Training 5 - Dynamic Programming - 20192

A. 05. GOLD MINING

1 second, 256 megabytes

The Kingdom ALPHA has n warehouses of golds located on a straight line and are numbered $1, 2, \ldots, n$. The warehouse i has amount of a_i (a_i is non-negative integer) and is located at coordinate i ($i = 1, \ldots, n$). The King of ALPHA opens a competition for hunters who are responsible to find a subset of gold warehouses having largest total amount of golds with respect to the condition that the distance between two selected warehouses must be greater than or equal to L_1 and less than or equal to L_2.

Input

- Line 1 contains n, L_1, and L_2 ($1 \leq n \leq 100000, 1 \leq L_1 \leq L_2 \leq n$)
- Line 2 contains n integers $a_1, a_2, \ldots, a_n$

Output

Contains only one single integer denoting the total amount of golds of selected warehouses.

input
```
6 2 3
3 5 9 6 7 4
```
output
```
19
```

B. NETWORKS

1 second, 256 megabytes

The network administrator of a company have to analyze the current state of their communication network all over the world. The communication network consists of servers and cable links between these servers, each link has a cost. A k-route is a sequence of $k+1$ different servers in which two consecutive servers are connected by a cable link. A cycle is a k-route (for any $k > 1$) such that the beginning and the terminating servers are connected by a cable link. The communication network contains no cycle. The cost of a k-route is the sum of cost of links between two consecutive servers of the k-route. One of the indicators of the analysis is the k-route having minimal cost of the network for a given value of k.

Given the communication network G and an integral value k, help the network administrator to find the k-route having minimal cost of G.

Input

The input consists of following lines

- Line 1: contains two integer n and k ($1 \leq n \leq 10000$, $1 \leq k \leq 2000$) in which n is the number of servers of the communication network G (servers are numbered 1,2,...,n)
- Line 2: contains an integer m ($1 \leq m \leq 10000$) which is the number of cable links between servers of G
- Line $i+2$: contains three integers u, v, and w: u and v are two end points of the i^{th} link of G ($\forall i = 1, \ldots, m$), w is the cost of this link.

Output

The output contains the cost of the k-route found.

input
```
6 2
5
1 2 4
1 4 1
1 5 3
1 6 2
2 3 1
```
output
```
3
```

C. GOLD MINING (HARD)

1 second, 256 megabytes

The Kingdom ALPHA has n warehouses of golds located on a straight line and are numbered 1, 2, ..., n. The warehouse i has amount of a_i (a_i is non-negative integer) and is located at coordinate i ($\forall i = 1, \ldots, n$). The King of ALPHA opens a competition for hunters who are responsible to find a subset of gold warehouses having largest total amount of golds with respect to the condition that the distance between two selected warehouses must be greater than or equal to L_1 and less than or equal to L_2.

Input

The input consists of the following lines:

Line 1 contains n, L_1, and L_2 ($1 \leq n \leq 1000000, 1 \leq L_1 \leq L_2 \leq n$).

Line 2 contains n integers $a_1, a_2, \ldots, a_n$

Output

Contains only one single integer denoting the total amount of golds of selected warehouses.

input
```
6 2 3
3 5 9 6 7 4
```
output
```
19
```

E. 05. MACHINE

1 second, 256 megabytes

An engineer needs to schedule a machine to run on some given periods $1, \ldots, n$ to produce a chemical product C. Each period i is represented by a starting time point s_i and terminating time point t_i ($s_i < t_i$). Due to a technical constraint, the machine must run on exactly two periods that are not overlap (two periods i and j are not overlap if $t_i < s_j$ or $t_j < s_i$). If the machine is runned on the period i, then the amount of C it will produce is equal to the duration of the period i (which is equal to $t_i - s_i$). Help the engineer to select two not-overlap periods to run the machine such that the amount of C produced is maximal.

Input

- Line 1: contains the positive integer n ($2 \leq n \leq 2 \cdot 10^6$)
- Line $i+1$: contains two positive integer s_i and t_i ($1 \leq s_i < t_i \leq 2 \cdot 10^6$)

Output

The output consists of only one single integer which is the amount of product C the machine will produce in the two selected periods. In case there is no solution (there does not exist two periods that are not overlap), the output contains the value -1.

input
```
5
8 12
6 11
3 9
2 5
1 4
```
output
```
8
```

F. 05. NURSE

1 second, 256 megabytes

The director of a hospital want to schedule a working plan for a nurse in a given period of N consecutive days 1,..., N. Due to the policy of the hospital, each nurse cannot work all the days 1,..., N. Instead, there must be days off in which the nurse need to take a rest. A working plan is a sequence of disjoint working periods. A working period of a nurse is defined to be a sequence of consecutive days on which the nurse must work and the length of the working period is the number of consecutive days of that working period. The hospital imposes two constraints:

- Each nurse can take a rest only one day between two consecutive working periods. it means that if the nurse takes a rest today, then she has to work tomorrow (1)
- The length of each working period must be greater or equal to K_1 and less than or equal to K_2 (2)

The director of the hospital want to know how many possible working plans satisfying above constraint?

Input

The input consists of one line which contains 3 positive integers $N, K_1, K_2 (N \leq 1000, K_1 < K_2 \leq 400)$

Output

The output consists of only one single integer M modulo $10^9 + 7$ where M is the total working plans satisfying the above constraints.

input

```
6 2 3
```

output

```
4
```

G. 05. MARBLE

1 second, 256 megabytes

Phong là một nhà điêu khắc, ông có một tấm đá cẩm thạch hình chữ nhật kích thước $W \times H$. Ông ta muốn cắt tấm đá thành các miếng hình chữ nhật kích thước $W_1 \times H_1, W_2 \times H_2, \ldots, W_N \times H_N$. Ông ta muốn cắt đến tối đa các mẫu kích thước có thể. Tấm đá có những vân đá cho nên không thể xoay khi sử dụng, có nghĩa là không thể cắt ra miếng $B \times A$ thay cho miếng $A \times B$ trừ khi $A = B$. Các miếng phải được cắt tại các điểm nguyên trên hàng cột và mỗi nhát cắt phải cắt đến hết hàng hoặc hết cột. Sau khi cắt sẽ còn lại những mẫu đá còn thừa bỏ đi, nghĩa là những mẫu đá không thể cắt thành miếng kích thước cho trước nào.

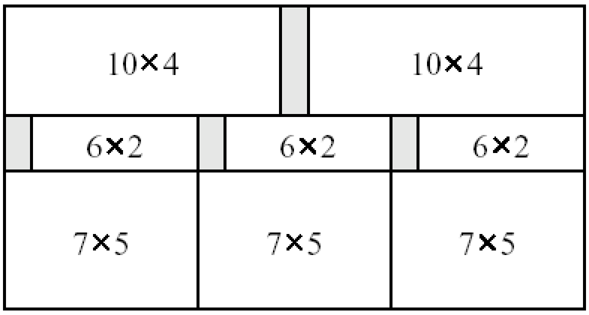
Yêu cầu: Hãy tìm cách cắt sao cho còn ít nhất diện tích đá thừa bỏ đi.

Hình dưới minh họa cách cắt các phiến đá trong ví dụ với diện tích thừa ít nhất tìm được là 10.

Famous sculptor Phong is making preparations to build a marvelous monument. For this purpose he needs rectangular marble plates of sizes $W_1 \times H_1, W_2 \times H_2, \ldots, W_N \times H_N$.

Recently, Phong has received a large rectangular marble slab. He wants to cut the slab to obtain plates of the desired sizes. Any piece of marble (the slab or the plates cut from it) can be cut either horizontally or vertically into two rectangular plates with integral widths and heights, cutting completely through that piece. This is the only way to cut pieces and pieces cannot be joined together. Since the marble has a pattern on it, the plates cannot be rotated: if Phong cuts a plate of size $A \times B$ then it cannot be used as a plate of size $B \times A$ unless $A = B$. He can make zero or more plates of each desired size. A marble plate is wasted if it is not of any of the desired sizes after all cuts are completed. Phong wonders how to cut the initial slab so that as little of it as possible will be wasted.

As an example, assume that in the figure below the width of the original slab is 21 and the height of the original slab is 11, and the desired plate sizes are $10 \times 4, 6 \times 2, 7 \times 5$, and 15×10. The minimum possible area wasted is 10, and the figure shows one sequence of cuts with total waste area of size 10.



Your task is to write a program that, given the size of the original slab and the desired plate sizes, calculates the minimum total area of the original slab that must be wasted.

Input

Dòng đầu tiên chứa hai số nguyên: W và H.

Dòng thứ hai chứa một số nguyên N. N dòng tiếp theo mỗi dòng chứa hai số nguyên W_i và H_i.

The first line of input contains two integers: first W, the width of the original slab, and then H, the height of the original slab;

The second line contains one integer N: the number of desired plate sizes. The following N lines contain the desired plate sizes. Each of these lines contains two integers: first the width W_i and then the height H_i of that desired plate size ($1 \leq i \leq N$).

Output

Kết quả ghi ra duy nhất một số nguyên là tổng diện tích nhỏ nhất các miếng thừa bỏ đi.

For each dataset, write in one line a single integer: the minimum total area of the original slab that must be wasted.

Scoring

- $1 \leq W \leq 600, 1 \leq H \leq 600, 0 < N \leq 200, 1 \leq W_i \leq W$, and $1 \leq H_i \leq H$.
- Có 50% số test ứng với $W \leq 20, H \leq 20$ và $N \leq 5$.

input

```
21 11
4
10 4
6 2
7 5
15 10
```

output

```
10
```

I. 05. TOWER

1 second, 256 megabytes

Perhaps you have heard of the legend of the Tower of Babylon. Nowadays many details of this tale have been forgotten. So now, in line with the educational nature of this contest, we will tell you the whole story:

The babylonians had n types of blocks, and an unlimited supply of blocks of each type. Each type-i block was a rectangular solid with linear dimensions (x_i, y_i, z_i). A block could be reoriented so that any two of its three dimensions determined the dimensions of the base and the other dimension was the height. They wanted to construct the tallest tower possible by stacking blocks. The problem was that, in building a tower, one block could only be placed on top of another block as long as the two base dimensions of the upper block were both strictly smaller than the corresponding base dimensions of the lower block. This meant, for example, that blocks oriented to have equal-sized bases couldn't be stacked.

Your job is to write a program that determines the height of the tallest tower the babylonians can build with a given set of blocks.

Input

The input file will contain one or more test cases. The first line of each test case contains an integer n, representing the number of different blocks in the following data set. The maximum value for n is 30. Each of the next n lines contains three integers representing the values x_i, y_i and z_i.

Input is terminated by a value of zero (0) for n.

Output

For each test case, print one line containing the case number (they are numbered sequentially starting from 1) and the height of the tallest possible tower in the format "Case *case*: maximum height = height"

input

```
1
10 20 30
2
6 8 10
5 5 5
7
1 1 1
2 2 2
3 3 3
4 4 4
5 5 5
6 6 6
7 7 7
5
31 41 59
26 53 58
97 93 23
84 62 64
33 83 27
0
```

output

```
Case 1: maximum height = 40
Case 2: maximum height = 21
Case 3: maximum height = 28
Case 4: maximum height = 342
```

J. 05. WAREHOUSE

1 second, 256 megabytes

A truck is planned to arrive at some stations among N stations $1, 2, \ldots, N$ located on a line. Station i $(i = 1, \ldots, N)$ has coordinate i and has following information

- a_i: amount of goods
- t_i: pickup time duration for taking goods

The route of the truck is a sequence of stations $x_1 < x_2 < \ldots < x_k$ ($1 \leq x_j \leq N, j = 1, \ldots, k$). Due to technical constraints, the distance between two consecutive stations that the truck arrives x_i and x_{i+1} is less than or equal to D and the total pickup time duration cannot exceed T. Find a route for the truck such that total amount of goods picked up is maximal.

Input

- Line 1: N, T, D $(1 \leq N \leq 1000, 1 \leq T \leq 100, 1 \leq D \leq 10)$
- Line 2: $a_1, \ldots, a_N (1 \leq a_i \leq 10)$
- Line 3: $t_1, \ldots, t_N (1 \leq t_i \leq 10)$

Output

Write the total amount of goods that the truck picks up in the route.

input

```
6 6 2
6 8 5 10 11 6
1 2 2 3 3 2
```

output

```
24
```

K. 05. RETAIL OUTLETS

1 second, 256 megabytes

A distribution company distribute goods to M retail outlets $1, 2, \ldots, M$. It has N branches $1, 2, \ldots, N$. Each branch i has a_i salesman. The company have to assign M retail outlets to N branches: each branch is responsible to distribute goods to some retail outlets, each retail outlet is distributed by one branch. In order to balance among salesman, the number of retail outlets assigned to each branch i must be positive and divisible by a_i. Compute the total number Q of ways of such assignment.

Example, N = 2, M = 20, a_1 = 3, a_2 = 2. There are 3 ways:

- Branch 1 is assigned to 6 retail outlets, branch 2 is assigned to 14 retail outlets
- Branch 1 is assigned to 12 retail outlets, branch 2 is assigned to 8 retail outlets
- Branch 1 is assigned to 18 retail outlets, branch 2 is assigned to 2 retail outlets

Input

- Line 1: N and M $(1 \leq N \leq 100, 1 \leq M \leq 500)$
- Line 2: N positive integers $a_1, \ldots, a_N$ ($1 \leq a_i \leq 10, i = 1, \ldots, N$)

Output

Write the value Q modulo (10^9+7)

input

```
2 20
3 2
```

output

```
3
```

L. 05. DRONE PICKUP

1 second, 256 megabytes

A Drone is planned to fly from point 1 to point N in which it can stop at some points among $1, 2, \ldots, N$. These points are located on a line, point i has coordinate i. Each point i is associated with

- c_i: amount of good
- a_i: amount of energy

When the drone stops at point i, it takes the amount c_i of good and the amount a_i of energy. After that, it can fly farthest to point $i + a_i$ (it can stops at some point among $i + 1, i + 2, \ldots, i + a_i$). Due to technical constraints, the Drone can only stop at at most $K + 1$ point. Compute the route (sequence of points at which the drone stops to take goods and energy) starting from point 1 and terminating at point N such that the total amount of goods is maximal (points 1 and N are considered as points that the Drone stop).

Input

- Line 1: N and K $(1 \leq N \leq 3000, 1 \leq K \leq 100)$
- Line 2: N positive integers $c_1, c_2, \ldots, c_N$ ($1 \leq c_i \leq 20, \forall i = 1, \ldots, N$)
- Line 3: N positive integers $a_1, a_2, \ldots, a_N$ ($1 \leq a_i \leq 50, \forall i = 1, \ldots, N$)

Output

Total amount of goods that the Drone takes during the route, or value -1 if no route exists.

input

```
6 3
3 1 4 2 2 2
2 3 1 1 3 1
```

output

```
8
```

P. INCREASE SUBSEQ

1 second, 256 megabytes

Given a sequence of integers $a_1, \ldots, a_n$. Compute the longest subsequence (elements are not necessarily continous) in which elements are in an increasing orders.

input
```
10
7 2 3 7 5 4 2 1 9 6
```
output
```
4
```

Q. LONGEST COMMON SUBSEQUENCE

1 second, 512 megabytes

A subsequence of a given sequence $S = S_1, \ldots, S_n$ is obtained by removing some elements from S. Given two sequence of integers $X = (X_1, \ldots, X_n)$ and $Y = (Y_1, \ldots, Y_m)$. Find the longest common subsequence of X and Y.

Input

- Line 1 contains n and m $(1 \leq n, m \leq 10^4)$
- Line 2 contains $X_1, \ldots, X_n$
- Line 3 contains $Y_1, \ldots, Y_m$

Output
Write the length of the longest subsequence of the given sequences

input
```
7 10
3 7 2 5 1 4 9
4 3 2 3 6 1 5 4 9 7
```
output
```
5
```

R. MAX SUBSEQUENCE

1 second, 256 megabytes

Given an integers sequence $a = (a_1, a_2, \ldots, a_n)$. A subsequence of a is defined to be $a_i, a_{i+1}, \ldots, a_j$. The weight of a subsequence is the sum of its elements. Find the subsequence having the highest weight.

Input

- Line 1 contains n $(1 \leq n \leq 10^6)$
- Line 2 contains $a_1, \ldots, a_n$ $(-500 \leq a_i \leq 500)$

Output
Write the weight of the highest subsequence found.

input
```
10
3 -1 -3 5 2 5 0 -1 5 4
```
output
```
20
```

S. RMQ

1 second, 256 megabytes

Given a sequence of n integers $a_0, \ldots, a_{n-1}$. We denote $rmq(i, j)$ the minimum element of the sequence $a_i, a_{i+1}, \ldots, a_j$. Given m pairs $(i_1, j_1), \ldots, (i_m, j_m)$, compute the sum $Q = \sum_{k=1}^{m} rmq(i_k, j_k)$

Input

- Line 1: n $(1 \leq n \leq 10^6)$
- Line 2: $a_0, \ldots, a_{n-1}$ $(1 \leq a_i \leq 10^6)$
- line 3: m $(1 \leq m \leq 10^6)$
- Line $k + 3$ $(k = 1, \ldots, m)$: i_k, j_k $(0 \leq i_k < j_k < n)$

Output
Write the value Q

input
```
16
2 4 6 1 6 8 7 3 3 5 8 9 1 2 6 4
4
1 5
0 9
1 15
6 10
```
output
```
6
```