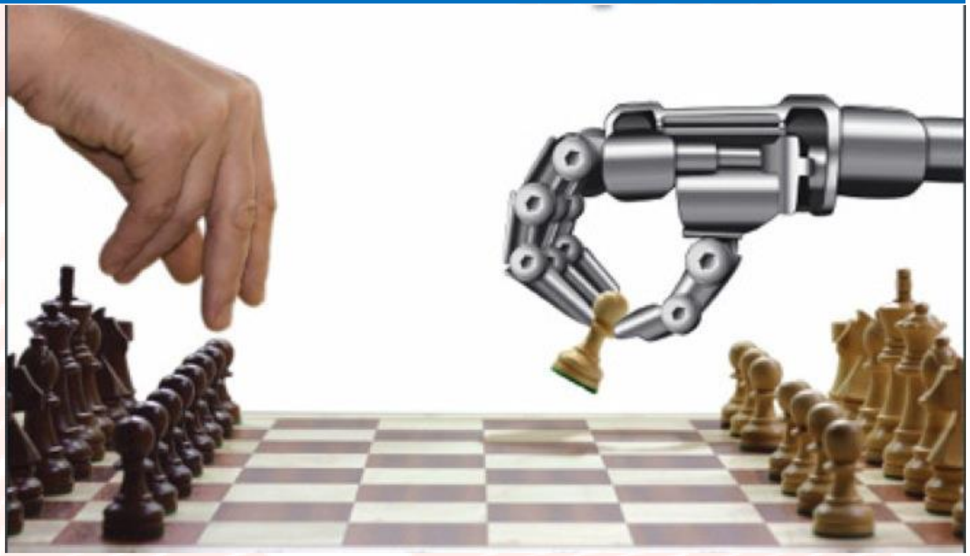
FACULTY OF INFORMATION TECHNOLOGY

Artificial Intelligence Fundamentals (NM TTNT)

Semester 1, 2023/2024

Chapter 5. Adversarial Search



Content

- What are games?
- Optimal decisions in games
  - Which strategy leads to success?
- $\alpha-\beta$ pruning
- Games of imperfect information
- Games that include an element of chance

What are and why study games?

- Games are a form of multi-agent environment
  - What do other agents do and how do they affect our success?
  - Cooperative vs. competitive multi-agent environments.
  - Competitive multi-agent environments give rise to adversarial problems a.k.a. games
- Game playing is a good problem for AI research

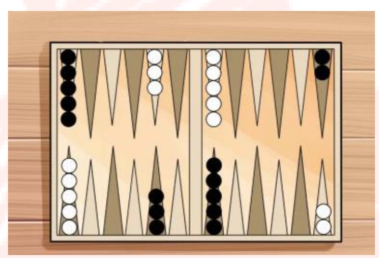
What are and why study games?

- Game playing is non-trivial
  - Players need "human-like" intelligence
  - Games can be very complex (e.g. chess, go)
  - Requires decision making within limited time
- Games usually are:
  - Well-defined and repeatable
  - Limited and accessible

Types of Games

- **Perfect Information** (fully observable)
  - **Deterministic**
    - Chess,
    - Checkers,
    - Go,
    - Othello

  - **Chance**
    - Backgammon,
    - Monopoly



Types of Games

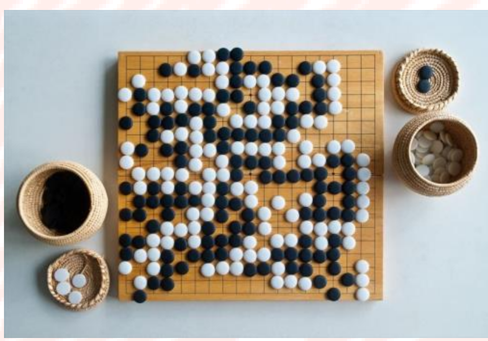
	Deterministic	Chance
Perfect Information (fully observable)	Chess, Checkers, Go, Othello	Backgammon, Monopoly
Inperfect Information (partially observable)	 	Brigde, Poker, Scrabble, Nuclear War

Types of Games

- Imperfect Information (partially observable)
  - **Deterministic**
    - Stratego
    - Battleship
  - **Chance**
    - Brigde,
    - Poker,
    - Scrabble,
    - Nuclear War

Types of Games

- This course focuses on:
  - **Perfect Information** (fully observable)
  - **Deterministic**



Relation of Games to Search

- Solution is (heuristic) method for finding goal
- Heuristics and CSP (Constraint Satisfaction Problems) techniques can find *optimal* solution
- Evaluation function: estimate of cost from start to goal through given node
- Examples: path planning, scheduling activities

Search – no adversary

- Solution is strategy (specifies move for every possible opponent reply).
- Time limits force an approximate solution
- Evaluation function: evaluate "**goodness**" of game position
- Examples: chess, checkers, Othello, backgammon

Games– adversary

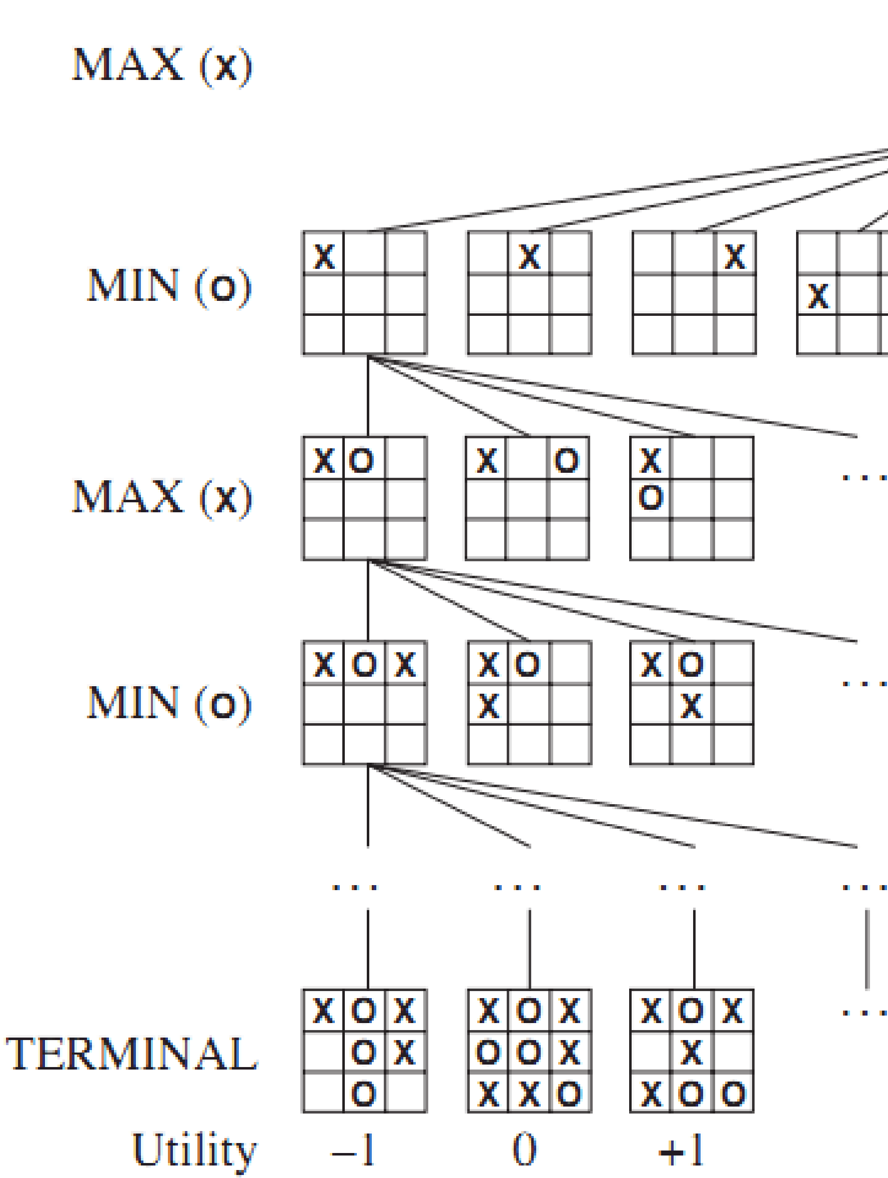
Game setup

- Two players: *MAX* and *MIN*
- *MAX* moves first and they take turns until the game is over.
  - Winner gets award,
  - Looser gets penalty.
- MAX uses search tree to determine next move.

Game Search

- Problem Formulation:
  - States: board configuration of chess
  - Successor function: legal moves a player can make.
  - Goal test: determines when the game is over.
  - initial state: start board configuration
  - Utility function: measures the outcome of the game and its desirability
- Search objective:
  - Find the sequence of player's decisions (moves) maximizing its utility
  - Consider the opponent's moves and **their utility**

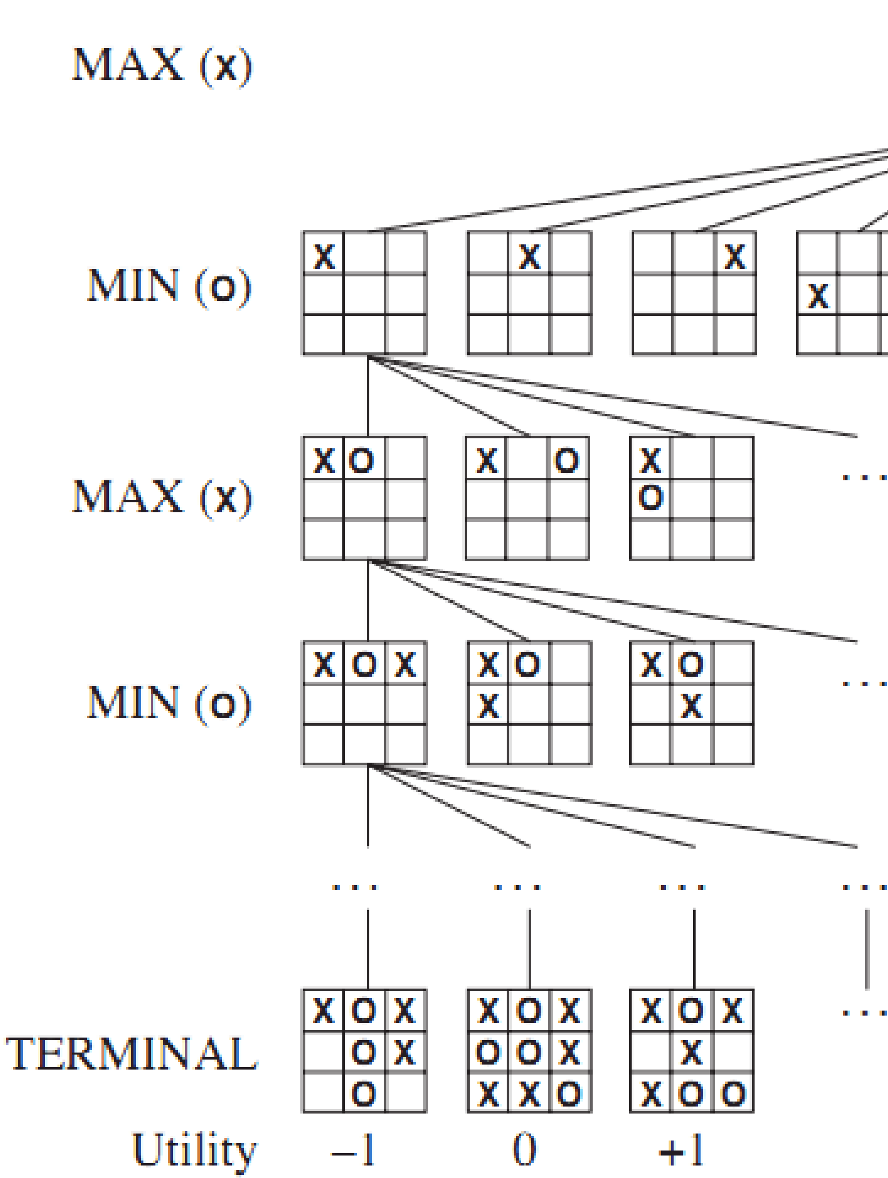
Game Tree



- MAX and MIN are the 2 players
- MAX goes first
- Players then take turns

- **MAX has 9 possible legal first moves**

Game Tree



- Utility of terminal states (when game is over) is from MAX's point of view

- Points are awarded to both players at the end of the game
  - -1 is a loss
  - 0 is a draw
  - 1 is a win

Game Playing as Search: Complexity

- Assume the opponent's moves *can* be predicted given the computer's moves.
- How **complex** would search be in this case?
  - Worst case: $O(b^d)$, branching factor, depth
  - Tic-Tac-Toe: ~5 legal moves, max of 9 moves
    - $5^9 = 1,953,125$ states
  - Chess: ~35 legal moves, ~100 moves per game
    - $35^{100} \sim 10^{154}$ states (but "only" $\sim 10^{40}$ legal states)

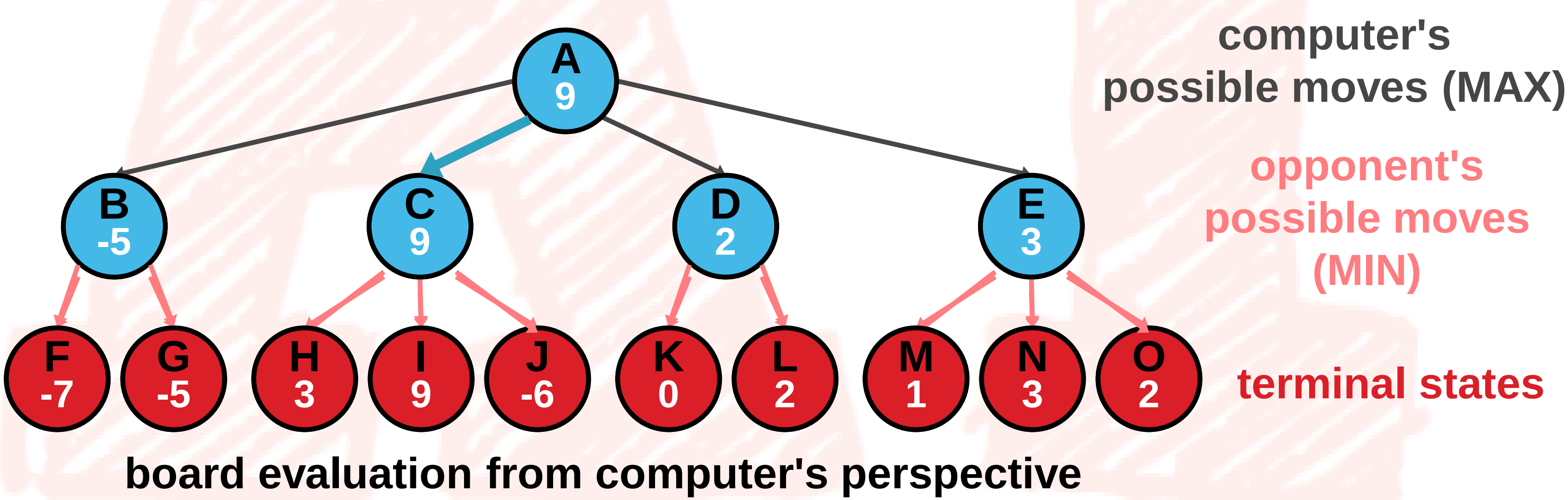
Common games produce enormous search trees!!

Greedy Search Game Playing

- A *utility* function maps each terminal state of the board to a numeric value corresponding to the value of that state to the computer.
  - positive for winning, large positive value: means better for computer (MAX)
  - negative for losing, large negative value: means better for opponent (MIN)
  - zero for a draw
  - typical values (lost to win):
    - -infinity to +infinity
    - -1.0 to +1.0

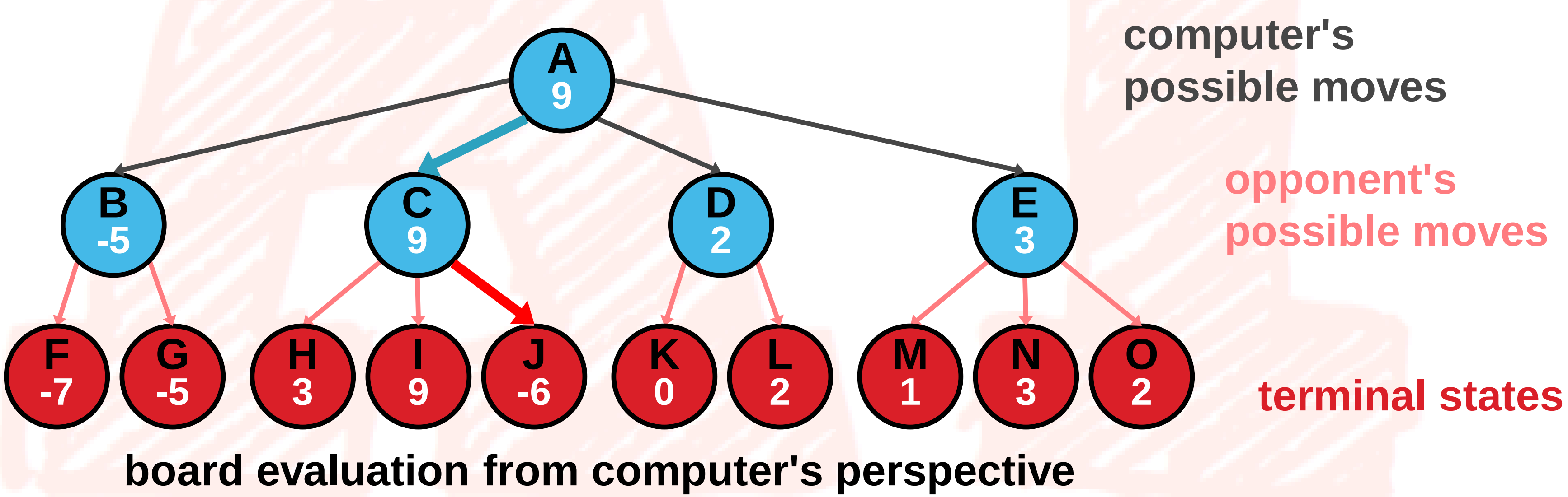
Greedy Search Game Playing

- Expand each branch to the terminal states
- Evaluate the utility of each terminal state
- Choose the move that results in the board configuration with the maximum value



Greedy Search Game Playing

- Assuming a reasonable search space, what's the problem with greedy search?
  - It ignores what the opponent might do!
  - e.g. MAX (computer) chooses C.
    MIN (opponent) chooses J and defeats computer.



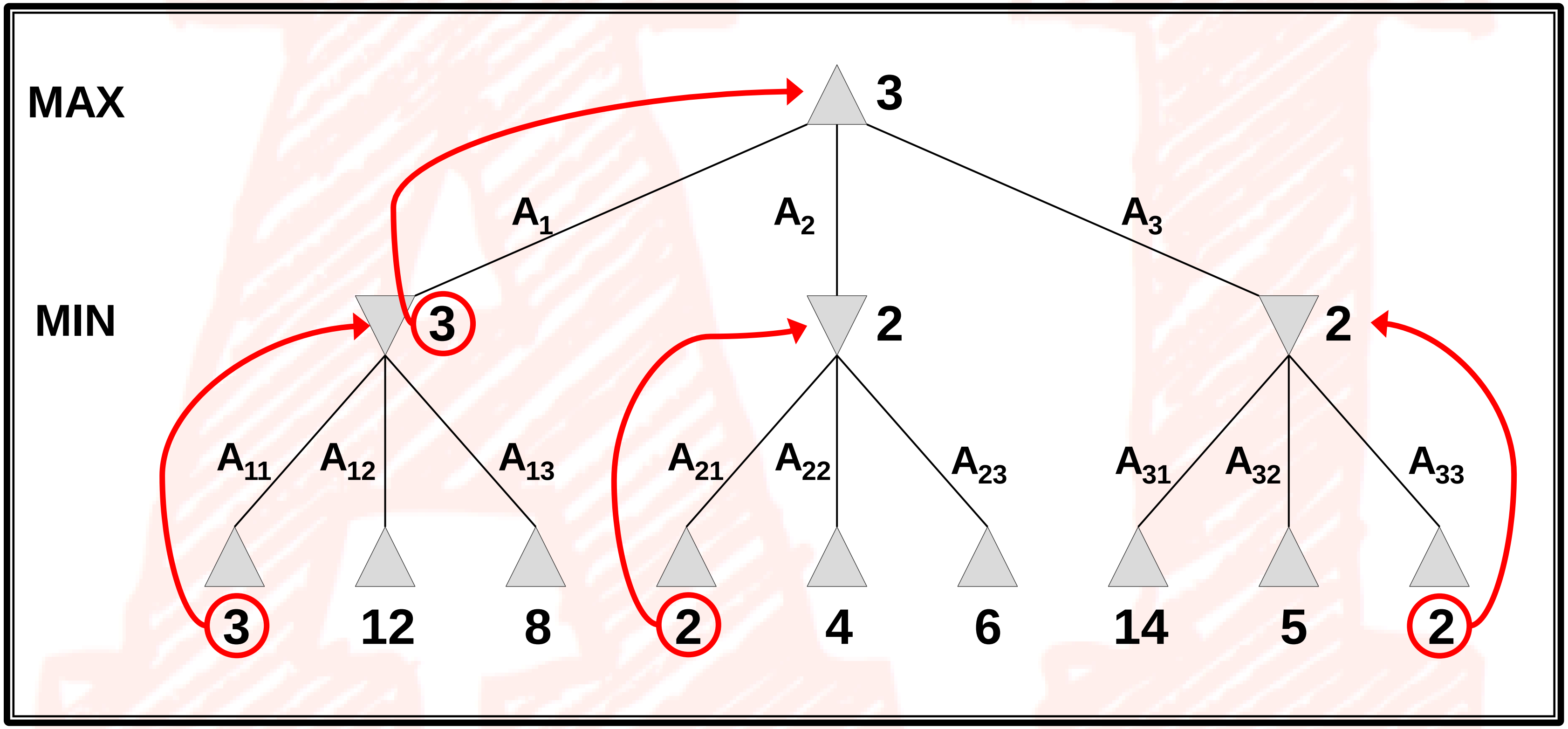
Minimax principle – Optimal strategies

- Chooses the best move considering both its move and the opponent's best move
- Assumption: Both players play optimally!!
  - **MAX** (computer) **maximizing** the utility under the assumption after it moves **MIN** (opponent) will choose the **minimizing** move.
- Given a game tree, the optimal strategy can be determined by using the minimax value of each node:

$$
\text{MINIMAX-VALUE}(n) = \begin{cases} \text{UTILITY}(n) & \text{If } n \text{ is a terminal} \\ \max_{s \in \text{successors}(n)} \text{MINIMAX-VALUE}(s), & \text{If } n \text{ is a max node} \\ \min_{s \in \text{successors}(n)} \text{MINIMAX-VALUE}(s), & \text{If } n \text{ is a min node} \end{cases}
$$

Two-Players Game Tree

The minimax decision



Minimax maximizes the worst-case outcome for max.

What if MIN does not play optimally?

- Definition of optimal play for MAX assumes MIN plays optimally: maximizes worst-case outcome for MAX.

- But if MIN does not play optimally, MAX will do even better. [Can be proved.]

Minimax: Direct Algorithm

For each move by the MAX (computer):

- Perform depth-first search to a terminal state
- Evaluate each terminal state
- Propagate upwards the minimax values
  - if opponent's move minimum value of children backed up
  - if computer's move maximum value of children backed up
- choose move with the maximum of minimax values of children
- Note:
  - minimax values gradually propagate upwards as DFS proceeds: i.e., minimax values propagate up in “left-to-right” fashion
  - minimax values for sub-tree backed up “as we go”, so only **O(bd)** nodes need to be kept in memory at any time

Minimax Algorithm

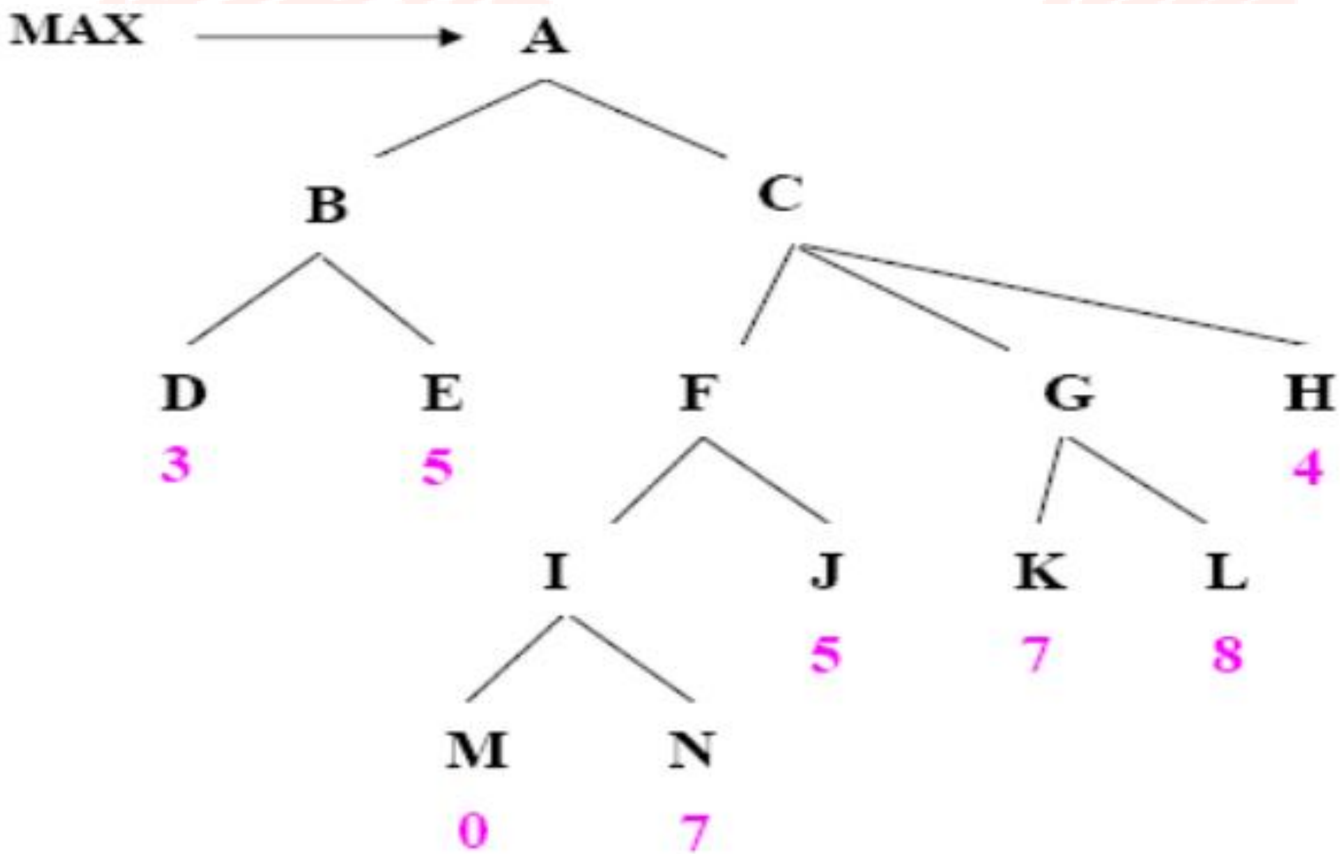
```
function MINIMAX-DECISION(state) returns an action
    inputs: state, current state in game
    v ← MAX-VALUE(state)
    return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← -∞
    for each s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s))
    return v
```

```
function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← +∞
    for each s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s))
    return v
```

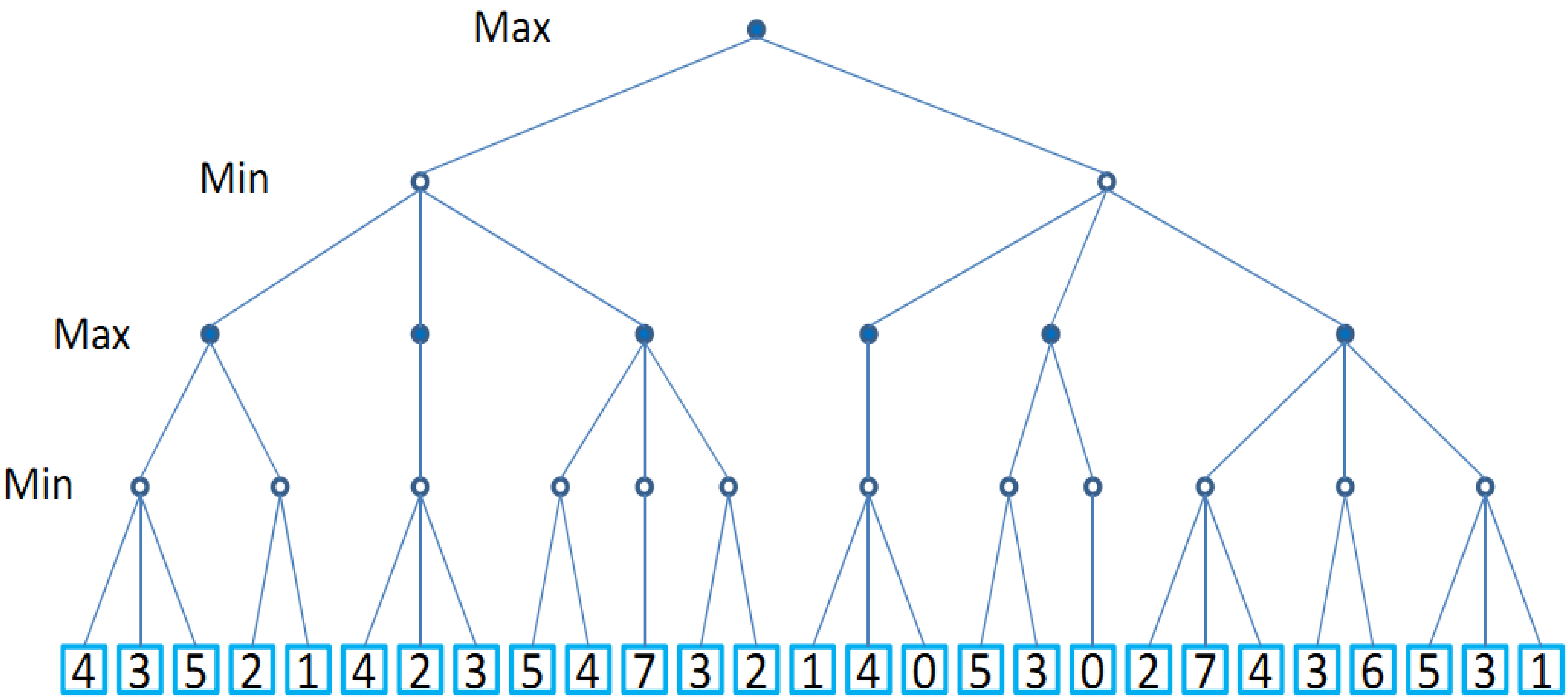
Exercise 1

- Perform the minimax algorithm on the figure below.



Exercise 2

- Perform the minimax algorithm on the figure below.



Properties of Minimax

Criterion	Minimax
Complete?	Yes (against an optimal opponent)
Time complexity	given branching factor b, $O(b^m)$
Space complexity	O(bm) (depth-first exploration)
Optimal?	Yes (if tree is finite)

- Time complexity is a major problem!
  Player typically only has a finite amount of time to make a move!!
- For chess, b ≈ 35, m ≈100 for "reasonable" games
  → exact solution completely infeasible

Problem of minimax search

- Number of games states is exponential to the number of moves.
- Some of the branches of the game tree won't be taken if playing against an intelligent opponent
- Solution: can "prune" those branches from the tree ==> Alpha-beta pruning
- While doing DFS of game tree, keep track of:
  - Alpha = Highest value found so far at any choice point along the MAX path
    - Lower bound on node's utility
  - Beta = Lowest value found so far at any choice point along the MIN path
    - Higher bound on node's utility

Alpha–Beta pruning

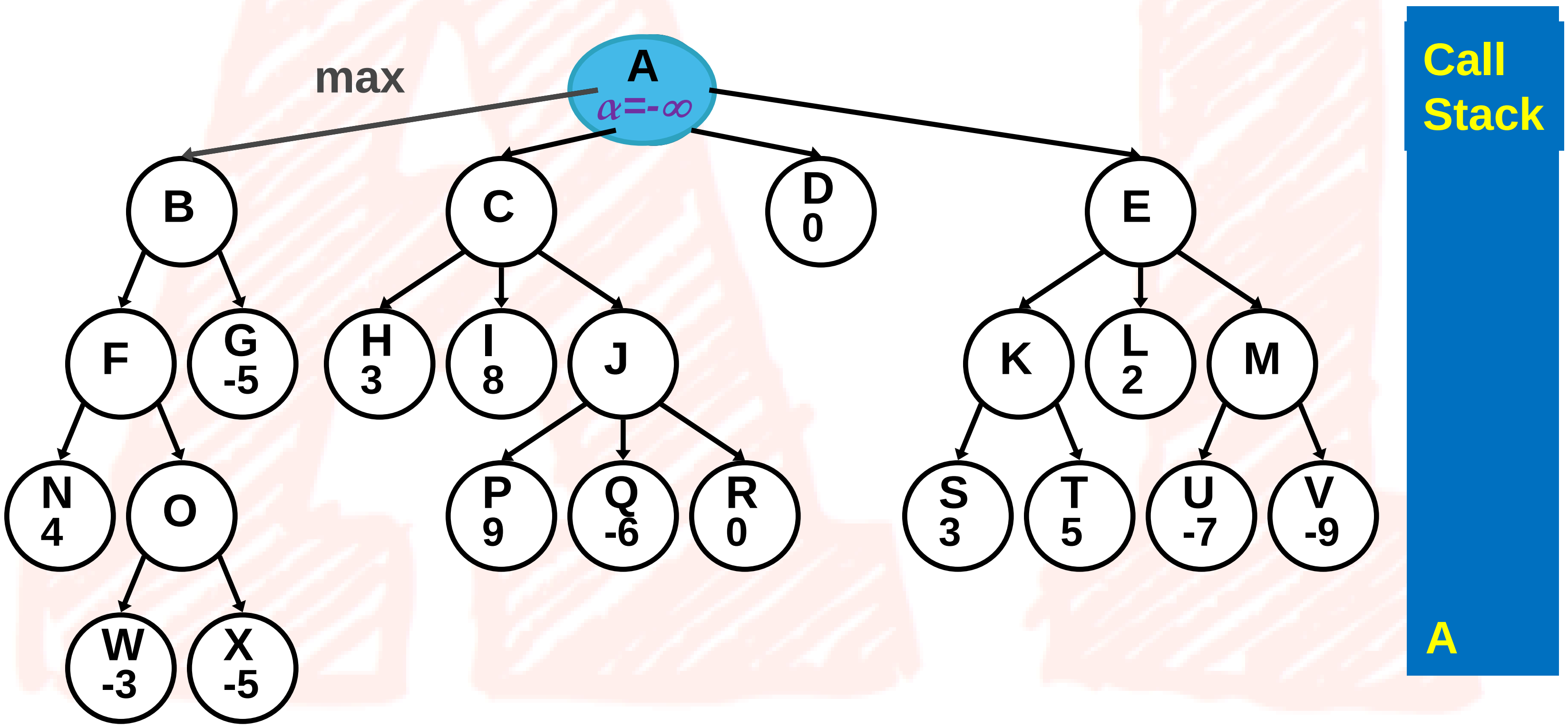
- Beta cutoff pruning
  occurs when maximizing (**MAX's turn**):
  - If **alpha ≥ parent's beta**, stop expanding
  - Why stop expanding children?
    - Opponent shouldn't allow the MAX to make this move

- Alpha cutoff pruning
  occurs when minimizing (**MIN's turn**):
  - If **beta ≤ parent's alpha**, stop expanding
  - Why stop expanding children?
    - MAX shouldn't take this route

Alpha–Beta Search Example

`minimax(A,0,4)` alpha initialized to -infinity

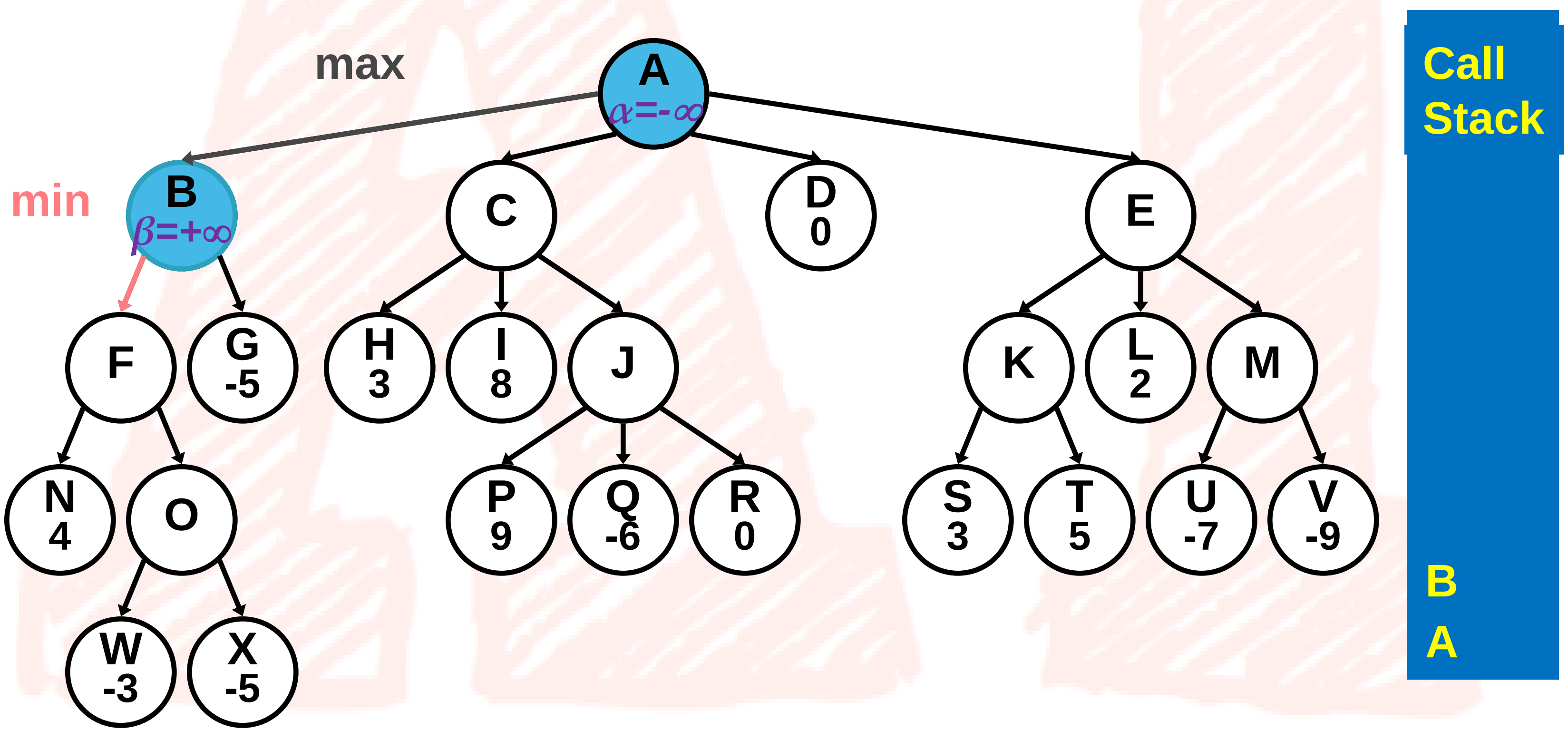
Expand A? **Yes** since there are successors, no cutoff test for root



Alpha-Beta Search Example

`minimax(B,1,4)` beta initialized to +infinity

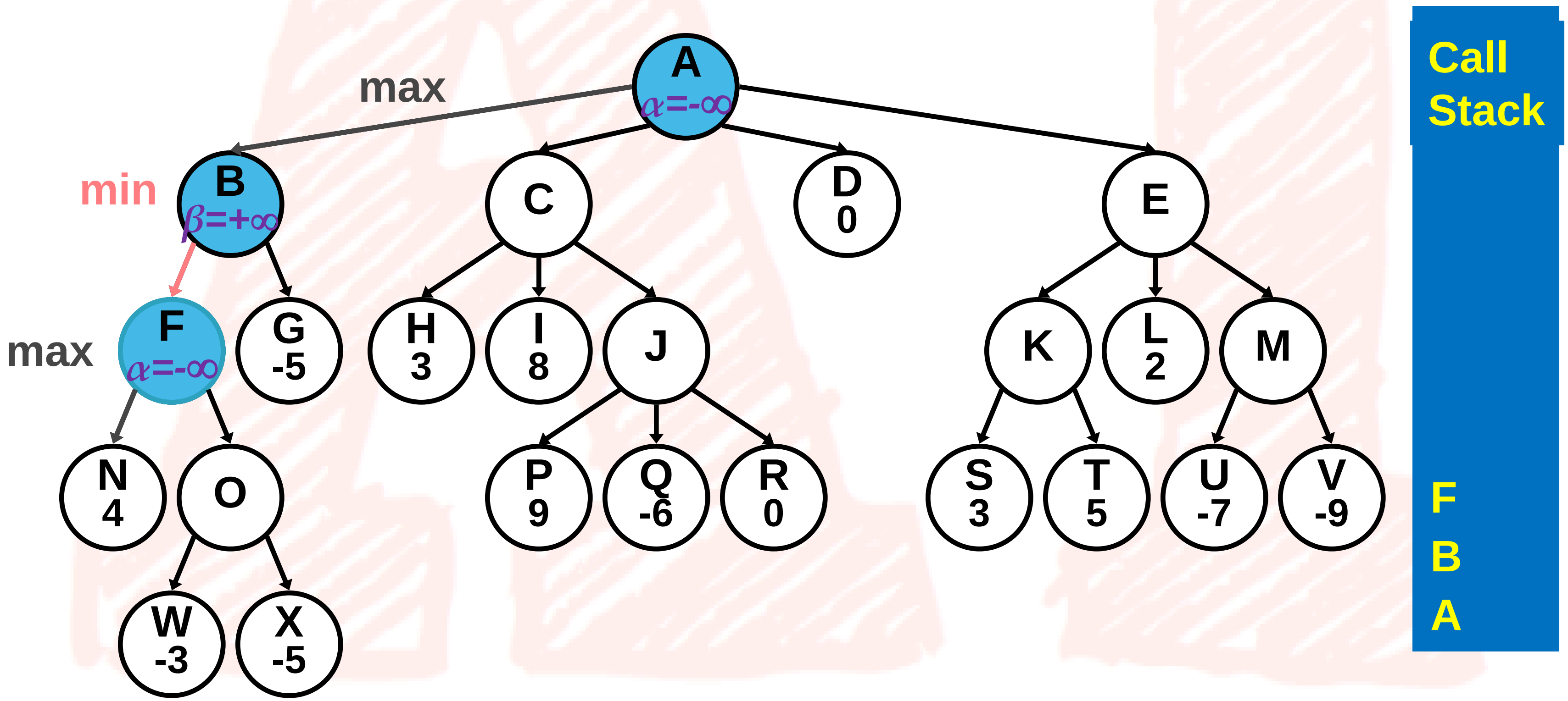
Expand B? **Yes** since A's alpha >= B's beta is false, no alpha cutoff



Alpha-Beta Search Example

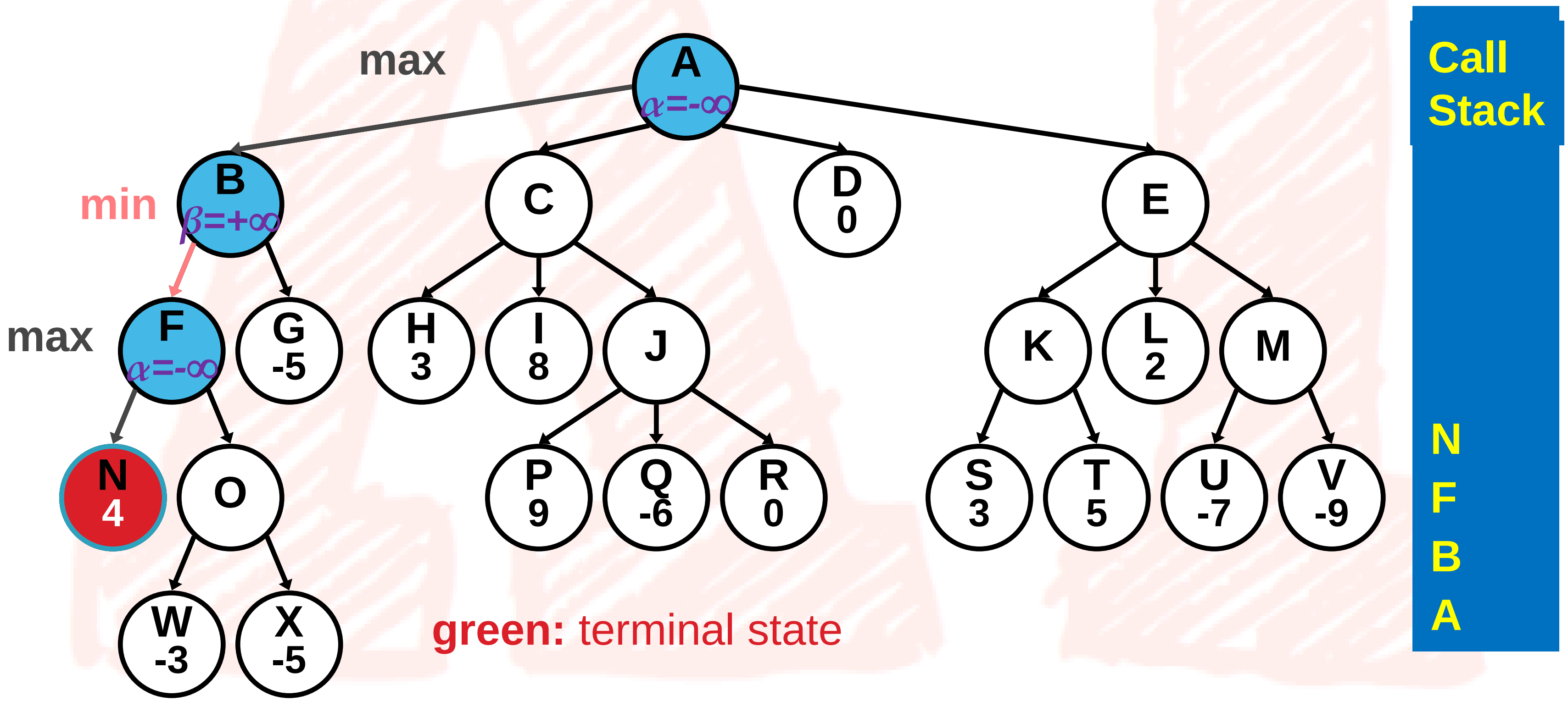
`minimax(F,2,4)` alpha initialized to -infinity

Expand F? **Yes** since F's alpha >= B's beta is false, no beta cutoff



Alpha-Beta Search Example

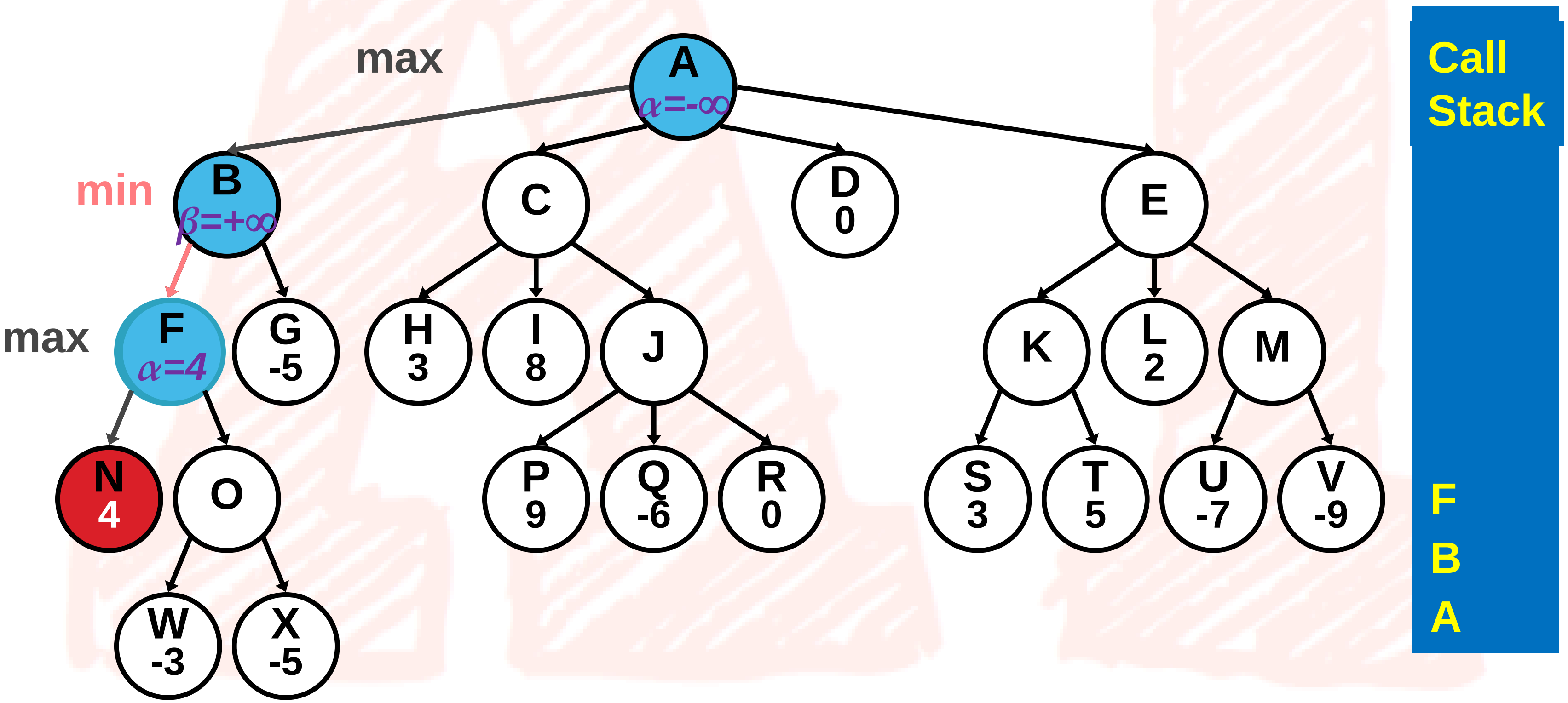
`minimax(N,3,4)` evaluate and return SBE value



Alpha-Beta Search Example

back to alpha = 4, since 4 >= -infinity (maximizing)
`minimax(F,2,4)`

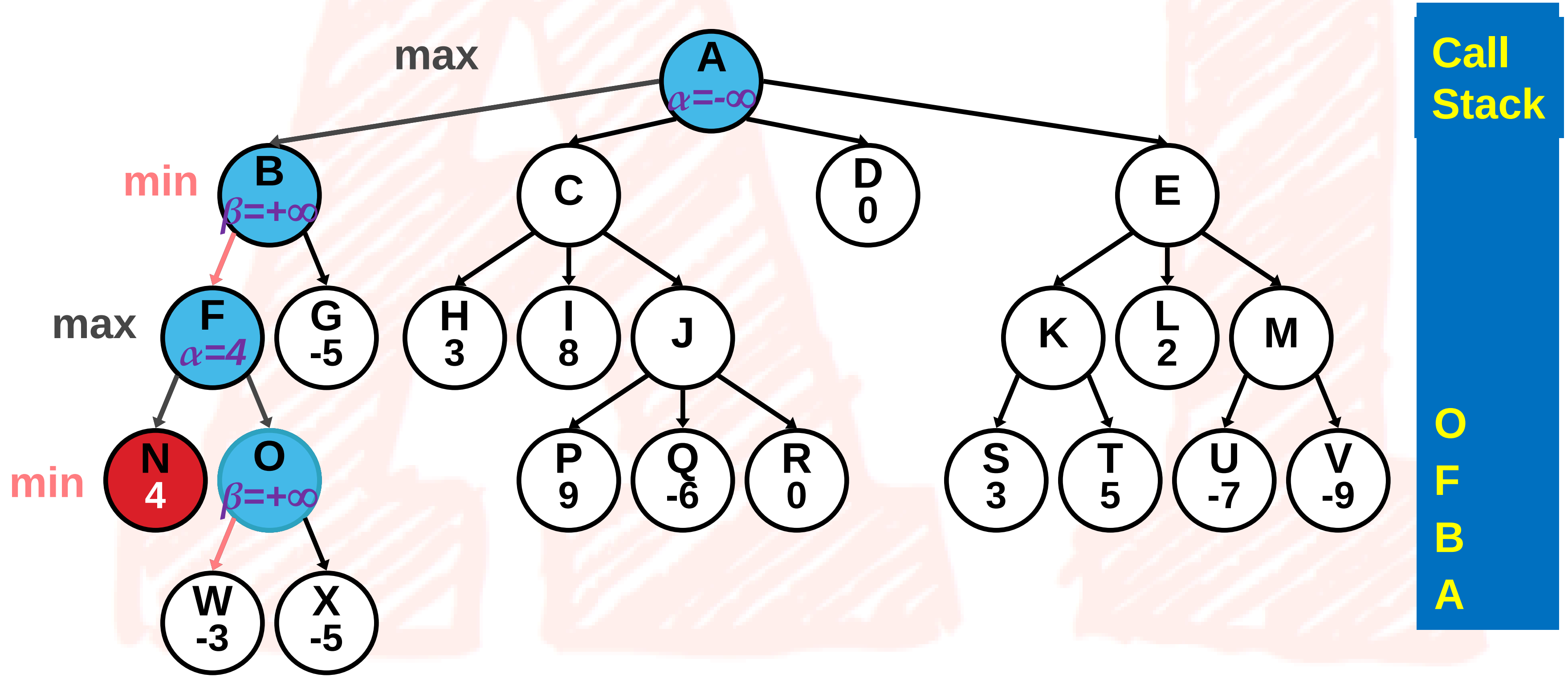
Keep expanding F? **Yes** since F's alpha >= B's beta is false, no beta cutoff



Alpha-Beta Search Example

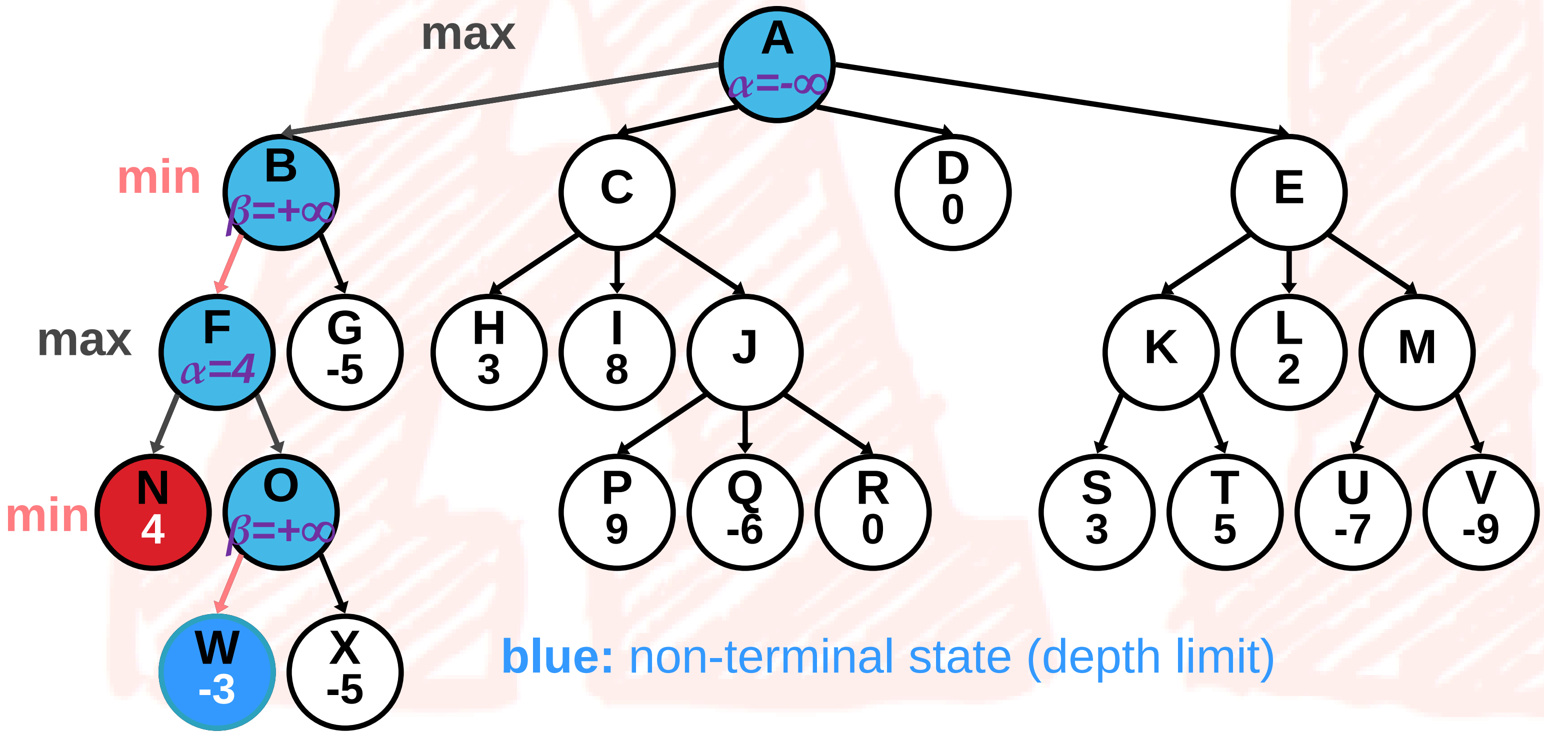
`minimax(O,3,4)` beta initialized to +infinity

Expand O? **Yes** since F's alpha >= O's beta is false, no alpha cutoff



Alpha-Beta Search Example

`minimax(W,4,4)` evaluate and return SBE value



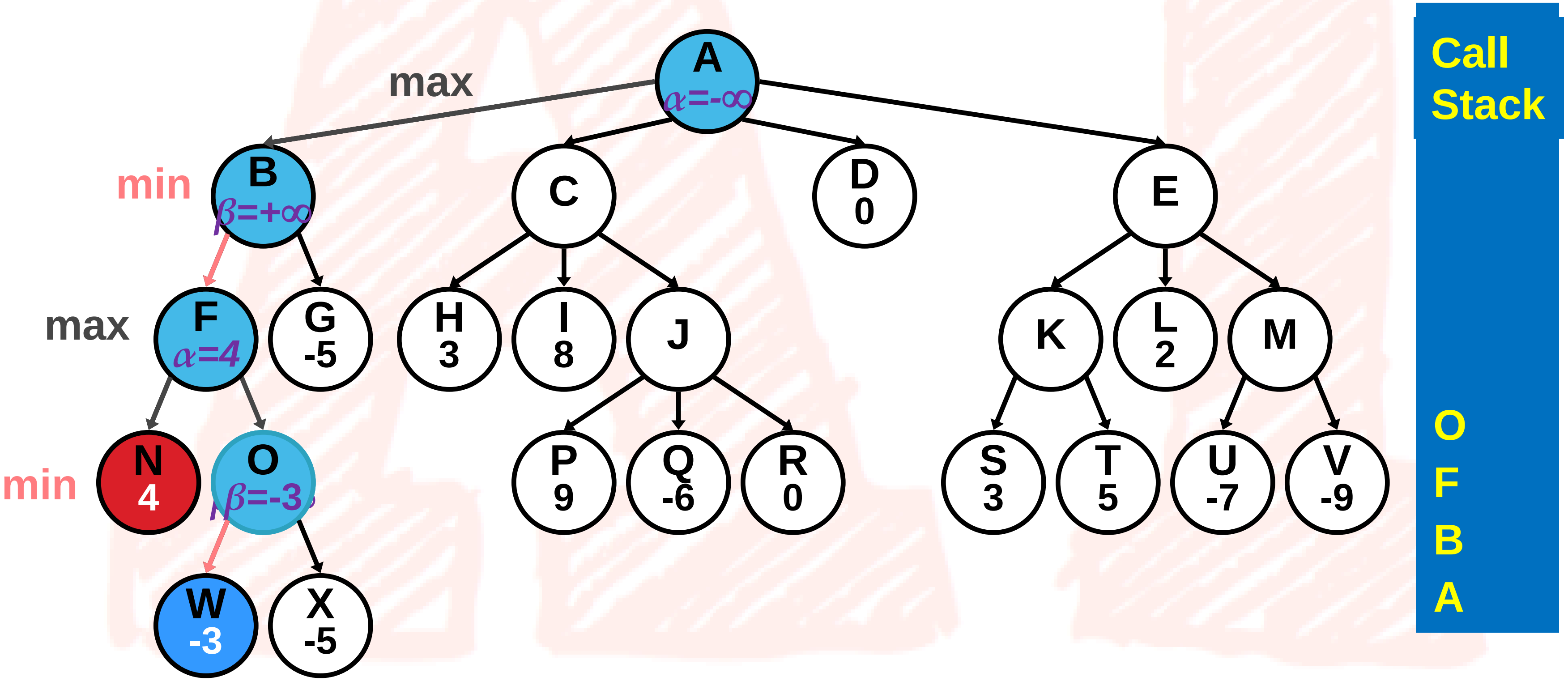
Call Stack

W
O
F
B
A

Alpha-Beta Search Example

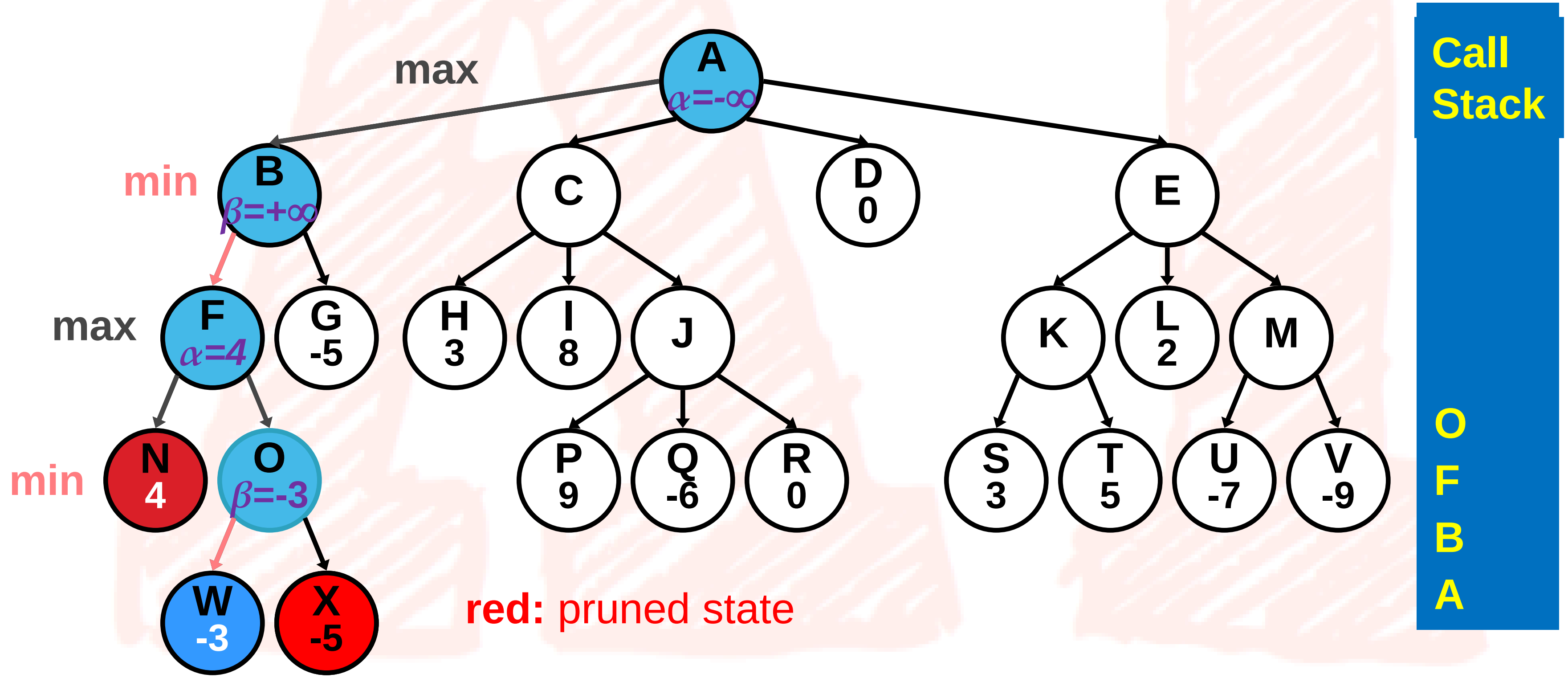
back to `minimax(O,3,4)` beta = -3, since -3 <= +infinity (minimizing)

Keep expanding O? No since F's alpha >= O's beta is true: **alpha cutoff**



Alpha-Beta Search Example

- Why?
  - Smart opponent will choose W or worse, thus O's upper bound is –3.
    Computer already has better move at N.

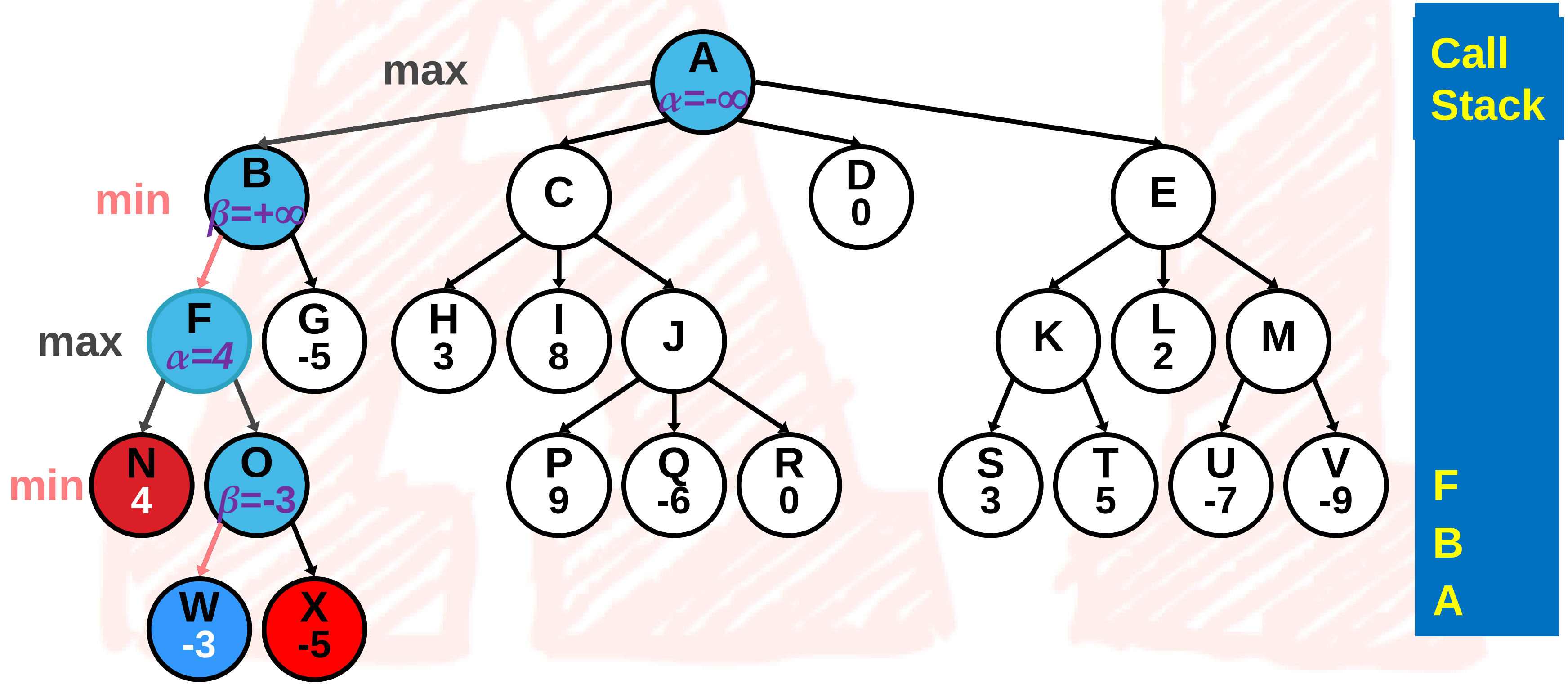


Alpha-Beta Search Example

back to `minimax(F,2,4)`

alpha doesn't change, since -3 < 4 (maximizing)

Keep expanding F? No since no more successors for F

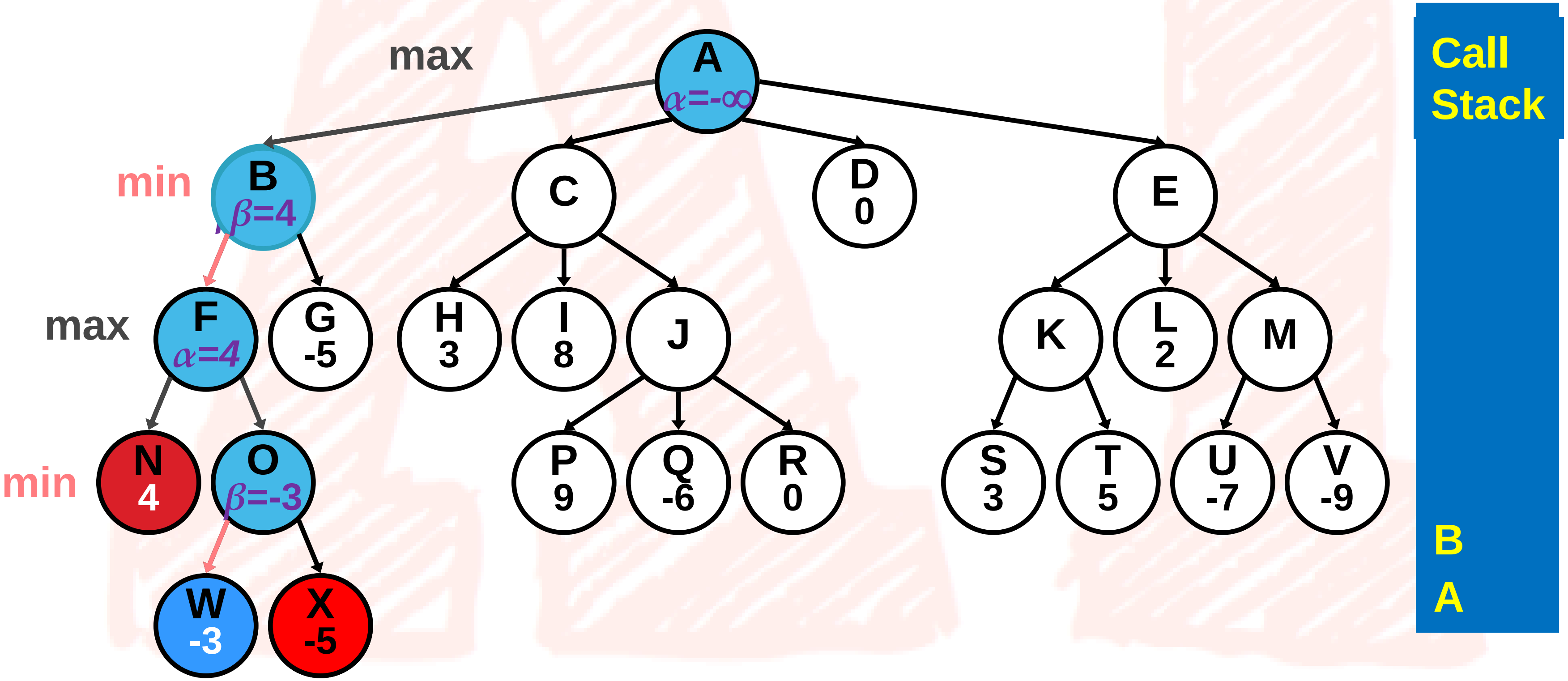


Alpha-Beta Search Example

back to `minimax(B,1,4)`

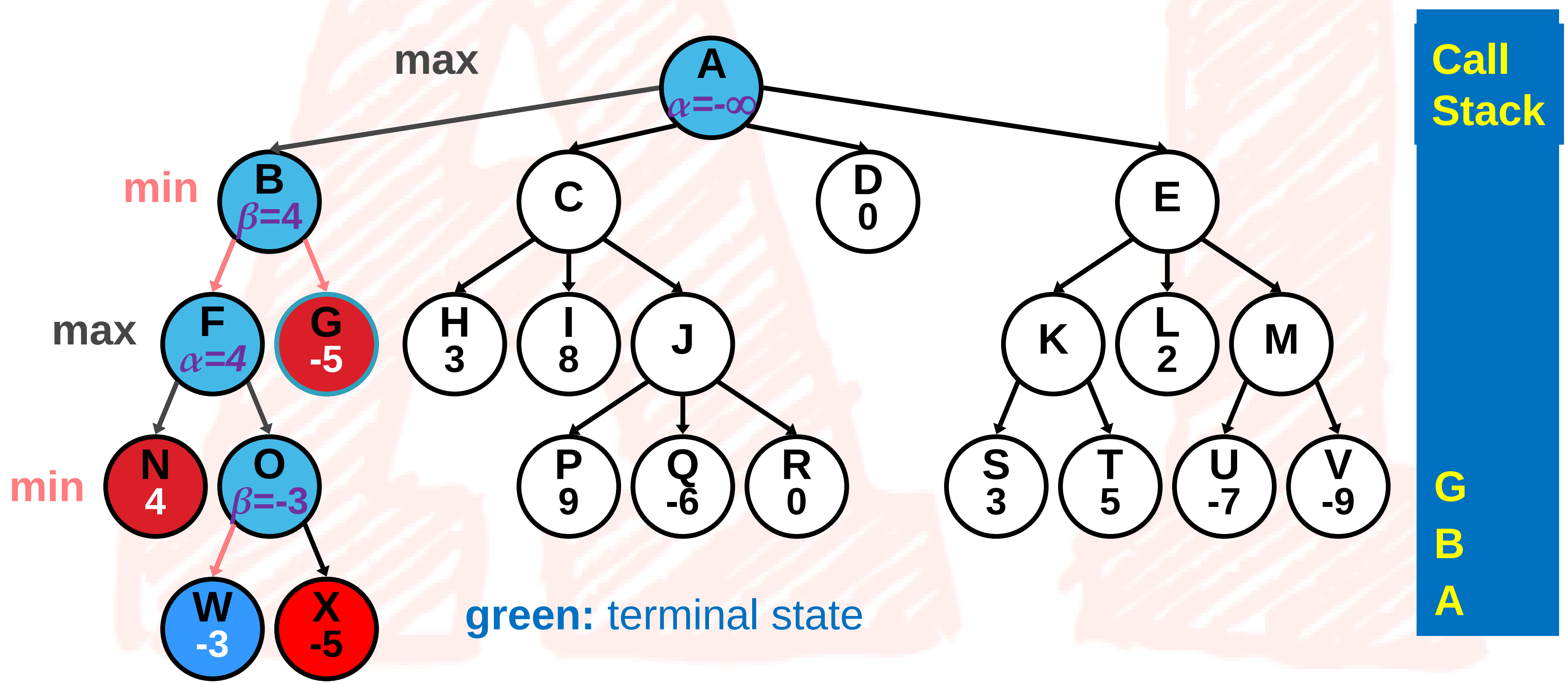
beta = 4, since 4 <= +infinity (minimizing)

Keep expanding B? Yes since A's alpha >= B's beta is false, no alpha cutoff



Alpha-Beta Search Example

`minimax(G,2,4)` evaluate and return SBE value



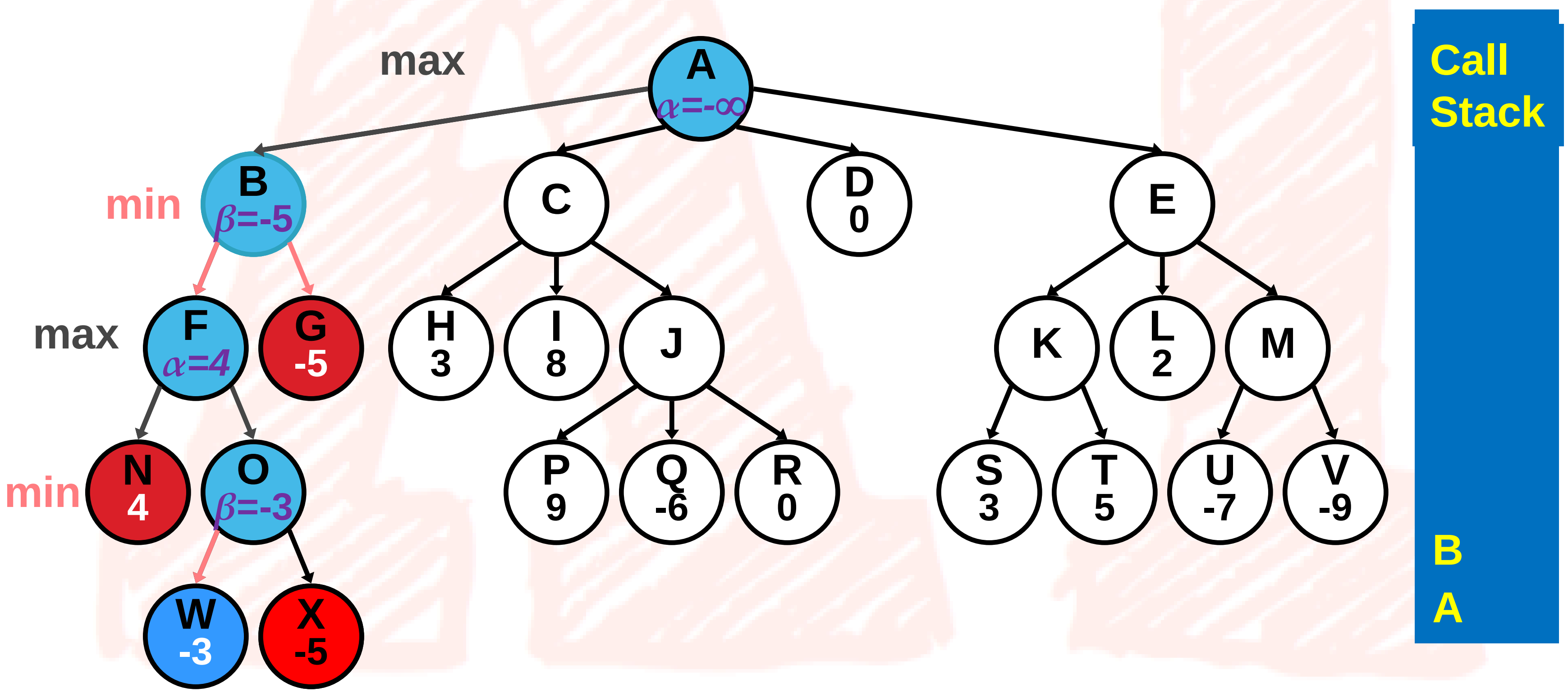
green: terminal state

Alpha-Beta Search Example

back to `minimax(B,1,4)`

beta = -5, since -5 <= 4 (minimizing)

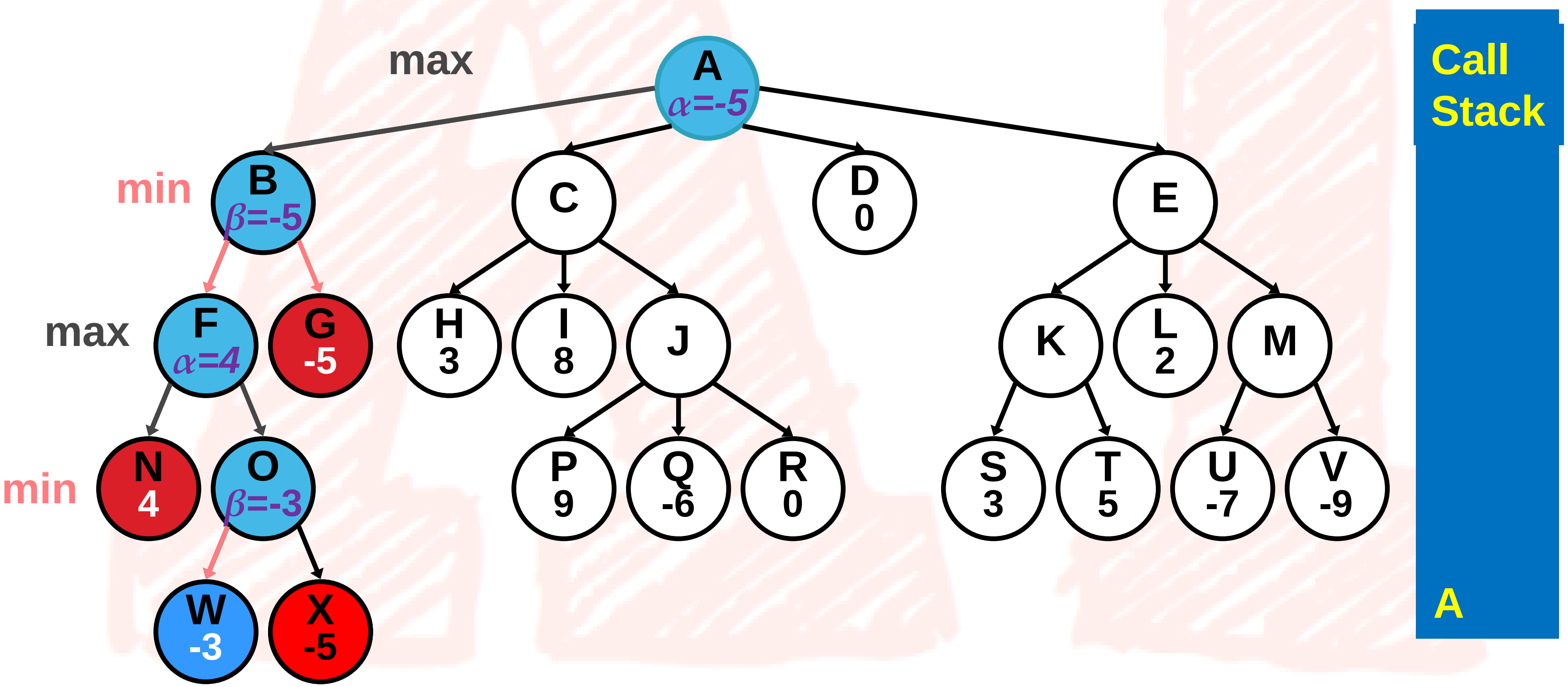
Keep expanding B? No since no more successors for B



Alpha-Beta Search Example

back to `minimax(A,0,4)` alpha = -5, since -5 >= -infinity (maximizing)

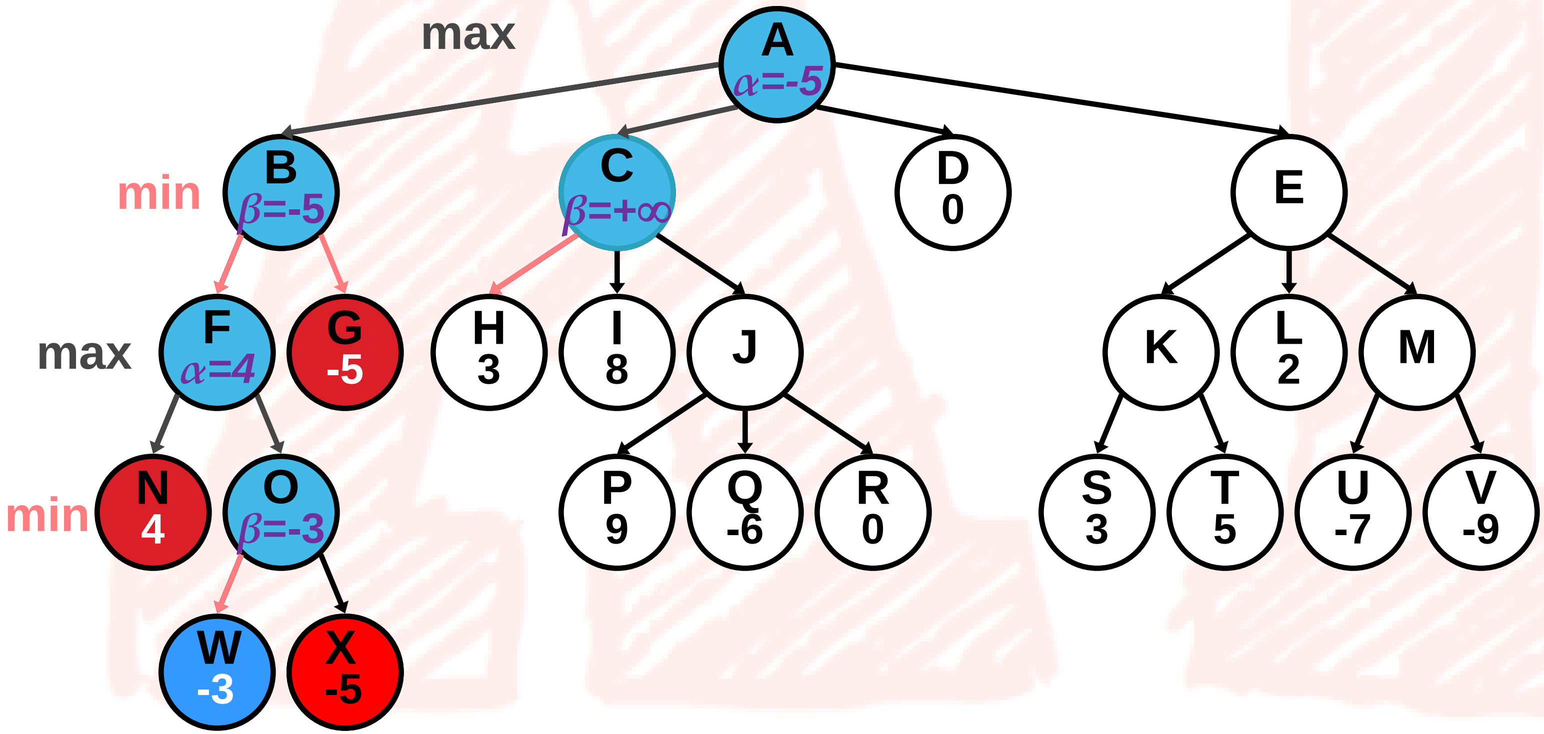
Keep expanding A? **Yes** since there are more successors, no cutoff test



Alpha-Beta Search Example

`minimax(C,1,4)` beta initialized to +infinity

Expand C? Yes since A's alpha >= C's beta is false, no alpha cutoff



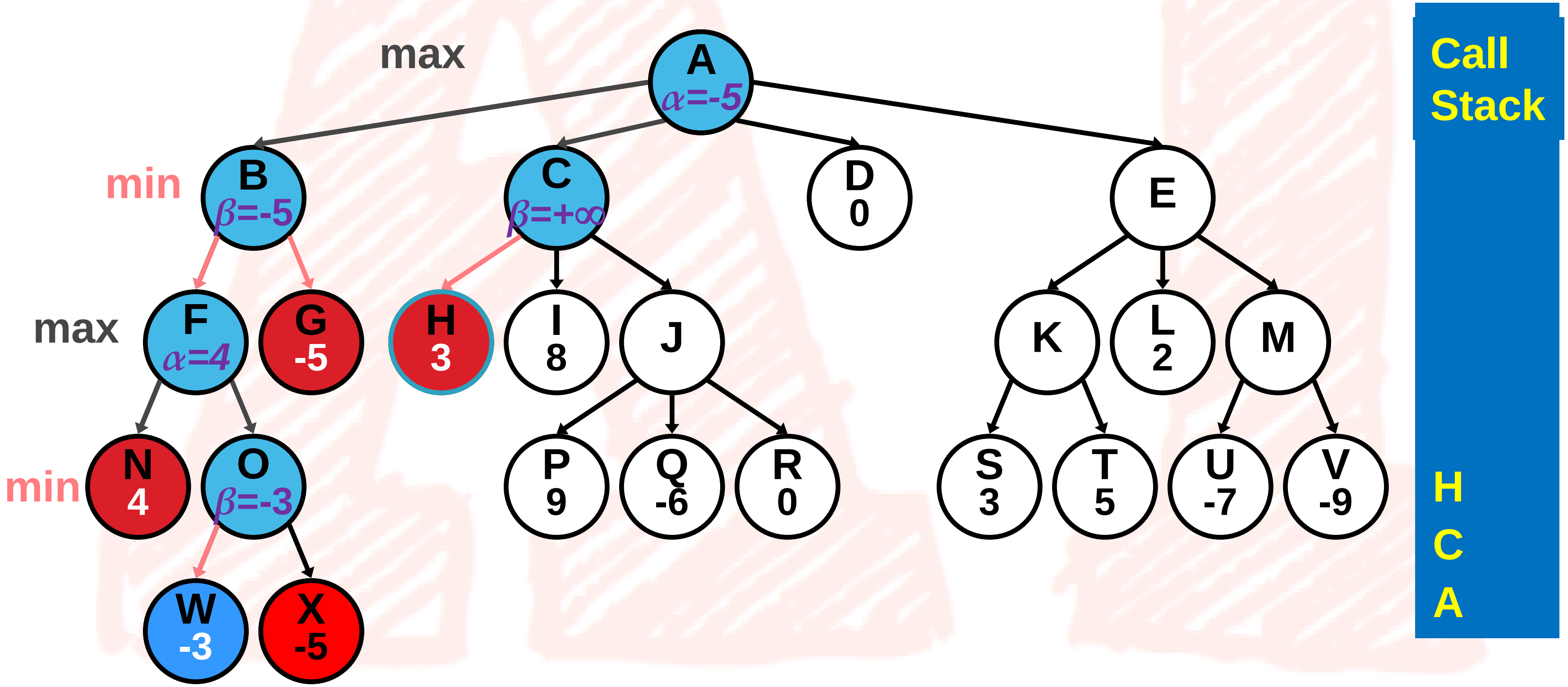
Call Stack

C

A

Alpha-Beta Search Example

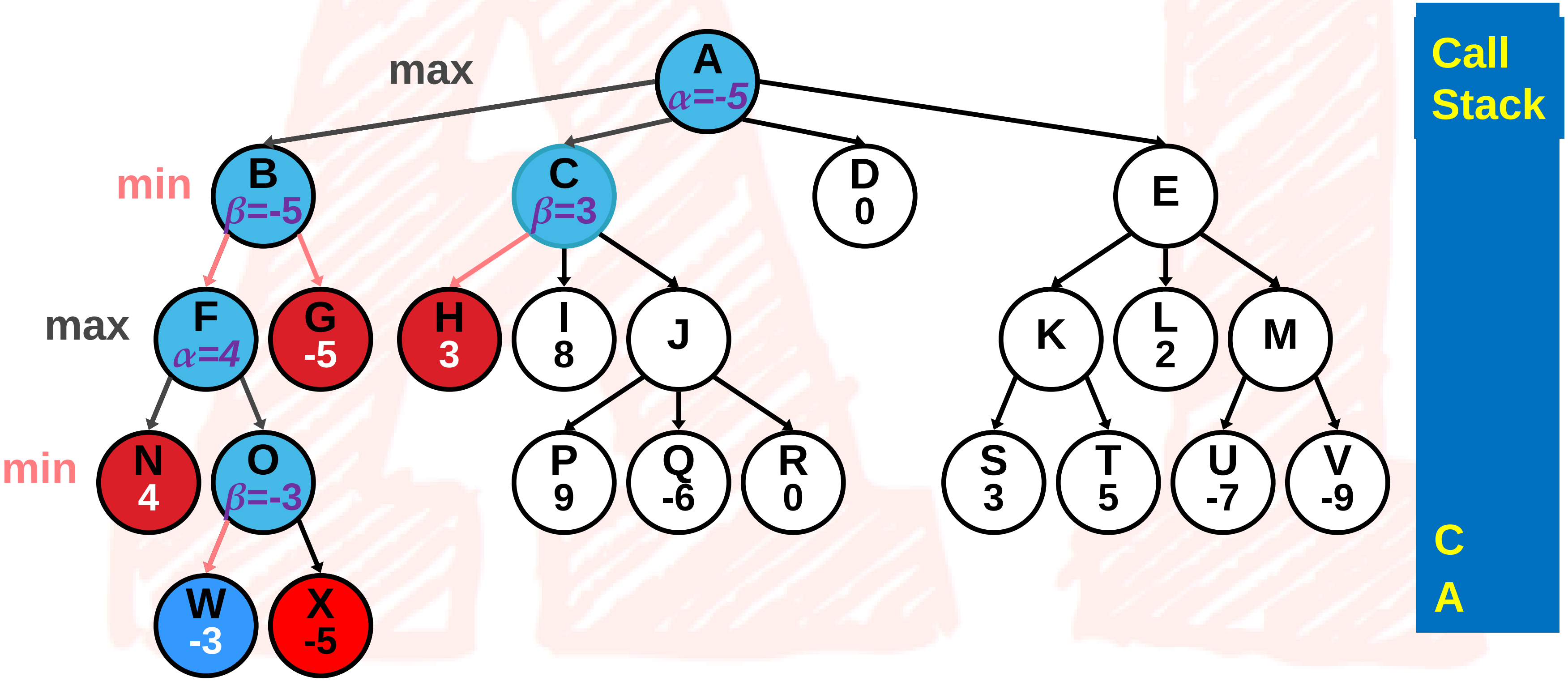
`minimax(H,2,4)` evaluate and return SBE value



Alpha-Beta Search Example

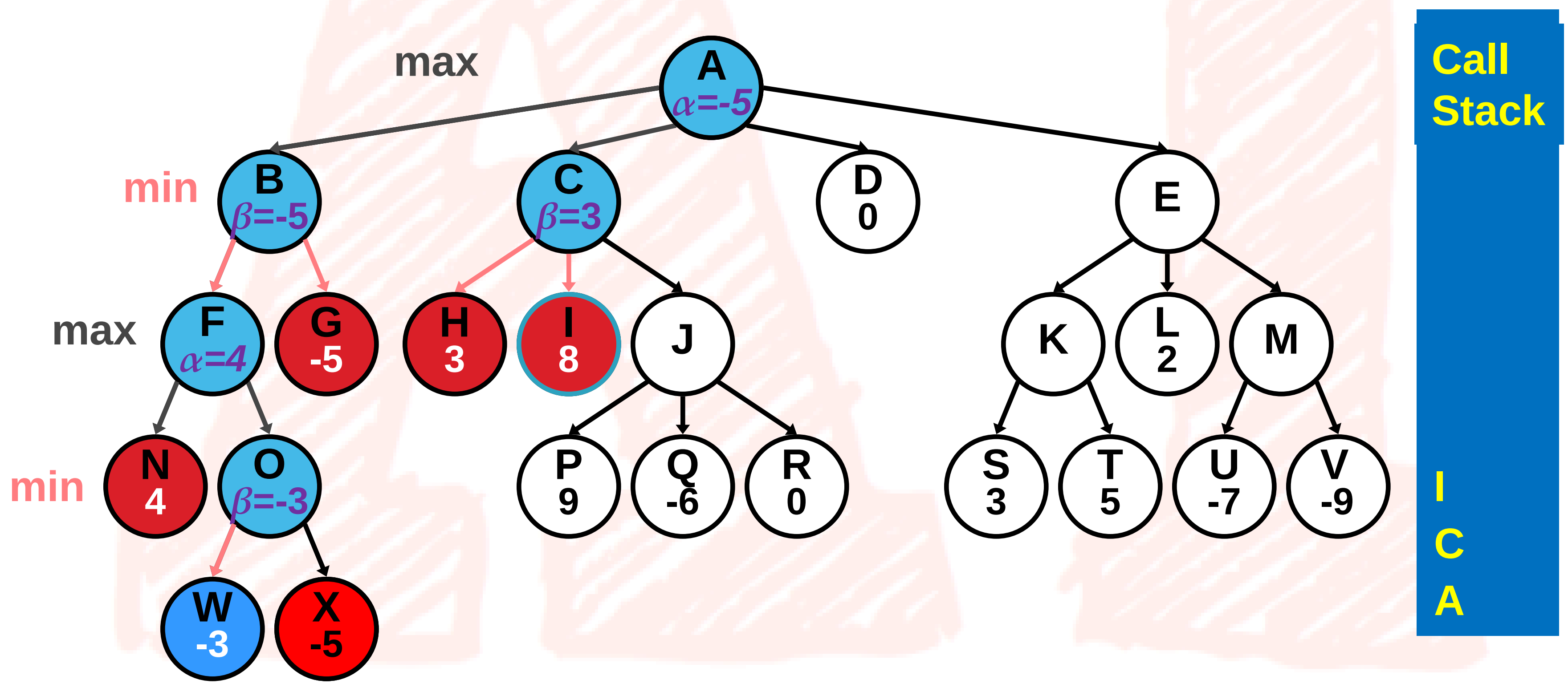
back to `minimax(C,1,4)` beta = 3, since 3 <= +infinity (minimizing)

Keep expanding C? Yes since A's alpha >= C's beta is false, no alpha cutoff



Alpha-Beta Search Example

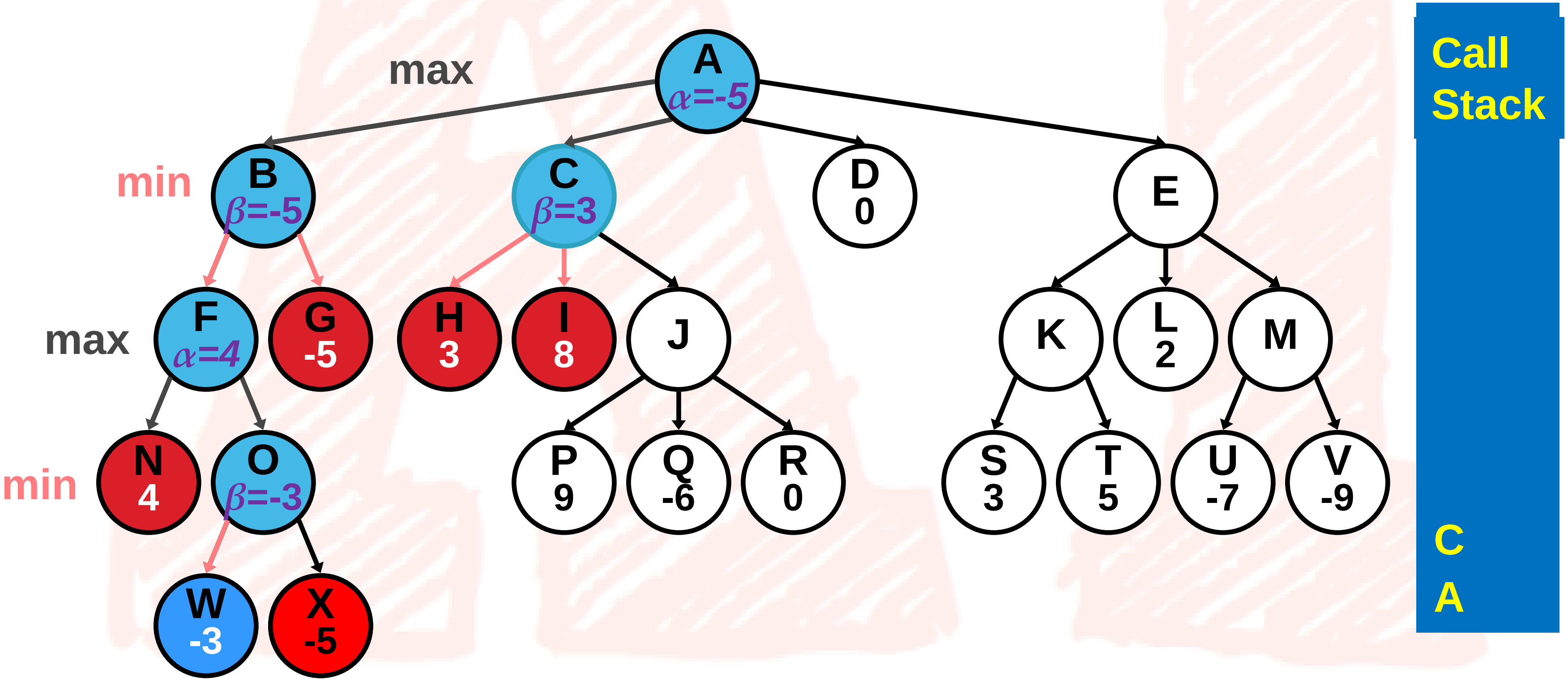
`minimax(I,2,4)` evaluate and return SBE value



Alpha-Beta Search Example

back to `minimax(C,1,4)` beta doesn't change, since 8 > 3 (minimizing)

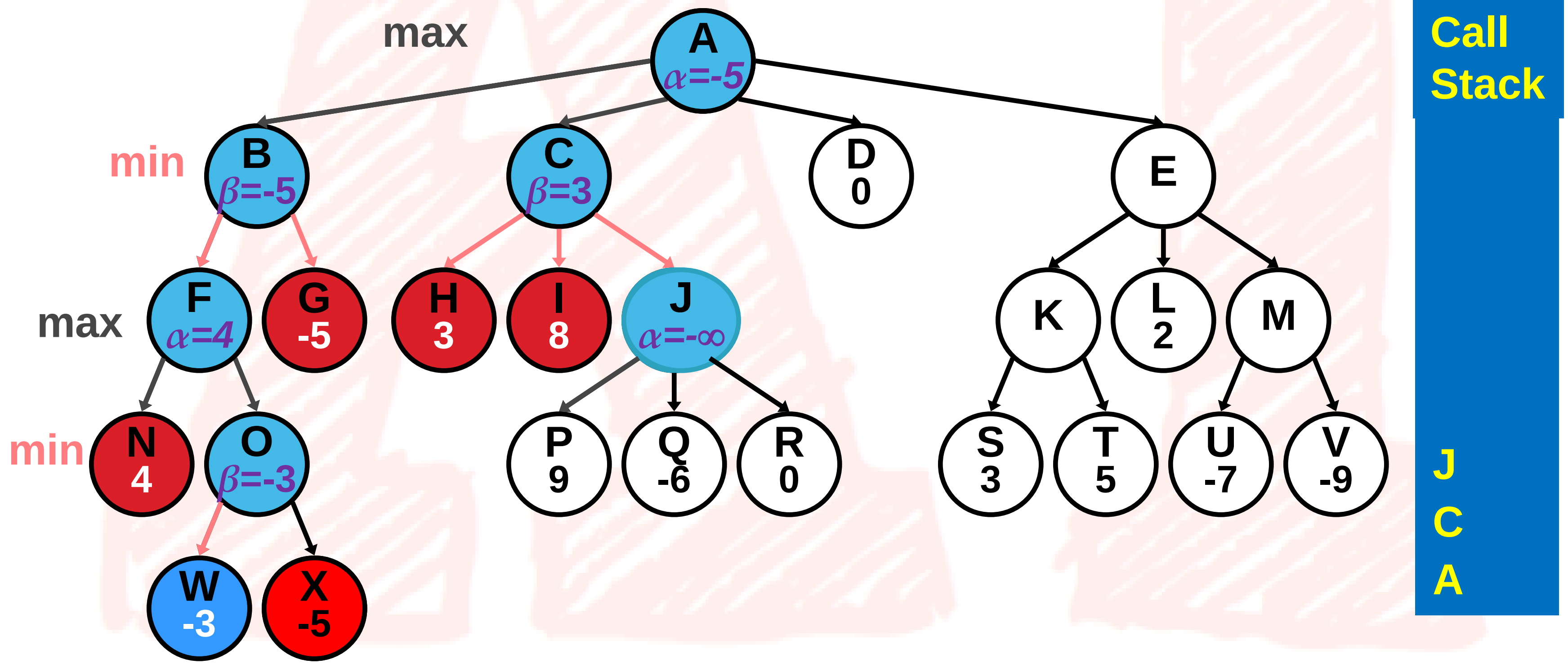
Keep expanding C? Yes since A's alpha >= C's beta is false, no alpha cutoff



Alpha-Beta Search Example

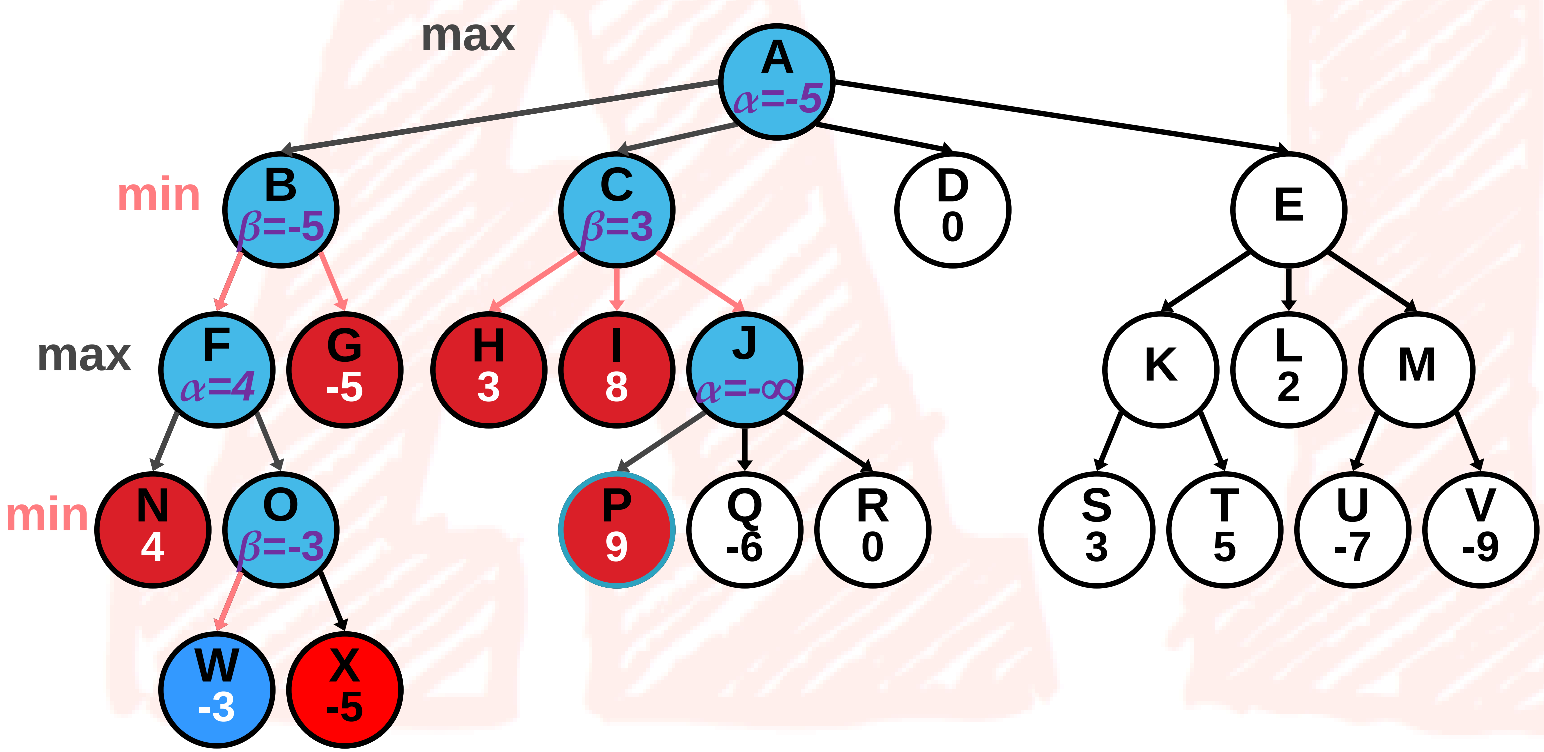
`minimax(J,2,4)`   alpha initialized to -infinity

Expand J? **Yes** since J's alpha >= C's beta is false, no beta cutoff



Alpha-Beta Search Example

`minimax(P,3,4)` evaluate and return SBE value



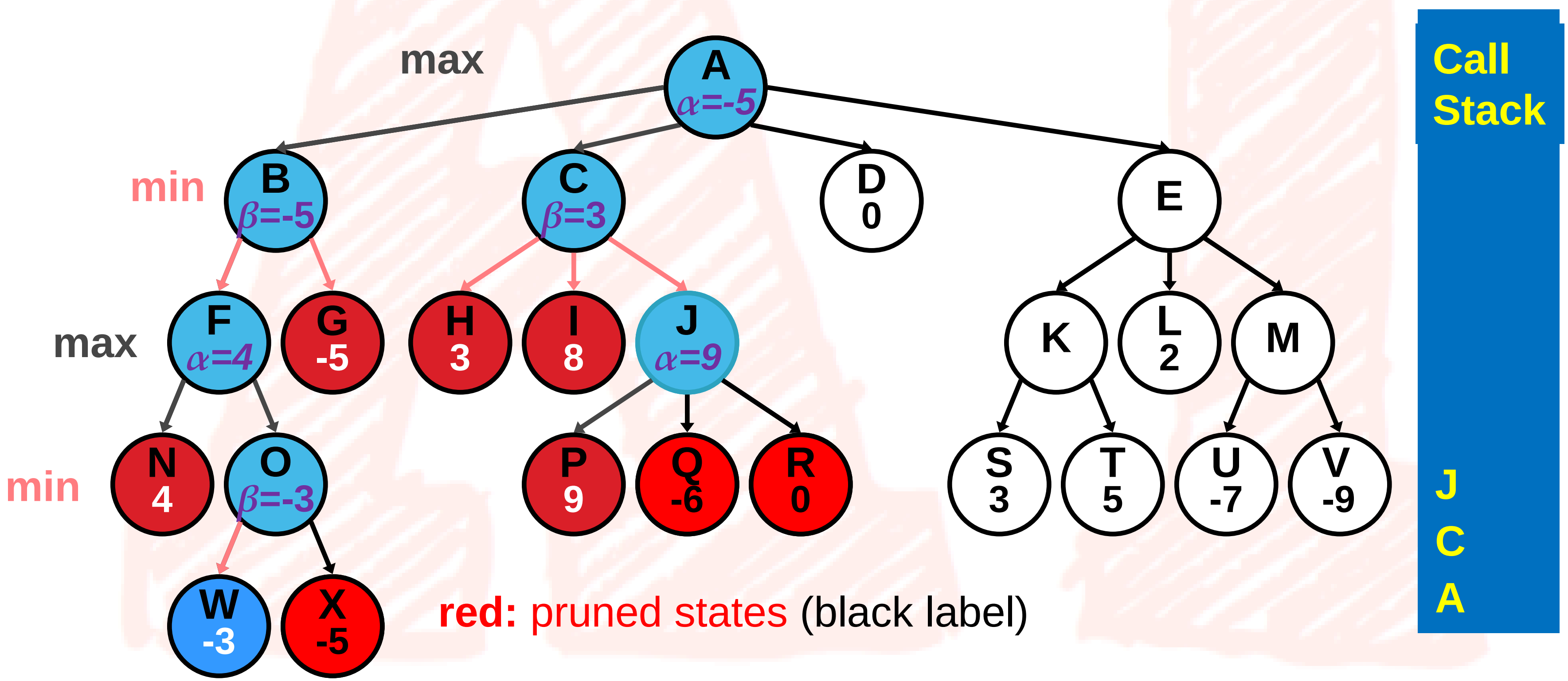
Call Stack

P
J
C
A

Alpha-Beta Search Example

back to alpha = 9, since 9 >= -infinity (maximizing)
`minimax(J,2,4)`

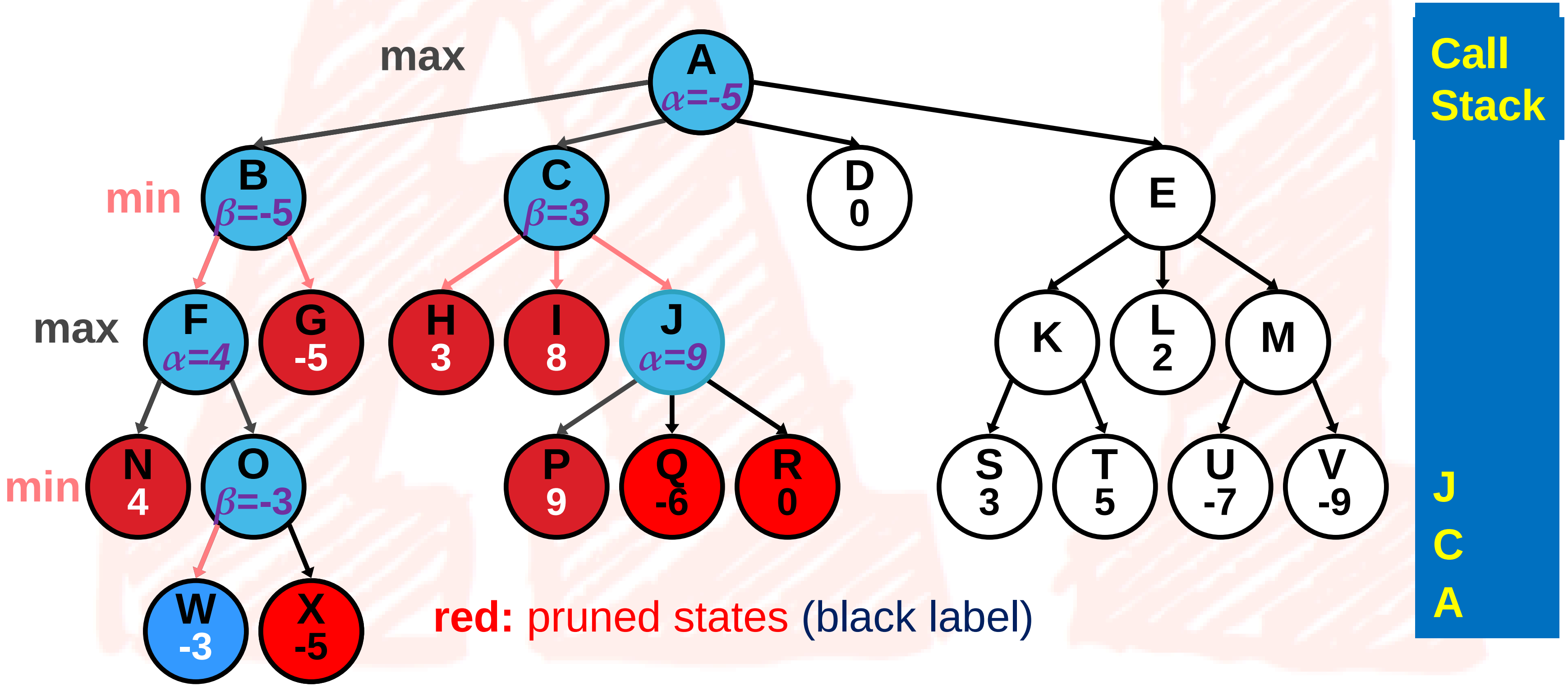
Keep expanding J? No since J's alpha >= C's beta is true: **beta cutoff**



red: pruned states (black label)

Alpha-Beta Search Example

- Why?
  - Computer will choose P or better, thus J's lower bound is 9.
    Smart opponent won't let computer take move to J (since opponent already has better move at H).

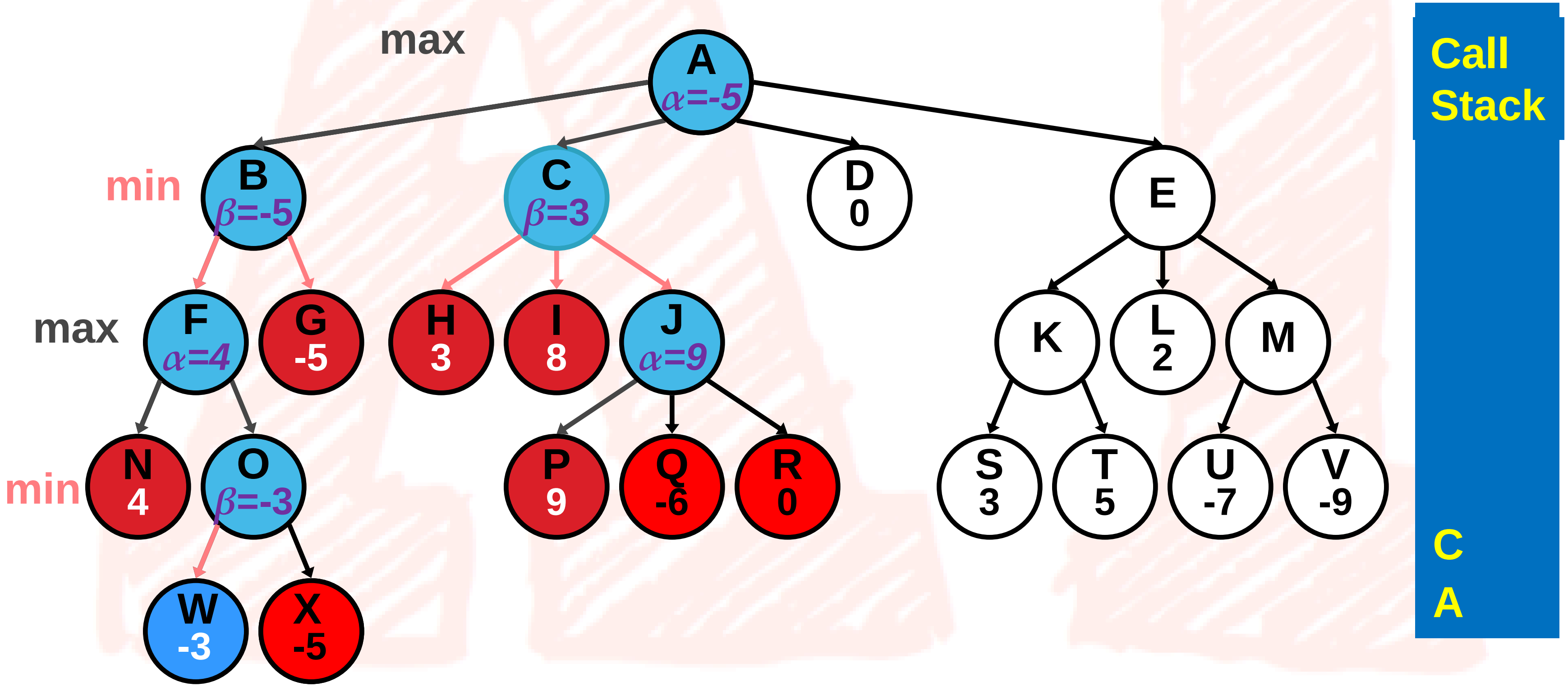


Alpha-Beta Search Example

back to `minimax(C,1,4)`

beta doesn't change, since 9 > 3 (minimizing)

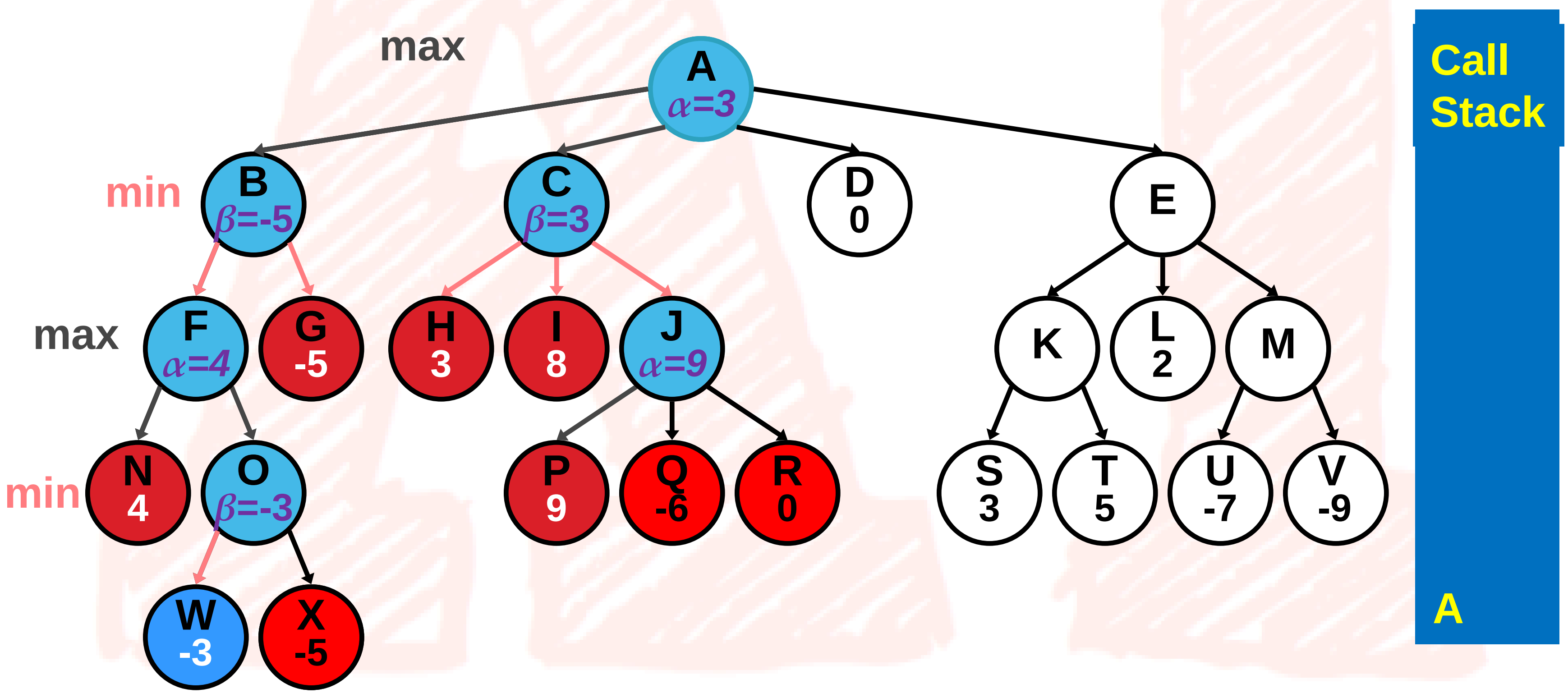
Keep expanding C? No since no more successors for C



Alpha-Beta Search Example

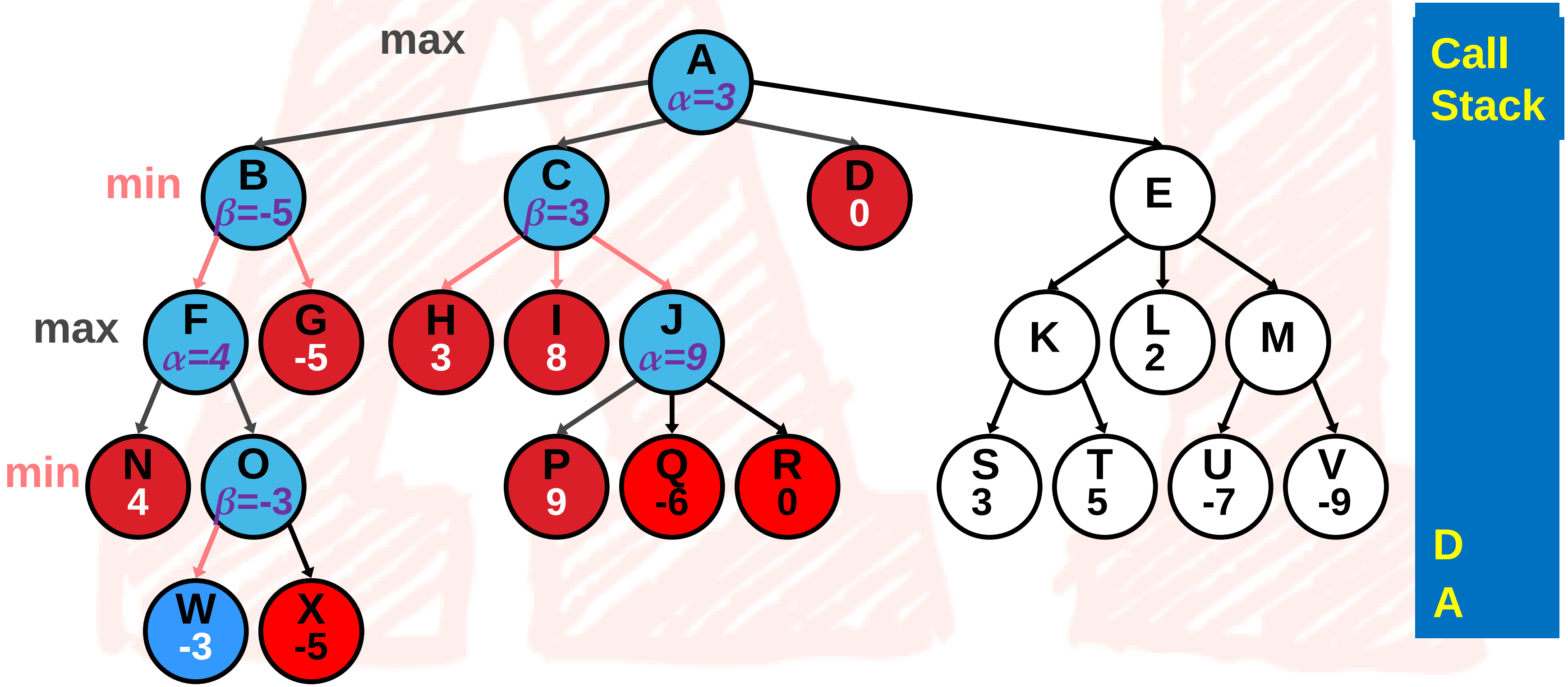
back to `minimax(A,0,4)` alpha = 3, since 3 >= -5 (maximizing)

Keep expanding A? **Yes** since there are more successors, no cutoff test



Alpha-Beta Search Example

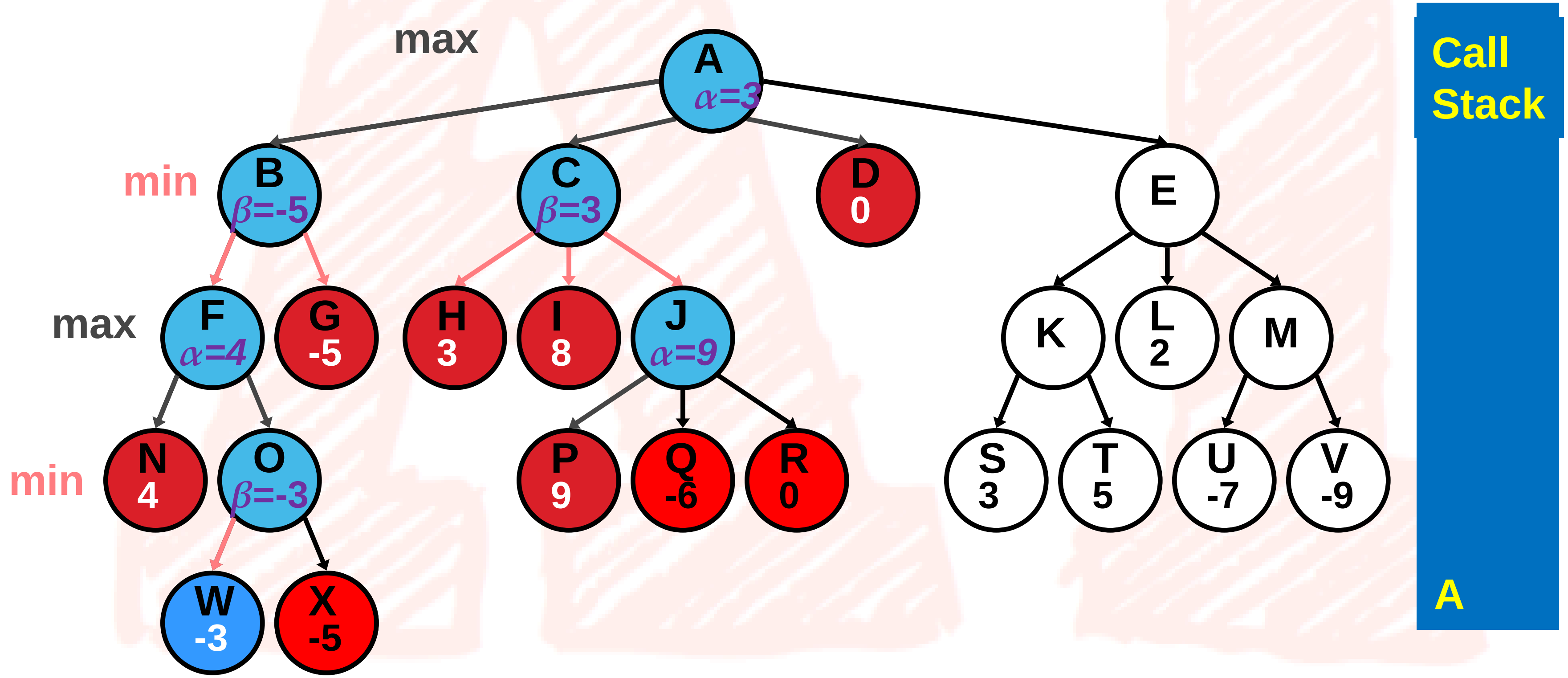
`minimax(D,1,4)` evaluate and return SBE value



Alpha-Beta Search Example

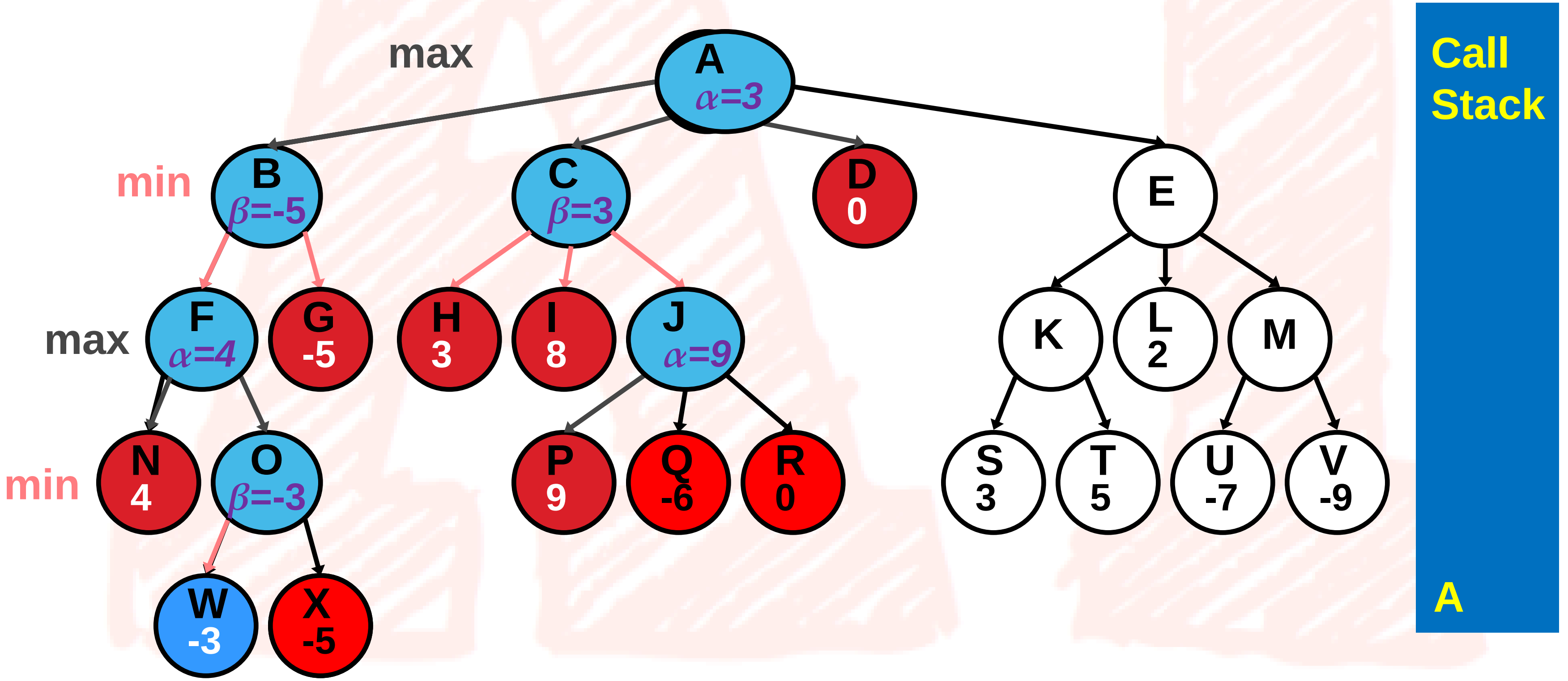
back to alpha doesn't change, since 0 < 3 (maximizing)
`minimax(A,0,4)`

Keep expanding A? Yes since there are more successors, no cutoff test



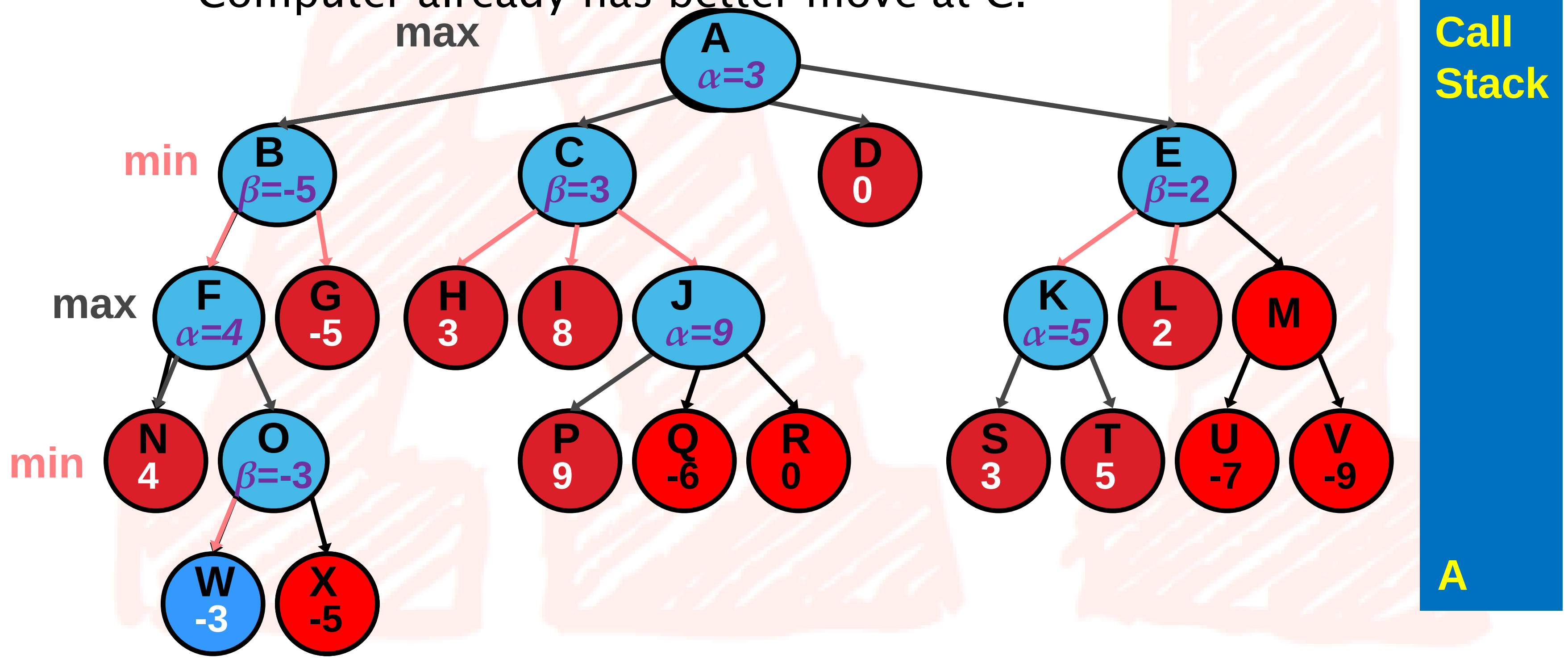
Alpha-Beta Search Example

- How does the algorithm finish searching the tree?



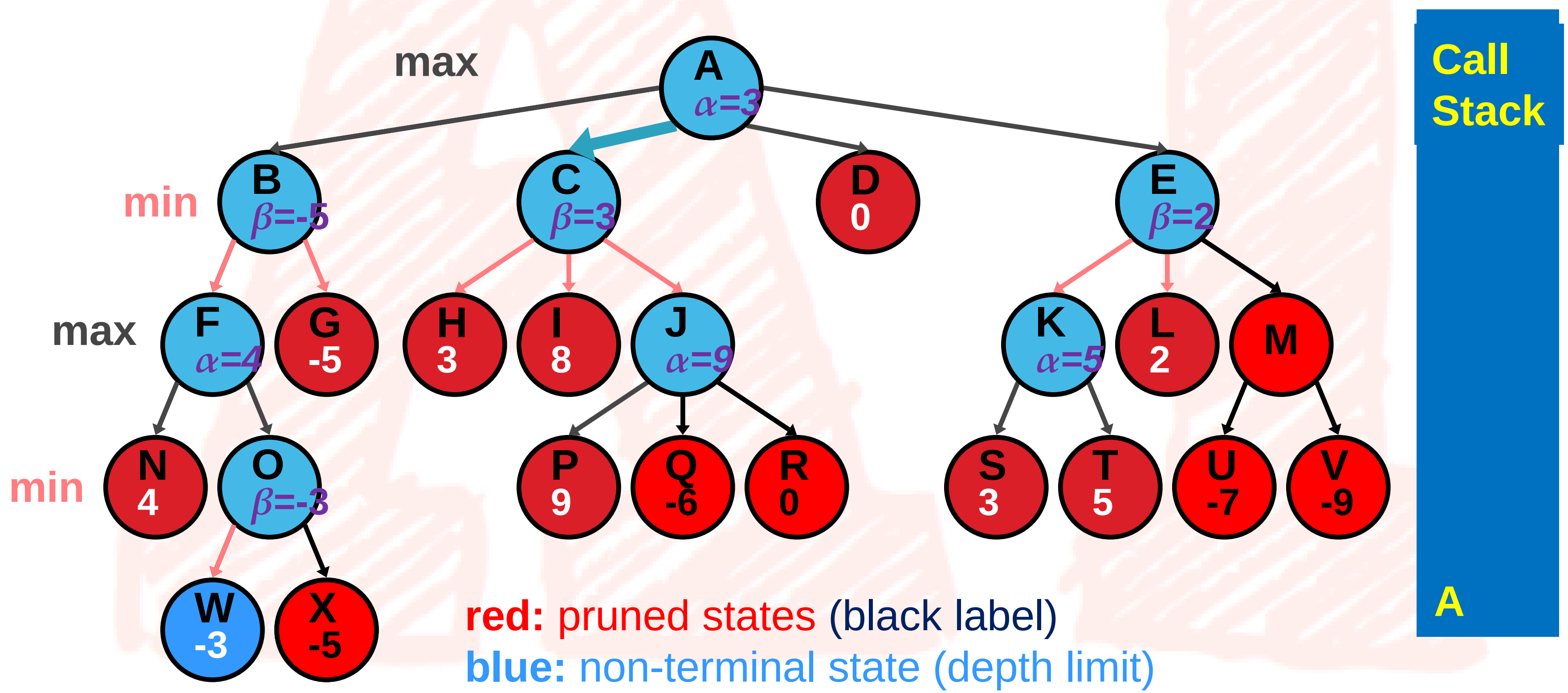
Alpha-Beta Search Example

- Stop Expanding E since A's alpha >= E's beta is true: alpha cutoff
- Why?
  - Smart opponent will choose L or worse, thus E's upper bound is 2.
  - Computer already has better move at C.



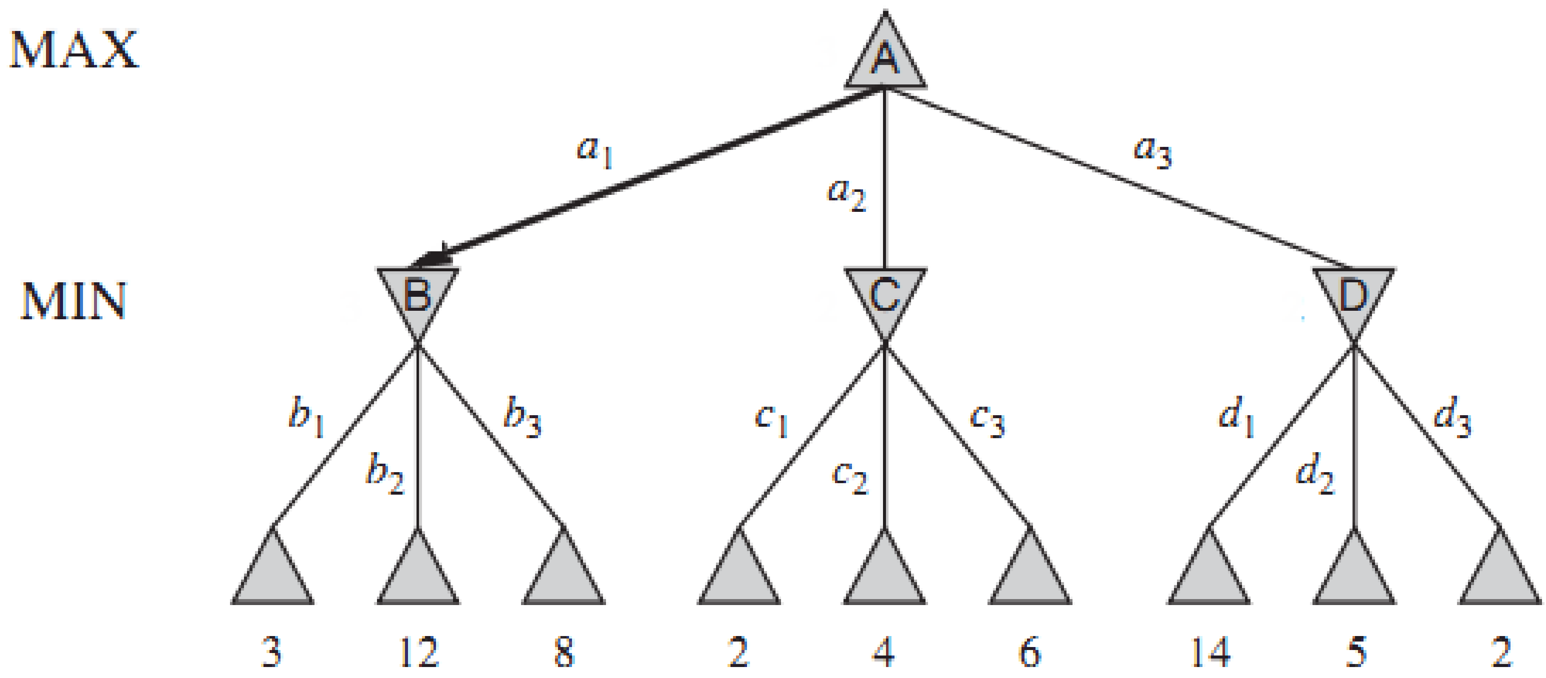
Alpha-Beta Search Example

- Result: Computer chooses move to C.



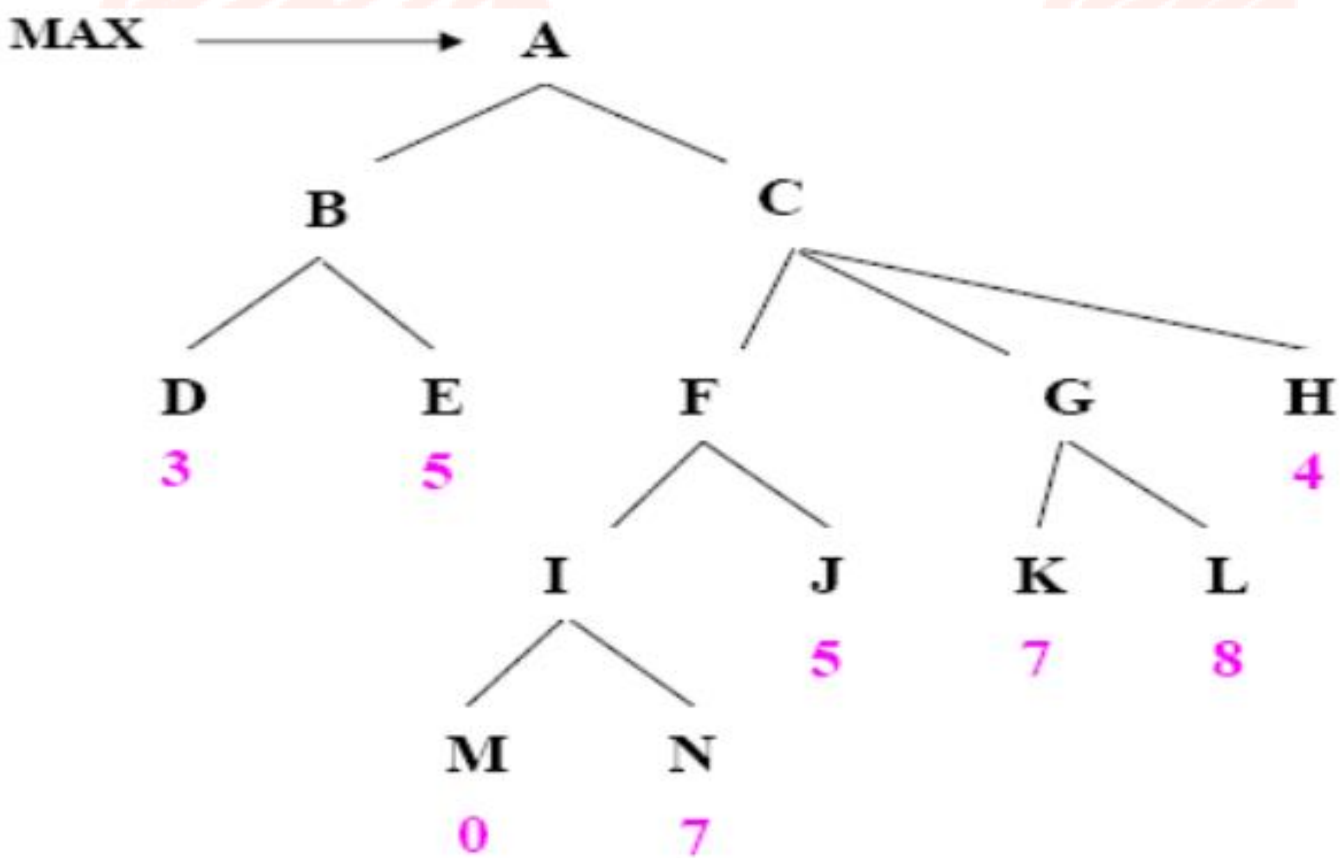
Exercise 3

- Perform the minimax algorithm on the figure below with Alpha-Beta-pruning



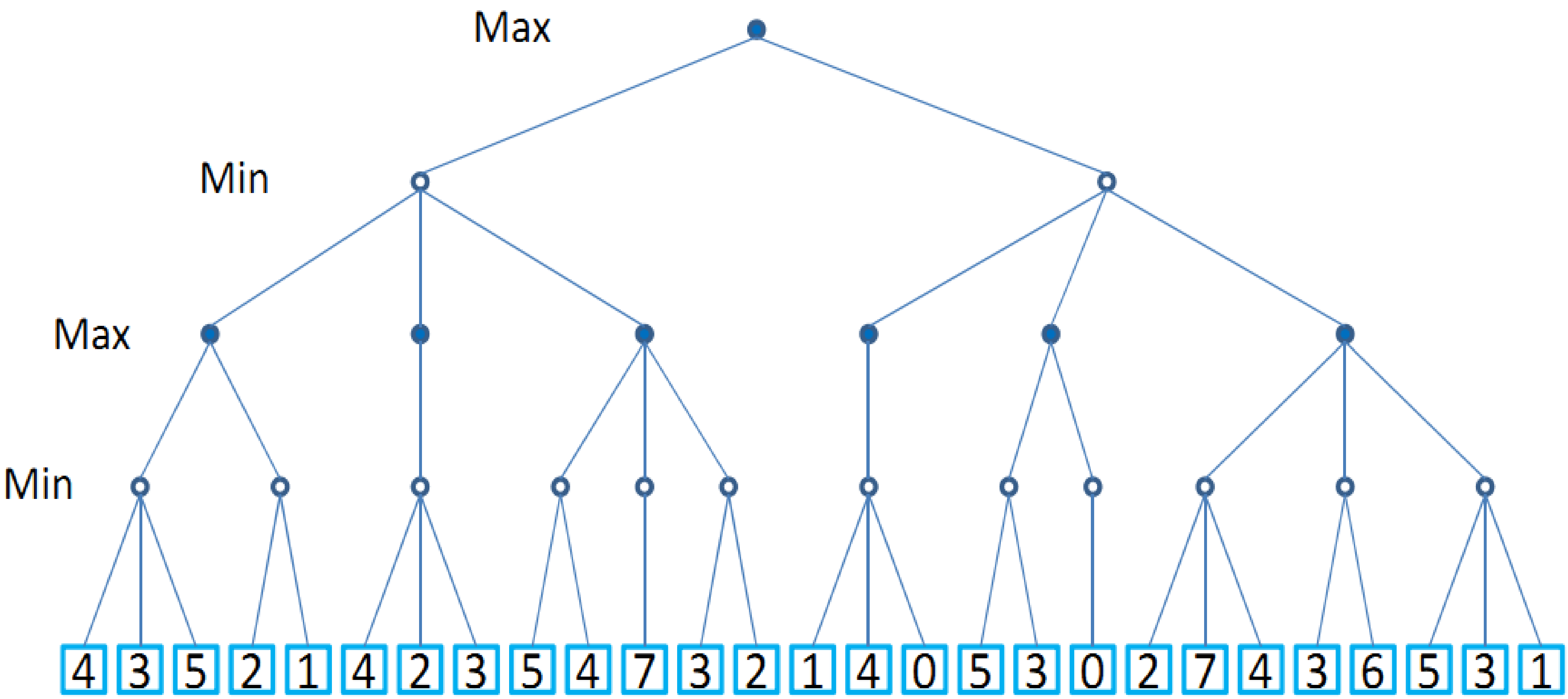
Exercise 4

- Perform the minimax algorithm on the figure below with Alpha-Beta-pruning



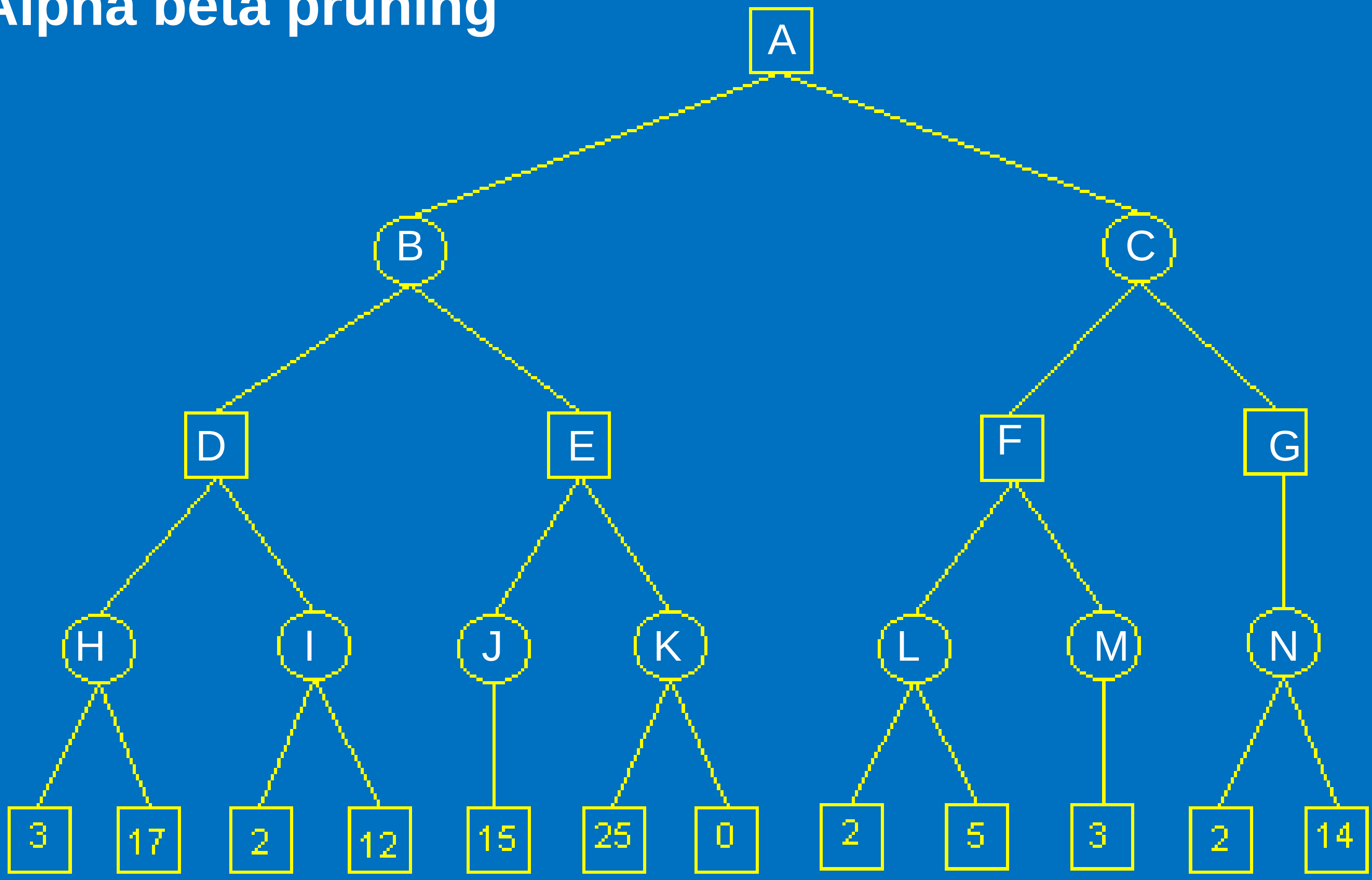
Exercise 5

- Perform the minimax algorithm on the figure below with Alpha-Beta-pruning



Exercise 6

- **Alpha beta pruning**



Alpha-Beta Algorithm

```
function ALPHA-BETA-SEARCH(state) returns an action
   inputs: state, current state in game
   v ← MAX-VALUE(state, - ∞ , +∞)
   return the action in SUCCESSORS(state) with value v
```

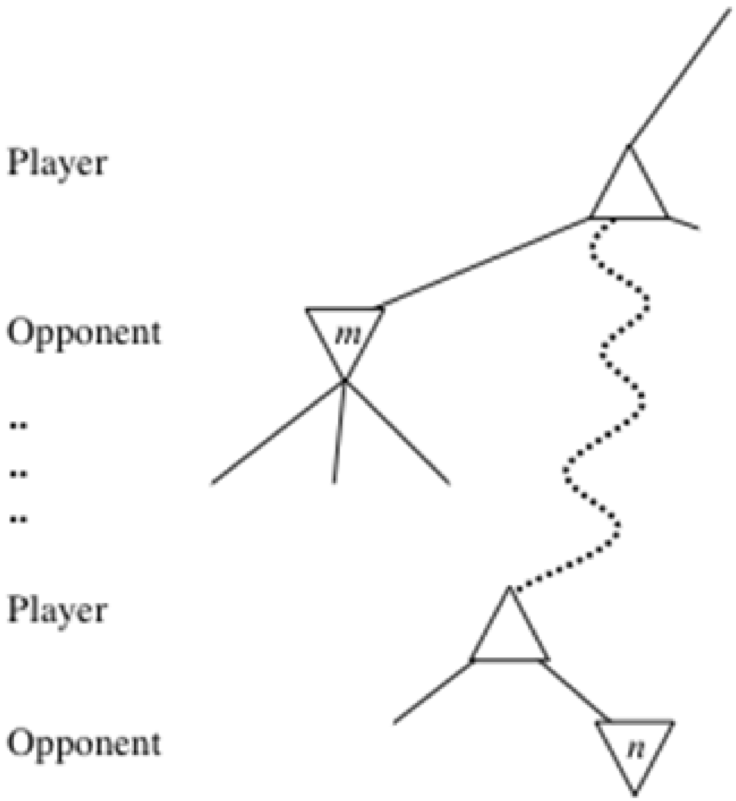
```
function MAX-VALUE(state, α, β) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← - ∞
   for each s in SUCCESSORS(state) do
      v ← MAX(v, MIN-VALUE(s, α, β))
      if v ≥ β then return v
      α ← MAX(α, v)
   return v
```

Alpha-Beta Algorithm

```
function MIN-VALUE(state, α , β) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← + ∞
    for each s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s, α , β))
        if v ≤ α then return v
        β ← MIN(β ,v)
    return v
```

General alpha-beta pruning

- Consider a node *n* somewhere in the tree
- If player has a better choice at
  - Parent node of n
  - Or any choice point further up
- *n* will never be reached in actual play.
- Hence when enough is known about *n*, it can be pruned.



Final Comments: Alpha-Beta Pruning

- Pruning does not affect final results
- Entire subtrees can be pruned.
- Good move *ordering* improves effectiveness of pruning
- With "perfect ordering" time complexity is $O(b^{m/2})$
  - Branching factor of sqrt(b) !!
  - Alpha-beta pruning can look twice as far as Minimax in the same amount of time
- Repeated states are again possible.
  - Store them in memory = transposition table

Imperfect Real-Time Decisions

- Minimax require too much leaf-node evaluations.
- May be impractical within a reasonable amount of time.
- SHANNON (1950):
  - Cut off search earlier (replace TERMINAL-TEST by CUTOFF-TEST)
  - Apply heuristic evaluation function EVAL (replacing utility function of alpha-beta)

Cutting off search

- Change:

```
if TERMINAL-TEST(state) then
    return UTILITY(state)
```

into

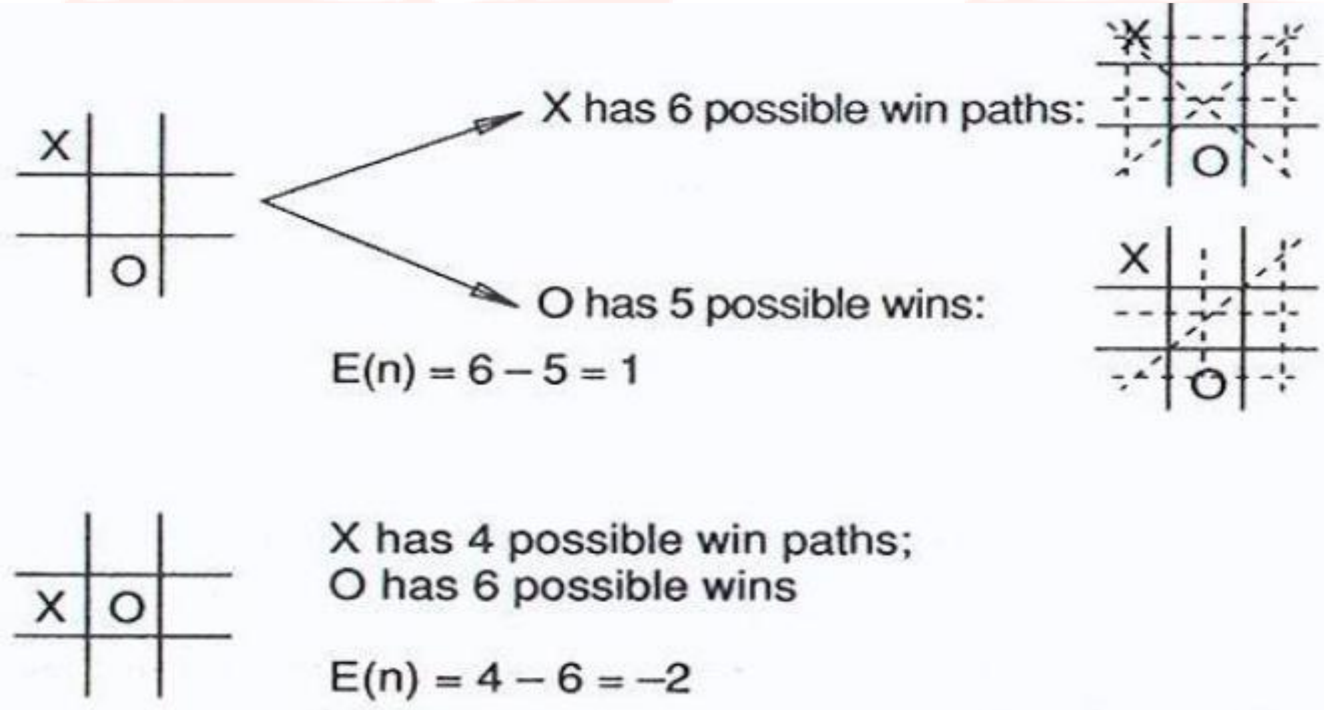
```
if CUTOFF-TEST(state, depth) then
    return EVAL(state)
```

- Introduces a fixed-depth limit depth
  - Is selected so that the amount of time will not exceed what the rules of the game allow.
- When cuttoff occurs, the evaluation is performed.

Heuristic EVAL

- EVAL function retuns an estimate of the expected utility of the game from a given position.
- Performance of game playing depends on quality of EVAL.
- Requirements:
  - EVAL must agree with terminal-nodes in the same way as UTILITY.
  - Computation may not take too long.
  - For non-terminal states the EVAL should be strongly correlated with the actual chance of winning.
- Only useful for quiescent (no wild swings in value in near future) states

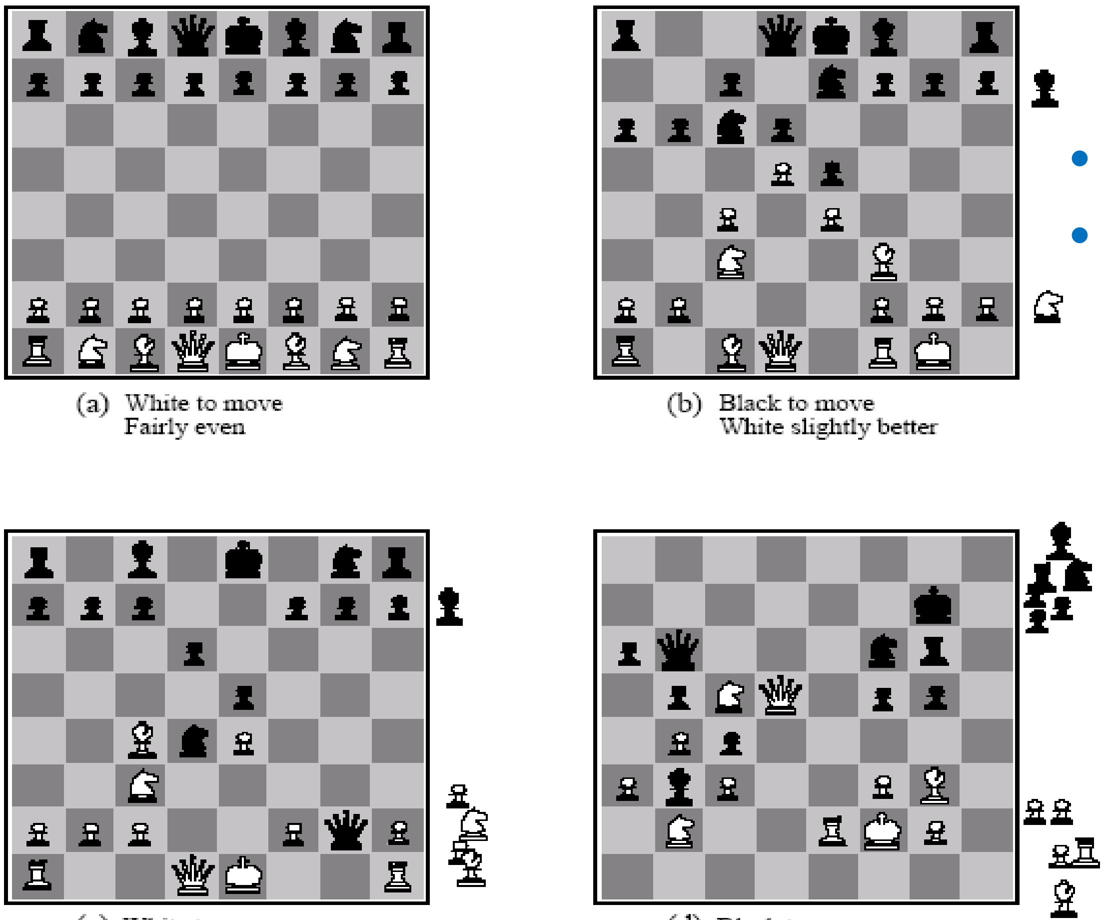
Heuristic EVAL example



- Heuristic: $E(n) = M(n) - O(n)$
  - M(n): total win paths of X,
  - O(n): total win paths of O

Heuristic EVAL example

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$



(a) White to move
Fairly even

(b) Black to move
White slightly better

(c) White to move
Black winning

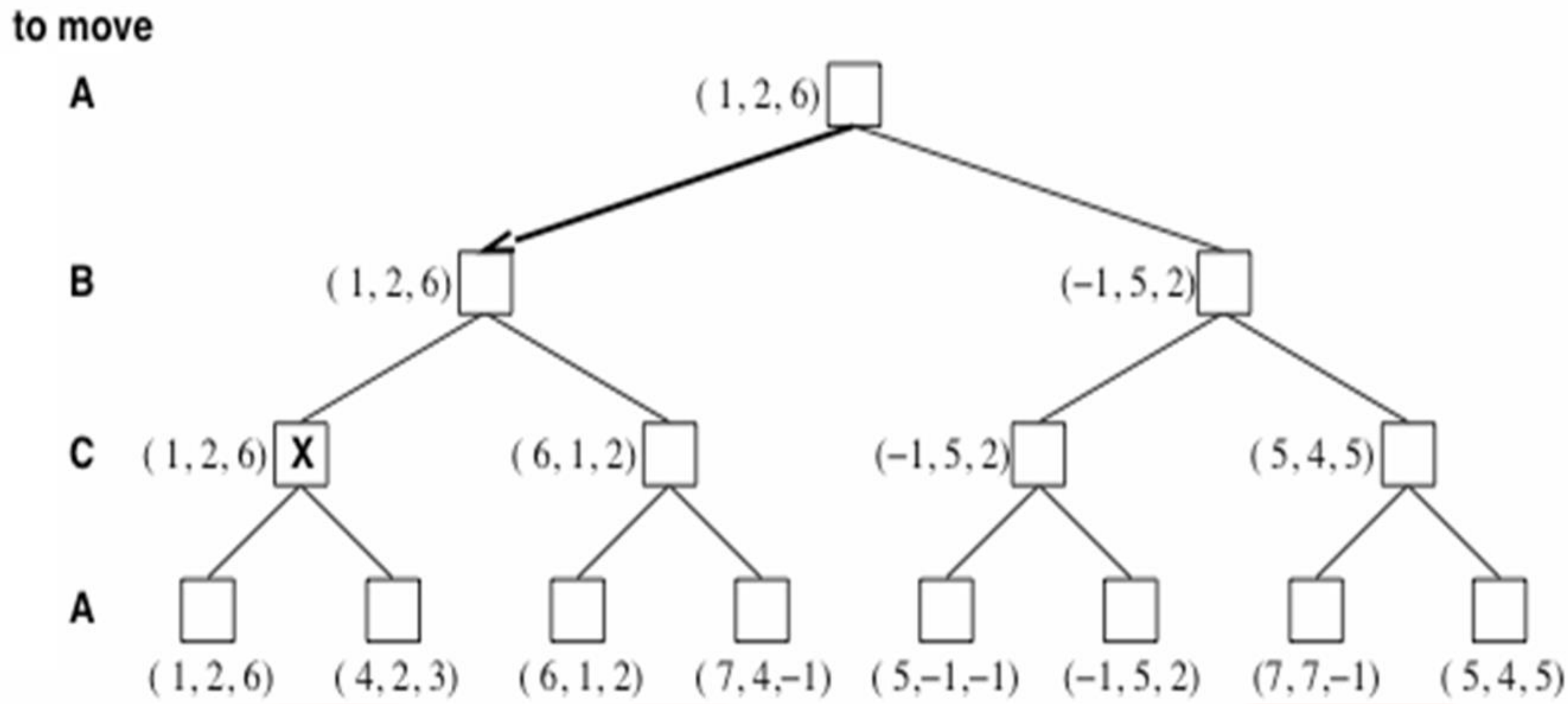
(d) Black to move
White about to lose

- w_i: a weight,
- f_i: a feature of the position

- Pawn: 1,
- Knight: 3,
- Rook: 5,
- Queen: 9

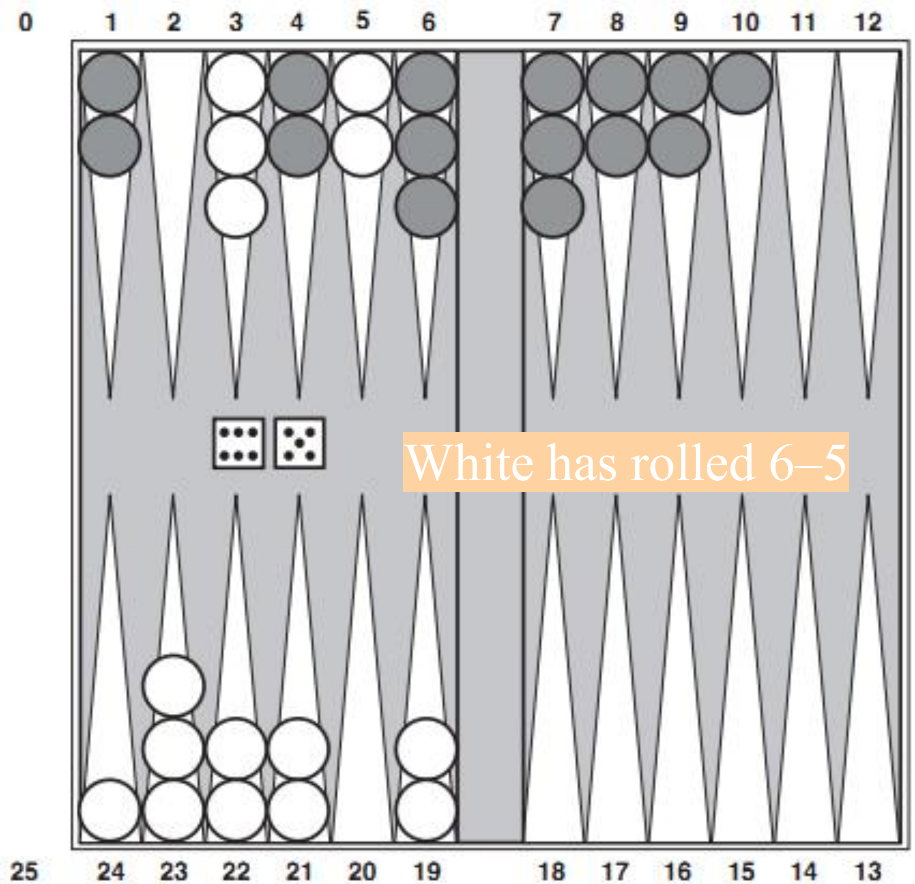
Multiplayer games

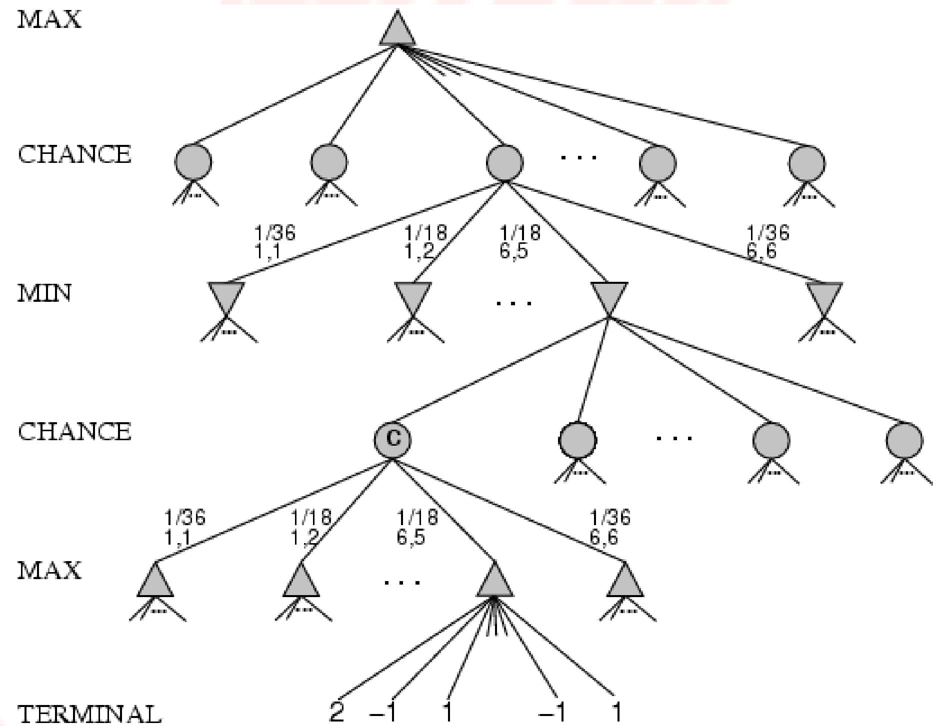
- Games allow more than two players
- Single minimax values become vectors



Games that include chance

Backgammon

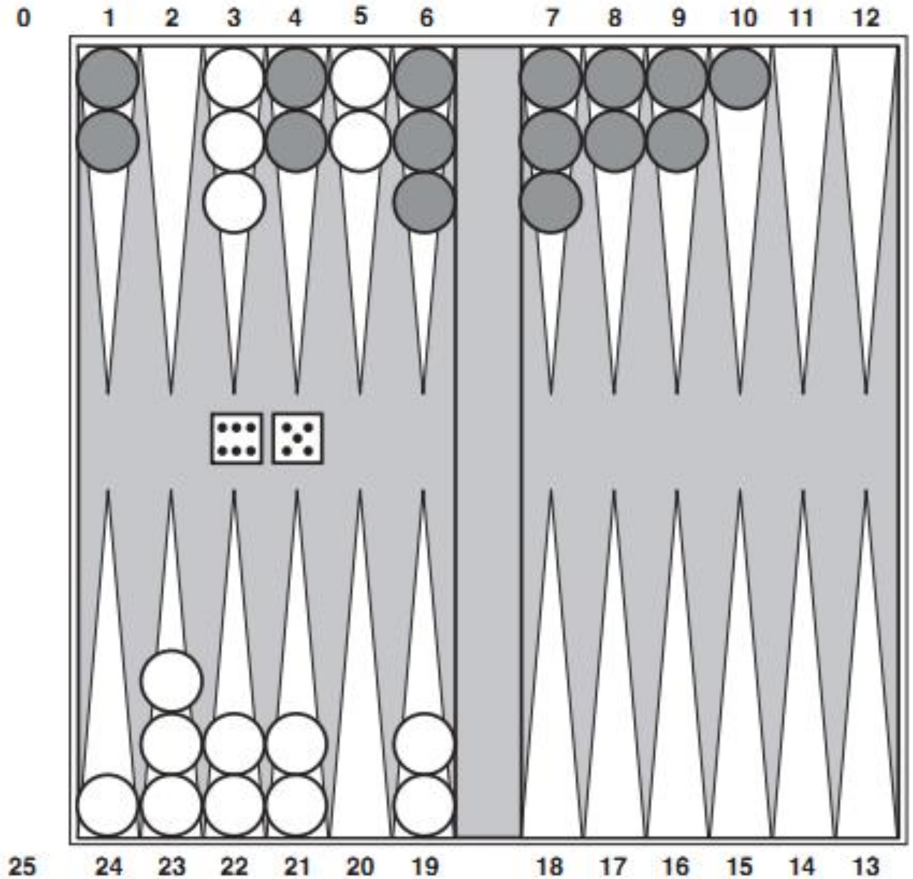


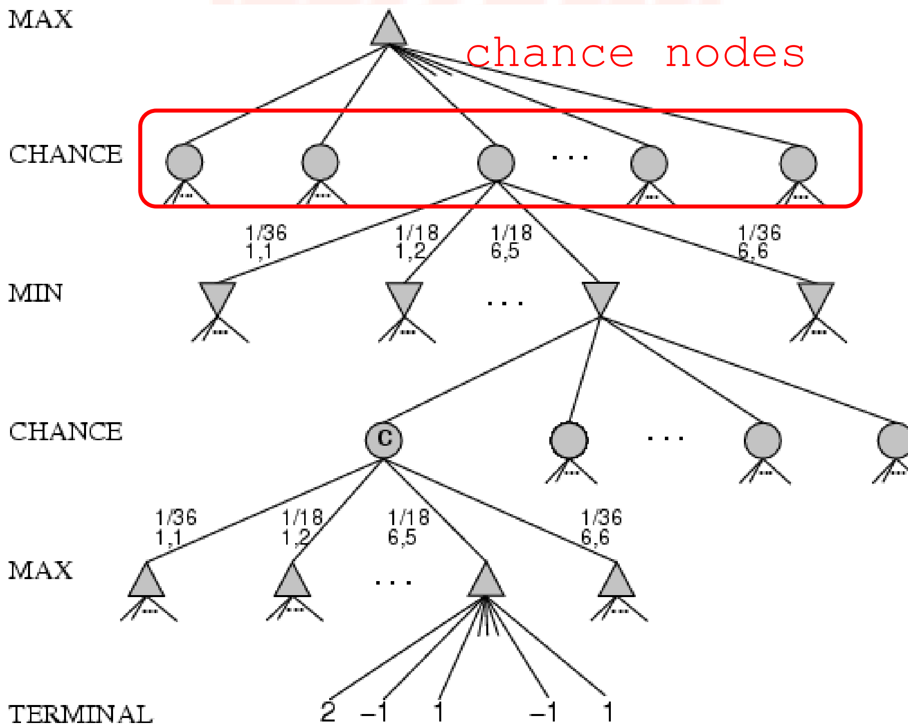


- Possible moves (5–10, 5–11), (5–11, 19–24), (5–10, 10–16) and (5–11, 11–16)

Games that include chance

Backgammon

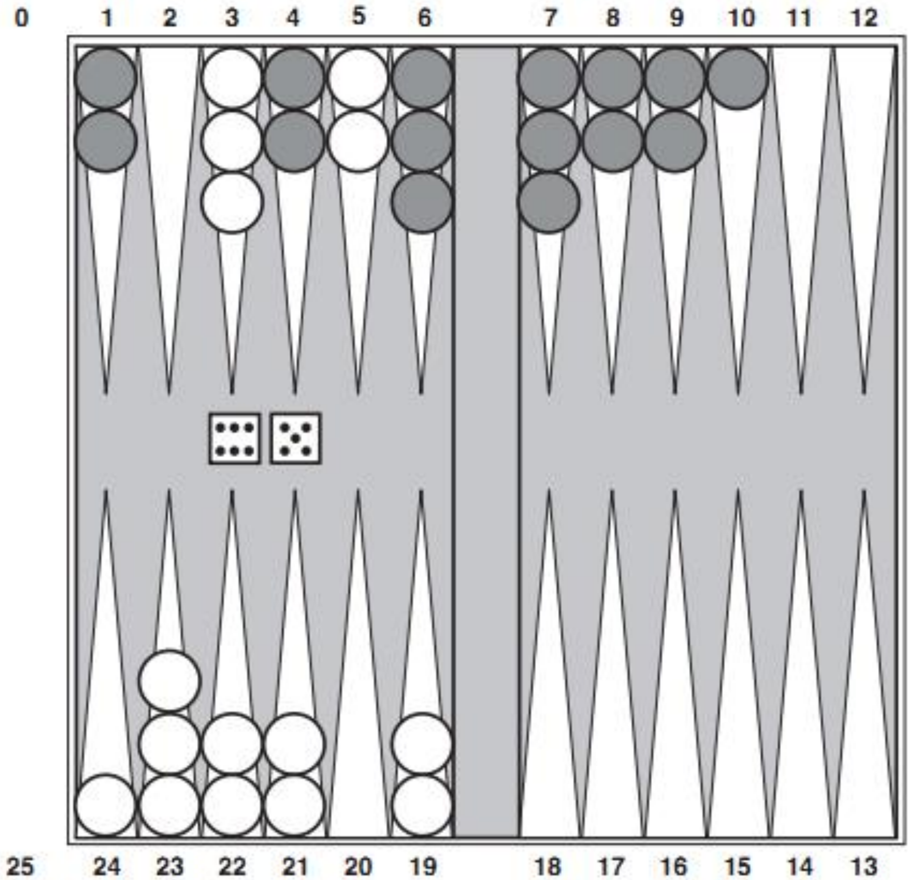


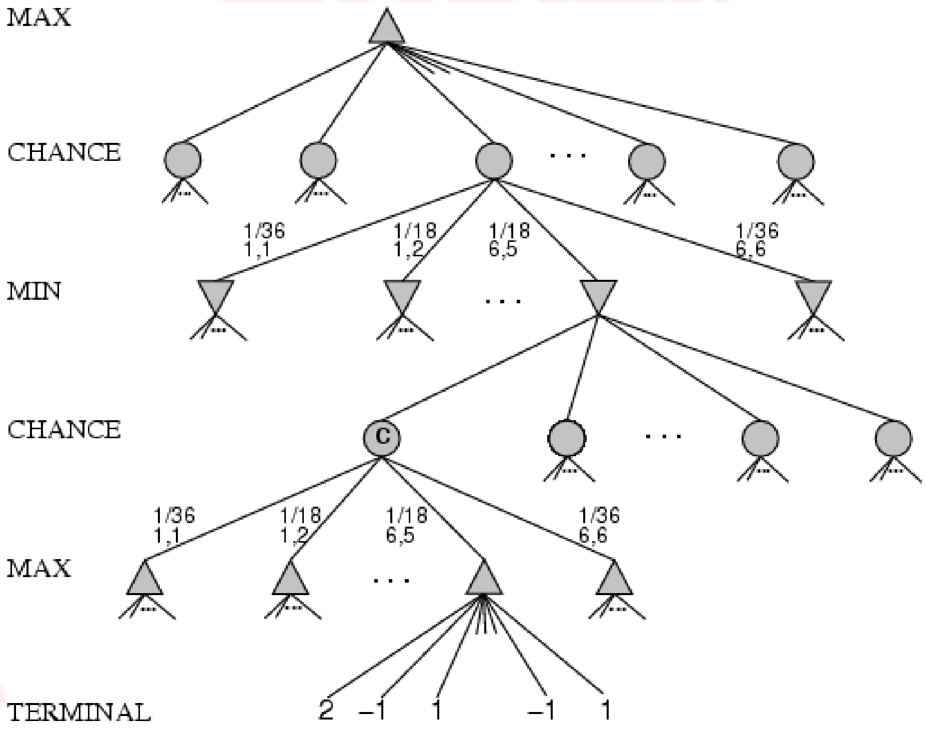


- Possible moves (5–10, 5–11), (5–11, 19–24), (5–10, 10–16) and (5–11, 11–16)
- [1,1], …, [6,6] chance 1/36, all other chance 1/18

Games that include chance

Backgammon





- [1,1], …, [6,6] chance 1/36, all other chance 1/18
- Can not calculate definite minimax value, only expected value

Expected minimax value

EXPECTED-MINIMAX-VALUE(n) =

UTILITY(n) If n is a terminal

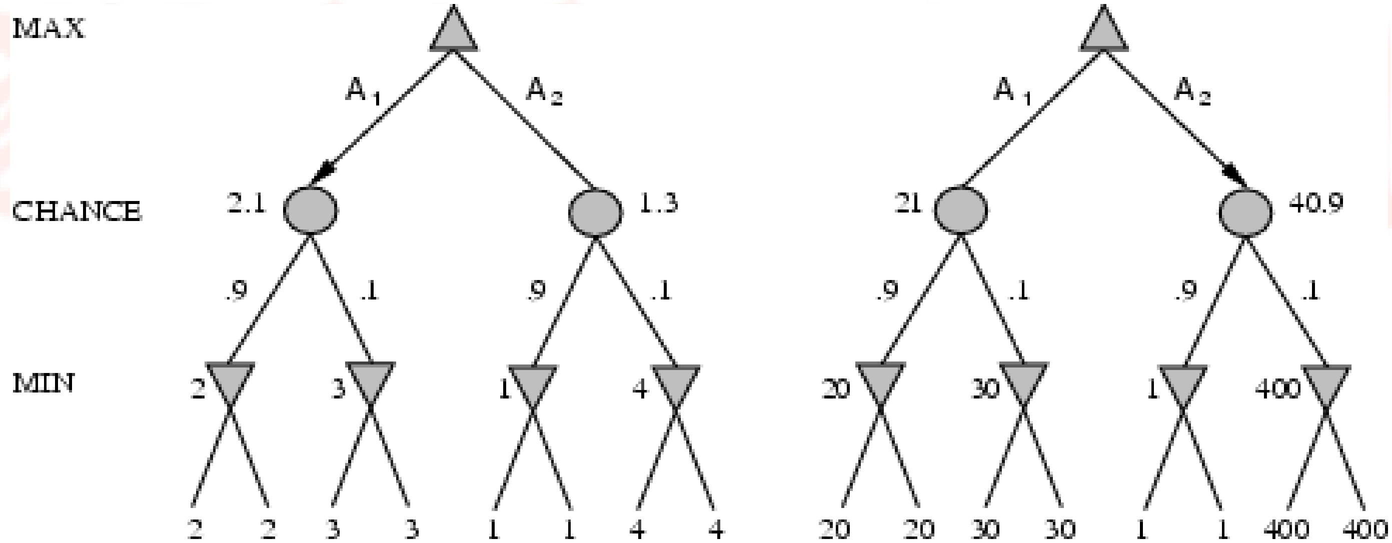
$\max_{s \in successors(n)}$ MINIMAX-VALUE(s) If n is a max node

$\min_{s \in successors(n)}$ MINIMAX-VALUE(s) If n is a min node

$\sum_{s \in successors(n)}$ P(s) . EXPECTEDMINIMAX(s) If n is a chance node

- These equations can be backed-up recursively all the way to the root of the game tree.

Position evaluation with chance nodes



- Left, A1 wins
- Right, A2 wins
- Outcome of evaluation function may not change when values are scaled differently.
- Behavior is preserved only by a positive linear transformation of EVAL.

Summary

- Games are fun (and dangerous)
- They illustrate several important points about AI
  - Perfection is unattainable -> approximation
  - Good idea what to think about
  - Uncertainty constrains the assignment of values to states
- Games are to AI as grand prix racing is to automobile design.

Thank you for your attention!