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# Nonlinear data reconciliation in gold processing plants

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**Abstract**

Data reconciliation in hydrometallurgy process is less common than in mineral processing, due to the more complex mathematical formulation of the problem to describe the process balances that can include multiphase and multi-component reactions and mass transfer. In this case, to avoid thermodynamic violations, the use of inequality constraints, in addition to the classical equality mass and energy balances, can be mandatory. This paper presents two case studies of data reconciliation for a gold extraction plant using the direct optimization of a nonlinear objective function and nonlinear constraints that includes equality and inequality equations. The problems are solved using a sequential quadratic programming algorithm and the results demonstrate the applicability of this approach to data reconciliation in hydrometallurgical processes generating reliable results, without mass balance or thermodynamic violations.
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## 1. Introduction

Data reconciliation is a classical technique that has been widely applied in processing engineering to improve the accuracy of measurements by reducing the effect of random errors. In spite the indubitable benefit of this systematic technique in process monitoring, analysis, control and optimization, the application of data reconciliation in hydrometallurgical process and particularly in gold processing plants has received much less attention and is the object of this study.

Since the seminal paper of Kuehn and Davidson (1961), the data reconciliation problem has been improved and applied for a large number of processes including petrochemical, chemical, biochemical, mineral and metallurgical as summarized by Mah (1990), Crowe (1996), Narasimhan and Jordache (2000) and Romagnoli and Sánchez (2000).

The mineral industry and academia recognized earlier the applicability of data reconciliation to improve plant audits based on more accurate data; as a result, this technique has been an active research area in mineral processing for more than thirty years (Wiegel, 1972; Smith and Ichiyen, 1973; Mular et al., 1976; Cutting, 1976; White et al., 1977; Hodouin and Everell, 1980; Reid et al., 1982; Laplante, 1984; Wills, 1986; Simpson et al., 1991).

Even though data reconciliation is a classical technique, nowadays a lot of attention is still given to simultaneous data reconciliation and parameter estimation, dynamic data reconciliation, detection of gross error data, and the use of robust optimization schemes to deal with large problems and the nonlinear constraints, as in the case of the problem analyzed in this work (Tjoa and Biegler, 1991; Liebman et al., 1992; Crowe, 1996; Kim et al., 1997; Narasimhan and Jordache, 2000; Soderstrom et al., 2000).

Data reconciliation in hydrometallurgical processes is not widely used as in mineral processing and only a very few applications can be found in the literature. This lack of application in such an important area seems to be related to the difficulty in the problem formulation and especially in the reliability of the reconciled results. A problem that can

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arise in hydrometallurgical processes data reconciliation is the violation of thermodynamic constraints with the reconciled data; for instance, in solvent extraction, the concentration of the chemical species extracted from the aqueous phase after the reconciliation must be higher at the organic phase exit than in the entrance, while the concentration in the aqueous phase must be lower at the exit than the entrance, which in presence of larger random errors may not hold. An analogous problem is found in heat exchangers data reconciliation where the high and low temperature levels in the cooling fluid and in the hot fluid cannot be modified by the reconciliation, as exemplified by Narasimhan and Jordache (2000).

Data reconciliation in gold processing plants is particularly challenging because the metal ore content is very low and its distribution is non-homogeneous; in addition, many plants treat ores from different sites with very different characteristics and steady-state operation cannot always be guaranteed. Only two systematic studies of data reconciliation in gold processing plants are available in the literature. In the first case, Laplante (1984) presented the basic concepts that a gold leaching material balance should take into account and briefly presented a steady-state reconciliation. In the second case, Cimon et al. (1987) and Cimon (1989) developed an algorithm to perform the steady-state reconciliation of a gold leaching plant, which is an extension of the mineral processing hierarchical algorithm developed by Hodouin and Everell (1980). The reconciled results for this case obey the material balance constraints, however, controversial results were found, such as an apparent gold precipitation in the thickener and in the primary and secondary filters and consequently an ore gold content in the exit of these operations higher than the entrance, which clearly is a numerical artifact (see the solid gold content in the fluxes 2 and 4, 8 and 10, and 11 and 13 in Table 1; the stream and nodes numbers are defined in Figs. 1 and 2 and discussed in Section 3.2). These previous studies provided the foundation for the data reconciliation in gold processing plant; however, in spite the fact that the material balance constraints hold, the results indicated that the conventional data reconciliation approach might not assure that all thermodynamic constraints will hold.

Smith and Ichiyen (1973) stated that the use of a direct search methods to solve the reconciliation problems found in mineral processing applications was not justified in the 1970s due to the computational and programming limitation to solve the first or second-order gradient methods that arise in the optimization. Therefore, these aspects motivated the development of analytical solutions for linear and bilinear constraints using linear algebra standard manipulations, and in some cases the previous linearization of the constraints. Nowadays, however, with the wide availability of high performance computers, it is clear that other more robust and complex approaches can be used to solve this classical problem (Liebman, 1991; Liebman et al., 1992; Weiss et al., 1996; Kim et al., 1997; Soderstrom et al., 2000; Eksteen et al., 2002; Poku et al., 2004).

This paper presents the first study on nonlinear data reconciliation of industrial hydrometallurgical plants that deals simultaneously with nonlinear equality and inequality constraints and upper and lower bounds for the variables. The main objective of this paper is to provide an alternative

Table 1
Corrected values for case study 1 according to Cimon et al. (1987) (see Figs. 1 and 2)

| Flux | Solid flow rates (t/h) | Liquid flow rates (t/h) | Solid concentration (%) | Solid gold content (mg/kg) | Liquid gold content (mg/L) |
|---|---|---|---|---|---|
| 1 | 64.62 | – | – | 4.98 | – |
| 2 | NA | NA | 31.20 | 2.61 | 1.56 |
| 3 | – | NA | – | – | 0.49 |
| 4 | NA | NA | 52.38 | 2.80 | 1.76 |
| 5 | – | NA | – | – | 2.91 |
| 6 | – | 147.28 | – | – | 1.97 |
| 7 | – | NA | – | – | 0.60 |
| 8 | NA | NA | 52.38 | 0.78 | 3.98 |
| 9 | – | 39.50 | – | – | 0.08 |
| 10 | NA | NA | 53.47 | 1.01 | 0.45 |
| 11 | NA | NA | 53.47 | 0.46 | 1.07 |
| 12 | – | NA | – | – | 0.60 |
| 13 | NA | NA | 53.19 | 0.64 | 0.23 |
| 14 | – | 17.90 | – | – | 0.08 |
| 15 | – | NA | – | – | 0.61 |
| 16 | NA | 54.80 | 52.54 | 0.34 | 0.05 |
| 17 | – | NA | – | – | 0.00 |
| 18 | – | NA | – | – | 0.08 |
| 19 | – | NA | – | – | 0.60 |
| 20 | – | NA | – | – | 0.00 |
| 21 | – | 147.28 | – | – | 0.08 |
| 22 | – | NA | – | – | 0.08 |
| 23[a] | – | – | – | 4.59 | – |

[a] Metallic gold reduced by cementation with zinc.

approach for steady-state data reconciliation in hydrometallurgical processes that gives accurate and reliable results. The method is illustrated with two case studies that correspond to different configurations of a cyanidation plant. In the first case, the process includes precipitation and filtration and more emphasis is given to the leaching-recovery section of the plant. In the second case, the process includes recovery by activated carbon and the grinding-classification-leaching section of the plant is analyzed in more detail.

This paper is organized as follows. Section 2 presents an overview of the theory of data reconciliation and nonlinear programming. Sections 3 and 4 present respectively case studies 1 and 2, for which the studied plant and the data acquisition process are explained, then the material balance equations and the results of the data reconciliation are presented. Section 5 presents the conclusions. Appendices A and B present the mass balance equations and the inequality constraints used in the two case studies.

## 2. Overview of data reconciliation formulation and nonlinear programming

The steady-state data reconciliation can be defined as a nonlinear programming problem, which means an optimization problem with a nonlinear criterion and linear and nonlinear constraints with equality and inequalities. The weighted least square is a widely used criterion for data reconciliation, and the constraints are the material and energy balance equations and other physical relationships. Mathematically, the steady data reconciliation problem can be expressed as follows (Mah, 1990; Narasimhan and Jordache, 2000; Romagnoli and Sánchez, 2000):

$$\begin{cases} \underset{\hat{\mathbf{x}}}{\mathrm{Min}} & J(\hat{\mathbf{x}}) = (\mathbf{x}-\hat{\mathbf{x}})^T\mathbf{W}^{-1}(\mathbf{x}-\hat{\mathbf{x}}) \\ \text{s.t.} & \mathbf{f}(\hat{\mathbf{x}}) = 0 \\ & \mathbf{g}(\hat{\mathbf{x}}) \geqslant 0 \\ & \hat{\mathbf{x}}_b \leqslant \hat{\mathbf{x}} \leqslant \hat{\mathbf{x}}_h \end{cases} \tag{1}$$

where $\hat{\mathbf{x}}$ is the vector of the adjusted variables, $\mathbf{x}$ is the vector of the measured variables, $\mathbf{W}$ is the variance–covariance matrix where each element of the diagonal is the measurement variances ($\sigma_{x_i}^2$) and the other elements are assumed as zero, $\mathbf{f}$ are the equality constraints, $\mathbf{g}$ are the inequality constraints, $\hat{\mathbf{x}}_b$ and $\hat{\mathbf{x}}_h$ are respectively the low and high bound values of the adjusted variables. This criterion assumes that the measurements are contaminated only by Gaussian errors of zero average and covariance $\mathbf{W}$, which also normalize the least square criterion and avoid the dominance of high value variables. The criterion $J(\hat{\mathbf{x}})$ can be rewritten as follows:

$$J(\hat{\mathbf{x}}) = \sum_i (x_i - \hat{x}_i)^2 \frac{1}{\sigma_{x_i}^2} \tag{2}$$

where the measurement variances can be given by

$$\sigma_{x_i}^2 = (\mathrm{RSD}_{x_i} x_i)^2 \tag{3}$$

where RSD is the relative standard deviation for the measurements.

If there are no inequality constraints the problem given by Eq. (1) can be solved using the classical Lagrange multipliers ($\boldsymbol{\lambda}$) method, where the Lagrangian ($\Phi$) contains the least squares criterion ($J$) and the physical constraints ($\mathbf{f}$) (Avriel, 1976; Gill et al., 1981; Nocedal and Wright, 1999):

$$\Phi(\hat{\mathbf{x}}, \boldsymbol{\lambda}) = J(\hat{\mathbf{x}}) + \sum_l \lambda_l f_l(\hat{\mathbf{x}}) \tag{4}$$

In this case the solution of the problem given by the vector ($\hat{\mathbf{x}}$) is obtained by looking for the saddle point of Lagrangian, i.e. by solving the following system of equations:

$$\begin{cases} \dfrac{\partial \Phi(\hat{\mathbf{x}}, \boldsymbol{\lambda})}{\partial \hat{x}_i} = -2(x_i - \hat{x}_i)\dfrac{1}{\sigma_{x_i}^2} + \dfrac{\partial}{\partial \hat{x}_i}\left(\sum_j \lambda_j f_l(\hat{\mathbf{x}})\right) = 0 \\ \dfrac{\partial \Phi(\hat{\mathbf{x}}, \boldsymbol{\lambda})}{\partial \lambda_l} = f_l(\hat{\mathbf{x}}) = 0 \end{cases} \tag{5a,b}$$

In the case of linear or bilinear constraints (i.e. the product of two variables) the problem has an analytical solution (Narasimhan and Jordache, 2000), but, for the general case where the constraints are nonlinear, the most useful approach is the linearization of the constraints, the successive linearization of the constraints, the use of an iterative solution scheme, or the numerical solution of the system of nonlinear equations.

In the more general case where there are constraints of equalities and inequalities, this problem can be formulated using the augmented Lagrange function, defined as follows:

$$\Phi(\hat{\mathbf{x}}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = J(\hat{\mathbf{x}}) + \sum_l \lambda_l f_l(\hat{\mathbf{x}}) + \sum_j \mu_j g_j(\hat{\mathbf{x}}) \tag{6}$$

where $\boldsymbol{\lambda}$ is the vector of the Lagrange multipliers and $\boldsymbol{\mu}$ is the vector of the Kuhn–Tucker multipliers.

Nonlinear programming, also known as mathematical programming or general constrained optimization, is a generic designation that includes the optimization problems that have nonlinear objective functions ($F$) and constraint functions, which can be both equalities ($\mathbf{c}$) and inequalities ($\mathbf{d}$), mathematically defined as follows (Avriel, 1976; Gill et al., 1981; Biegler et al., 1997; Nocedal and Wright, 1999):

$$\begin{cases} \underset{\mathbf{x}}{\min} & F(\mathbf{x}) \\ \text{s.t.} & \mathbf{c}(\mathbf{x}) = 0 \\ & \mathbf{d}(\mathbf{x}) \geqslant 0 \end{cases} \tag{7}$$

where $F$, $\mathbf{c}$ and $\mathbf{d}$ are smooth, real-valued functions. This method has experienced major developments in the last 10 years including the emphasis on large-scale problems, that allowed its application to some data reconciliation problems of chemical processes (Liebman, 1991; Liebman et al., 1992; Weiss et al., 1996; Narasimhan and Jordache, 2000; Poku et al., 2004).

The solution of the nonlinear programming problem given by Eq. (6) ($\hat{\mathbf{x}}^*$) must satisfy the following first order Kuhn–Tucker conditions (Biegler et al., 1997; Nocedal and Wright, 1999):

$$\nabla\Phi(\hat{\mathbf{x}}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*) = \nabla J(\hat{\mathbf{x}}^*) + \boldsymbol{\lambda}^*\nabla\mathbf{f}(\hat{\mathbf{x}}^*) + \boldsymbol{\mu}^*\nabla\mathbf{g}(\hat{\mathbf{x}}^*) = 0 \quad (8a)$$

$$\mathbf{g}(\hat{\mathbf{x}}^*) \leqslant 0 \quad \text{and} \quad \mathbf{f}(\hat{\mathbf{x}}^*) = 0 \quad (8b)$$

$$\boldsymbol{\mu}^* \geqslant 0 \quad (8c)$$

$$\text{If } \mu_i^* = 0 \text{ or } g_i(\hat{\mathbf{x}}^*) = 0 \quad \text{then } \boldsymbol{\mu}^{*T}\mathbf{g}(\hat{\mathbf{x}}^*) = 0 \quad (8d)$$

The problem given by Eqs. (1) or (7) can be solved using the generalized reduced gradient (GRG) method that solves a series of successive linear programming problems, composed by the linearization objective function and constraints (Biegler et al., 1997; Nocedal and Wright, 1999). Another effective method to solve this problem is the successive quadratic programming (SQP) that uses the method of Newton to solve the system of nonlinear equations generated by the simplification of the first two conditions of Kuhn–Tucker (see Eqs. (8a) and (8b)); in this case, the optimization problem is replaced by a succession of problems of quadratic programming that use the Hessian matrix of the Lagrange function, the linearization of the nonlinear constraints using Taylor expansion, the gradient of the objective function, and the gradient of the constraints (Biegler et al., 1997; Nocedal and Wright, 1999).

The solution of nonlinear programming problems can be challenging because the feasible regions can be non-convex or discontinuous and the objective function can have local minima (Kokossis and Floudas, 1990). As a result, the optimal point is not necessarily on the constraints nor in the intersection of the constraints as in the case of linear programming. In the present study, the data reconciliation problem is solved using the nonlinear programming algorithm NPSOL v.5.0 that is based on the SQP method (see Gill et al., 1986).

In the next sections, the two case studies of data reconciliation in gold hydrometallurgy related to the Doyon Mine plant (Cambior, Inc.) are presented. This plant is located in Northwestern Quebec (Canada) and treats an ore from the Abitibi mineralized zone. The first case corresponds to the plant 1980s configuration for which a previous data reconciliation study was done by Cimon (1989) and Cimon et al. (1987), while the second case corresponds to the 1990s configuration of the same plant for which a sampling campaign was performed (de Andrade Lima, 2001).

## 3. Case study 1: Doyon Mine plant 1980s configuration

### 3.1. Plant description and data acquisition

The data set used in this case study is one of the data sets presented by Cimon (1989), who performed several sampling campaigns in the plant (see Fig. 1), including the grinding, dewatering, filtering, zinc precipitation, primary and secondary leaching sections. The specific data set used here was collected during a sampling campaign on October 1985 and was also used in the data reconciliation presented by Cimon et al. (1987). The reconciled results of this previous study are presented in Table 1.

The measured variables and the corresponding assumed relative standard deviation of the measures ($\text{RDS}_x$) used for calculation of the variances of the variables in the objective function (see Eq. (2)), which reflects the level of confidence of the variables, are summarized in Table 2. The flux numbers 1–23 correspond to the plant streams and will be discussed in Section 3.2. Note that for the non-measured variables, identified by "NA", the absolute standard deviation ($\sigma_x$) is used in the calculation instead of the conventional relative standard deviation ($\text{RSD}_x$). Also note that the solid gold content is assumed as a very sensible measure due to the low values and the heterogeneity of the distribution of the gold in ore particles. One notes that the assumed relative standard deviations were slightly modified in the present work to better represent the level of confidence of the measured variables. Finally, note that in the original study, due to the difficulties to achieve reliable reconciled results, the three drum filters were divided in two sections: the first was filtering and washing, and the second was re-pulping (Cimon, 1989); in the present study these sections were merged and the filters were represented only by one node.

### 3.2. Mass balance equations and constraints

The studied sections of the Doyon plant 1980s configuration (see Fig. 1) are represented as a 9 node and 23 stream graph diagram, see Fig. 2. The node numbers in Fig. 2 stand for—I: grinding and classification, II: thickener, III: primary leaching tanks, IV: primary filter, V: secondary leaching tanks, VI: secondary filter, VII: tertiary filter, VIII: gold precipitation, IX: mill head tank, X: solution division, and XI: Barren solution tank. The fluxes in Fig. 2 stand for—1: plant ore entrance, 2: grinding-classification circuit exit, 3: clarified solution, 4: thickener underflow, 5: clarified solution, 6: thickener overflow, 7: clarified solution, 8: primary leaching tanks exit, 9: Barren solution addition, 10: re-pulped ore, 11: secondary leaching tanks exit, 12: clarified solution, 13: re-pulped ore, 14: Barren solution addition 15: clarified solution, 16: re-pulped ore, 17: fresh water addition, 18: Barren solution addition, 19: clarified solution, 20: fresh water addition. 21: Barren solution, 22: Barren solution losses, and 23: recovered gold.

The mass balances for the main species in each node of the graph (see Fig. 2) are used as constraints for the data reconciliation problem. The constraints include: (i) the ore mass balance, (ii) the solution mass balance, (iii) the physical constraints of equality of ore concentrations in the leaching tanks, (iv) the gold mass balance in the ore and in the solution, (v) the gold mass balance for the fastest nodes, in which the residence time of the ore is very short and no leaching take place, (vi) the physical constraints of equality of gold concentrations, (vii) the inequality

Table 2
Measured values and standard deviation for case study 1 (see Figs. 1 and 2)

| Flux | Solid flow rate (t/h) | $RSD_{Qs}$ (%) | Solid concentration (%) | $RSD_{Cw}$ (%) | Liquid flow rate (t/h) | $RSD_{Qs}$ (%) | Solid gold content (mg/kg) | $RSD_{Cs}$ (%) | Liquid gold content (mg/L) | $RSD_{Cl}$ (%) | Gold mass (g/h) | $RSD_M$ (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 55.98 | 20 | – | – | – | – | 16.459 | 90 | – | – | NA | – |
| 2 | NA | – | 29.00 | 20 | NA | – | 3.733 | 70 | 1.7685 | 20 | NA | – |
| 3 | – | – | – | – | 139.50 | 70 | – | – | 0.8175 | 20 | NA | – |
| 4 | NA | – | 55.29 | 20 | NA | – | 3.5036 | 70 | 1.8603 | 20 | NA | – |
| 5 | – | – | – | – | 28.59 | 70 | – | – | 2.7946 | 20 | NA | – |
| 6 | – | – | – | – | 154.94 | 70 | – | – | 1.7351 | 20 | NA | – |
| 7 | – | – | – | – | 121.19 | 70 | – | – | 0.5506 | 20 | NA | – |
| 8 | NA | – | 52.65 | 20 | NA | – | NA | 0.00001[a] | 3.9791 | 20 | NA | – |
| 9 | – | – | – | – | 29.17 | 70 | – | – | 0.0792 | 20 | NA | – |
| 10 | NA | – | 52.37 | 20 | NA | – | 1.2930 | 70 | 0.4672 | 20 | NA | – |
| 11 | NA | – | 51.51 | 20 | NA | – | 0.5839 | 70 | 1.3097 | 20 | NA | – |
| 12 | – | – | – | – | 145.91 | 70 | – | – | 0.5506 | 20 | NA | – |
| 13 | NA | – | 52.31 | 20 | NA | – | 0.4171 | 70 | 0.2086 | 20 | NA | – |
| 14 | – | – | – | – | 144.22 | 70 | – | – | 0.0792 | 20 | NA | – |
| 15 | – | – | – | – | NA | 0.00001[a] | – | – | NA | 1 | NA | – |
| 16 | NA | – | 51.43 | 20 | NA | – | 0.3545 | 70 | 0.0459 | 20 | NA | – |
| 17 | – | – | – | – | NA | 0.00001[a] | – | – | – | – | NA | – |
| 18 | – | – | – | – | NA | 0.00001[a] | – | – | 0.0792 | 20 | NA | – |
| 19 | – | – | – | – | 24.72 | 70 | – | – | 0.5506 | 20 | NA | – |
| 20 | – | – | – | – | NA | 0.00001[a] | – | – | – | – | NA | – |
| 21 | – | – | – | – | 154.92 | 70 | – | – | 0.0792 | 20 | NA | – |
| 22 | – | – | – | – | NA | 0.00001[a] | – | | 0.0792 | 20 | NA | – |
| 23 | – | – | – | – | – | – | – | – | – | – | NA | 0.00001[a] |

[a] Absolute standard deviation.

Table 3
Corrected and estimated values for case study 1 (see Figs. 1 and 2)

| Flux | Solid flow rate (t/h) | Solid concentration (%) | Liquid flow rate (t/h) | Solid gold content (mg/kg) | Liquid gold content (mg/L) | Gold mass (g/h) |
|---|---|---|---|---|---|---|
| 1 | 54.03 | – | – | 5.0060 | – | 270.5 |
| 2 | 54.03 | 37.09 | 91.63 | 3.0724 | 1.7283 | 324.4 |
| 3 | – | – | 91.63 | – | 0.5881 | 53.9 |
| 4 | 54.03 | 57.24 | 40.37 | 2.7241 | 1.8340 | 221.2 |
| 5 | – | – | 41.49 | – | 3.0437 | 126.3 |
| 6 | – | – | 167.52 | – | 1.6214 | 271.6 |
| 7 | – | – | 74.78 | – | 0.5643 | 42.2 |
| 8 | 54.03 | 57.24 | 40.37 | 1.3986 | 3.6080 | 221.2 |
| 9 | – | – | 50.27 | – | 0.0793 | 4.0 |
| 10 | 54.03 | 52.36 | 49.15 | 1.3986 | 0.4754 | 98.9 |
| 11 | 54.03 | 52.36 | 49.15 | 0.6247 | 1.3262 | 98.9 |
| 12 | – | – | 115.53 | – | 0.5643 | 65.2 |
| 13 | 54.03 | 58.22 | 38.77 | 0.6247 | 0.2148 | 42.1 |
| 14 | – | – | 105.14 | – | 0.0793 | 8.3 |
| 15 | – | – | 29.91 | – | 1.0328 | 30.9 |
| 16 | 54.03 | 62.18 | 32.86 | 0.1972 | 0.0456 | 12.2 |
| 17 | – | – | 11.89 | – | – | – |
| 18 | – | – | 12.11 | – | 0.0793 | 1.0 |
| 19 | – | – | 40.75 | – | 0.5643 | 23.0 |
| 20 | – | – | 20.97 | – | – | – |
| 21 | – | – | 167.52 | – | 0.0793 | 13.3 |
| 22 | – | – | 0.0 | – | 0.0793 | – |
| 23 | – | – | – | – | – | 258.3 |

physical constraints, (viii) the positive value constraints for all variables, and (ix) the low and high value constraints.

In this work the original nonlinear constraints are used directly without transformation, and the complete set of equality and inequality constraints used is presented in Appendix A, where Qs is the ore flow rate, Cw is the weight solid concentration in the pulp, Ql is the solution flow rate, Cs is ore gold content, Cl is the liquid gold content, X stands for all set of variables, and the subscripts stand for the stream number. Note that the bound constraints

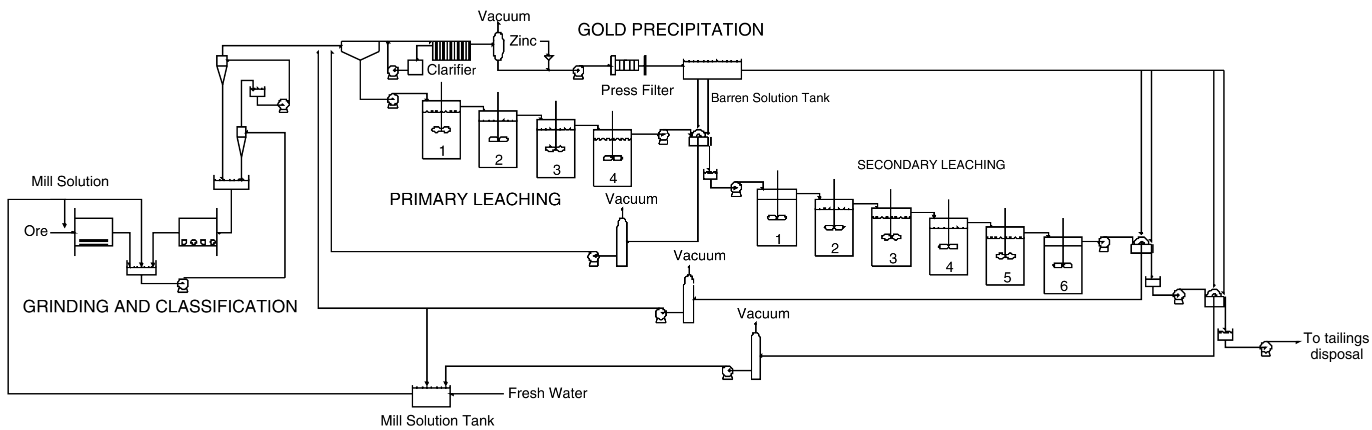


Fig. 1. Gold extraction plant flowsheet of Doyon Mine (1980s configuration) showing the comminution, classification, dewatering primary and secondary leaching, primary, secondary and tertiary filtration, and gold precipitation stages (adapted from Cimon et al., 1987; Cimon, 1989).

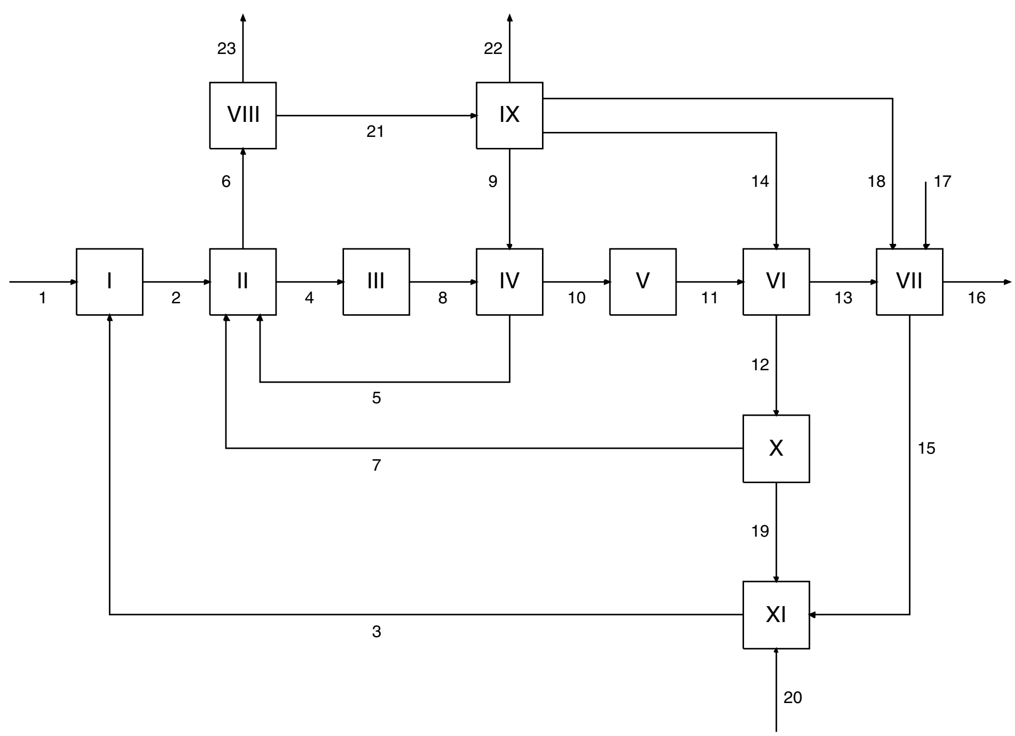


Fig. 2. Conceptual flowsheet with nodes and stream for the gold extraction plant of Doyon Mine (1980s configuration) presented at Fig. 1 (see explanation in the text).

(Eqs. A9a–f) were chosen based on the expected average process behavior.

### 3.3. Data reconciliation results

Data reconciliation of the measured variables and the estimation of the unmeasured variables were performed by solving the system of nonlinear equations given by the objective function (Eq. (2)) and all constraints (Eqs. A1a–f).

The measured flow rate, solid concentration and ore and liquid gold content in the plant streams were used as preliminary estimates for non-measured ore and solution flow rates throughout the plant using the ore and solution mass balance for each node. The resulting system of nonlinear equations was solved using the Gauss–Newton method. The data reconciliation problem, which has 57 variables, 28 linear constraints and 18 nonlinear constraints, was numerically solved using the nonlinear programming software NPSOL v.5.0 (Gill et al., 1986), in which the objective function, and all constraints, and gradients of the Lagrangian function were provided. The solution of this problem converged quickly and took a few seconds in a conventional personal computer.

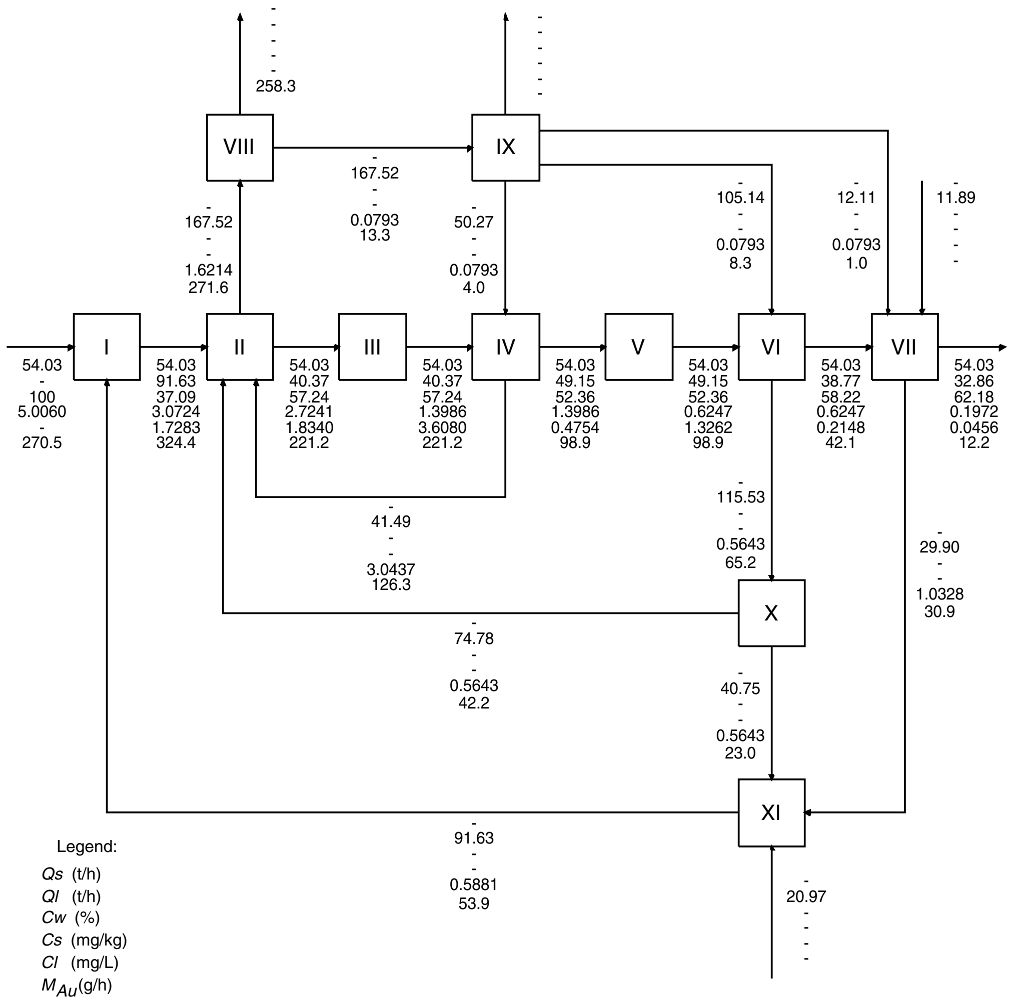


Fig. 3. Conceptual flowsheet for Doyon Mine 1980s configuration with the reconciled results; the nodes are described in Fig. 2.

Table 3 presents the reconciled values, which are summarized in Fig. 3 where the flow rates (Qs and Ql), the percentage of solids (Cw), the concentrations (Cs and Cl), and the gold masses ($M_{\mathrm{Au}}$) are presented on the plant graph. Note that the reconciled results in general give weak correction, except to the plant entrance gold content, and the liquid flow rates, and most of the values obtained show the expected trends. Note also that the corrected values are in general close to that reported by Cimon (1989) (see Table 1); however, the inconsistency in the ore gold content at the exit and entrance of the thickener and the primary and secondary filters previously found (see Fig. 2 and Table 1, nodes II, IV, and VI, and streams 2 and 4, 8 and 10, and 11 and 13) does not exist and the expected behavior of progressive dissolution through the plant now holds.

## 4. Case study 2: Doyon Mine plant 1990s configuration

### *4.1. Plant description and data acquisition*

The plant configuration used in the second case study is presented in Fig. 4. The ore is initially ground in a semi-autogeneous mill (SAG) followed by a spiral mechanical classifier. Then the slurry is pumped to a secondary grinding-classification stage composed of two ball mills and two hydrocyclone packs. The ore leaving this section is 80% finer than 37 μm. The slurry is then dewatered to about 50% of solids and pumped to the primary leaching circuit. Lead nitrate is added at the SAG mill discharge at a rate of 100 mg/kg of ore, in order to lower cyanide consumption by sulphides. The pH in the leaching section is set to about 12 by adding lime at the ball mill discharges. Sodium cyanide and alkalinized water are added in the SAG mill feed and in other points of the grinding circuit to promote the dissolution of the gold particles in the early stages. In the primary leaching stage, sodium cyanide is also added to the pulp, and the concentration of free cyanide ion ($CN^-$) is adjusted to about 250 mg/L.

The data set used in this second case study was collected in the plant during a sampling campaign performed on December 1998 at the grinding, classification, dewatering and primary leaching sections (see de Andrade Lima, 2001). The samples were taken manually throughout the circuit, and the gold content in the solution and in the

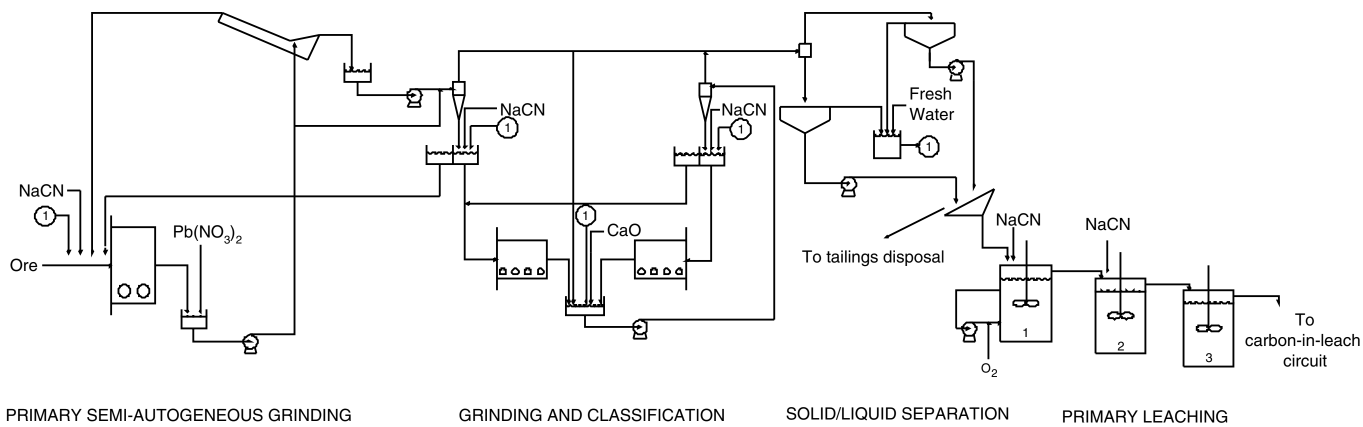


Fig. 4. Gold extraction plant flowsheet of Doyon Mine (1990s configuration) showing the comminution, classification, dewatering and primary leaching stages.

ore were analyzed by atomic absorption spectroscopy and fire assay, respectively. The ore fed rate in the plant and the solid concentration in the slurry during the period of the sampling campaign were provided by the plant operation. The samples were collected as two increments and the reaction stopped by oxidizing the free cyanide. Note that in the audit of gold processing plants, the use of more increments in the samples collection can be mandatory for unsteady-state streams. Also, note that the steady-state data reconciliation approach can be applied to naturally unsteady-state plant sections, such as the carbon-in-pulp or carbon-in-leach adsorption processes with repeated carbon transfer sequences, only if a very large number of increments in the samples collection are used.

The measured variables and the corresponding assumed standard deviation of the measures ($RDS_x$), which reflects the level of confidence of the variables, are summarized in Table 4. Note that in this case the solid gold content is also assumed as a very sensible measure due to the low values and the heterogeneity of the distribution of the gold in ore particles. The flux numbers 1–23 correspond to the plant streams and will be discussed in Section 4.2.

Note that for most of the non-measured variables, identified by "NA", the relative standard deviation ($RSD_x$) is chosen as a high value (90%) which allows a strong correction of these values without impacting the criterion given by Eq. (2). Also note that, as in the previous case, the solid

Table 4
Measured values and standard deviation for case study 2 (see Figs. 4 and 5)

| Flux | Solid flow rate (t/h) | $RSD_{Qs}$ (%) | Solid concentration (%) | $RSD_{Cw}$ (%) | Liquid flow rate (t/h) | $RSD_{Qs}$ (%) | Solid gold content (mg/kg) | $RSD_{Cs}$ (%) | Liquid gold content (mg/L) | $RSD_{Cl}$ (%) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 158.50 | 1 | 95.0 | 10 | NA | – | 6.5 | 1 | 0.0 | 1 |
| 2 | NA | 90 | 55.3 | 10 | NA | – | 17.0 | 1 | 3.1 | 1 |
| 3 | NA | 90 | 73.0 | 10 | NA | – | 14.0 | 1 | 3.0 | 1 |
| 4 | NA | 90 | 30.5 | 10 | NA | – | 3.9 | 1 | 3.0 | 1 |
| 5 | NA | 90 | 73.0 | 10 | NA | – | 14.0 | 1 | 3.0 | 1 |
| 6 | NA | 90 | 73.0 | 10 | NA | – | 14.0 | 1 | 3.0 | 1 |
| 7 | NA | 90 | 69.3 | 10 | NA | – | 82.0 | 1 | 3.8 | 1 |
| 8 | NA | 90 | 69.3 | 10 | NA | – | 65.0 | 1 | 4.7 | 1 |
| 9 | NA | 90 | NA | 90 | NA | – | NA | 90 | NA | 90 |
| 10 | NA | 90 | 72.3 | 10 | NA | – | 70.0 | 1 | 5.0 | 1 |
| 11 | NA | 90 | 74.5 | 10 | NA | – | 72.0 | 1 | 4.1 | 1 |
| 12 | NA | 90 | 24.7 | 10 | NA | – | 4.9 | 1 | 3.2 | 1 |
| 13 | NA | 90 | 74.5 | 10 | NA | – | 72.0 | 1 | 4.1 | 1 |
| 14 | NA | 90 | 74.5 | 10 | NA | – | 72.0 | 1 | 4.1 | 1 |
| 15 | NA | 90 | 72.3 | 10 | NA | – | 66.0 | 1 | 3.6 | 1 |
| 16 | NA | 90 | NA | 90 | NA | – | 9.1 | 1 | 3.3 | 1 |
| 17 | – | – | – | – | NA | 90 | – | – | 2.7 | 1 |
| 18 | NA | 90 | 48.9 | 10 | NA | – | 2.3 | 1 | 3.4 | 1 |
| 19 | – | – | – | – | NA | 90 | – | – | NA | 90 |
| 20 | – | – | – | – | NA | 90 | – | – | NA | 90 |
| 21 | – | – | – | – | NA | 90 | – | – | NA | 90 |
| 22 | – | – | – | – | NA | 90 | – | – | NA | 90 |
| 23 | – | – | – | – | NA | 90 | | | | |

gold content is assumed as a very noisy measure, for the same reason.

### 4.2. Mass balance equations and constraints

The studied sections of the plant (see Fig. 4) are represented as a 12 node and 23 stream graph diagram, see Fig. 5. The node numbers in Fig. 5 stand for—I: semi-autogeneous grinding and mechanical classification, II: primary hydrocyclone, III: division, IV: junction, V: thickener and primary leaching (tank 1), VI: junction, VII: primary ball mill, VIII: junction, IX: secondary hydrocyclone, X: division, XI: junction, and XII: secondary ball mill. The fluxes in Fig. 5 stand for: 1: plant ore entrance, 2: mechanical classifier overflow, 3: primary hydrocyclone underflow, 4: primary hydrocyclone overflow, 5: slurry splitter exit, 6: recycling, 7: primary ball mill entrance, 8: primary ball mill exit, 9: secondary hydrocyclone entrance, 10: secondary ball mill exit, 11: recycling, 12: secondary hydrocyclone overflow, 13: secondary hydrocyclone underflow, 14: slurry splitter exit, 15: secondary ball mill entrance, 16: thickener entrance, 17: thickener overflow, 18: first leaching tank exit, 19–22: process water additions, and 23: fresh water addition.

The mass balances for the main species in each node of the graph (see Fig. 5) are used as constraints for the data reconciliation problem. As in the previous case, the constraints include: (i) the ore mass balance, (ii) the solution mass balance, (iii) the physical constraints of equality of ore concentration in the leaching tanks, (iv) the gold mass balance in the ore and in the solution, (v) the gold mass balance in the ore for the fastest nodes, without leaching, (vi) the gold mass balance in the ore and in the solution for the fastest nodes, without leaching, (vii) the physical constraints of equality of gold concentrations, (viii) the inequality physical constraints, (ix) the positive value constraints for all variables, and (x) the low and high value constraints. The complete set of equality and inequality constraints is presented in Appendix B. Note that in this case the bound constraints (Eqs. B10a–r) were chosen also based on the expected average process behavior.

### 4.3. Data reconciliation results

Initially the measured flow rate, solid concentration and ore and liquid gold content in the plant streams were used for a preliminarily estimation of the non-measured ore and solution flow rates throughout the plant using the ore and solution mass balance for each node. The resulting system of nonlinear equations was solved using the Gauss–Newton method.

Data reconciliation of the measured variables and estimation of the unmeasured variables were performed by solving

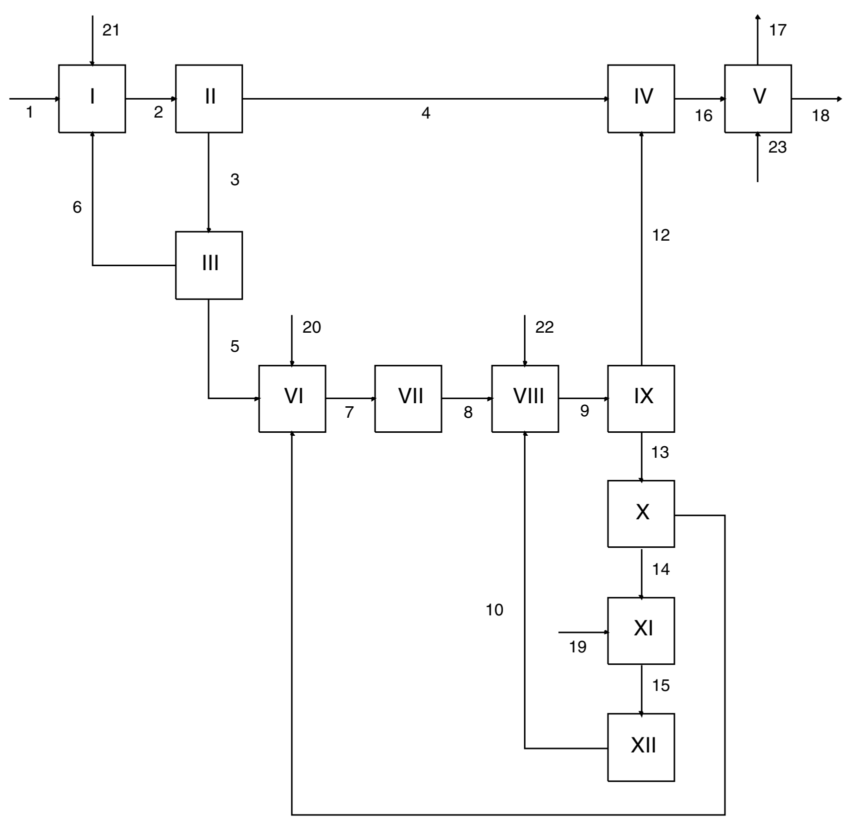


Fig. 5. Conceptual flowsheet with nodes and stream for the Gold extraction plant of Doyon Mine (1990s configuration) presented in Fig. 4 (see explanation in the text).

the system of nonlinear equations given by the objective function (Eq. (2)) and all constraints (Eqs. B1a–r). The data reconciliation problem, which has 79 variables, 41 linear constraints and 29 nonlinear constraints, was also numerically solved using the nonlinear programming software NPSOL v.5.0 (see Gill et al., 1986), in which the objective

Table 5
Corrected and estimated values for case study 2 (see Figs. 4 and 5)

| Flux | Solid flow rate (t/h) | Solid concentration (%) | Liquid flow rate (t/h) | Solid gold content (mg/kg) | Liquid gold content (mg/L) |
|---|---|---|---|---|---|
| 1 | 163.24 | 98.67 | 2.20 | 7.65 | 0.17 |
| 2 | 213.69 | 53.08 | 188.89 | 7.66 | 3.00 |
| 3 | 158.76 | 65.48 | 83.70 | 8.92 | 3.00 |
| 4 | 54.93 | 34.31 | 105.17 | 4.04 | 3.00 |
| 5 | 108.30 | 65.48 | 57.09 | 8.92 | 3.00 |
| 6 | 50.45 | 65.48 | 26.59 | 8.92 | 3.00 |
| 7 | 110.75 | 65.01 | 59.61 | 10.30 | 3.00 |
| 8 | 110.75 | 65.01 | 59.61 | 8.60 | 6.16 |
| 9 | 291.14 | 46.62 | 333.36 | 47.20 | 3.75 |
| 10 | 180.39 | 70.07 | 77.05 | 70.90 | 4.67 |
| 11 | 2.45 | 76.95 | 0.73 | 71.44 | 3.75 |
| 12 | 108.30 | 27.99 | 278.62 | 6.29 | 3.75 |
| 13 | 182.84 | 76.95 | 54.77 | 71.44 | 3.75 |
| 14 | 180.39 | 76.95 | 54.04 | 71.44 | 3.75 |
| 15 | 180.39 | 70.07 | 77.05 | 71.44 | 3.42 |
| 16 | 163.24 | 29.84 | 383.81 | 5.53 | 3.54 |
| 17 | – | – | 203.60 | – | 2.66 |
| 18 | 163.24 | 45.18 | 198.07 | 3.46 | 5.84 |
| 19 | – | – | 23.03 | – | 2.66 |
| 20 | – | – | 1.79 | – | 2.66 |
| 21 | – | – | 160.08 | – | 2.66 |
| 22 | – | – | 196.68 | – | 2.66 |
| 23 | – | – | 17.84 | | |

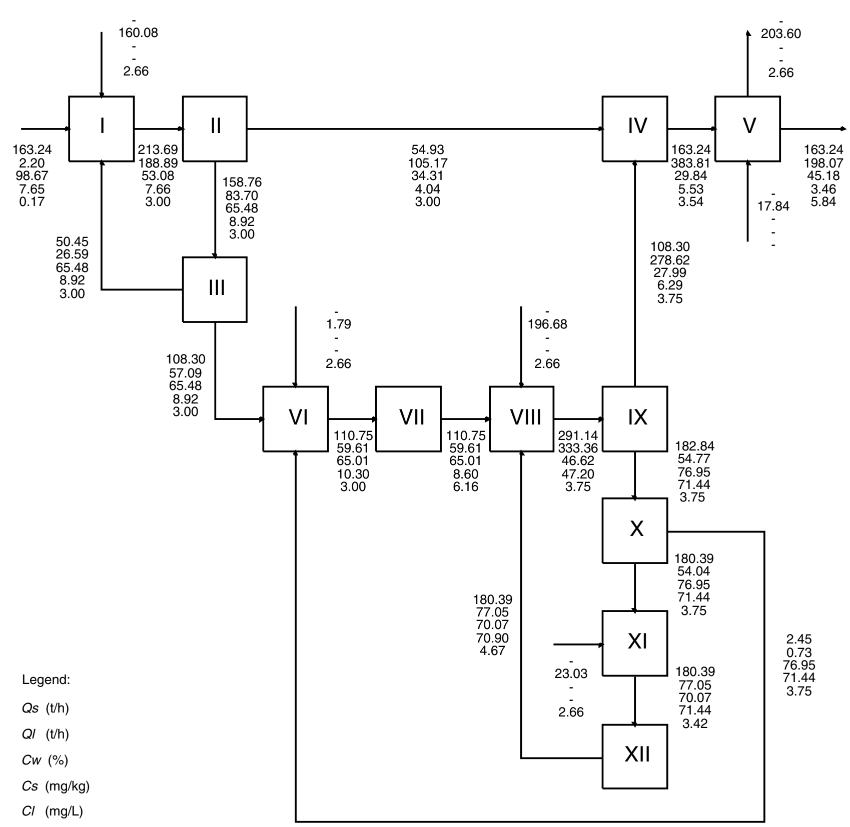


Fig. 6. Conceptual flowsheet for Doyon Mine 1990s configuration with the reconciled results; the nodes are described in Fig. 5.

function, and all constraints, and gradients of the Lagrangian function were provided. The solution of this problem also converged quickly and took a few seconds in a conventional personal computer.

Table 5 presents the reconciled values, which are also shown in Fig. 6, where the flow rates, the percentage of solids, and the concentrations are presented on the plant graph. Note that the reconciled results show that some measurements had a strong correction, particularly gold concentrations in the solids for flows 5–8, and that all values obtained showed the expected trends. The strong corrections in the values of ore gold content in streams 7 and 8 can be explained by the intermittent recycling streams given by flows 6 and 11. Note that the ore flow rate in flow 11 is very small compared to other flows but this specific stream is responsible for the enhancement in the gold content of flows 7 and 8. Note also that the expected behavior of progressive dissolution through the plant holds and there is no thermodynamic violation. Finally, note that in this particular case there is a weak dissolution of gold in the ball mills, a strong dissolution for the thickener and primary leaching tank and strong gravity concentration of gold in the hydrocyclone that closes the loop of the grinding classification section.

## 5. Conclusions

This paper presents two case studies of nonlinear data reconciliation for two different configurations of a gold extraction plant. In the first case the plant includes grinding, leaching, thickener, filtering, and precipitation sections while in the second case a more detailed description of the grinding, classification and primary leaching sections is done. The data in both cases were measured during sampling campaigns and reconciled by using the direct optimization of a nonlinear objective function and nonlinear constraints that included equality and inequality equations. The problems were readily solved using a sequential quadratic programming algorithm and the results demonstrated the applicability of this approach to provide reliable and accurate results in gold processing plants data reconciliation without thermodynamic violations. It is expected that the technique illustrated in this work will be useful to solve other data reconciliation problems in gold extraction plants and also to solve data reconciliation in other complex hydrometallurgical processes.

## Acknowledgements

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## Appendix A. Case study 1: Doyon Mine plant 1980s configuration (see Figs. 1 and 2)

(i) Ore mass balance:

$$\begin{aligned}
&\mathrm{Qs}_1 - \mathrm{Qs}_2 = 0\\
&\mathrm{Qs}_2 - \mathrm{Qs}_4 = 0\\
&\mathrm{Qs}_4 - \mathrm{Qs}_8 = 0\\
&\mathrm{Qs}_8 - \mathrm{Qs}_{10} = 0\\
&\mathrm{Qs}_{10} - \mathrm{Qs}_{11} = 0\\
&\mathrm{Qs}_{11} - \mathrm{Qs}_{13} = 0\\
&\mathrm{Qs}_{13} - \mathrm{Qs}_{16} = 0
\end{aligned} \tag{A1a–g}$$

(ii) Solution mass balance:

$$\begin{aligned}
&\mathrm{Ql}_3 - \mathrm{Qs}_2(\mathrm{Cw}_2^{-1} - 1) = 0\\
&\mathrm{Qs}_2(\mathrm{Cw}_2^{-1} - 1) + \mathrm{Ql}_7 + \mathrm{Ql}_5\\
&\quad - \mathrm{Ql}_6 - \mathrm{Qs}_4(\mathrm{Cw}_4^{-1} - 1) = 0\\
&\mathrm{Qs}_4(\mathrm{Cw}_4^{-1} - 1) - \mathrm{Qs}_8(\mathrm{Cw}_8^{-1} - 1) = 0\\
&\mathrm{Qs}_8(\mathrm{Cw}_8^{-1} - 1) + \mathrm{Ql}_9 - \mathrm{Ql}_5\\
&\quad - \mathrm{Qs}_{10}(\mathrm{Cw}_{10}^{-1} - 1) = 0\\
&\mathrm{Qs}_{10}(\mathrm{Cw}_{10}^{-1} - 1) - \mathrm{Qs}_{11}(\mathrm{Cw}_{11}^{-1} - 1) = 0\\
&\mathrm{Qs}_{11}(\mathrm{Cw}_{11}^{-1} - 1) + \mathrm{Ql}_{14} - \mathrm{Ql}_{12}\\
&\quad - \mathrm{Qs}_{13}(\mathrm{Cw}_{13}^{-1} - 1) = 0\\
&\mathrm{Qs}_{13}(\mathrm{Cw}_{13}^{-1} - 1) + \mathrm{Ql}_{18} + \mathrm{Ql}_{17}\\
&\quad - \mathrm{Ql}_{15} - \mathrm{Qs}_{16}(\mathrm{Cw}_{16}^{-1} - 1) = 0\\
&\mathrm{Ql}_{12} - \mathrm{Ql}_7 - \mathrm{Ql}_{19} = 0\\
&\mathrm{Ql}_{19} + \mathrm{Ql}_{15} + \mathrm{Ql}_{20} - \mathrm{Ql}_3 = 0\\
&\mathrm{Ql}_6 - \mathrm{Ql}_{21} = 0\\
&\mathrm{Ql}_{21} - \mathrm{Ql}_{22} - \mathrm{Ql}_9 - \mathrm{Ql}_{14} - \mathrm{Ql}_{18} = 0
\end{aligned} \tag{A2a–k}$$

(iii) Physical constraints—equality of ore concentrations:

$$\begin{aligned}
&\mathrm{Cw}_8 - \mathrm{Cw}_4 = 0\\
&\mathrm{Cw}_{11} - \mathrm{Cw}_{10} = 0
\end{aligned} \tag{A3a, b}$$

(iv) Gold mass balance in the ore and in the solution:

$$\begin{aligned}
&\mathrm{Qs}_1\mathrm{Cs}_1 + \mathrm{Ql}_3\mathrm{Cl}_3 - \mathrm{Qs}_2\mathrm{Cs}_2 - \mathrm{Qs}_2(\mathrm{Cw}_2^{-1} - 1) = 0\\
&\mathrm{Qs}_2\mathrm{Cs}_2 + \mathrm{Qs}_2(\mathrm{Cw}_2^{-1} - 1)\mathrm{Cl}_2 + \mathrm{Ql}_7\mathrm{Cl}_7\\
&\quad + \mathrm{Ql}_5\mathrm{Cl}_5 - \mathrm{Ql}_6\mathrm{Cl}_6 - \mathrm{Qs}_4\mathrm{Cs}_4 - \mathrm{Qs}_4(\mathrm{Cw}_4^{-1} - 1)\mathrm{Cl}_4 = 0\\
&\mathrm{Qs}_4\mathrm{Cs}_4 + \mathrm{Qs}_4(\mathrm{Cw}_4^{-1} - 1)\mathrm{Cl}_4 - \mathrm{Qs}_8\mathrm{Cs}_8\\
&\quad - \mathrm{Qs}_8(\mathrm{Cw}_8^{-1} - 1)\mathrm{Cl}_8 - \mathrm{Qs}_8\mathrm{Cs}_8 = 0\\
&\mathrm{Qs}_8\mathrm{Cs}_8 + \mathrm{Qs}_8(\mathrm{Cw}_8^{-1} - 1)\mathrm{Cl}_8 + \mathrm{Ql}_9\mathrm{Cl}_9 - \mathrm{Ql}_5\mathrm{Cl}_5\\
&\quad - \mathrm{Qs}_{10}(\mathrm{Cw}_{10}^{-1} - 1)\mathrm{Cl}_{10} - \mathrm{Qs}_{10}\mathrm{Cs}_{10} = 0\\
&\mathrm{Qs}_{10}\mathrm{Cs}_{10} + \mathrm{Qs}_{10}(\mathrm{Cw}_{10}^{-1} - 1)\mathrm{Cl}_{10} - \mathrm{Qs}_{11}\mathrm{Cs}_{11}\\
&\quad - \mathrm{Qs}_{11}(\mathrm{Cw}_{11}^{-1} - 1)\mathrm{Cl}_{11} = 0\\
&\mathrm{Qs}_{11}\mathrm{Cs}_{11} + \mathrm{Qs}_{11}(\mathrm{Cw}_{11}^{-1} - 1)\mathrm{Cl}_{11} + \mathrm{Ql}_{14}\mathrm{Cl}_{14}\\
&\quad - \mathrm{Ql}_{12}\mathrm{Cl}_{12} - \mathrm{Ql}_{13}\mathrm{Cl}_{13} - \mathrm{Qs}_{13}(\mathrm{Cw}_{13}^{-1} - 1)\mathrm{Cl}_{13} = 0\\
&\mathrm{Qs}_{13}\mathrm{Cs}_{13} + \mathrm{Qs}_{13}(\mathrm{Cw}_{13}^{-1} - 1)\mathrm{Cl}_{13} + \mathrm{Qs}_{18}\mathrm{Cs}_{18}\\
&\quad - \mathrm{Ql}_{15}\mathrm{Cl}_{15} - \mathrm{Qs}_{16}\mathrm{Cs}_{16} - \mathrm{Qs}_{16}(\mathrm{Cw}_{16}^{-1} - 1)\mathrm{Cl}_{16} = 0
\end{aligned} \tag{A4a–g}$$

(v) Gold mass balance for the fastest nodes (without leaching):

$$
\begin{aligned}
&\mathrm{Ql}_{12}\mathrm{Cl}_{12}-\mathrm{Ql}_{7}\mathrm{Cl}_{7}-\mathrm{Ql}_{19}\mathrm{Cl}_{19}=0\\
&\mathrm{Ql}_{19}\mathrm{Cl}_{19}+\mathrm{Ql}_{15}\mathrm{Cl}_{15}-\mathrm{Ql}_{3}\mathrm{Cl}_{3}=0\\
&\mathrm{Ql}_{6}\mathrm{Cl}_{6}-\mathrm{Ql}_{21}\mathrm{Cl}_{21}-M_{\mathrm{Au}_{23}}=0\\
&\mathrm{Ql}_{21}\mathrm{Cl}_{21}-\mathrm{Ql}_{22}\mathrm{Cl}_{22}-\mathrm{Ql}_{9}\mathrm{Cl}_{9}\\
&\quad-\mathrm{Ql}_{14}\mathrm{Cl}_{14}-\mathrm{Ql}_{18}\mathrm{Cl}_{18}=0
\end{aligned}
\tag{A5a–d}
$$

(vi) Physical constraints—equality of gold concentrations:

$$
\begin{aligned}
&\mathrm{Cl}_{21}-\mathrm{Cl}_{22}=0\\
&\mathrm{Cl}_{21}-\mathrm{Cl}_{9}=0\\
&\mathrm{Cl}_{21}-\mathrm{Cl}_{14}=0\\
&\mathrm{Cl}_{21}-\mathrm{Cl}_{18}=0\\
&\mathrm{Cl}_{12}-\mathrm{Cl}_{7}=0\\
&\mathrm{Cl}_{12}-\mathrm{Cl}_{19}=0
\end{aligned}
\tag{A6a–f}
$$

(vii) Inequality physical constraints:

$$
\begin{aligned}
&\mathrm{Cl}_{8}-\mathrm{Cl}_{4}\geqslant 0\\
&\mathrm{Cl}_{11}-\mathrm{Cl}_{10}\geqslant 0\\
&\mathrm{Cs}_{1}-\mathrm{Cs}_{2}\geqslant 0\\
&\mathrm{Cs}_{2}-\mathrm{Cs}_{4}\geqslant 0\\
&\mathrm{Cs}_{4}-\mathrm{Cs}_{8}\geqslant 0\\
&\mathrm{Cs}_{8}-\mathrm{Cs}_{10}\geqslant 0\\
&\mathrm{Cl}_{10}-\mathrm{Cl}_{11}\geqslant 0\\
&\mathrm{Cl}_{11}-\mathrm{Cl}_{13}\geqslant 0\\
&\mathrm{Cl}_{13}-\mathrm{Cl}_{16}\geqslant 0
\end{aligned}
\tag{A7a–i}
$$

(viii) Positive value constraints:

$$X_i \geqslant 0 \tag{A8}$$

(ix) Low and high value constraints:

$$
\begin{aligned}
&1000\leqslant \mathrm{Qs}_1\leqslant 5000\\
&0\leqslant \mathrm{Qs}_i\leqslant 10000\\
&0\leqslant \mathrm{Ql}_i\leqslant 10000\\
&20\leqslant \mathrm{Cw}_i\leqslant 70\\
&0\leqslant \mathrm{Cs}_i\leqslant 20\\
&0\leqslant \mathrm{Cl}_i\leqslant 15
\end{aligned}
\tag{A9a–f}
$$

## Appendix B. Case study 2: Doyon Mine plant 1990s configuration (see Figs. 4 and 5)

(i) Ore mass balance:

$$
\begin{aligned}
&\mathrm{Qs}_1+\mathrm{Qs}_6-\mathrm{Qs}_2=0\\
&\mathrm{Qs}_2-\mathrm{Qs}_3-\mathrm{Qs}_4=0\\
&\mathrm{Qs}_3-\mathrm{Qs}_6-\mathrm{Qs}_5=0\\
&\mathrm{Qs}_4+\mathrm{Qs}_{12}-\mathrm{Qs}_{16}=0\\
&\mathrm{Qs}_{16}-\mathrm{Qs}_{18}=0\\
&\mathrm{Qs}_5+\mathrm{Qs}_{11}-\mathrm{Qs}_7=0\\
&\mathrm{Qs}_7-\mathrm{Qs}_8=0\\
&\mathrm{Qs}_8+\mathrm{Qs}_{10}-\mathrm{Qs}_9=0\\
&\mathrm{Qs}_9-\mathrm{Qs}_{12}-\mathrm{Qs}_{13}=0\\
&\mathrm{Qs}_{13}-\mathrm{Qs}_{14}-\mathrm{Qs}_{11}=0\\
&\mathrm{Qs}_{14}-\mathrm{Qs}_{15}=0\\
&\mathrm{Qs}_{15}-\mathrm{Qs}_{10}=0
\end{aligned}
\tag{B1a–l}
$$

(ii) Solution mass balance:

$$
\begin{aligned}
&\mathrm{Qs}_1(\mathrm{Cw}_1^{-1}-1)+\mathrm{Qs}_6(\mathrm{Cw}_6^{-1}-1)\\
&\quad-\mathrm{Qs}_2(\mathrm{Cw}_2^{-1}-1)+\mathrm{Ql}_{21}=0\\
&\mathrm{Qs}_2(\mathrm{Cw}_2^{-1}-1)-\mathrm{Qs}_3(\mathrm{Cw}_3^{-1}-1)\\
&\quad-\mathrm{Qs}_4(\mathrm{Cw}_4^{-1}-1)=0\\
&\mathrm{Qs}_4(\mathrm{Cw}_4^{-1}-1)+\mathrm{Qs}_{12}(\mathrm{Cw}_{12}^{-1}-1)\\
&\quad-\mathrm{Qs}_{16}(\mathrm{Cw}_{16}^{-1}-1)=0\\
&\mathrm{Qs}_{16}(\mathrm{Cw}_{16}^{-1}-1)-\mathrm{Qs}_{18}(\mathrm{Cw}_{18}^{-1}-1)\\
&\quad-\mathrm{Ql}_{17}+\mathrm{Ql}_{23}=0\\
&\mathrm{Qs}_5(\mathrm{Cw}_5^{-1}-1)+\mathrm{Qs}_{11}(\mathrm{Cw}_{11}^{-1}-1)\\
&\quad-\mathrm{Qs}_7(\mathrm{Cw}_7^{-1}-1)+\mathrm{Ql}_{20}=0\\
&\mathrm{Qs}_8(\mathrm{Cw}_8^{-1}-1)+\mathrm{Qs}_{10}(\mathrm{Cw}_{10}^{-1}-1)\\
&\quad-\mathrm{Qs}_9(\mathrm{Cw}_9^{-1}-1)+\mathrm{Ql}_{22}=0\\
&\mathrm{Qs}_9(\mathrm{Cw}_9^{-1}-1)-\mathrm{Qs}_{12}(\mathrm{Cw}_{12}^{-1}-1)\\
&\quad-\mathrm{Qs}_{13}(\mathrm{Cw}_{13}^{-1}-1)=0\\
&\mathrm{Qs}_{14}(\mathrm{Cw}_{14}^{-1}-1)-\mathrm{Qs}_{15}(\mathrm{Cw}_{15}^{-1}-1)+\mathrm{Ql}_{19}=0
\end{aligned}
\tag{B2a–h}
$$

(iii) Physical constraints—equality of ore concentrations:

$$
\begin{aligned}
&\mathrm{Cw}_{15}-\mathrm{Cw}_{10}=0\\
&\mathrm{Cw}_{14}-\mathrm{Cw}_{13}=0\\
&\mathrm{Cw}_{11}-\mathrm{Cw}_{13}=0\\
&\mathrm{Cw}_{5}-\mathrm{Cw}_{3}=0\\
&\mathrm{Cw}_{6}-\mathrm{Cw}_{3}=0\\
&\mathrm{Cw}_{7}-\mathrm{Cw}_{8}=0
\end{aligned}
\tag{B3a–f}
$$

(iv) Gold mass balance in the ore and in the solution:

$$
\begin{aligned}
&\mathrm{Qs}_1\mathrm{Cs}_1+\mathrm{Qs}_1(\mathrm{Cw}_1^{-1}-1)\mathrm{Cl}_1+\mathrm{Qs}_6\mathrm{Cs}_6+\mathrm{Qs}_6(\mathrm{Cw}_6^{-1}-1)\\
&\quad-\mathrm{Qs}_2\mathrm{Cs}_2-\mathrm{Qs}_2(\mathrm{Cw}_2^{-1}-1)+\mathrm{Ql}_{21}\mathrm{Cl}_{21}=0\\
&\mathrm{Qs}_2\mathrm{Cs}_2+\mathrm{Qs}_2(\mathrm{Cw}_2^{-1}-1)\mathrm{Cl}_2-\mathrm{Qs}_3\mathrm{Cs}_3-\mathrm{Qs}_3(\mathrm{Cw}_3^{-1}-1)\mathrm{Cl}_3\\
&\quad-\mathrm{Qs}_4\mathrm{Cs}_4-\mathrm{Qs}_4(\mathrm{Cw}_4^{-1}-1)\mathrm{Cl}_4=0\\
&\mathrm{Qs}_4\mathrm{Cs}_4+\mathrm{Qs}_4(\mathrm{Cw}_4^{-1}-1)\mathrm{Cl}_4+\mathrm{Qs}_{12}\mathrm{Cs}_{12}\\
&\quad+\mathrm{Qs}_{12}(\mathrm{Cw}_{12}^{-1}-1)\mathrm{Cl}_{12}-\mathrm{Qs}_{16}\mathrm{Cs}_{16}-\mathrm{Qs}_{16}(\mathrm{Cw}_{16}^{-1}-1)\mathrm{Cl}_{16}=0\\
&\mathrm{Qs}_{16}\mathrm{Cs}_{16}+\mathrm{Qs}_{16}(\mathrm{Cw}_{16}^{-1}-1)\mathrm{Cl}_{16}-\mathrm{Qs}_{18}\mathrm{Cs}_{18}\\
&\quad-\mathrm{Qs}_{18}(\mathrm{Cw}_{18}^{-1}-1)\mathrm{Cl}_{18}-\mathrm{Ql}_{17}\mathrm{Cl}_{17}=0\\
&\mathrm{Qs}_5\mathrm{Cs}_5+\mathrm{Qs}_5(\mathrm{Cw}_5^{-1}-1)\mathrm{Cl}_5+\mathrm{Qs}_{11}\mathrm{Cs}_{11}\\
&\quad+\mathrm{Qs}_{11}(\mathrm{Cw}_{11}^{-1}-1)\mathrm{Cl}_{11}-\mathrm{Qs}_7\mathrm{Cs}_7-\mathrm{Qs}_7(\mathrm{Cw}_7^{-1}-1)\mathrm{Cl}_7\\
&\quad+\mathrm{Ql}_{20}\mathrm{Cl}_{20}=0
\end{aligned}
$$

$$Qs_7Cs_7 + Qs_7(Cw_7^{-1} - 1)Cl_7 - Qs_8Cs_8 - Qs_8(Cw_8^{-1} - 1)Cl_8 = 0$$
$$Qs_8Cs_8 + Qs_8(Cw_8^{-1} - 1)Cl_8 + Qs_{10}Cs_{10} + Qs_{10}(Cw_{10}^{-1} - 1)Cl_{10} - Qs_9Cs_9 - Qs_9(Cw_9^{-1} - 1)Cl_9 + Ql_{22}Cl_{22} = 0$$
$$Qs_9Cs_9 + Qs_9(Cw_9^{-1} - 1)Cl_9 - Qs_{12}Cs_{12} - Qs_{12}(Cw_{12}^{-1} - 1)Cl_{12} - Qs_{13}Cs_{13} - Qs_{13}(Cw_{13}^{-1} - 1)Cl_{13} = 0$$
$$Qs_{14}Cs_{14} + Qs_{14}(Cw_{14}^{-1} - 1)Cl_{14} - Qs_{15}Cs_{15} - Qs_{15}(Cw_{15}^{-1} - 1)Cl_{15} + Ql_{19}Cl_{19} = 0$$
$$Qs_{15}Cs_{15} + Qs_{15}(Cw_{15}^{-1} - 1)Cl_{15} - Qs_{10}Cs_{10} - Qs_{10}(Cw_{10}^{-1} - 1)Cl_{10} = 0 \qquad \text{(B4a–j)}$$

(v) Gold mass balance in the ore for the fastest nodes (without leaching):

$$\begin{aligned} &Qs_2Cs_2 - Qs_3Cs_3 - Qs_4Cs_4 = 0 \\ &Qs_4Cs_4 + Qs_{12}Cs_{12} - Qs_{16}Cs_{16} = 0 \\ &Qs_5Cs_5 + Qs_{11}Cs_{11} - Qs_7Cs_7 = 0 \\ &Qs_8Cs_8 + Qs_{10}Cs_{10} - Qs_9Cs_9 = 0 \\ &Qs_9Cs_9 - Qs_{12}Cs_{12} - Qs_{13}Cs_{13} = 0 \end{aligned} \qquad \text{(B5a–e)}$$

(vi) Gold mass balance in the ore and in the solution for the fastest nodes (without leaching):

$$\begin{aligned} &Qs_2(Cw_2^{-1} - 1)Cl_2 - Qs_3(Cw_3^{-1} - 1)Cl_3 - Qs_4(Cw_4^{-1} - 1)Cl_4 = 0 \\ &Qs_4(Cw_4^{-1} - 1)Cl_4 + Qs_{12}(Cw_{12}^{-1} - 1)Cl_{12} - Qs_{16}(Cw_{16}^{-1} - 1)Cl_{16} = 0 \\ &Qs_5(Cw_5^{-1} - 1)Cl_5 + Qs_{11}(Cw_{11}^{-1} - 1)Cl_{11} - Qs_7(Cw_7^{-1} - 1)Cl_7 + Ql_{20}Cl_{20} = 0 \\ &Qs_8(Cw_8^{-1} - 1)Cl_8 + Qs_{10}(Cw_{10}^{-1} - 1)Cl_{10} - Qs_9(Cw_9^{-1} - 1)Cl_9 + Ql_{22}Cl_{22} = 0 \\ &Qs_9(Cw_9^{-1} - 1)Cl_9 - Qs_{12}(Cw_{12}^{-1} - 1)Cl_{12} - Qs_{13}(Cw_{13}^{-1} - 1)Cl_{13} = 0 \\ &Qs_{14}(Cw_{14}^{-1} - 1)Cl_{14} - Qs_{15}(Cw_{15}^{-1} - 1)Cl_{15} + Ql_{19}Cl_{19} = 0 \end{aligned} \qquad \text{(B6a–f)}$$

(vii) Physical constraints—equality of gold concentrations:

$$\begin{aligned} &Cs_3 - Cs_6 = 0 \\ &Cs_3 - Cs_5 = 0 \\ &Cs_{13} - Cs_{14} = 0 \\ &Cs_{13} - Cs_{11} = 0 \\ &Cs_{14} - Cs_{15} = 0 \\ &Cl_3 - Cl_6 = 0 \\ &Cl_3 - Cl_5 = 0 \\ &Cl_{13} - Cl_{14} = 0 \\ &Cl_{13} - Cl_{11} = 0 \\ &Cl_9 - Cl_{13} = 0 \\ &Cl_2 - Cl_3 = 0 \\ &Cl_3 - Cl_4 = 0 \\ &Cl_9 - Cl_{12} = 0 \\ &Cl_{19} - Cl_{20} = 0 \\ &Cl_{19} - Cl_{21} = 0 \\ &Cl_{19} - Cl_{22} = 0 \\ &Cl_{19} - Cl_{17} = 0 \end{aligned} \qquad \text{(B7a–q)}$$

(viii) Inequality physical constraints:

$$\begin{aligned} &Qs_3 - Qs_4 \geqslant 0 \\ &Qs_5 - Qs_6 \geqslant 0 \\ &Cs_7 - Cs_8 \geqslant 0 \\ &Cs_{15} - Cs_{10} \geqslant 0 \\ &Cs_{16} - Cs_{18} \geqslant 0 \\ &Cl_{18} - Cl_{16} \geqslant 0 \end{aligned} \qquad \text{(B8a–f)}$$

(ix) Positive value constraints:

$$X_i \geqslant 0 \qquad \text{(B9)}$$

(x) Low and high value constraints:

$$\begin{aligned} &140 \leqslant Qs_1 \leqslant 180 \\ &90 \leqslant Cw_1 \leqslant 99 \\ &50 \leqslant Cw_2 \leqslant 57 \\ &65 \leqslant Cw_3 \leqslant 77 \\ &27 \leqslant Cw_4 \leqslant 35 \\ &0 \leqslant Cw_5 \leqslant 100 \\ &0 \leqslant Cw_6 \leqslant 100 \\ &65 \leqslant Cw_7 \leqslant 80 \\ &0 \leqslant Cw_8 \leqslant 100 \\ &0 \leqslant Cw_9 \leqslant 100 \\ &0 \leqslant Cw_{10} \leqslant 100 \\ &0 \leqslant Cw_{11} \leqslant 100 \\ &20 \leqslant Cw_{12} \leqslant 28 \\ &72 \leqslant Cw_{13} \leqslant 77 \\ &0 \leqslant Cw_{14} \leqslant 100 \\ &70 \leqslant Cw_{15} \leqslant 80 \\ &0 \leqslant Cw_{16} \leqslant 100 \\ &45 \leqslant Cw_{18} \leqslant 50 \end{aligned} \qquad \text{(B10a–r)}$$

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