2}{1+\lambda_2}\right) \sim \chi^2_{2N}
\end{aligned}
$$

where λ_1 and λ_2 are eigenvalues of $\hat{H}\hat{G}^{-1}$.

3.1 Application 1: International Diversification from Domestic assets

The idea in **Erruza, Hogan, Hung (1999)** is that if we hold a home-made diversification from US assets (domestic assets), adding a foreign asset (foreign market indices) does not add benefits to the domestic assets.

We have:

- Basis/benchmark assets: home-made (i.e., domestic traded) Diversification portfolios using US market indices, 12 US industry portfolios, 30 multinational corporations (MNCs), close-end country funds (CFs), and American Depositary Recepts (ADRs).
- Test assets: 7 developed markets (DMs) and 9 emerging markets (EMs)

To select the basis assets ("benchmark assets" in paper), they propose some restrictions when select the assets:

1. Limit the number of assets in each set to 4
2. Choose 4 assets that maximize the probability of not rejecting spanning (null hypothesis)

The regression of the test:

$$R_{I,t} = \alpha_i + \beta_1 R_{e1,t} + \beta_2 R_{e2,t} + \beta_3 R_{e3,t} + \beta_4 R_{e4,t} + e_{I,t}$$

where R_I is return on Ith foreign index, R_e is 4 benchmark assets.

The null hypothesis is:

$$\begin{aligned} \alpha_i &= 0 \\ \sum_{i=1}^{4} \beta_i &= 1 \end{aligned}$$

They call the spanning test here is Huberman-Kandel test.

- The higher $p-value$, we cannot reject null hypothesis. It means that adding the foreign assets does not enhance the benefits to the existing home-made diversification portfolio.

- However, it the $p-value$ is small and we could reject the null hypothesis. It means that adding the foreign assets improve the benefits to the existing home-made diversification portfolio.

Moreover, while spanning test checks the statistical significance, the economic significance could be tested from the shift in Sharpe ratio. The idea is if the spanning test is rejected, but the Sharpe ratio increases too small, it will not economic significance. Bekaert and Urias use Monte Carlo simulation and find that changes in Sharpe ratio of less than 0.057 are not economic significant.

If applied this cut-points, only adding Chile and Thailand indices to domestic traded diversification portfolio could bring benefits. Based on this result, Erruza, Hogan, Hung (1999) conclude that the economic gains from international diversification that cannot be obtained with domestically traded securities, with few exceptions, are minimal.

3.2 Application 2: Diversification from Small-Caps

Eun, Huang, Lai (2008) propose that adding the small-cap funds could benefits the diversification portfolios of 10 MSCI indices. In other words, they want to test if small-cap funds (CBF) can be spanned by MSCI country indices.

- Basis assets: 10 MSCI indices

- Test assets: small-cap funds from countries

The model:

$$R_i = \alpha_i + \beta_i^{AU} MSCI^{AU} + ... + \beta_i^{US} MSCI^{US} + e_i$$

where R_i is return on the small-cap fund from i th country, $MSCI^{AU}$ denotes the return from MSCI Australia country index, $MSCI^{US}$ for US, and so on.

The null hypothesis:

$$\begin{aligned} \alpha_i &= 0 \\ \sum_{i=1}^{10} \beta_i &= 1 \end{aligned}$$

The results show that almost in all case, the spanning test is rejected at conventional significance levels. It means that small-ap funds are indeed unique and investors may benefit from adding small-cap funds to their portfolio of country indices.

4 Mean-variance intersection test

Using the same setting as in Spanning test, the intersection is when the original mean-variance frontier and the new mean-variance frontier have only **one point** in common. The null is:

$$H_0 : \alpha - \eta(1 - \beta 1_K) = 0$$

where η is risk-free rate.

The idea here is whether the new mean-variance frontier has *higher maximum attainable Sharpe ratio (or enhance the mean-variance efficiency)* than the original mean-variance frontier.

The test statistic is: $F = T\frac{\hat{\theta}_2(\eta)^2 - \hat{\theta}_1(\eta)^2}{1+\hat{\theta}_1(\eta)^2}$

where T is number of observations, $\hat{\theta}_1$ and $\hat{\theta}_2$ are maximum Sharpe ratios attainable by benchmark assets (original frontier) and benchmark plus test assets (new frontier).

The null hypothesis is maximum Sharpe ratio attainable with new assets is the same as that attainable with the benchmark assets. Thus, if we could reject the null, it means that adding new assets tends to enhance the maximum Sharpe ratio of the portfolio of original assets.

4.1 Application in Small-Caps

In Eun, Huang, Lai (2008), they use the mean-variance intersection test to check whether adding small-cap funds could help to enhance the mean-variance efficiency of a benchmark diversification portfolio. The benchmark portfolio includes MSCI country indices. While the new portfolio (augmented diversification portoflio) add the small-cap funds.

Their intersection test statistic is a little different: $F = \frac{T-(K+N)}{N}\frac{\hat{\theta}_2(\eta)^2 - \hat{\theta}_1(\eta)^2}{1+\hat{\theta}_1(\eta)^2}$

where K is number of benchmark assets (number of MSCI indices equals 10) and N is number of new assets.

The asymptotic distribution of the test statistic is:

- If short sale is allowed: $F \sim F\,dist(T-K-N, N)\,d.f$
- If short sale is *not allowed*: they use simulation to approximate the distribution

In the results, they find that the adding the small-cap funds could improve the Sharpe ratio. In every case, they can reject the null hypothesis (small p-value of F test in last two rows of Table 5) in both short sale allowed and not allowed.

References

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