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Omitted debt risk, financial distress and the cross-section of expected equity returns

Kevin Aretz *, Mark B. Shackleton

Department of Accounting & Finance, Lancaster University, Bailrigg, Lancaster LA1 4YX, UK

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ABSTRACT

The study of Ferguson and Shockley (2003) shows that, if the Merton (1974) model can reflect reality, the omission of debt claims from the market portfolio proxy may explain the poor pricing ability of the CAPM in empirical tests. We critically re-assess this argument by first reviewing existing, but also new avenues through which the Merton (1974) model can point to a systematic bias in market beta estimates. However, we also show that some avenues are diversifiable, and that they all rely on excessive economy-wide default risk to create a non-negligible bias. We then use the Merton (1974) model to proxy for the total debt portfolio, but find that its application in empirical tests cannot improve pricing performance. We conclude that there are (so far) no valid theoretical reasons to believe that omitted debt claims undermine CAPM tests.

1. Introduction

In an interesting article, Ferguson and Shockley (2003) offer evidence that the inability of Sharpe's (1964) capital asset pricing model (CAPM) to price characteristic-sorted portfolios in empirical tests may stem from the omission of debt claims from the market portfolio proxy.[1] Extending the seminal work of Roll (1977), the authors analyze a multiple securities market economy based on Merton's (1974) assumptions, some of the more important ones being that a firm can finance itself through equity and debt, and that the assets of the firm – and hence claims on them – are correctly priced by the CAPM.

Under mild conditions, they then show that the ratio of a firm's equity beta calculated two ways with (i) the market portfolio consisting of all equity and debt claims and (ii) the total equity portfolio alone increases in its leverage ratio. Empirical studies usually rely on an equity beta on the total equity portfolio to measure market risk, and we can hence interpret the former ratio as the market beta bias, i.e., the spread between beta to the true market and to its proxy (i.e., the market without debt claims). As firm characteristics often capture default risk (Chan and Chen, 1991; Shumway, 2001), the positive relation between market beta bias and leverage, a determinant of default risk, could suggest that equity portfolios mispriced by the empirical CAPM suffer from downward-biased beta estimates.

In this paper, we offer new insights on the impact of omitted debt risk on market beta bias. Our study contributes to the literature in four ways. First, we review existing, but also new avenues through which the Merton (1974) model suggests that beta estimates in existing studies testing the CAPM could suffer from a systematic bias. Second, we point out two intuitive reasons as to why these market beta biases may not be large. Third, we offer a calibration exercise of the Merton (1974) model economy to test our suspicions. Finally, we employ the structural model to derive a proxy for the total US debt portfolio, enabling us to test the marginal pricing ability of exposure to this portfolio.

Our study explicitly considers market beta biases induced through two avenues, the well-known one being leverage and the new one being dynamic asset return correlations. Concerning the second avenue, we reveal that higher asset return correlations with distressed firms, but lower ones with safe firms generate a more downward-biased proxy beta. This finding is relevant, as the equity values of firms with similar characteristics tend to move together (Fama and French, 1993), e.g., a small firm should have high (low) asset return correlations with other small (large) firms. In turn, if the size of firms associates negatively with default risk,

* Corresponding author. Tel.: +44 1524 593 402; fax: +44 1524 847 321.
E-mail addresses: k.aretz@lancs.ac.uk (K. Aretz), m.shackleton@lancs.ac.uk (M.B. Shackleton).

[1] Other explanations offered in the literature are that firm characteristics can proxy for exposure to macroeconomic risk factors omitted from the CAPM (e.g., Aretz et al., 2010) or that the firm characteristics' impact on equity prices can be explained by cognitive biases on behalf of investors (e.g., Asem, 2009).

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the proxy beta estimate of the small firm should be biased downwards. We then show that, as the leverage ratio has no *systematic* impact on covariances, its relation with the market beta bias can be diversified away in an economy with many firms. In contrast, the effect of asset return correlations on the market beta bias cannot be diversified away. Notwithstanding these conclusions, our findings also suggest that a strong market beta bias induced through any possible avenue requires a substantial amount of total asset value being exposed to extreme default risk.

To shed more light on the magnitude of the market beta biases, we next calibrate the model economy using either oversimplified adhoc model values (the textbook case) or model values estimated from CRSP and COMPUSTAT data at the end of December 2006 and at some other dates. Using these sets of model values and various permutations, we show that, unless we set the default risk of all firms to excessive levels, the market beta bias, i.e., the ratio of true to proxy equity beta of one single firm, increases only insignificantly with (i) its leverage ratio and (ii) the strength of its asset return correlations with high relative to low default risk firms. Consistent with intuition, we find almost no relation between market beta bias and the leverage ratio, but still one between market beta bias and asset return correlations in an economy with 5000 firms using either the simulated or the real-world data.

We finally employ the Merton (1974) model to imply the debt values of all US firms, enabling us to compute a total debt portfolio. We can interpret this new variable in two ways, either as the theoretically-sound total debt portfolio within the model economy or as an estimate of the real-world one. The second interpretation is less heroic than it may sound: the return of the total debt portfolio is a value-weighted average of those of all debt claims. The evidence reported in Eberhart (2005, p. 420) suggests that the value-weighted average debt pricing error from the Merton (1974) model is less than 1.00%, with the time-invariant component of the error term being differenced away by computing returns, implying that debt returns should be even more accurately estimated than values. Given a realistic total debt portfolio volatility, the pricing error should hence not too strongly impact the return of the debt portfolio. In line with this argument, our implied equity betas on the total debt portfolio are similar to those derived from traded debt quotes in Sweeney et al. (1997, p. 16).

Under both interpretations, the equity beta on the total debt portfolio of one firm depends more strongly on its asset return correlations with other firms than on its default risk, suggesting that asset return correlations are a more important driver of betas than default risk. We then perform asset pricing tests on an empirical version of the CAPM featuring both equity and debt claims. As the expected equity returns are not determined by the structural model, these tests only make sense, if we believe that our implied total debt portfolio is close to the unobservable true one. Unfortunately, our evidence indicates that the implied total debt portfolio cannot improve the pricing ability of the CAPM.

Section 2 reviews the theoretical model and market beta biases. In Sections 3 and 4, we offer the results from our calibration exercise and our analysis using the implied total debt portfolio, respectively. Section 5 reports robustness checks. Section 6 concludes. Proofs are in the Appendix A.

2. The Merton (1974) securities market economy

2.1. Assumptions

Consistent with Ferguson and Shockley (2003), we assume a securities market economy with continuous trading. A firm can finance its assets through equity and debt claims. The indenture of all debt contracts specifies that principle plus interest (F_i, with i being the firm subscript) must be repaid in a single instalment at contract maturity (T). At maturity T, firms are liquidated following strict priority rules. Other assumptions (e.g., asset dynamics are governed by Geometric Brownian motion (GBM), no market imperfections exist, etc.) are equivalent to those reported in Merton (1974). Black and Scholes (1973) were then the first to show that the market value of equity equals:

$$S_i = V_i N(d_1)_i - F_i e^{-r_f T} N(d_2)_i, \tag{1}$$

where V_i and S_i are the asset and equity value of firm i, respectively, r_f is the riskfree rate of return, $(d_1)_i = \left[\ln(V_i/F_i) + \left(r_F + 0.5\sigma_i^2\right)T\right]/\sigma_i\sqrt{T}$ and $(d_2)_i = (d_1)_i - \sigma_i\sqrt{T}$; σ_i is asset volatility and $N(.)$ indicates the standard normal cumulative density function. We can interpret $N(d_2)_i$ as the risk-neutral probability of default. Other properties of the model economy are shown in the Appendix A.

The model economy so far imposes no restriction on the drift term of GBM of firm i, and hence also not on its expected asset return. For the purpose of the study, we however assume that the expected asset return and hence also the expected returns of the two claims follow the CAPM equation. The expected equity return can therefore be written as:

$$E[r_{Si} - r_f] = \beta_{Si} E[r_M - r_f] = \left[\frac{E}{M}\frac{\sigma_E^2}{\sigma_M^2}\beta_{Si}^E + \frac{D}{M}\frac{\sigma_D^2}{\sigma_M^2}\beta_{Si}^D\right] E[r_M - r_f], \tag{2}$$

where r is the return of either firm i's equity claims (S_i) or the market portfolio (M) and β_{Si} is the equity beta on the market portfolio. The second equality splits a firm's equity beta on the market portfolio into its beta on the total equity portfolio $\left(\beta_{Si}^E\right)$ and into its beta on the total debt portfolio $\left(\beta_{Si}^D\right)$, with E,D and M being the total market values of the equity claims, debt claims and assets, respectively, and, obviously, $M = E + D$. Next, σ^2 is the variance of the total equity portfolio (subscript E), the total debt portfolio (subscript D) or the market portfolio (subscript M). Most CAPM tests rely on estimates of β_{Si}^E as proxy for market risk, and we hence call this beta the proxy equity beta.

Using β_{Si}^E as our measure of systematic risk, we induce a bias into the equity beta on the true market portfolio (β_{Si}). We can quantify this bias in the following way:

$$\frac{\beta_{Si}}{\beta_{Si}^E} = \frac{E}{M}\frac{\sigma_E^2}{\sigma_M^2}\left[\frac{\beta_{Si}}{\beta_{Si} - \frac{D}{M}\frac{\sigma_D^2}{\sigma_M^2}\beta_{Si}^D}\right] = \frac{E}{M}\frac{\sigma_E^2}{\sigma_M^2}\left[\frac{\beta_i}{\beta_i - \frac{D}{M}\frac{\sigma_D^2}{\sigma_M^2}\beta_i^D}\right], \tag{3}$$

where the second equality follows from relations shown in the Appendix A. Eq. (3) suggests that a firm's asset beta on the market portfolio (β_i) and that on the debt portfolio $\left(\beta_i^D\right)$ affect the market beta bias. Moreover, the firm's contributions to the market values and volatilities of the total equity, debt and market portfolios also, but probably less strongly influence the bias. One interesting question is whether the market beta bias relates to firm characteristics, e.g., if small firms exhibit stronger biases than large firms, then it would not be surprising that the empirical CAPM fails to price size portfolios.

2.2. Market beta bias and firm characteristics

The main finding of Ferguson and Shockley (2003) is:

Proposition 1. *Assuming the Merton (1974) economy with positive equity betas, the market beta bias of firm i* $\left(\beta_{Si}/\beta_{Si}^E\right)$ *relates positively to the debt instalment* (F_i)*, if and only if:*

$$\frac{2\rho_{S_i,E}^2 - 1}{\sigma_E \rho_{S_i,E}\sqrt{T}} < \frac{N(d_2)_i}{n(d_2)_i}, \tag{4}$$

where $\rho_{S_i,E}$ is the correlation coefficient of the firm's equity claim with the total equity portfolio and n(.) is the standard normal density function. Inequality constraint (4) is fulfilled, if $\rho_{S_i,E} < 0.707$.

Proof. See Ferguson and Shockley (2003). □

As the great majority of firms have empirical correlation coefficients of less than 0.707 with the total equity portfolio, the market beta bias of most firms increases in their leverage ratio and, ceteris paribus, in their default risk. Eq. (3) reveals that, as β_i and F_i are independent, this relation must mainly be driven by β_i^D. In fact, as the equity beta on the total debt portfolio increases monotonically in the debt instalment (Ferguson and Shockley, 2003), there is a positive relation *unless* the positive effect induced through the debt instalment is counteracted by a negative effect induced through the market values and volatilities of the total equity, debt and market portfolios. Inequality (4) is a sufficient condition for the positive effect to prevail over the negative effect.

A priori, Proposition 1 is appealing. Its main message is that the proxy equity betas of firms with high leverage ratios and hence high default risk should experience the strongest downward bias. Moreover, as firm characteristics often reflect default risk (Chan and Chen, 1991; Shumway, 2001), the proposition could explain the failure of the CAPM to price characteristic portfolios in empirical tests. On second thought, one may however doubt the importance of this effect, as the leverage ratio has an idiosyncratic and not a systematic impact on $(\beta_{Si}/\beta_{Si}^E)$. We return to this point.

We next analyze another firm characteristic with a more systematic impact on the market beta bias, i.e., a firm's pattern of asset return correlations with other firms. Our main finding regarding the relation between the market beta bias and firm *i*'s asset return correlation with firm *k* ($\rho_{i,k}$) is:

Proposition 2. *Assuming the Merton (1974) economy with positive equity betas, the market beta bias of firm i $\left(\beta_{Si}/\beta_{Si}^E\right)$ relates positively (negatively) to its asset correlation with firm k ($\rho_{i,k}$), if and only if:*

$$\frac{E}{M}\frac{\sigma_E^2}{\sigma_M^2}\frac{\beta_i^E}{\beta_i}\frac{\left[1-2\frac{V_i}{M}\beta_i\right]}{\left[1-2\frac{V_i}{E}N[d_1]_i\beta_i^E\right]} > (<)N[d_1]_k. \tag{5}$$

If the terms in square parentheses are positive, the ratio on the left-hand side (LHS) of inequality (5) is positive and can, but does not need to be below unity.

Proof. See Appendix A. □

Empirical estimates of the LHS of inequality (5) suggest that it is normally between 0.90 and 1.00.

Although less obvious, Proposition 2 could also entail a relation between market beta bias and firm characteristics. To see this, assume that the LHS of inequality (5) is 0.95 for firm $i = 1$, and that we analyze the asset return correlation of this firm with two other firms, for simplicity $k = 2$ or 3. Firm $k = 2$ is entirely equity financed, implying that its default risk is zero, whereas firm $k = 3$ has a high leverage ratio and hence high default risk. As $N[d_1]_k$ is close to the risk-neutral survival probability of firm *k*, especially for firms with extremely high or low survival probabilities,[2] it is probable that inequality (5) holds with a < sign for firm $k = 2$ and with a > sign for firm $k = 3$. Lowering firm 1's asset return correlation with the low default risk firm ($k = 2$), but raising that with the high default risk firm ($k = 3$) then increases the market beta bias of firm 1. In greater generality, it is hence not unlikely that the strength of asset return correlations with high compared to low default risk firms positively affects the market beta bias.

Proposition 2 may explain the importance of spread portfolios on a default risk proxy for equity pricing. Empirical studies, such as, e.g., Chen et al. (1986), suggest that exposure to the yield change of a corporate bond portfolio long on BAA issues and short on AAA issues, to the change in an equally-weighted economy-wide default risk measure, and to the equity return of an equally-weighted portfolio long on high default risk and short on low default risk firms help to explain the cross-section of average US equity returns. As yield changes are negatively related to returns, exposure to these pricing factors may measure the strength of a firm's correlations with high compared to low default risk firms – and may hence correct for market beta biases in empirical asset pricing tests.

2.3. The strength of market beta biases

We have already cast doubt on the strength of the relation between market beta bias and leverage, as the leverage ratio does not systematically affect the market beta bias. To see this, recall that the market beta bias is the ratio of β_{Si} and β_{Si}^E, i.e., the scaled covariance of firm *i*'s equity return (r_{S_i}) with the return of the market portfolio (r_M) and that with the return of the total equity portfolio (r_E). A change in the leverage ratio (or any other idiosyncratic characteristic) scales the equity return by an identical factor in both numerator and denominator, implying that the market beta bias can also be written as the ratio of β_i and β_i^E, i.e., the scaled covariance between firm *i*'s asset return with the return of the market portfolio and that with the return of the total equity portfolio. However, the asset return does not depend on the leverage ratio: it is exogenously determined. A change in the leverage ratio of firm *i* does therefore only affect one variable in the market beta bias, namely its own equity return within the total equity portfolio return. As the number of firms increases, the contribution of firm *i* to the total equity portfolio is likely to decrease to zero, implying that the dependence of the market beta bias on leverage disappears.

More formally, we can state the relation between leverage and market beta bias as follows:

Corollary 1. *Assuming the Merton (1974) economy with positive equity betas, only non-dominant firms (i.e., each firm's contribution to the total portfolios tends to zero with the number of firms) and firm characteristics drawn from a stable multivariate distribution with finite moments, the partial derivative of the market beta bias of firm i $\left(\beta_{Si}/\beta_{Si}^E\right)$ with respect to its debt instalment (F_i) converges to zero, as the number of firms tends to infinity.*

Proof. See Appendix A. □

For the same reason, the impact of one single asset return correlation of firm *i* with one other firm also disappears in an economy with a large number of firms. However, the relation between market beta bias and asset return correlations does not depend on one single correlation, but instead on firm *i*'s pattern of correlations with all other firms. In other words, we are now not interested in one partial derivative, but in the total derivative of the market beta bias with respect to all correlation pairs. Although the strength of each partial derivative declines with the number of firms, the number of partial derivatives within the total derivative increases, implying that the total derivative does not necessarily converge to zero.

[2] As the spread between d_1 and d_2 equals $\sigma\sqrt{T}$, a positive number, $N[d_1]_k$ overstates a firm's survival probability.

We can formally state the relation between asset return correlations and market beta bias as:

Corollary 2. *Assuming the Merton (1974) economy with positive equity betas, only non-dominant firms (i.e., each firm's contribution to the total portfolios tends to zero with the number of firms) and firm characteristics drawn from a stable multivariate distribution with finite moments, the partial derivative of the market beta bias of firm i* $\left(\beta_{Si}/\beta^E_{Si}\right)$ *with respect to its asset return correlation with one single firm k* ($\rho_{i,k}$) *converges to zero, as the number of firms tends to infinity. However, the total derivative with respect to all possible correlation pairs does not converge to zero.*

Proof. See Appendix A. □

So far, our analysis focuses exclusively on the strength of the impact of two firm characteristics (i.e., the leverage ratio and asset return correlations) on the market beta bias. Instead of analyzing other firm characteristics with possible relations to the market beta bias, we could however also examine whether under realistic conditions scaled debt risk (i.e., $(D/M)\left(\sigma^2_D/\sigma^2_M\right)\beta^D_{Si}$) in Eq. (2) should comprise a significant part of market risk (i.e., β_{Si}) for at least some firms in the model economy. If not, then the relation between market beta bias and any possible firm characteristic must be weak.

To this end, let us define the contribution of scaled debt risk to market risk as:

$$\frac{\frac{D}{M}\frac{\sigma^2_D}{\sigma^2_M}\beta^D_{S_i}}{\beta_{Si}} = \frac{\frac{D}{M}\frac{\sigma^2_D}{\sigma^2_M}\sum_{j=1}^{N}\frac{B_j}{D}\frac{\sigma_{Si,Bj}}{\sigma^2_D}}{\sum_{j=1}^{N}\frac{V_j}{M}\frac{\sigma_{Si,j}}{\sigma^2_M}} = \sum_{j=1}^{N}\frac{V_j\rho_{i,j}\sigma_j}{\sum_{j=1}^{N}V_j\rho_{i,j}\sigma_j}N[-d_1]_j$$
$$= \sum_{j=1}^{N}\omega_{i,j}(V_j,\rho_{i,j},\sigma_j)N[-d_1]_j, \tag{6}$$

where we employ the definition of the equity betas in the first equality, and several relations shown in the Appendix A in the second. Moreover, we define $\omega_{i,j}(V_j,\rho_{i,j},\sigma_j)$ as $(V_j\rho_{i,j}\sigma_j)/\left(\sum_{j=1}^{N}V_j\rho_{i,j}\sigma_j\right)$, whose sum over the N firms is unity. As $d_1 > d_2$ and as cumulative density functions monotonically increase in their argument, we can re-write the last term in Eq. (6) as:

$$\sum_{j=1}^{N}\omega_{i,j}(V_j,\rho_{i,j},\sigma_j)N[-d_1]_j < \sum_{j=1}^{N}\omega_{i,j}(V_j,\rho_{i,j},\sigma_j)N[-d_2]_j, \tag{7}$$

with $N[-d_2]_j$ being firm i's risk-neutral default probability. As a result, the contribution of debt risk to market risk is bounded by a weighted average of risk-neutral default risk in the Merton (1974) economy, implying that the importance of debt risk increases with the number of firms with substantial default risk. If all correlation pairs ($\rho_{i,j}$) and standard deviations (σ_j) were equal to unity, then the bound would reduce to the asset value-weighted average of default risk, probably a very small number. However, if firm i has high asset correlations with high default risk firms and high default risk firms have high standard deviations, then the bound could be higher than the asset value-weighted average.

As a final step, we hence analyze whether realistic magnitudes of default risk can produce a significant debt risk component. Assuming that the risk-neutral default probability is equal (or at least close) to the real-world one, we can estimate the upper bound in Eq. (7) from:

$$\widehat{\mathrm{UB}} = \sum_{j=1}^{N}\omega_{i,j}(V_j,\rho_{i,j},\sigma_j)I(\mathrm{def})_j, \tag{8}$$

where $I(\mathrm{def})_j$ is a dummy variable equal to one, if firm j defaults in the current time period, and zero otherwise. The expectation of the estimator is equal to the true upper bound, and its maximum standard error under independent defaults is: $\sqrt{(1/4)\sum_{j=1}^{N}\omega_{i,j}(V_j,\rho_{i,j},\sigma_j)^2}$.[3] Independence also implies that the estimator is asymptotically normally distributed (Davidson and MacKinnon, 2004, p. 149).

We obtain bankruptcy data from July 2001 to June 2002 (the 12-month period with the greatest number of defaults in our sample period) from bankruptcydata.com. We assume that we can proxy for asset values at the beginning of the period by equity values plus book values of debt. We therefore over-estimate the asset values of distressed firms, artificially boosting the significance of debt risk. We extract market data and accounting data from CRSP and COMPUSTAT, respectively. To be even more conservative, we also increase the product of asset return correlation and standard deviation ($\rho_{i,j}\sigma_j$) uniformly from 0.20 to 0.80 from the largest to the smallest firm. Our estimate of *UB* is then 0.0048, and the upper end of the 95% confidence interval is 0.0878.[4] Our main conclusion is therefore that realistic magnitudes of default risk produce a very low upper bound on the importance of debt risk, even for firms with high asset return correlations with small firms and low asset return correlations with large firms.

One can obviously quibble with our assumptions. However, our main point is that, almost independent of auxiliary assumptions, the Merton (1974) model does *under realistic default risk estimates*, as, e.g., those in Altman (2007), not lead to the prediction that debt risk should significantly matter for equity pricing. We provide further support for our intuition in the remainder of the study.

3. Calibration exercise

We now use a calibration exercise of the Merton (1974) model to more closely examine the relation between firm characteristics and either the scaled equity beta on the total debt portfolio or the market beta bias. To this end, we choose values for a subset of the model parameters, and then derive the values of the remaining model parameters via the relations shown in the Appendix A. To gauge the strength of the relations, we let the leverage ratio of one firm (for simplicity firm 1) or its asset return correlations with high compared to low default risk firms increase from initially low to then high numbers. The change in firm 1's equity beta (or market beta bias) over these numbers then approximates the partial derivative of the equity beta (or market beta bias) with respect to the two firm characteristics.

Our results are in Table 1. In Panels A and B, we choose oversimplified adhoc model values (textbook case) and values estimated from real-world firm data, respectively. Considering the adhoc model values, we select 100 firms, whose asset values are all equal to 1000. In contrast, we let the book values of debt range evenly from 0.1 to 1200 to induce cross-sectional variation in default risk. The standard deviations of all asset returns are 0.25 p.a., and all asset return correlation pairs are likewise 0.25. We assume that all times-to-maturity are 10 years, and that the riskfree rate is 0.04. Regarding the estimated model values, we obtain these using data

[3] The standard error is the square root of $\mathrm{var}\left(\sum_{j=1}^{N}\omega_{i,j}(V_j,\rho_{i,j},\sigma_j)(I(\mathrm{def})_j - N[-d_2]_j)\right)$, where $N[-d_2]_j$ equals the expectation of the bankruptcy indicator variable and therefore the default probability of firm j. Under independence, the variance can be re-written as: $\sum_{j=1}^{N}\omega_{i,j}(V_j,\rho_{i,j},\sigma_j)^2\mathrm{var}(I(\mathrm{def})_j - N[-d_2]_j)$. It is easy to show that the former variance term equals $N[-d_2]_j(1 - N[-d_2]_j)$, whose maximum is 0.25 for $N[-d_2]_j = 0.50$. Although there are some contagion effects in bankruptcy risk (e.g., Jorion and Zhang, 2009), these effects do not seem large, and independent defaults may therefore not be a too strong assumption.

[4] Although not in our sample period, the estimate of *UB* for the time period from July 2008 to June 2009 (featuring a record number of US bankruptcy filings) is 0.0184, with the upper end of the 95% confidence interval being 0.0943.

Table 1
Market beta bias and the Merton (1974) economy.[a]

Panel A: Hypothetical model values (textbook case)								
Summary statistics		# firms	V_i	F_i	σ_i	$\rho_{i,j}$	T	r_F
		100	1000	U[0.1; 1200]	0.25	0.25	10	0.04
Case 1: Variation in the debt instalment of firm 1								
	Min (0.1)	10th	25th	Median	75th	90th	Max (1200)	Spread
Scaled β_{Si}^{D}	0.08	0.09	0.10	0.13	0.15	0.17	0.18	0.10
$\left(\beta_{Si}/\beta_{Si}^{E}\right)$	1.40	1.40	1.41	1.41	1.42	1.42	1.42	0.02
Default risk (in %)	0.00	0.18	4.72	22.08	40.20	49.41	54.77	54.77
Case 2: Variation in the correlations of firm 1 with other firms (F_1 = 600)								
	0.1	0.2	0.4	0.5	0.6	0.8	0.9	Spread
Scaled β_{Si}^{D}	0.09	0.11	0.16	0.19	0.21	0.27	0.29	0.20
$\left(\beta_{Si}/\beta_{Si}^{E}\right)$	1.35	1.37	1.41	1.42	1.44	1.48	1.50	0.14
Default risk (in %)	22.46	22.46	22.46	22.46	22.46	22.46	22.46	0.00
Panel B: Estimated model values (December 2006)								
Summary statistics	Min	10th	25th	Median	75th	90th	Max	Spread
S_i (in millions)	1.36	36.16	132.06	545.74	2282.10	8702.50	446,940	446,939
F_i (in millions)	0.00	0.00	0.08	45.08	494.10	1783.30	305,360	305,360
σ_{S_i}	0.02	0.06	0.08	0.11	0.16	0.22	0.62	0.60
ρ_{S_i,S_j}	−0.56	−0.07	0.02	0.13	0.24	0.33	0.88	1.45
Leverage (F_i/S_i)	0.00	0.00	0.00	0.10	0.35	0.73	176.35	176.35
Default risk (in %)	0.00	0.00	0.00	0.36	13.88	42.39	99.91	99.91
Case 1: Variation in the debt instalment of firm 1 ($S_1 = 2500,\ \sigma_{S_1} = 0.25$)								
	100	250	500	1000	2000	4000	6000	Spread
Scaled β_{Si}^{D}	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
$\left(\beta_{Si}/\beta_{Si}^{E}\right)$	1.10	1.10	1.10	1.10	1.10	1.10	1.10	0.00
Default risk (in %)	2.23	7.24	13.91	22.50	31.75	40.35	44.70	42.48
Case 2: Variation in the correlations of firm 1 with other firms (S_1 = 2500, F_1 = 1500, and $\sigma_{S_1} = 0.25$)								
	0.2	0.3	0.4	0.5	0.6	0.7	0.8	Spread
Scaled β_{Si}^{D}	0.01	0.01	0.02	0.02	0.03	0.03	0.03	0.02
$\left(\beta_{Si}/\beta_{Si}^{E}\right)$	1.10	1.10	1.10	1.10	1.11	1.11	1.11	0.02
Default risk (in %)	27.92	27.92	27.92	27.92	27.92	27.92	27.92	0.00

[a] In this table, we report the impact of a change in firm 1's debt instalment (F_1; Case 1) or in its correlations with other firms ($\rho_{1,j}$; Case 2) on the scaled beta of its equity claim with respect to the total debt portfolio (scaled β_{Si}^{D}) and on its market beta bias $\left(\beta_{Si}/\beta_{Si}^{E}\right)$ within the Merton (1974) economy. The equity claim's beta is scaled by the ratio of total debt value over total market value times the ratio of total debt portfolio variance over market portfolio variance. We also report the Merton (1974) default risk of firm 1 (default risk (in %)). To compute these measures, we require model values for firm 1 and all other firms within the Merton (1974) economy. In Panel A, we use a simple adhoc choice for these model values, i.e., there are 100 firms (# firms), whose asset values (V_i) all equal 1000 and whose debt values (F_i) range uniformly from 0.1 to 1200. Other assumptions are shown under summary statistics, with σ_i being the asset standard deviation, $\rho_{i,j}$ the asset correlation, T the time-to-maturity, and r_F the riskfree rate of return. In Panel B, we use estimates of 1000 random firms from December 2006 to calibrate the Merton (1974) economy, with S_i and F_i being the equity value and the book value of debt in December 2006, respectively. We compute the equity standard deviation (σ_{S_i}) and equity correlation (ρ_{S_i,S_j}) using monthly data from 2003 to 2006. We show the distribution of these estimates under summary statistics. To gauge the impact of a change in the debt value (Case 1), we let the debt value of firm 1 range from a low to a high number, i.e., from 0.1 to 1200 or from 100 to 6000. To gauge the impact of a change in correlations (Case 2), we set the correlations of firm 1 with the 20% of firms with the highest default risk to the number in the table, and those with the 20% of firms with the lowest default risk to one minus the number in the table.

from CRSP and COMPUSTAT from December 2006.[5] In particular, the equity value and the book value of debt are directly available in these databases. We compute equity variances and correlations from the prior 48 months of data. The riskfree rate is from Factset's Economic Database. As we have no information on debt re-payment times, we set the time-to-maturity equal to 10 years. We further assume that firm 1's market value of equity and its equity volatility are close to the mean values for these variables in December 2006 (2500 and 0.25, respectively), but we leave the equity correlations of firm 1 with the other firms identical to those estimated. To keep the calibration exercises manageable, we choose a random sample of 1000 firms when analyzing the real-world data.

We can extract three conclusions from Panel A of Table 1. First, the proxy equity beta is systematically lower than the true market beta (i.e., the market beta bias exceeds unity). Second, an increase in firm 1's debt instalment from 0.1 to 1200 and a resulting increase in default risk from 0.00% to 55.77% raises the scaled equity beta on the total debt portfolio from 0.08 to 0.18 and the market beta bias from 1.40 to 1.42. Consistent with our theoretical analysis, we therefore conclude that, although scaled debt risk is to some extent affected by the leverage ratio, the impact of the leverage ratio on the market beta bias is negligible. Finally, if we set firm 1's debt instalment to 600, a change in the asset return correlation pattern from a 0.10 correlation with the 20% highest, but a 0.90 correlation with the 20% lowest default risk firms to a 0.90 correlation with the 20% highest, but a 0.10 correlation with the 20% lowest default risk firms (the number shown in the table is the correlation with high default risk firms; one minus this number is the one with low default risk firms) increases scaled debt risk from 0.09 to 0.29 and the market beta bias from 1.35 to 1.50. While not influencing default risk, a change in asset return correlations can hence have a more notable impact on both scaled debt risk and market beta bias.

[5] Our findings for several other dates were similar, and are therefore suppressed.

Our conclusions could be significantly affected by our choice of model values. In unreported tests,[6] we hence also separately varied the values of the single model parameters from low to high numbers, e.g., we let the number of firms range from 100 to 5000, we scale up the book values of debt by a factor ranging from 0.25 to 10, and so on. Consistent with our corollaries, the spread in the market beta bias induced through the leverage ratio converges to zero with the number of firms, with it being below 0.001 in an economy with 5000 firms. By contrast, the spread in the market beta bias induced through asset return correlations seems unaffected by the number of firms. Moreover, when we scale up the book values of debt by a factor of ten, then the spread in the market beta biases can become significant, i.e., 0.13 if induced through leverage, but 2.50 if induced through asset return correlations. Neither variation in asset return variance nor correlation appears able to raise spreads in the market beta bias, although extremely short times-to-maturity, e.g., one year, or low riskfree rates of return, e.g., 1%, can raise it. In general, asset return correlations seem more able to create large spreads in the market beta bias than leverage.

Although our initial analysis offers some useful first insights on simple relations within the Merton (1974) economy, the oversimplified adhoc model values fail to accommodate cross-sectional variation in firm characteristics, which may be responsible for the often small spreads in the market beta bias. As a result, we now turn to model values estimated from real-world data in Panel B to analyze the spreads in scaled debt risk and the market beta bias. In the first rows of this panel, we offer summary statistics on the estimated model values. We note that both the equity value and the book value of debt are strongly skewed to the right. Our estimators of equity volatility and correlations can create some extreme numbers, e.g., one firm has an equity volatility of only 2.3%, whereas the equity return correlation between two firms is 0.885. These extreme estimates are probably an artifact of stock illiquidity and the short estimation period. The default probability of a quarter of all firms is above 13.77%, possibly creating a large spread in the market beta bias across the leverage or asset return correlation values.

Using the estimated model values, the magnitude of the market beta bias is an order of magnitude lower than that obtained from the adhoc model values, i.e., the market beta bias is around 1.40 in Panel A, but only around 1.10 in Panel B. Spreads in scaled debt risk and the market beta bias are also lower, e.g., an increase in the debt instalment from 100 to 6000 and a resulting increase in the default probability from 2.23% to 44.70% fails to significantly affect scaled debt risk and the market beta bias, i.e., both variables remain close to 0.01 and 1.10, respectively. If we set the debt instalment to 1500, a change in the asset return correlation pattern from a 0.20 correlation with the 20% highest, but a 0.80 correlation with the 20% lowest default risk firms to a 0.80 correlation with the 20% highest, but a 0.20 correlation with the 20% lowest default risk firms raises scaled debt risk from 0.01 to 0.03 and the market beta bias from 1.10 to 1.11. Hence, although the spreads induced through asset return correlations are larger than those induced through leverage, they are also negligible under the estimated model values.

As a result, we can conclude that, especially in the presence of a large number of firms, the spread in the market beta bias induced through the leverage ratio is of almost no consequence in the Merton (1974) economy. In contrast, that induced through asset return correlations has at least theoretically the potential to significantly distort beta estimates in asset pricing tests. To this end, it however requires a substantial amount of asset value exposed to high default risk.

[6] The findings from these tests, available in an earlier version, can now be obtained upon request.

4. Empirical tests using an implied total debt portfolio

4.1. Data construction and summary statistics

As our calibration exercise abstracts from time-series dynamics in our analysis variables, we now also employ the Merton (1974) model to imply the total debt portfolio from real-world data.[7] We can interpret the implied new variable as either the theoretically-sound total debt portfolio within the model economy or as a proxy for the real-world total debt portfolio. The latter interpretation may not be as heroic as it appears. For example, although empirical studies often suggest that structural models create significant debt pricing (or yield spread) errors for single issues (e.g., Jones et al., 1984; Lyden and Saranati, 2000; Eom et al., 2004), the *average* one implied from them is normally small. In this context, the results of Eberhart (2005) are interesting, revealing that the average and median debt valuation error produced by the Merton (1974) model is only −0.96% and 0.98%, respectively.[8] Although Eberhart (2005) includes non-investment grade bonds in his analysis, we admit that his findings are mostly based on larger and more successful firms, e.g., Jones et al. (1984) indicate that average debt valuation errors can approach 10% for non-investment grade bonds in periods of high interest rates. However, as we eliminate the constant component of the pricing error by taking returns and as the return of the total debt portfolio is a *value-weighted* average of those of the single debt issues (and hence underemphasizes non-investment grade bonds), we are confident that the error in the implied debt portfolio return is not too large for the purpose of our tests.

We use the return of the implied total debt portfolio to analyze both the determinants of the equity beta on the total debt portfolio and its incremental pricing ability in cross-sectional tests of the CAPM. As the former test relies exclusively on variables endogenously specified by the structural model, its findings are not only valid within the model economy, but also within the real-world, if the implied total debt portfolio is a good proxy for the true total debt portfolio. In contrast, the Merton (1974) model does not endogenously specify the expected return. The findings from the latter tests can hence not be interpreted within the context of the model economy,[9] but only within a real-world context, at least if the implied total debt portfolio is close to the real-world total debt portfolio.

To construct the total debt portfolio return, we retrieve daily equity values (S_i) and monthly equity returns (r_{Si}) from CRSP. Annual book values of long-term plus short-term debt (F_i) are from COMPUSTAT. We obtain a proxy for the time-to-maturity (T) of a firm's debt claims via a book value-weighted average of the times-to-maturity of its short-term and its long-term debt, where we assume that the times-to-maturity of short-term debt and long-term debt are one and ten years, respectively. We obtain the 3-month US T-Bill rate (r_f) from Factset's Economic Database. We then use an iterative approach proposed by Vassalou and Xing (2004) to compute each firm's implied asset value and volatility at the end of each month. This approach relies on rolling windows of

[7] It is important to consider time-series variation in our analysis variables, as the evidence in Bruche and González-Aguado (2010) and Tang and Yan (2010) suggests that default risk (and hence probably also the equity betas on the total debt portfolio) co-move with macroeconomic variables, as, e.g., GDP growth. If the market risk premium also relates to these macroeconomic variables, then this may multiply the effect of omitted debt risk on equity prices.

[8] In personal communications with Allan Eberhart, he states that he is confident that the low whole-sample mean debt valuation error produced by the Merton (1974) model is not driven by large positive mean errors in some years, and large negative mean errors in other years.

[9] In the model economy, the relation between expected return and the equity beta on the total debt portfolio can be whatever we want it to be. It hence makes little sense to analyze the model-implied relation.

daily data over the prior twelve months. More concretely, we initially set a firm's asset volatility to its equity volatility calculated over the rolling window. Using this initial guess and other proxy variables, we then imply asset values from the Merton (1974) model for each trading day over the prior twelve months. The time-series of asset values allows us to obtain an updated estimate of asset volatility, which can in turn be employed to update the time-series of asset values. We stop the iteration once the change in asset volatility drops below 0.001.

A firm's debt value (B_i) equals its asset value minus its equity value. We compute the (continuously compounded) return of firm i's debt claims through:

$$r_{Bi,t} = \ln[k \cdot (B_{i,t} + C_{i,t}/12)/B_{i,t-1}], \quad k = (F_{i,t-1}/F_{i,t}), \tag{9}$$

where $C_{i,t}$ denotes the annual interest expense from COMPUSTAT.[10] In Eq. (9), we assume that we can adjust for new debt issuances through the gross percent change in the book value of debt, i.e., we treat a doubling in the book value of debt from $t-1$ to t as equivalent to a doubling in the number of debtholders, leaving the old debtholders with 50% of the debt claims. The return of the total debt portfolio is then a value-weighted average of the returns of all debt claims in the economy.

We use standard methods to form value-weighted characteristic decile portfolios on size, BM and momentum (Fama and French, 1993; Carhart, 1997), with the only exception being that we apply firms from all US exchanges to compute decile breakpoints. Our modification guards us against greatly different numbers of assets in the single decile portfolios. We compute a proxy for a firm's equity (=asset)[11] return correlation with high compared to low default risk firms by first constructing a portfolio long on the 20% of firms with the highest and short on the 20% of firms with the lowest 2-month lagged Merton (1974) default risk in each month (DRSP), and by then calculating a firm's equity return correlation with this portfolio over the prior 60 months. The 2-month gap ensures that equity return and default risk proxy are not mechanically related. Using the first 60 months to construct initial rolling window estimates, our sample period ranges from January 1968 to December 2006 (468 monthly observations).

In Table 2, we report summary statistics on the total equity, debt and market portfolios (Panel A) plus the characteristic portfolios (Panel B). Panel A reveals that total equity value is on average around four times larger than total implied debt value. In line with intuition, the total equity portfolio attracts a higher average return than the total debt portfolio (1.57% vs. 1.33%) with a – surprisingly – slightly lower volatility per month (4.37% vs. 4.80%). Upon further investigation, we find that the high volatility of the total debt portfolio is largely driven by low diversification, i.e., a small number of large debt issues dominate this portfolio. Finally, we also obtain a much lower correlation coefficient between the total equity and the total debt portfolio (0.22) than other studies.

One crucial issue is to establish whether the implied total debt portfolio can capture variation in the true total debt portfolio. The total debt portfolio consists of both the private and public debt claims of large established firms and the (mostly) private debt claims of small risky firms. As a result, if the implied total debt portfolio is an effective proxy variable, then we should find a substantial, but far from perfect correlation between it and a broad US bond index. Using data from the Barclays US aggregate bond index between March 1973 and December 2006 from DataStream International (406 monthly observations), the correlation coefficient equals 0.41. In stable economic periods, we should not experience great volatility in the debt values of large firms, possibly biasing the correlation coefficient towards zero. Hence, we also repeat our computations using only observations whose absolute US bond index return is above the 90th percentile. We then find a correlation coefficient of 0.59.[12]

In Panel B of Table 2, we report time-series averages of several attributes of the characteristic deciles, i.e., their equity return, leverage, Merton (1974) default risk and correlation with DRSP. These portfolio attributes are all computed through value-weighted averages over firms in the portfolios. Although our stock universe and hence our decile portfolios differ from those in other studies, i.e., we only include a firm in this study, if we can compute its Merton (1974) default risk, the average equity return still decreases significantly over the size deciles (−0.88%), and it increases significantly over the BM deciles (0.80%). We also discover a large, but insignificant spread over the momentum deciles (0.48%).[13] Leverage, default risk and correlation with DRSP relate negatively to size, but positively to BM, all at the 99% confidence level. As these three firm attributes should theoretically reflect market beta biases, the betas of firms in size decile 1 or in BM decile 10 should be most strongly downward biased in CAPM tests, offering a possible reason for the inability of the CAPM to price these portfolios in empirical studies. Regarding the momentum deciles, both leverage and default risk decrease significantly over the deciles. Hence, the betas of the loser firms in decile 1 should be more strongly downward biased than those of the winner firms in decile 10, unfortunately only deepening the momentum puzzle.

4.2. Determinants of the equity beta on the total debt portfolio

Our hypothesis that omitted debt risk can explain the inability of the empirical CAPM to price characteristic portfolios relies on the assumption that some firm characteristics have a systematic and profound impact on the equity beta on the total debt portfolio. As a result, we now study the determinants of the equity beta on the total debt portfolio. Not only does this analysis allow us to establish the importance of a firm's default risk and its asset return correlation pattern with other firms for explaining variation in this beta, it also helps us to find out whether default risk and asset correlation patterns can capture – and hence drive out – links between firm characteristics and beta.[14]

We use three methods to estimate a firm's conditional equity beta on the total debt portfolio. First, we run OLS regressions of the equity return on the return of the total debt portfolio over rolling windows of 60 months. We then employ the slope coefficient as the beta estimate for the last month in the rolling window. While intuitive, it is well-known that this approach often suffers from large estimation errors, biasing the findings from subsequent cross-sectional pricing tests. To attenuate this bias, we also employ

[10] Our empirical findings are not sensitive to the inclusion of coupons.

[11] This follows from relations shown in the initial part of the Appendix A.

[12] A cleaner test would be to compute the correlation between the US bond index and a total debt portfolio only implied from the index firms. Unfortunately, DataStream contains no information on the firms in the bond index.

[13] We compare our characteristic effects with those reported on Kenneth French's website (−0.25%, 0.61% and 1.45% over their size, BM and short-term momentum deciles, respectively). The conclusion is that our data creates relatively strong size and relatively weak momentum effects. One reason for the differences could be that we use firms from all US exchanges instead of only NYSE firms to construct the portfolio breakpoints. Using only NYSE firms, our characteristic effects become equal to −0.27%, 0.81% and 0.48%, respectively, rendering the size effect similar to that on Kenneth French's website. However, the momentum effect remains weak. Upon further investigation, we find that the large average equity return of their momentum spread portfolio is driven by an unusual low average equity return of their loser portfolio (0.09% compared to 0.73–1.55% for momentum deciles 2–10). Our loser portfolio attracts a more realistic average equity return (0.78%), thereby explaining the much weaker momentum effect in our study.

[14] To save space, we no longer consider the leverage ratio in this and subsequent tests. In our test findings, the leverage ratio is always of lower importance than default risk and equity correlation with other firms.

Table 2
Summary statistics on the equity and debt data.[a]

Panel A: Summary statistics on equity and implied debt portfolios

	Mean	StD	Skew	Kurt	P1	P10	Q1	M	Q3	P90	P99
Total equity value	3793	4511	1.25	0.07	186.4	297.3	629.0	1475	5437	12,041	14,881
Total debt value	1022	1305	1.64	1.61	26.72	44.79	162.2	361.9	1315	3517	4643
Equity return	1.57	4.37	−0.22	1.78	−10.23	−3.54	−1.15	1.67	4.48	6.69	12.61
Implied debt return	1.33	4.80	0.46	5.38	−11.23	−3.58	−1.12	1.23	3.60	6.36	18.23
Implied market return	1.55	3.84	−0.22	2.76	−7.19	−2.89	−0.72	1.59	3.79	5.95	11.07

Panel B: Summary statistics on characteristic-based portfolios

Decile portfolios	1	2	3	4	5	6	7	8	9	10	Spread
Market capitalization (size) deciles:											
Ret (in %)	1.80***	1.34***	1.35***	1.25***	1.35***	1.21***	1.15***	1.14***	1.10***	0.92***	−0.88***
Lev (D/E)	1.01***	0.80***	0.76***	0.69***	0.65***	0.63***	0.60***	0.55***	0.55***	0.41***	−0.60***
Def risk (in %)	36.05***	31.45***	27.90***	25.38***	22.58***	20.43***	18.76***	16.88***	15.51***	9.06***	−26.99***
Corr with DRSP	0.173***	0.161***	0.164***	0.16***	0.149***	0.14***	0.124***	0.112***	0.096***	0.060***	−0.113***
Book-to-market (BM) deciles:											
Ret (in %)	0.75***	0.90***	1.04***	1.05***	1.05***	1.10***	1.22***	1.28***	1.42***	1.55***	0.80***
Lev (D/E)	0.14***	0.25***	0.37***	0.48***	0.56***	0.67***	0.72***	0.87***	1.09***	1.78***	1.64***
Def risk (in %)	6.95***	8.47***	10.36***	11.57***	13.66***	14.76***	14.45***	17.13***	20.90***	27.77***	20.82***
Corr with DRSP	0.038**	0.056***	0.086***	0.11***	0.123***	0.133***	0.141***	0.155***	0.173***	0.198***	0.159***
Momentum deciles:											
Ret (in %)	0.78**	0.76***	0.91***	0.87***	0.89***	0.94***	0.98***	1.01***	1.06***	1.26***	0.48
Lev (D/E)	0.90***	0.74***	0.54***	0.51***	0.47***	0.50***	0.42***	0.39***	0.40***	0.33***	−0.57***
Def risk (in %)	41.99***	31.78***	21.15***	17.39***	13.51***	11.34***	8.44***	7.17***	6.37***	7.26***	−34.73***
Corr with DRSP	0.136***	0.136***	0.135***	0.128***	0.126***	0.117***	0.114***	0.117***	0.111***	0.113***	−0.023

[a] In this table, we report summary statistics on both portfolios comprising either all equity claims, debt claims or assets (i.e., equity plus debt claims) (Panel A) and on portfolios sorted on several firm characteristics (Panel B). We compute the market value and hence the return of all debt claims from the Merton (1974) model. In Panel A, we report mean, standard deviation (StD), skewness (Skew), kurtosis (Kurt), the first percentile (P1), the tenth percentile (P10), the first quartile (Q1), the median (M), the third quartile (Q3), the 90th percentile (P90) and the 99th percentile (P99) over our sample period for both the market value and the value-weighted return. In Panel B, we offer the time-series mean of several characteristics of value-weighted decile portfolios sorted on size, book-to-market and momentum. These characteristics are their equity return (ret), leverage ratio (lev), Merton (1974) default risk (def risk), and their correlation with a default risk spread portfolio (corr with DRSP). The leverage ratio is defined as the book value of total debt over the market value of equity. The default risk spread portfolio is long on the 20% of firms with the highest 2-month lagged Merton (1974) default risk, and short on those with the lowest. We re-balance this portfolio on a monthly basis. We compute the correlation with the default risk spread portfolio using a rolling window containing the prior 60 months of equity return data. Standard errors are corrected via the approach of Newey and West (1987), with the lag parameter set equal to 12. Our sample period ranges from January 1968 to December 2006.
* Denote statistical significance at the 90% confidence level.
** Denote statistical significance at the 95% confidence level.
*** Denote statistical significance at the 99% confidence level.

two other methods to estimate conditional betas. Under the first alternative method, we assume that conditional betas are an affine function of lagged economy-wide instruments including a constant, and obtain the conditional beta from full-sample OLS regressions of the equity return onto the return of the total debt portfolio interacted with the instruments (e.g., Ferson and Harvey, 1999). Our one-month lagged instruments are the S&P 500 dividend yield, the default yield spread, the term yield spread and the riskfree rate.[15] Under the second alternative method, we assume that a firm's beta is equal to that of a large characteristic portfolio containing the firm. This method can be recommended, if the estimation error from the portfolio regression plus the difference in beta between firm and portfolio is smaller than the estimation error from the firm regression (e.g., Fama and French, 1992). To employ this method, we first form 100 portfolios sequentially sorted on size and then BM. We then run full sample OLS regressions of each portfolio return onto the return of the total debt portfolio. We finally assign the resulting beta of each portfolio to all firms included in the portfolio.

In Table 3, we reveal the results from Fama and MacBeth (1973) (FM) regressions of a firm's conditional equity beta on the total debt portfolio on size, BM, momentum, default risk and correlation with DRSP, with the beta estimates from the three methods used in Panels A–C, respectively. Plain numbers are estimates, and the stars denote their significance levels. Significance levels are computed following Newey and West (1987), with the lag parameter

[15] The default yield spread is the difference between the yield of a corporate bond portfolio containing BBB-rated issues and that of a corporate bond portfolio containing AAA-rated issues. The term yield spread is the difference between the yield of a 10-year and that of a 1-year Treasury Bond. The riskfree rate of return is the return of a 3-month Treasury Bill. We retrieve the dividend yield from Robert Shiller's website, the yield on the bond portfolios from the website of the Board of Governors and the riskfree rate from Factset's Economic Database.

Table 3
Drivers of the equity beta on the total debt portfolio.[a]

Ind. vars.	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6		Model 7		Model 8	
	est sl	%p	est sl	%p	est sl	%p	est sl	%p	est sl	%p	est sl	%p	est sl	%p	est sl	%p
Panel A: 60-period rolling window regressions:																
Constant	0.37***	0.98	0.29***	0.99	0.30***	1.00	0.28***	1.00	0.26***	0.95	0.31***	0.96	0.26***	0.94	0.25***	0.95
Size	−0.03***	0.80									−0.02***	0.65				
BM			0.06	0.22									−0.16***	0.04		
Mom					−0.38**	0.31									−0.19	0.37
pDD							0.98***	0.58			0.42**	0.34	0.74***	0.45	0.63***	0.36
Corr (DRSP)									0.40***	0.71	0.35***	0.63	0.37***	0.70	0.38***	0.70
Mean(R^2) (%)	2.35		0.26		1.49		0.92		3.99		5.68		4.62		5.85	
Panel B: Instrumental variables regressions:																
Constant	0.44***	0.96	0.37***	0.94	0.34***	0.95	0.34***	0.95	0.30***	0.93	0.39***	0.94	0.31***	0.93	0.28***	0.92
Size	−0.03***	0.77									−0.02***	0.68				
BM			−0.15***	0.09									−0.23***	0.06		
Mom					0.23	0.49									0.41***	0.48
pDD							0.56***	0.43			−0.03	0.17	0.48***	0.37	0.72***	0.34
Corr (DRSP)									0.16**	0.56	0.11	0.52	0.15*	0.57	0.14*	0.55
Mean(R^2) (%)	2.27		0.31		1.33		0.76		3.00		5.01		3.87		4.80	
Panel C: Whole-period portfolio regressions:																
Constant	0.30***	1.00	0.26***	1.00	0.26***	1.00	0.26***	1.00	0.25***	1.00	0.30***	1.00	0.24***	1.00	0.24***	1.00
Size	−0.01***	1.00									−0.02***	1.00				
BM			0.08***	0.96									0.12***	0.95		
Mom					−0.03*	0.30									0.05***	0.54
pDD							0.02***	0.92			−0.01***	0.03	0.01***	0.70	0.02***	0.90
Corr (DRSP)									0.05***	0.75	0.01***	0.23	0.04***	0.72	0.05***	0.75
Mean(R^2) (%)	39.74		2.86		1.10		2.50		6.04		47.74		11.75		9.02	

[a] In this table, we report the findings from Fama and MacBeth (1973) (FM) regressions of the conditional equity claim beta on debt $\left(\beta_{Si}^{D}\right)$ onto several explanatory variables. In Panel A, we construct the conditional beta by, first, performing 60-month rolling window regressions of the equity return onto a constant and the return of the total debt portfolio and by, second, assigning the resulting slope coefficient to the last month of the rolling window. In Panel B, we construct the conditional beta from instrumental variables regressions, modeling the conditional beta as a linear function of lagged economy-wide indicators. In Panel C, we set the conditional beta of a firm equal to the full sample beta of the portfolio containing the firm at that point in time. To facilitate the last method, we sort firms into 100 two-way sorted size and book-to-market portfolios. Our independent variables are various combinations of size, book-to-market (BM), momentum (Mom), default risk (pDD) and correlation with a default risk spread portfolio (corr (DRSP)). See the footnote of Table 2 for more information on these variables. We compute standard errors using the method of Newey and West (1987), with the lag parameter set equal to 59 (Panel A) and 12 (Panels B and C). %p shows the percent of cross-sectional OLS regressions featuring a significant slope coefficient (at the 5% error level) with the hypothesized sign ('−' for size and '+' for all other variables). Mean(R^2) is the average adjusted R^2 from the OLS regressions. Our sample period ranges from January 1968 to December 2006.
* Denote statistical significance (sl) at the 90% confidence level.
** Denote statistical significance (sl) at the 95% confidence level.
*** Denote statistical significance (sl) at the 99% confidence level.

l set equal to 59 in Panel A and to 12 in Panels B and C. We also report the proportion of OLS regressions in which the slope coefficient on the explanatory variable is significant and has the correct sign (%p) (i.e., beta should decrease in size, but increase in all other variables) and the average adjusted R^2 from the OLS regressions (Mean(R^2)).

We can extract several conclusions from Table 3. First, we obtain a strong relation between size and the equity beta on the total debt portfolio across Panels A–C, suggesting that the proxy equity betas of small firms are more strongly downward biased than those of large firms. However, the relation between beta and BM or momentum is less convincing, e.g., the relation with BM is insignificant in Panel A, significant with the wrong sign in Panel B and significant with the right sign in Panel C. Given that Table 2 reveals that BM and default risk are significantly positively related and that beta estimates are constructed from the Merton (1974) model, the weak association between beta estimates and BM is remarkable. In general, it seems that omitted debt risk is of no great importance for the pricing of BM and momentum deciles. Second, our data confirm that default risk and correlation with DRSP both explain variation in beta estimates. However, although correlation with DRSP is a crude proxy for a firm's correlation pattern with other firms, it is the more important determinant of variation in beta, e.g., see the proportion of slope coefficients with the right sign and the mean R^2. Finally, the inclusion of either size, BM or momentum together with default risk and correlation with DRSP in the regression model indicates that these variables capture only partially overlapping information, i.e., in the majority of cases neither set of variables can drive out the explanatory power of the other set.

Our general conclusion is hence that of the three variables used to form equity portfolios only a firm's size can consistently explain variation in the equity beta on the total debt portfolio. Interestingly, some fraction of the variation explained by size does not appear to be related to either default risk and asset return correlations, but instead to other unidentified variables.

4.3. Cross-sectional pricing ability

We finally also study the relation between a firm's average equity return, a proxy for the expected equity return, and its equity betas on the total equity portfolio (equity risk), on the total debt portfolio (debt risk) and on the market portfolio (market risk). Since the expected equity return is not endogenously specified by the Merton (1974) model, these tests are only meaningful, if we believe that the implied total debt portfolio is a reasonable proxy for the true total debt portfolio.

In Table 4, we use FM regressions of single firms' excess equity return, i.e., raw return minus riskfree rate, onto our risk proxies. We also separately include each firm's size, BM and momentum

Table 4
Pricing ability on single equity claims.[a]

Ind. vars.	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6		Model 7		Model 8		Model 9	
	est	sl	est	sl	est	sl	est	sl	est	sl	est	sl	est	sl	est	sl	est	sl
Panel A: 60-period rolling window regressions:																		
Constant	0.59	***	0.57	***	0.58	***	1.10	***	1.10	***	1.09	***	1.13	***	1.13	***	1.12	***
β_{Si}^{E}	0.29		0.35				0.30		0.37				0.30		0.36			
β_{Si}^{D}			0.12						−0.01						−0.03			
β_{Si}					0.27						0.26						0.26	
Size							−0.14	***	−0.14	***	−0.14	***	−0.16	***	−0.16	***	−0.16	***
BM							0.24	***	0.23	***	0.24	***	0.23	***	0.22	***	0.23	***
Mom													0.71	***	0.64	***	0.71	***
Average(R^2) (%)	4.58		6.55		4.55		6.45		8.30		6.42		7.09		8.91		7.07	
Panel B: Instrumental variables regressions:																		
Constant	−0.04		0.01		−0.11		0.49		0.54	*	0.41		0.57	*	0.60	**	0.49	
β_{Si}^{E}	0.92	***	0.81	***			0.97	***	0.89	***			0.96	***	0.88	***		
β_{Si}^{D}			0.63	***					0.50	***					0.50	***		
β_{Si}					0.87	***					0.90	***					0.88	***
Size							−0.18	***	−0.17	***	−0.17	***	−0.20	***	−0.19	***	−0.19	***
BM							0.35	***	0.32	***	0.35	***	0.33	***	0.31	***	0.34	***
Mom													0.58	***	0.49	***	0.56	***
Average(R^2) (%)	4.63		6.66		4.58		6.46		8.32		6.38		7.04		8.88		6.97	
Panel C: Whole-period portfolio regressions:																		
Constant	3.10	***	2.13	***	3.01	***	4.38	***	4.84	***	4.54	***	4.86	***	5.37	***	5.05	***
β_{Si}^{E}	−2.04	***	−2.54	***			−2.28	***	−2.12	***			−2.66	***	−2.47	***		
β_{Si}^{D}			5.76	***					−1.98	***					−2.26	***		
β_{Si}					−1.74	***					−2.13	***					−2.49	***
Size							−0.22	***	−0.25	***	−0.23	***	−0.25	***	−0.28	***	−0.26	***
BM							0.04		0.04		0.04		−0.01		0.00		0.00	
Mom													0.64	***	0.65	***	0.64	***
Average(R^2) (%)	0.71		1.41		0.70		2.52		2.57		2.52		3.34		3.39		3.34	

[a] In this table, we report the findings from Fama and MacBeth (1973) (FM) regressions of excess equity returns (i.e., equity return minus riskfree rate) on various combinations of pricing factors. These pricing factor are the conditional equity beta on the total equity portfolio $\left(\beta_{Si}^{E}\right)$, that on the total debt portfolio $\left(\beta_{Si}^{D}\right)$, that on the market portfolio (β_{Si}), size, book-to-market (BM) and momentum (Mom). In Panel A, we construct the conditional beta by, first, performing 60-month rolling window regressions of the equity return onto a constant and the return of the total debt portfolio and by, second, assigning the resulting slope coefficient to the last month of the rolling window. In Panel B, we construct the conditional beta from instrumental variables regressions, modeling the conditional beta as a linear function of lagged economy-wide indicators. In Panel C, we set the conditional beta of a firm equal to the full sample beta of the portfolio containing the firm at that point in time. To facilitate the last method, we sort firms into 100 two-way sorted size and book-to-market portfolios. We compute standard errors using the method of Newey and West (1987), with the lag parameter set equal to 59 (Panel A) and 12 (Panels B and C). Average(R^2) is the average adjusted R^2 from the cross-sectional OLS regressions. Our sample period ranges from January 1968 to December 2006.

* Denote statistical significance (sl) at the 90% confidence level.

** Denote statistical significance (sl) at the 95% confidence level.

*** Denote statistical significance (sl) at the 99% confidence level.

to analyze whether our risk proxies can drive out the pricing ability of these variables. Similar to Table 3, we apply betas estimated from the rolling window, the instrumental variables and the portfolio methods in Panels A–C, respectively. Plain numbers are estimates; the corresponding stars indicate their significance levels computed from the approach of Newey and West (1987), with the lag parameter l set equal to 59 in Panel A and to 12 in Panels B and C. At the bottom of each panel, we report the average adjusted R^2 over the cross-sectional OLS regressions as a model diagnostic. In the absence of debt risk (models 1, 4 and 7), the CAPM performs badly in our tests, i.e., the constant (alpha) is statistically and economically significant and size, BM and momentum explain equity returns even after controlling for risk. Two exceptions are that the constant is always insignificant at the 95% confidence level in Panel B (but the estimates on size, BM and momentum are significant) and that equity risk can drive out the explanatory power of BM in Panel C (but the estimates on the constant, size and momentum are significant).

If we separately add debt risk (models 2, 5 and 8), its risk premium is insignificant in Panel A, but significant in Panels B and C, although the risk premia estimates in Panel C appear somewhat large in magnitude and can be negative. However, in no single case can the separate inclusion of debt risk reduce the constant to zero or drive out the pricing ability of one of the three firm characteristics. Even worse, if we combine equity and debt risk into market risk following Eq. (2) and then use market risk as our single pricing factor (models 3, 6 and 9), our empirical findings collapse to those obtained from the CAPM with only equity risk. One reason for this is that total equity value is far larger than total debt value. In addition, the spreads in the equity beta on the total debt portfolio over firm characteristics are almost negligible, so that the market beta estimate no longer reflects them.

Our pricing tests so far rely on instrumental variables and portfolio approaches to alleviate the impact of estimation error in beta estimates. However, as the betas of characteristic portfolios are in general more accurately estimated and more stable over time than those of firms, another method is to use portfolios instead of single firms as test assets. As a result, Fig. 1 offers the full-sample OLS equity betas on the total equity, the total debt and the market portfolio of decile portfolios sorted on size (Panel A), BM (Panel B) and momentum (Panel C). In line with other studies, the relations between equity risk and the firm characteristics are too weak for equity risk to be able to explain the spreads in average equity returns over the decile portfolios in Table 2. Debt risk declines with size (good) and momentum (bad) and increases with BM (good). Although the decrease in the equity beta on the total debt portfolio over size is significant, its value is only −0.12 (not shown). The decline over BM (0.07) is insignificant, while that over momentum

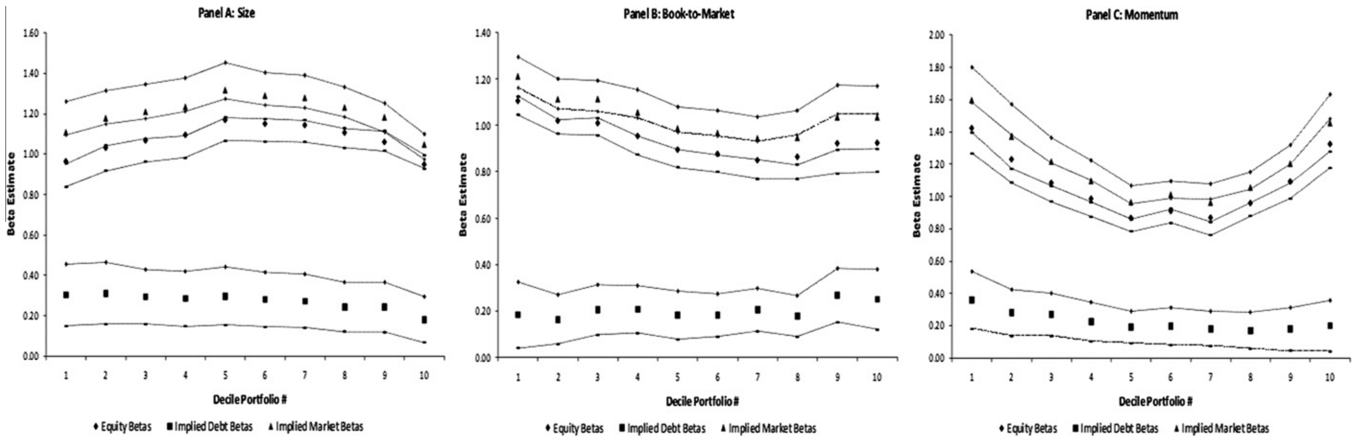

Fig. 1. In this figure, we show the exposures of equity decile portfolios sorted on size (Panel A), book-to-market (Panel B) and momentum (Panel C) on the total equity portfolio (diamond), the total implied debt portfolio (square), and the implied market portfolio containing both equity and debt claims (triangle). The two lines above (+) and below (−) each set of exposures represent the 95% confidence intervals of the estimate.

Table 5
Pricing ability on equity portfolios.[a]

	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6	
	est	sl	est	sl	est	sl	est	sl	est	sl	est	sl
r_E												
sdf-est	3.86 ***		−4.42 ***				4.46 ***		−0.88			
rp-est	0.78 ***		0.37				0.53 ***		0.25			
r_D												
sdf-est			24.39 ***						14.78 ***			
rp-est			7.25 ***						4.35 ***			
r_M												
sdf-est					4.52 ***						5.26 ***	
rp-est					0.71 ***						0.55 ***	
r_{HML}												
sdf-est							3.64 ***		1.48		3.39 ***	
rp-est							0.33 ***		0.26		0.33***	
r_{SMB}												
sdf-est							10.03 ***		7.08 ***		9.77 ***	
rp-est							0.59 ***		0.78 ***		0.59 ***	
J-test	0.17		0.31		0.18		0.23		0.28		0.25	
R^2 (%)	−59.48		64.64		−55.26		42.30		54.38		40.56	

[a] In this table, we report the findings from Cochrane (2001) stochastic discount factor/GMM estimations of various empirical asset pricing models on 25 two-way sorted size and book-to-market portfolios. Our pricing factors consist of the return of the total equity portfolio (r_E), that of the total debt portfolio (r_D), that of the market portfolio (r_M), SMB (r_{SMB}) and HML (r_{HML}). Our test assets and SMB and HML are from Kenneth French's website. Their definitions can also be found on the website. We denote the loadings on the stochastic discount factor by 'sdf-est' and the risk premia by 'rp-est'. At the bottom of the table, we also report the R^2 from an OLS regression of the average portfolio return onto the model forecast (R^2) and the test of the over-identifying restrictions (*J*-test). Standard errors are corrected using the approach of Newey and West (1987), with the lag parameter set equal to 12. The sample period ranges from January 1963 to December 2006.
**, * Denote statistical significance (sl) at the 95% anf 90% confidence level, respectively.
*** Denote statistical significance (sl) at the 99% confidence level.

(−0.16) is significant with the wrong sign. Due to the low spreads in debt risk, the patterns of market risk over the decile portfolios are almost equivalent to those of equity risk, suggesting that debt risk should not greatly modify the findings of asset pricing tests.

In Table 5, we report the findings from Cochrane's (2001) stochastic discount factor/GMM approach on 25 equity portfolios two-way sorted on size and BM. We report loadings on the stochastic discount factor and monthly risk premia together with significance levels. Model fit is evaluated with a *J*-test and the unadjusted R^2 from an OLS regression of average equity returns on the model forecast. We further test whether the risk factors can drive out the explanatory power of SMB and HML. Our findings suggest that, in the absence of debt risk, the CAPM cannot price the 25 portfolios, i.e., although equity risk is significant, the unadjusted R^2 is highly negative (model 1). Separate inclusion of debt risk improves model fit considerably, with both loadings on the stochastic discount factor and the debt premium being significant and the R^2 shooting up to 64.64% (model 2), suggesting that debt risk is related to the average equity returns of the 25 portfolios. However, as debt risk increases only weakly over (e.g.) size (see Fig. 1), the model requires an unreasonable debt risk premium (7.25% per month) to fit equity prices. In fact, if we combine equity and debt risk into market risk and then use market risk as pricing factor (model 3), the pricing ability of the CAPM drops back to that of the CAPM with only equity risk. Consistent with these observations, only the CAPM separately featuring equity and debt risk can drive out the explanatory power of one firm characteristic, BM (model 5). In conclusion, the portfolio tests also fail to support the notion that omitted debt risk matters substantially for equity pricing.

5. Robustness checks

At the very least, our study shows that the Merton (1974) model should not lead us to believe that the omission of debt risk from the market portfolio proxy can be responsible for the low pricing ability of the CAPM in empirical tests. One obvious question then is whether more complex structural models should lead us to this belief. To offer some preliminary evidence, we also construct an implied total debt market portfolio from the more sophisticated models of Leland (1994) and Leland and Toft (1996), allowing for more realistic debt structures, market imperfections and an endogenous bankruptcy trigger.

We use similar techniques to imply the total debt portfolios from these models as from the Merton (1974) model. More specifically, we rely on rolling windows of daily data over the prior 12 months to estimate asset value and volatility for each firm at the end of each month. To this end, we require several new variables: a firm's coupon payment, tax rate, fraction of value lost in bankruptcy and the cash outflow. We proxy for the tax rate via a rolling time-series average of taxes over pre-tax income, excluding observations with negative income, and we bound this number by 0.20 and 0.80. Our proxy for the recovery rate (i.e., one minus the value lost in bankruptcy) is a weighted average of the recovery rate of firms in various credit rating classes, where the weight equals the conditional chance that a firm ends up in the relevant credit rating class. We proxy for the cash outflow rate using a weighted average of a firm's payouts to equityholders and creditors. Equity outflows include common and preferred dividends and stock repurchases, while the debt outflows include the coupon and a partial amortization. We rely on the market value of equity and the book value of total debt as weights.

All new variables are from COMPUSTAT and Standard & Poor's. We form five broader classes out of all credit rating classes from Standard & Poor's, and these are AAA, AA-A, BBB-BB, B and <B. As only a small fraction of US firms are rated, we use an approach similar to that of Barth et al. (2008) to compute pseudo credit ratings, i.e., we regress existing credit ratings (AAA = 1, D = 23) on total assets, the ROA, long-term debt over total assets (book leverage), cash dividends and a dummy variable equal to one, if the ROA is positive, and zero otherwise. Using 14,758 observations, we find only estimates with absolute t-statistics above 20, with those of the estimates on total assets and cash dividends around 50. The adjusted R^2 of the regression is 54.62%. Estimates of recovery rates and transition probabilities can be obtained from the website of Standard and Poor's.

Repeating the tests in Tables 2–5 with our new implied debt market portfolios, we find that, although the market beta bias can be related to firm characteristics, especially size, the importance of debt claims for equity pricing is too low for omitted debt claims to materially affect empirical CAPM tests.[16]

6. Conclusion

Prior research reveals that the Merton (1974) model suggests that two firm characteristics, leverage and default risk, capture bias in the beta estimates in empirical tests of the CAPM relying on a stock index as the market portfolio proxy (Ferguson and Shockley, 2003). We add to this literature in the following ways. First, we identify other firm characteristics through which the Merton (1974) model can point to a bias in empirical studies. However, we then also offer intuitive reasons for why these biases are unlikely to be strong, i.e., we show that the relation between leverage and beta bias can be diversified away in an economy with a large number of firms. We also discuss that a strong beta bias requires a substantial fraction of total asset value exposed to significant default risk. Third, we offer a calibration exercise of the Merton (1974) model economy to test our suspicions. Finally, we also employ the structural model to obtain a proxy for the total US debt portfolio, which we then use in empirical asset pricing tests. In neither case do we find that omitted debt risk materially influences the validity of empirical tests of the CAPM. As a result, even if one doubts the reasonability of our implied total debt portfolio, our general conclusion is that the Merton (1974) model should not lead us to believe that the omission of debt risk can explain the CAPM's poor empirical performance. Our robustness checks show that our conclusions continue to hold under the more sophisticated models of Leland (1994) and Leland and Toft (1996).

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Appendix A. Proofs

A.1. Some relations in the Merton (1974) model economy

We initially repeat some properties of the Merton (1974) model economy originally shown by Galai and Masulis (1976). The value of firm i's claim C_i, with C_i being either the equity claim (S_i) or the debt claim (B_i), is a function of its asset value and time-to-maturity. As the dynamics of the asset value process are modeled by GBM with drift, we can apply Ito's lemma to derive the differential of the claim:

$$\Delta C_i = \frac{\partial C_i}{\partial V_i}\Delta V_i + \frac{\partial C_i}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 C_i}{\partial V_i^2}(\Delta V_i)^2$$
$$= \frac{\partial C_i}{\partial V_i}\Delta V_i + \frac{\partial C_i}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 C_i}{\partial V_i^2}\sigma^2 V^2 \Delta t,$$

with all variables being defined as in the main text.

The claim's instantaneous return can then be found from dividing by C_i and letting Δt go to zero:

$$r_{C_i} \equiv \frac{\Delta C_i}{C_i} = \frac{\partial C_i}{\partial V_i}\frac{1}{C_i}\Delta V_i = \frac{\partial C_i}{\partial V_i}\frac{V_i}{C_i}\frac{\Delta V_i}{V_i} = \frac{\partial C_i}{\partial V_i}\frac{V_i}{C_i}r_i,$$

where r_i is firm i's asset return. For the equity claim (S_i), this equation becomes:

$$r_{Si} = N(d_1)_i \frac{V_i}{S_i} r_i = \eta_{Si} r_i, \quad \text{with } \eta_{Si} \equiv N(d_1)_i (V_i/S_i). \tag{10}$$

For the debt claim (B_i), it becomes:

$$r_{Bi} = N(-d_1)_i \frac{V_i}{B_i} r_i = \eta_{Bi} r_i, \quad \text{with } \eta_{Bi} \equiv N(-d_1)_i (V_i/B_i). \tag{11}$$

From Eqs. (10) and (11), we can easily derive variances and covariances of the instantaneous equity or debt return. In particular, the variance of the instantaneous equity return is:

$$\sigma_{Si}^2 = E[(r_{Si} - E[r_{Si}])^2] = \eta_{Si}^2 E[(r_i - E[r_i])^2] = \eta_{Si}^2 \sigma_i^2,$$

and that of the instantaneous debt return is $\eta_{Bi}^2\sigma_i^2$. Following the same logic, the covariance between the instantaneous return of firm i's equity claim and that of firm j, between the instantaneous return of firm i's debt claim and that of firm j, and between the

[16] These results are also available upon request.

instantaneous return of firm i's equity claim and that of firm j's debt claim are $\eta_{Si}\,\eta_{Sj}\sigma_{i,j}$, $\eta_{Bi}\eta_{Bj}\sigma_{i,j}$ and $\eta_{Si}\eta_{Bj}\sigma_{i,j}$, respectively.

Finally, the risk-neutral default risk of firm i equals:

$$\text{Risk-neutral default risk} = N\left[-\frac{\ln(V_i/F_i) + (r_f - 0.5\sigma_i^2)T}{\sigma_i\sqrt{T}}\right].$$

A.2. Proof of Proposition 2

In this section, we analyze the sign of the partial derivative of the market beta bias of firm i with respect to its asset return correlation with firm k, i.e., the sign of $\partial(\beta_{Si}/\beta_{Si}^E)/\partial\rho_{i,k}$. Using Eq. (10), Ferguson and Shockley (2003) note that the market beta bias can be simplified as follows:

$$\frac{\beta_{Si}}{\beta_{Si}^E} = \frac{\eta_{Si}\beta_i}{\eta_{Si}\beta_i^E} = \frac{\beta_i}{\beta_i^E},$$

i.e., firm i's asset beta on the market portfolio over its asset beta on the total equity portfolio. Using the quotient rule, the partial derivative of this ratio with respect to the asset return correlation equals:

$$\frac{\partial(\beta_i/\beta_i^E)}{\partial\rho_{i,k}} = \frac{\frac{\partial\beta_i}{\partial\rho_{i,k}}\beta_i^E - \frac{\partial\beta_i^E}{\partial\rho_{i,k}}\beta_i}{(\beta_i^E)^2}. \tag{12}$$

For ease of exposition, we treat the two single partial derivatives in the numerator of the last term in Eq. (12) separately. We start with the left one:

$$\frac{\partial\beta_i}{\partial\rho_{i,k}} = \frac{\partial\frac{1}{\sigma_M^2}\sum_{j=1}^N\frac{V_j}{M}\sigma_{i,j}}{\partial\rho_{i,j}} = \frac{\left[\frac{V_k}{M}\sigma_i\sigma_k\sigma_M^2 - \frac{\partial\sigma_M^2}{\partial\rho_{i,k}}\sum_{j=1}^N\frac{V_j}{M}\sigma_{i,j}\right]}{\sigma_M^4}, \tag{13}$$

where we use the definition of the asset beta on the market portfolio in the first equation, and the quotient rule in the second equation. The partial derivative in the numerator term in Eq. (13) is:

$$\frac{\partial\sigma_M^2}{\partial\rho_{i,k}} = \frac{\partial\sum_{j=1}^N\sum_{h=1}^N\frac{V_j}{M}\frac{V_h}{M}\sigma_j\sigma_h\rho_{j,h}}{\partial\rho_{i,k}} = 2\frac{V_i}{M}\frac{V_k}{M}\sigma_i\sigma_k,$$

with the first equality following from the definition of variance. Plugging the solution of the partial derivative back into Eq. (13) and simplifying, we can write:

$$\frac{\partial\beta_i}{\partial\rho_{i,k}} = \frac{V_k}{M}\frac{\sigma_i\sigma_k}{\sigma_M^4}\left[\sigma_M^2 - 2\frac{V_i}{M}\sum_{j=1}^N\frac{V_j}{M}\sigma_{i,j}\right] = \frac{V_k}{M}\frac{\sigma_i\sigma_k}{\sigma_M^2}\left(1 - 2\frac{V_i}{M}\beta_i\right).$$

We next deal with the right partial derivative in the last term in Eq. (12):

$$\frac{\partial\beta_i^E}{\partial\rho_{i,k}} = \frac{\partial\frac{1}{\sigma_E^2}\sum_{j=1}^N\frac{S_j}{E}\sigma_{i,Sj}}{\partial\rho_{i,k}} = \frac{\partial\frac{1}{\sigma_E^2}\sum_{j=1}^N\frac{V_j}{E}N[d_1]_j\sigma_i\sigma_j\rho_{i,j}}{\partial\rho_{i,k}}$$
$$= \frac{1}{\sigma_E^4}\left[\frac{V_k}{E}N[d_1]_k\sigma_i\sigma_k\sigma_E^2 - \frac{\partial\sigma_E^2}{\partial\rho_{i,k}}\sum_{j=1}^N\frac{V_j}{E}N[d_1]_j\sigma_{i,j}\right], \tag{14}$$

where the first equality follows from the definition of the asset beta on the total equity portfolio, and the second one from the fact that $\sigma_{i,Sj} = N(d_1)_j(V_j/S_j)\sigma_{i,j}$. The final equality then follows from another application of the quotient rule. The partial derivative in the last term in Eq. (14) is:

$$\frac{\partial\sigma_E^2}{\partial\rho_{i,k}} = \frac{\partial\sum_{j=1}^N\sum_{h=1}^N\frac{S_j}{E}\frac{S_h}{E}\sigma_{S_j,S_h}}{\partial\rho_{i,k}} = \frac{\partial\sum_{j=1}^N\sum_{h=1}^N\frac{V_j}{E}\frac{V_h}{E}N[d_1]_jN[d_1]_h\sigma_{j,h}}{\partial\rho_{i,k}}$$
$$= 2\frac{V_i}{E}\frac{V_k}{E}N[d_1]_iN[d_1]_k\sigma_i\sigma_k,$$

where we apply the definition of the volatility of the total equity portfolio in the first equation and the relation that $\sigma_{Si,Sj} = N(d_1)_i(V_i/S_i)N(d_1)_j(V_j/S_j)\sigma_{i,j}$ in the second one. Plugging back into Eq. (14) and simplifying, we can write:

$$\frac{\partial\beta_i^E}{\partial\rho_{i,k}} = \frac{1}{\sigma_E^4}\left[\frac{V_k}{E}N[d_1]_k\sigma_i\sigma_k\sigma_E^2 - 2\frac{V_i}{E}\frac{V_k}{E}N[d_1]_iN[d_1]_k\sigma_i\sigma_k\sum_{j=1}^N\frac{V_j}{E}N[d_1]_j\sigma_{i,j}\right]$$
$$= \frac{V_k}{E}\frac{\sigma_i\sigma_k}{\sigma_E^2}N[d_1]_k\left[1 - 2\frac{V_i}{E}N[d_1]_i\beta_i^E\right].$$

Substituting the two partial derivatives back into Eq. (12):

$$\frac{\frac{V_k}{M}\frac{\sigma_i\sigma_k}{\sigma_M^2}\left[1 - 2\frac{V_i}{M}\beta_i\right]\beta_i^E - \frac{V_k}{E}\frac{\sigma_i\sigma_k}{\sigma_E^2}N[d_1]_k\left[1 - 2\frac{V_i}{E}N[d_1]_i\beta_i^E\right]\beta_i}{(\beta_i^E)^2}.$$

The last equation is positive, if and only if:

$$\frac{V_k}{M}\frac{\sigma_i\sigma_k}{\sigma_M^2}\left[1 - 2\frac{V_i}{M}\beta_i\right]\beta_i^E > \frac{V_k}{E}\frac{\sigma_i\sigma_k}{\sigma_E^2}N[d_1]_k\left[1 - 2\frac{V_i}{E}N[d_1]_i\beta_i^E\right]\beta_i,$$

or, assuming that $\left[1 - 2(V_i/E)N[d_1]_i\beta_i^E\right] > 0$, if and only if:

$$\Upsilon \equiv \frac{E}{M}\frac{\sigma_E^2}{\sigma_M^2}\frac{\beta_i^E}{\beta_i}\frac{\left[1 - 2\frac{V_i}{M}\beta_i\right]}{\left[1 - 2\frac{V_i}{E}N[d_1]_i\beta_i^E\right]} > N[d_1]_k.$$

If the asset betas and the terms in square parentheses are positive, which seems reasonable in large economies with small (V_i/M) and (V_i/E), then Υ is positive. Moreover, we can show that Υ can be below unity. To this end, we set the debt instalments of all firms (F) equal to zero, and choose some arbitrary, but feasible values for all other model variables. Then $E = M$ and $N[d_1] = 1$. Hence:

$$\Upsilon|_{F_1=0,F_2=0,\ldots,F_N=0} = \frac{M}{M}\frac{\sigma_M^2}{\sigma_M^2}\frac{\beta_i}{\beta_i}\frac{\left[1 - 2\frac{V_i}{M}\beta_i\right]}{\left[1 - 2\frac{V_i}{M}\beta_i\right]} = 1.$$

What remains to be shown is that the partial derivative of Υ with respect to the debt instalment of firm k at $F_1 = 0, F_2 = 0, \ldots, F_N = 0$ can be negative, implying that an indefinitely small increase in firm k's debt instalment would force Υ below unity. Again using the quotient rule, the partial derivative of Υ with respect to the debt instalment of firm k can be written as:

$$\frac{\partial\Upsilon}{\partial F_k} = \frac{\left[1 - 2\frac{V_i}{M}\beta_i\right]}{M\sigma_M^2\beta_i}\frac{\frac{\partial E\sigma_E^2\beta_i^E}{\partial F_k}\left[1 - 2\frac{V_i}{E}N[d_1]_i\beta_i^E\right] - \frac{\partial\left[1-2\frac{V_i}{E}N[d_1]_i\beta_i^E\right]}{\partial F_k}E\sigma_E^2\beta_i^E}{\left[1 - 2\frac{V_i}{E}N[d_1]_i\beta_i^E\right]^2}.$$

Note that the multiplicative constant (i.e., $\left(1 - 2\frac{V_i}{M}\beta_i\right)/(M\sigma_M^2\beta_i)$) in the last equation is unaffected by a change in the debt instalment, as both asset value and return are exogenously determined by GBM. The term on the right-hand side (RHS) is negative, if and only if:

$$\frac{\partial E\sigma_E^2\beta_i^E}{\partial F_k}\left[1 - 2\frac{V_i}{E}N[d_1]_i\beta_i^E\right] - \frac{\partial\left[1 - 2\frac{V_i}{E}N[d_1]_i\beta_i^E\right]}{\partial F_k}E\sigma_E^2\beta_i^E < 0, \tag{15}$$

as $\left(1 - 2\frac{V_i}{M}\beta_i\right)$ and β_i are positive by assumptions made above.

We first deal with the left partial derivative in Eq. (15):

$$\frac{\partial E\sigma_E^2\beta_i^E}{\partial F_k} = \frac{\partial\sum_{j=1}^N V_jN[d_1]_j\sigma_{i,j}}{\partial F_k} = -\frac{V_k\sigma_{i,k}n[d_1]_k}{F_k\sigma_k\sqrt{T}},$$

with the first equality following from the definition of the asset beta on the equity portfolio and $\sigma_{i,Sj} = N(d_1)_j(V_j/S_j)\,\sigma_{i,j}$. We next deal with the right partial derivative in Eq. (15):

$$\frac{\partial\left[-\frac{2V_iN[d_1]_i\beta_i^E}{E}\right]}{\partial F_k} = \frac{\partial\left[-\frac{2V_iN[d_1]_i\sum_{j=1}^N V_jN[d_1]_j\sigma_{i,j}}{E^2\sigma_E^2}\right]}{\partial F_k} = -\left[\frac{-\frac{2V_iV_kE^2\sigma_E^2\sigma_{i,k}N[d_1]_in[d_1]_k}{F_k\sigma_k\sqrt{T}} - \frac{\partial E^2\sigma_E^2}{\partial F_k}2V_iN[d_1]_i\sum_{j=1}^N V_jN[d_1]_j\sigma_{i,j}}{E^4\sigma_E^4}\right], \quad (16)$$

which follows from the definition of the asset beta on the equity portfolio, $\sigma_{i,Sj} = N(d_1)_j(V_j/S_j)\sigma_{i,j}$ and the quotient rule. The final partial derivative in Eq. (16) is:

$$\frac{\partial E^2\sigma_E^2}{\partial F_k} = -2E\sigma_E^2e^{-r_FT}N[d_2]_k - 2V_kE\sigma_{k,E}\frac{n[d_1]_k}{F_k\sigma_k\sqrt{T}} + 2E\sigma_E^2e^{-r_fT}N[d_2]_k = -2V_kE\sigma_{k,E}\frac{n[d_1]_k}{F_k\sigma_k\sqrt{T}}.$$

(e.g., see Ferguson and Shockley, 2003). Plugging back into Eq. (16), we obtain:

$$\frac{1}{E^4\sigma_E^4}\frac{2V_iV_kE^2\sigma_E^2\sigma_{i,k}N[d_1]_in[d_1]_k - 4V_iV_kE\sigma_{k,E}N[d_1]_in[d_1]_k\sum_{j=1}^N V_jN[d_1]_j\sigma_{i,j}}{F_k\sigma_k\sqrt{T}}.$$

Writing out the individual partial derivatives in Eq. (15) and simplifying:

$$\begin{aligned}&\frac{1}{F_k\sigma_k\sqrt{T}}\Bigg(-V_k\sigma_{i,k}n[d_1]_k\left[1-2\frac{V_i}{E}N[d_1]_i\beta_i^E\right] \\ &-\frac{E\sigma_E^2\beta_i^E}{E^4\sigma_E^4}2V_iV_kE^2\sigma_E^2\sigma_{i,k}N[d_1]_in[d_1]_k \\ &+\frac{E\sigma_E^2\beta_i^E}{E^4\sigma_E^4}4V_iV_kE\sigma_{k,E}N[d_1]_in[d_1]_k\sum_{j=1}^N V_jN[d_1]_j\sigma_{i,j}\Bigg) \\ &=\frac{1}{F_k\sigma_k\sqrt{T}}\Bigg(-V_k\sigma_{i,k}n[d_1]_k+\frac{2}{E}V_iV_k\sigma_{i,k}N[d_1]_in[d_1]_k\beta_i^E \\ &-\frac{2}{E}V_iV_k\sigma_{i,k}N[d_1]_in[d_1]_k\beta_i^E+\frac{4}{E}V_iV_k\sigma_{k,E}N[d_1]_in[d_1]_k(\beta_i^E)^2\Bigg),\end{aligned}$$

which can then be written as:

$$\begin{aligned}&\frac{1}{F_k\sigma_k\sqrt{T}}\left(-V_k\sigma_{i,k}n[d_1]_k+\frac{4}{E}V_iV_k\sigma_{k,E}N[d_1]_in[d_1]_k(\beta_i^E)^2\right) \\ &=\frac{V_kn[d_1]_k}{F_k\sigma_k\sqrt{T}}\left(-\sigma_{i,k}+\frac{4}{E}V_i\sigma_{k,E}N[d_1]_i(\beta_i^E)^2\right). \qquad (17)\end{aligned}$$

The multiplicative constant is the scaled partial derivative of $N[d_1]_k$ with respect to F_k,[17] implying that for F_k indefinitely close to zero the multiplicative constant and therefore also the partial derivative of Υ with respect to F_k equal zero. For F_k slightly greater than zero, the multiplicative constant is positive and the partial derivative of Υ with respect to F_k is hence negative, if and only if:

$$\frac{4}{E}V_i\sigma_{k,E}N[d_1]_i(\beta_i^E)^2 < \sigma_{i,k}.$$

If we set $\sigma_{i,k} > 0$, then we can always make the inequality hold by choosing a sufficiently large total equity value (E), implying that Υ can be below unity for at least some set of model values.

A.3. Proof of Corollary 1

We now prove that, under our set of auxiliary assumptions, the relation between market beta bias and leverage (i.e., $\partial(\beta_{Si}/\beta_{Si}^E)/\partial F_i$) converges to zero with the number of firms in the model economy. We can more formally write the assumptions that the contribution of each firm to the market, the total equity and the total debt portfolios decreases to zero with the number of firms as:

$$\lim_{N\to\infty}\frac{V_i}{M} = \lim_{N\to\infty}\frac{S_i}{E} = \lim_{N\to\infty}\frac{B_i}{D} = 0.$$

As equal weights (i.e., $(1/N)$) also tend to a zero limit, they are indistinguishable from the non-dominant weights above in the asymptotic limit. Another one of our assumptions is that the firm characteristics (i.e., the model values) of new firms added to the model economy are independently drawn from a stable, but not necessarily analytic multivariate distribution. Independence across draws implies that we can use a law of large numbers (e.g., Davidson and MacKinnon, 2004) to show that the cross-sectional mean of one firm characteristic, e.g., the asset value, equals the expectation of the firm characteristic specified by the multivariate distribution in the asymptotic limit.

Ferguson and Shockley (2003) show that the partial derivative of the market beta bias with respect to a firm's debt instalment can be written as:

$$\frac{\partial\beta_{Si}/\beta_{Si}^E}{\partial F_i} = \frac{e^{-rT}}{E}\left[-\frac{Z(d_2)_i}{\sigma_i\sqrt{T}}\frac{\sigma_i^2}{\sigma_E^2} + 2(\beta_i^E)^2\frac{Z(d_2)_i}{\sigma_i\sqrt{T}} - \beta_i^EN(d_2)_i\right].$$

As the number of firms increases to infinity, total equity value (E) also increases to infinity (unless, or course, all firms are entirely debt financed, which we rule out), implying that the multiplicative factor (e^{-rT}/E) outside of the square parentheses converges to zero. The limit of the term in square parentheses depends on β_i^E and σ_E^2, the only model values affected by the number of firms in the Merton (1974) model economy. Recall that β_i^E is the ratio of σ_{iE} and σ_E^2.

Our assumptions now imply that:

$$\lim_{N\to\infty}\sigma_{iE} = \lim_{N\to\infty}\sum_{j=1}^N\frac{S_j}{E}\sigma_{i,Sj} = \lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^N\sigma_{i,Sj} = E[\sigma_{i,Sj}],$$

with the first equality following from the fact that the covariance operator is linear, the second one from the fact that non-dominant weights and equal weights are indistinguishable in the asymptotic limit, and the third one from the application of a law of large numbers. The assumption that beta is positive then implies that σ_{iE} converges to a positive constant in the asymptotic limit.

Similarly:

$$\lim_{N\to\infty}\sigma_E^2 = \lim_{N\to\infty}\sum_{i=1}^N\sum_{j=1}^N\frac{S_i}{E}\frac{S_j}{E}\sigma_{Si,Sj} = \lim_{N\to\infty}\frac{1}{N^2}\sum_{i=1}^N\sum_{j=1}^N\sigma_{Si,Sj} = E[\sigma_{Si,Sj}],$$

which must also be a positive constant. As both β_i^E and σ_E^2 converge to positive constants, the term in square parentheses also converges to a constant (which could be zero). By the product rule of limits, the partial derivative of the market beta bias with respect to the debt instalment must then necessarily tend to zero with the number of firms in the model economy.

A.4. Proof of Corollary 2

The proof that the partial derivative of the market beta bias of firm i with respect to asset correlation with firm k (i.e., $\partial(\beta_{Si}/\beta_{Si}^E)/\partial\rho_{i,k}$) converges to zero with the number of firms is almost identical to that of Corollary 1. Note that we can write this partial derivative as:

$$\frac{\partial\beta_{Si}/\beta_{Si}^E}{\partial\rho_{i,k}} = \frac{\frac{V_k}{M}\frac{\sigma_i\sigma_k}{\sigma_M^2}\left[1-2\frac{V_i}{M}\beta_i\right]\beta_i^E - \frac{V_k}{E}\frac{\sigma_i\sigma_k}{\sigma_E^2}N[d_1]_k\left[1-2\frac{V_i}{E}N[d_1]_i\beta_i^E\right]\beta_i}{(\beta_i^E)^2}. \qquad (18)$$

Using equivalent arguments as before, we can show that, as the number of firms increases to infinity, σ_M^2, σ_E^2, β_i and β_i^E tend to positive constants, whereas $(1/M)$ and $(1/E)$ tend to zero, again ruling

[17] That is, the scaled partial derivative is $-V_k\partial N[d_1]_k/\partial F_k$.

out that all firms are entirely debt financed. Hence, the partial derivative must converge to zero.

Next, the total impact of a simultaneous change in all correlation pairs on the market beta bias (the total derivative of the market beta bias with respect to all correlation pairs) can be stated as:

$$d(\beta_{Si}/\beta_{Si}^E) = \sum_{k=1}^{N} \frac{\partial \beta_{Si}/\beta_{Si}^E}{\partial \rho_{i,k}} d\rho_{i,k}.$$

Using Eq. (18) and assuming that $d\rho_{i,k}$=1 for all k, the total derivative becomes:

$$\frac{\sigma_i}{\sigma_M^2 \beta_i^E}\left[1 - 2\frac{V_i}{M}\beta_i\right]\frac{\sum_{k=1}^N V_k \sigma_k}{M} - \frac{\sigma_i \beta_i}{\sigma_E^2 (\beta_i^E)^2}\left[1 - 2\frac{V_i}{E}N[d_1]_i \beta_i^E\right]\frac{\sum_{k=1}^N V_k \sigma_k N[d_1]_k}{E}.$$

As the number of firms increases to infinity, not only (V_i/M), but also (V_i/E) tends to a limit of zero, rendering both pairs of weights indistinguishable from equal weights (i.e., $(1/N)$). Again applying similar arguments as in Corollary 1, the limit of the total derivative is then:

$$\frac{\sigma_i}{\sigma_M^{2*} \beta_i^{E*}} E[\sigma_k] - \frac{\sigma_i \beta_i^*}{\sigma_E^{2*} (\beta_i^{E*})^2} E[\sigma_k N[d_1]_k] \tag{19}$$

(i.e., $\lim_{N\to\infty} \sum_{k=1}^N (V_k/M)\sigma_k = \lim_{N\to\infty} 1/N \sum_{k=1}^N \sigma_k = E[\sigma_k]$ and also $\lim_{N\to\infty} \sum_{k=1}^N (V_k/E)\sigma_k N[d_1]_k = \lim_{N\to\infty} 1/N \sum_{k=1}^N \sigma_k N[d_1]_k = E[\sigma_k N[d_1]_k]$), with $\sigma_M^{2*}, \sigma_E^{2*}, \beta_i^*$ and β_i^{E*} being positive constants. As the additive terms in formula (19) converge to constants, the total impact of all correlation pairs cannot necessarily be diversified away in an economy with a large number of firms.

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