Mechanical Systems and Signal Processing 24 (2010) 1739–1755

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/jnlabr/ymssp

Optimum design of linear tuned mass dampers for structures with nonlinear behaviour

Sara Sgobba *, Giuseppe Carlo Marano

Department of Environmental Engineering and Sustainable Development, Tecnical University of Bari, Viale del Turismo 10, 74100 Taranto, Italy

ARTICLE INFO

Article history:
Received 15 February 2009
Received in revised form
22 September 2009
Accepted 30 January 2010
Available online 10 February 2010

Keywords:
Bouc–Wen hysteretic model
Covariance analysis
Design optimization
Dissipated energy
Seismic excitation
Tuned mass dampers

ABSTRACT

This paper investigates the optimum design of tuned mass dampers (TMDs) for the seismic protection of inelastic structures. A single linear TMD is treated and is assumed to be applied to a single nonlinear degree of freedom system, described by the Bouc–Wen hysteretic model. The seismic load is modelled by a stationary filtered stochastic process to consider its intrinsic stochastic nature. The optimization problem is set by taking into consideration three different possible objective functions (OFs): the maximum of the peak structural displacement standard deviation, the average hysteretic dissipated energy of a protected building with reference to an unprotected one, and a functional damage that considers the two indexes previously described. Different numerical examples and parametric analysis are shown to confront the three optimization criteria and to determine the best tuning frequency and damping ratio of the TMDs to be used in the structure. Results confirm that the application of a TMD system reduces the amount of the hysteretic dissipated energy, which is a direct measure of damage in the structure, and so it is beneficial for the protection of buildings that develop a nonlinear behaviour under severe dynamic loadings.

1. Introduction

The vibration control regarding structures subjected to environmental dynamic loads is a central theme in civil and mechanical structural designs. In this field, different strategies have been proposed, which also consider structural safety under random vibrations caused by natural or artificial loads (such as earthquakes, winds, traffic vibrations, sea waves. The passive control is a widely accepted strategy that is frequently adopted in civil structures. Among different passive control methods that are available now, the tuned mass damper (TMD) is used efficiently to check vibrations of mechanical and structural systems, as well as to reduce the dynamic structural response caused by wind action. Its use is rapidly increasing and a number of similar control devices are widely implemented in a large number of buildings throughout the world [1]. The system is based on an elasto-viscous mass linked to the main structure and generally designed to oscillate with the same period of the main system but in an opposite phase. If the natural frequency of a TMD is tuned in resonance with the fundamental mode of the primary structure, a large amount of structural vibrating energy is transferred to the TMD and then dissipated by damping when the primary structure is subject to external excitations such as earthquake ground motions. Its performance greatly depends on the characteristics of the ground motion, as demonstrated by several studies. For instance, it was found that TMDs become effective in reducing the seismic response of structures only when the ground motion exhibits a narrow band frequency and long duration [2].

* Corresponding author.
E-mail address: s.sgobba@poliba.it (S. Sgobba).

0888-3270/$ - see front matter
doi:10.1016/j.ymssp.2010.01.009

Most of these studies are concerned with the analysis of TMD effectiveness on systems with linear behaviour. In fact, in the literature, there are many references to the application of TMDs to linear structures. On the contrary, the literature speaks very little about TMD applications to inelastic structures. Regarding this topic, Soto-Brito and Ruiz [3], for the first time, studied the influence of ground motion intensity on the effectiveness of TMDs to consider the possibility of reducing the damage of a building during a high intensity earthquake. Their research shows that the effectiveness of TMDs in limiting the peak response of the structures is highly reduced for systems developing nonlinear behaviour, which generally occurs under high-intensity ground motions. Other authors have also examined the behaviour of an inelastic single degree of freedom (SDOF) structure-TMD system under random excitation [4,5]. In all these studies, reduction in the maximum displacement response was found to decrease with increase in inelastic excursion.

However, the peak displacement response reduction of the nonlinear structures was found to be insufficient to describe the TMD effectiveness, because this measure failed to take into account the effects of accumulated damage due to low cycle fatigue [6]. For this reason, a damage reduction strategy was proposed by Pinkaew et al. [7] as an index to evaluate the seismic effectiveness of the TMDs for an inelastic concrete building, modelled by an equivalent inelastic single-degree-of-freedom system. This study concluded that TMDs are very effective in reducing the peak displacement of the structure when the structure is vibrated within the elastic range but its effectiveness gradually decreases as the inelasticity in the structure increases. However, the same authors revealed that TMD reduces the structural damage for the entire range of peak ground acceleration (PGA) considered and provides positive effectiveness in damage reduction of the structure because the damage index is mainly governed by the energy term [7].

Other studies have also demonstrated that the use of TMDs on inelastic structures reduces the plastic energy dissipation and then the structural damage. Energy is indeed an excellent way of characterizing the dynamic behaviour of the structure [8]. The effectiveness of TMDs in reducing structural response from the point of view of energy was studied by Wong and Chee [9], which investigated the energy balance of the TMD-primary structure system. Also Miranda [10] performed a study on SDOF structures with TMDs. He found that the equivalent damping in the equivalent elastic structure is increased and therefore damping energy dissipation is also increased accordingly. More recently, Wong [8] analyzed various forms of energy in the structure during an earthquake in order to evaluate the effectiveness of using TMDs. His results showed that the use of TMDs enhances energy dissipation by the structure. Wong and Johnson [11] also researched the ability to use multiple TMDs for improving inelastic structural performance to dissipate earthquake input energy.

Even though all of the earlier mentioned studies have widely demonstrated that the use of TMD on inelastic structures enhances the ability of the system to reduce plastic energy dissipation [8–11], most of these researches were based on deterministic analysis and made use of a few number of earthquake time-histories for their numerical simulations, and a larger number of records should be considered in order to draw out more general conclusions. Moreover, in the same studies, the optimization problem of the design parameters of TMDs on inelastic structures and the sensitivity analysis to some of the main important hysteretic mechanical characteristics have never been treated. For these reasons, in order to examine the protection effectiveness of a TMD on a hysteretic structure from an energy perspective, in the present research, a stochastic approach is adopted to properly take into account the uncertainty related to the intrinsic nature of seismic excitation. An optimization procedure is applied to better investigate the performance of TMDs and to estimate their optimal mechanical parameters (i.e., the optimum tuning frequency and damping ratio) compared to that resulting from the protection of a linear structure. With this aim, three different indexes of protection performance have been defined. The first one is quite conventional and concerns the dimensionless peak of the protected system displacement with respect to the unprotected one. This index of the structural response is quite significant for linear systems, but it cannot give a valid indication of the TMD effectiveness for structures with an elasto-plastic behaviour [6]. Thus, the second objective function (OF) considered here takes into account the energy dissipation ratio of the protected system to that of the unprotected one. This ratio has been introduced to consider the hysteretic properties of structures in the nonlinear range. In fact, as discussed above, the hysteretic dissipated energy is a direct measure of the damage level in structural elements, especially when they are subject to severe ground motions in which the buildings are supposed to undergo inelastic deformations. Furthermore, a third objective function has been introduced to analyze a combination of both straining and dissipative effects.

In this paper, the unconstrained optimization problem is developed on an SDOF Bouc–Wen nonlinear hysteretical mechanical model [12,13]; it is used to attain a suitable mechanical description of system behaviour under strong earthquake loads in the inelastic range. Regarding other optimization procedures proposed for the design of TMD mechanical parameters, an important difference consists in adopting a stochastic approach to take into account the intrinsic stochastic nature of the earthquakes. More precisely, a Gaussian zero mean stationary Kanai–Tajimi filtered stochastic process is used to model the seismic action. Since the problem is nonlinear, an *equivalent stochastic linearization* is adopted to solve it. This approach allows the evaluation of the maximum displacement variance and the hysteretic energy mean value using a covariance analysis. A sensitivity analysis is carried out and the presented results are a useful tool for preliminary design of TMDs.

2. Nonlinear structural model

The evaluation of the performance of structures subject to earthquakes needs an accurate modelling of the nonlinear structural behaviour characterized by hysteresis. Hence, in structural mechanics, a great interest has been directed towards the research of some mathematical models useful to represent hysteretic deformation cycles and to reproduce the load history and the degradation depending on the dissipated energy. The nonlinear differential Bouc–Wen (BW) model has been and is still adopted in the field of seismic engineering to analyze the conventional reinforced concrete and steel structures and to study passive seismic protection systems. This kind of model has been already used by Zhu et al. [14] and Huang et al. [15] in order to examine the effectiveness of linear secondary systems attached to yielding primary structure. The BW model was originally formulated by Bouc [12] and later generalized by Wen [13]. All details of the BW model can be found in the work of Baber and Wen [16]. In some work, it has been pointed out that the Bouc–Wen class models are not in agreement with the requirements of the classical plasticity theory, such as Drucker's postulate [17], and may produce negative energy dissipation when the unloading–reloading process occurs without load reversal [18,19]. Nevertheless, the BW class models have been widely used in the field of structural engineering because they greatly facilitate deterministic and stochastic dynamic analyses of real structures with reasonable accuracy. Moreover, local violation of the plasticity theory is not seen as a particularly important factor in the random vibration analysis if the expected value of the restoring force is zero [20,21]. The BW model has the advantage of possessing mathematical simplicity (its definition is similar to that of a bilinear plastic model). At the same time it takes into account the amount of dissipated energy more accurately and realistically, showing a smoother hysteresis cycle that can differ from a bilinear trend significantly. Besides, it offers the possibility of expressing the linearization coefficients in a closed form. For these reasons and because the main aim of this paper is to assess the seismic protection effectiveness of a TMD, the BW model has been chosen to describe the nonlinear behaviour of the main structure.

In the BW model the restoring force $F(x,\dot{x},z)$ can be divided into two parts: the first $L(x,\dot{x};t)$ is caused by the linear viscous–elastic contribution while the second $F_h(x,\dot{x},z;t)$ is due to the hysteretic one, as follows:

$$L(x,\dot{x};t) = c\dot{x}(t) + \alpha_s kx(t) \tag{1}$$

$$F_h(x,\dot{x},z;t) = (1-\alpha_s)kz(t) \tag{2}$$

where c is the damping, k is the stiffness, and $x(t)$ and $z(t)$ are the quantities, which denote, respectively, the position and the hysteretic restoring force of the oscillator. The initial elastic stiffness k_i and the post-yielding stiffness k_p are defined as follows [22,23]) (Fig. 1):

$$k_i = \left(\frac{\partial F}{\partial x}\right)_{z=0} = \alpha_s k + (1-\alpha_s)k\lambda \tag{3}$$

$$k_p = \left(\frac{\partial F}{\partial x}\right)_{z_{max}} = \alpha_s k$$

where z_{max} is the maximum value of the internal variable z (for positive value of $\dot{x}$ and z) [24]

$$z_{max} = \left(\frac{\lambda}{\beta+\gamma}\right)^{1/\eta} \tag{4}$$

and α_s is the post-yielding stiffness ratio

$$\alpha_s = \frac{k_p}{k_i} \tag{5}$$

The variable $z(t)$ satisfies the following nonlinear equation [13]:

$$\dot{z}(t) = \dot{x}(t)[\lambda - |z(t)|^\eta \left(\beta + \gamma\,\mathrm{sgn}\{z(t)\}\mathrm{sgn}\{\dot{x}(t)\}\right)] \tag{6}$$

The model parameters β, γ, η, λ appearing in Eq. (6) are simple loop parameters that control the scale and the shape of hysteresis [23] characterized by positive energy dissipation in each cycle. Hence, thermodynamic laws require that $\gamma > 0$. The parameter η controls the smoothness sharpness of the yield. In fact, for $\eta \to \infty$, the model becomes bilinear.

Several mechanical quantities can be related to the above described parameters to model the structural elements more realistically.

The hysteretic restoring force $F_h = (1-\alpha_s)kz$ has an associated maximum value at z_{max}:

$$F_h^{max} = (1-\alpha_s)kz_{max} \tag{7}$$

Moreover, it is possible to define the elastic limit displacement X_Y [22,23]:

$$X_Y = \frac{F_h^{max}}{(1-\alpha_s)k\lambda} = \frac{z_{max}}{\lambda} = \lambda^{(1-\eta)/\eta}(\beta+\gamma)^{-(1/\eta)} \tag{8}$$

From these relations, it follows that if $\lambda = 1$ then $k = k_i$, and α_s is exactly the ratio of the final to the initial stiffness; hence

$$X_Y = (1/(\beta+\gamma))^{(1/\eta)} \tag{9}$$

Thus, the limit of elastic force per unit of mass is

$$f_Y = F_y/m = \omega^2 X_y \tag{10}$$

where ω is the natural frequency of the system.

Moreover, if the unloading stiffness is equal to the elastic one, as in a large number of mechanical elements, it is possible to assume that $\beta=\gamma$ [23]).

3. Equations of motion

The structural system is described by a single mass m, which is linked to the vibration support by a linear spring, a dashpot, and hysteretic properties whose mechanical characteristics are k, c, and z, as described above. The mass is connected to a secondary linear tuned mass m_T by an elastic spring k_T and a dashpot c_T (to introduce viscous–elastic characteristics), so that the global system is 2-DOF as shown in Fig. 2.

For such an SDOF system, which is modelled by means of the BW constitutive law and excited at the base by an acceleration $\ddot{x}_b(t)$, the equation of motion is

$$\ddot{x}(t)+2\xi_s\omega_s\dot{x}(t)+\alpha\omega_s^2x(t)+(1-\alpha)z(t)\omega_s^2=\ddot{x}_b(t) \tag{11}$$

The dynamic parameters of the main SDOF system are defined as follows:

the natural structural frequency is

$$\omega_s=\sqrt{\frac{k}{m}} \tag{12}$$

the structural damping ratio is

$$\xi_s=\frac{c}{2\omega_s m}=\frac{c}{2\sqrt{km}} \tag{13}$$

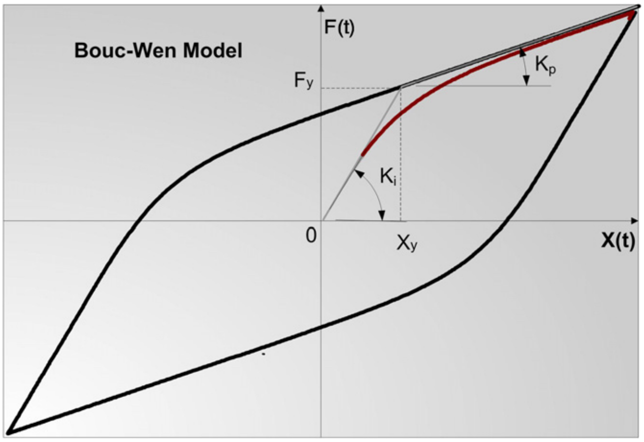

Fig. 1. Bouc–Wen constitutive law.

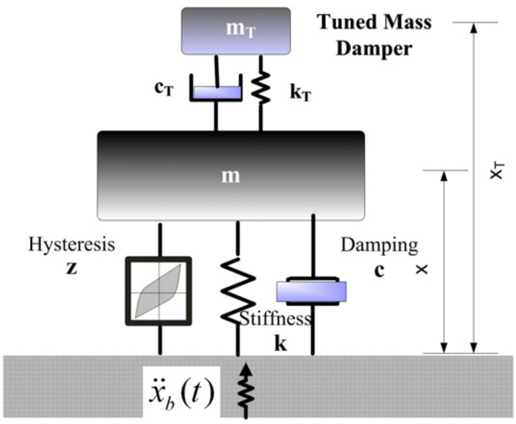

Fig. 2. SDOF hysteric model equipped with a TMD subjected to base acceleration.

the limit elastic force (per unit of mass) is

$$f_Y = F_y/m = \omega_s^2 X_y \tag{14}$$

where X_y is the elastic limit displacement as defined in Eq. (9). The hysteretic properties are described by the $z(t)$ variable [13,23] through Eq. (6). In this work, η is assumed equal to 1 as established in the classical formulation of the model [23].

The state vector $\overline{X} = (x, x_T)^T$ incorporates the displacements $x(t)$ of the main system and the displacement of the TMD, $x_T(t)$. For the base random accelerations modelling, a widely adopted model for both stationary and non-stationary cases is obtained by a simple linear second order filtering of the white noise process. The model can characterize the input frequency modulation for a wide range of practical situations and also amplitude and time variations of the frequency content in the case of non-stationary inputs.

3.1. Seismic model

In this work, the original stationary Kanai–Tajimi (K–T) [25] stochastic seismic model is applied to describe the earthquake ground acceleration. This model has had some wide applications in the random vibration analysis of structures because it provides a simple way to describe ground motions characterized by a single dominant frequency [26]. However, it should be observed that the TMD offers its major protection effectiveness when it is tuned to the first vibration mode of the system because, in this case, the maximum vibrational energy is transferred from the system to the TMD. For this reason, the choice of a single dominant frequency seismic model is considered an acceptable simplification. The K–T model is applied to the stochastic approach to deal with various uncertainties present in the nature of the ground motions, related to the parameters involving the geological conditions of the site, the distance from the source, the fault mechanism, etc. Moreover, the application of this model serves to provide some input excitations to the structural models for sites with no strong ground motion data. The K–T model is obtained using a simple filtered white noise linear oscillator, which treats the earthquakes as stationary random processes in its original formulation. In the present study a stationary Gaussian white noise process $w(t)$ is assumed to be generated at the bed-rock.

On the bases of the above mentioned considerations, the total acceleration $\ddot{x}_b(t)$ acting on the structure is given by adding the contribution of the inertial force $\ddot{x}_f(t)$ of the K–T filter and the white noise excitation function $w(t)$, as follows:

$$\begin{cases} \ddot{x}_f(t) + 2\xi_f\omega_f\dot{x}_f + \omega_f^2 x_f = -w(t) \\ \ddot{x}_b(t) = \ddot{x}_f(t) + w(t) = -2\xi_f\omega_f\dot{x}_f + \omega_f^2 x_f \end{cases} \tag{15}$$

where $w(t)$ is a stationary Gaussian zero mean white noise process whose intensity is given by S_0 representing the frequency content of excitation at the bed-rock, ω_f is the filter natural frequency, and ξ_f the filter damping. The white noise spectral intensity factor S_0 can be estimated from the PGA through the following relation [27]:

$$3\sigma_{\ddot{x}_b} = PGA \tag{16}$$

where σ_b is the standard deviation of the total acceleration $\ddot{x}_b(t)$:

$$\sigma_{\ddot{x}_b}^2 = \frac{\pi}{2}\frac{S_0\omega_f}{\xi_f}(1+4\xi_f) \tag{17}$$

Then

$$S_0 = \frac{2}{3^2}\frac{\xi_f(PGA)^2}{(1+4\xi_f^2)\omega_f} = 0.0707\frac{\xi_f(PGA)^2}{(1+4\xi_f^2)\omega_f} \tag{18}$$

where ξ_f and ω_f are the filter damping ratio and the natural frequency, respectively.

3.2. Statistical linearization and seismic covariance response

Combining Eqs. (11) and (15) with the dynamic equation of the TMD, the following can be obtained:

$$\begin{cases} \ddot{x}_T = \ddot{x} - \omega_T^2 x_T + \omega_T^2 x - 2\xi_T\omega_T\dot{x}_T + 2\xi_T\omega_T\dot{x} + (\omega_f^2 x_f + 2\xi_f\omega_f\dot{x}_f) \\ \ddot{x} = -\alpha_s\omega_s^2 x - (1-\alpha_s)\omega_s^2 z - 2\xi_s\omega_s\dot{x} + 2\mu\xi_T\omega_T\dot{x}_T - 2\mu\xi_T\omega_T\dot{x} + \mu\omega_T^2 x_T - \mu\omega_T^2 x + (2\xi_f\omega_f\dot{x}_f + \omega_f^2 x_f) \end{cases} \tag{19}$$

where the TMD mechanical parameters are as follows:

1. TMD frequency

$$\omega_T = \sqrt{\frac{k_T}{m_T}} \tag{20}$$

2. TMD damping

$$\xi_T = \frac{c_T}{2\sqrt{m_T k_T}} \tag{21}$$

3. mass ratio

$$\mu = \frac{m_T}{m} \tag{22}$$

The equations of motion (19) are nonlinear because of the hysteretic behaviour. To solve this differential problem, several mathematical methods can be applied [28]. The *equivalent stochastic linearization method* is the most commonly adopted approximation method to analyze the stochastic nonlinear structural dynamic problems (see, e.g., [29]). Adopting this approach, nonlinear equation (19) can be replaced with a linear one with the technique of stochastic linearization (see Appendix 1 for details). As is well known, this procedure provides an adequate approximation for unprotected linear structures. Nevertheless, when a TMD is added to the main system, the effects on it are so sensitive to the tuning condition that the quality of the approximation can be influenced. In such cases, a Monte Carlo simulation would be recommendable to validate the procedure. Regarding this aspect, a few authors have proved, through simulations, that the linearization technique is reliable for the nonlinear system under study. Among these, [14] deals with the response and reliability of an SDOF linear secondary system mounted on an SDOF yielding hysteretic primary structure subject to a Gaussian white noise horizontal ground acceleration. The authors obtained results to Monte Carlo simulations. Another reference for the present issue was also offered by Huang et al. [15], who analyzed the response and reliability of an SDOF linear secondary system attached to an MDOF, yielding a hysteretic primary structure subject to Gaussian white noise or coloured noise horizontal ground acceleration. They found that the results, using equivalent linearization, usually underestimate the response of the secondary system on primary structure yielding but can indicate the main trend of the response. Rudinger [30] investigated the behaviour of a TMD with a nonlinear viscous power law. The system was analyzed by statistical linearization and stochastic simulation via Monte Carlo's technique, with the objective of reducing the variance of the displacement of the primary structure. The stochastic simulation confirmed the accuracy of the results from statistical linearization in terms of the standard deviation of the response. Also Inaudi and Kelly [31] studied a nonlinear mass damper with friction dampers on an SDOF system. They compared results of the optimal parameters obtained by the statistical linearization approximation and numerical simulation. The authors found that an excellent accuracy is obtained with the statistical linearization method for both displacement and acceleration responses. The accuracy deteriorates with an increase in the mass ratio (for ratios larger than 0.1). In this work, the mass ratio is assumed to be μ=0.025, so the approximation technique can be considered acceptable.

Adopting stochastic linearization, the motion equation governing the internal variable $z(t)$ is substituted with a linear equivalent one, and Eq. (19) can be replaced with the following linear time-invariant representation of the input–output model:

$$\dot{\overline{Y}}(t) = \mathbf{A}^{eq}(\mathbf{R},t)\overline{Y}(t) + \overline{F}(t) \tag{23}$$

where $\overline{Y} = [x_T, x, x_f, z, \dot{x}_T, \dot{x}, \dot{x}_f]^T$ is the state space vector and $\overline{F}(t)$ is a vector of order 7, called the *transfer input vector*, having all the elements equal to zero except for the last one:

$$\overline{F}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & w(t) \end{bmatrix}^T \tag{24}$$

Assuming that the system is linear and the input function is a Gaussian zero mean process, the response is a Gaussian zero mean too. It follows that the knowledge of the covariance matrix $\mathbf{R}(t)$ can completely define the state space of the statistic response.

The $\mathbf{A}^{eq}(\mathbf{R},t)$ equivalent linearized matrix is a square matrix of order 7, representing the equivalent linearized state matrix (details are reported in Appendix 2) and contains the linearized coefficients, which are dependent on time. It depends on the covariance matrix defined as $\mathbf{R}(t) = \langle \overline{Y}\overline{Y}^T \rangle$. Then the analysis is performed by solving the following *Lyapunov type matrix differential* equation to determine the matrix covariance for the stationary case:

$$\mathbf{A}^{eq}\mathbf{R} + \mathbf{R}\mathbf{A}^{eq} + \mathbf{B} = 0 \tag{25}$$

In Eq. (25), $\mathbf{B}$ is a matrix of order 7 having all elements equal to zero except for

$$B_{7,7}(t) = 2\pi S_0 \tag{26}$$

where S_0 is the white noise spectral intensity factor of Eq. (18).

3.3. Stochastic definition of the hysteretic dissipated energy fraction

The energy balance equation is developed for a nonlinear system described by means of the Bouc–Wen differential law subject to a seismic motion $\ddot{x}_b(t)$. Each term of the motion equation is multiplied by the velocity $\dot{x}(t)$ and integrated in the time domain:

$$\ddot{x}(t)\dot{x}(t)+2\xi_s\omega_s\dot{x}^2(t)+\alpha_s\omega_s^2x(t)\dot{x}(t)+(1-\alpha_s)\omega_s^2z(t)\dot{x}(t)=-\ddot{x}_b(t)\dot{x}(t) \tag{27}$$

The terms that appear in (27), defined for unit of mass, are as follows:

$$e_k=\ddot{x}(t)\dot{x}(t) \quad \text{rate of the kinetic energy at time } t \tag{28}$$

$$e_d=2\xi_s\omega_s\dot{x}^2(t) \quad \text{rate of the energy dissipated by damping at time } t \tag{29}$$

$$e_e=\alpha_s\omega_s^2x(t)\dot{x}(t) \quad \text{rate of the elastic energy at time } t \tag{30}$$

$$e_h=(1-\alpha_s)\omega_s^2z(t)\dot{x}(t) \quad \text{rate of the hysteretic dissipated energy at time } t \tag{31}$$

$$e_I=\ddot{x}_b(t)\dot{x}(t) \quad \text{rate of the input energy at time } t \tag{32}$$

The power balance expressed through relation (27) is called the "*relative power* equation *for a unit mass*" because it supplies the "*relative energy* equation *for a unit mass*", which is obtained through the integration with reference to time t:

$$\int_0^t\ddot{x}_b(\tau)\dot{x}(\tau)d\tau+2\int_0^t\xi_s\omega_s\dot{x}^2(\tau)d\tau+\int_0^t\alpha\omega_s^2x(\tau)\dot{x}(\tau)d\tau+\int_0^t(1-\alpha)\omega_s^2z(\tau)\dot{x}(\tau)d\tau=-\int_0^t\ddot{x}_b(\tau)\dot{x}(\tau)d\tau \tag{33}$$

The relative energy balance at time t can be written as

$$e_I(t)=e_k(t)+e_d(t)+e_e(t)+e_h(t) \tag{34}$$

The latter term can be stochastically evaluated once the covariance response is obtained [22]:

$$e_h=\frac{E}{m}=(1-\alpha_s)\omega_s^2\int_0^t z(\tau)\dot{x}(\tau)d\tau \tag{35}$$

The average value is

$$\langle e_h\rangle=(1-\alpha_s)\omega_s^2\int_0^T\gamma_{\dot{x}z}(\tau)d\tau=(1-\alpha_s)\omega_s^2\gamma_{\dot{x}z}T \tag{36}$$

where T is the temporal duration of the exciting process and $\gamma_{\dot{x}z}$ the cross-covariance of the state variable of the hysteretic system.

4. Optimization criteria for the TMD mechanical parameters

To better investigate the potential effectiveness of the TMD control strategy, it is essential to define the *optimum mechanical parameters* of a TMD (i.e. the optimum tuning frequency, damping, and mass ratio). In fact, although the basic design concept of a TMD is quite simple, the parameters of the TMD systems must be obtained through some optimal design procedures. These procedures are important for better control of the performance. Many researchers developed some optimal design approaches for the TMD systems regarding the structures excited by seismic and wind loads. Several design formulae have also been proposed to obtain the optimum parameters of a TMD for different types of oscillations. For example, Brock [32] and Den Hartog [33] described evaluation of the optimum parameters of the TMDs for an undamped structure subject to a harmonic external excitation over a broad band of excitation frequencies. Since Den Hartog first proposed an optimal design theory of the TMD for an undamped SDOF structure, many optimal design methods have been developed to control the structural vibrations induced by various kinds of excitation sources [34–36]. Starting from Den Hartog's method, Warburton [35] and Warburton and Ayorinde [37] achieved optimal parameters of a TMD for an undamped structure under harmonic support excitation. In the literature, analytical developments for the optimal design of a TMD take into consideration several sorts of optimization procedures and different mathematical models for the primary system and the associated external loading [36]. More recently, an optimization procedure of the stochastic simulation [38] has been applied to analyze the effectiveness of the TMDs with nonlinear viscous dampers excited by white noise [39].

In the above cited studies, the optimal parameters are determined by means of some parametric analyses or by some optimal design methods. Moreover, the external excitations are normally limited to a white noise or a harmonic force over a frequency range, or random signals, such as earthquake excitations, as studied by Park and Reed [40].

4.1. Formulation of the problem

The optimization problem of the structures subject to random vibrations can be formulated as the search of a suitable set of variables (design parameters) and gathered in a design vector (DV) $\overline{b}\in\mathbf{\Omega}_{\overline{b}}$ where $\mathbf{\Omega}_{\overline{b}}$ is the set of admissible values.

In this study, some optimal TMD design mechanical parameters are collected in a two-dimensional DV:

$$\overline{b}=(\omega_T,\xi_T)^T \tag{37}$$

whereby the mass ratio μ is assumed to be known and equal to 0.025 as a reasonable and common value. The optimization of the mass ratio is rarely used in practice due to economic considerations as its optimal value is generally very high and inapplicable for real purposes [41].

This work proposes three optimization criteria in a dimensionless form. The first one is a traditional performance measure, evaluated as the minimum dimensionless peak displacement of the protected system in comparison with the unprotected one. Thus, OF_1 is defined as the ratio of σ_{XS} to $\sigma^0{}_{XS}$, which are standard deviations of the displacement of the protected (with TMD) and unprotected (without TMD) systems, respectively:

$$OF_1 = \frac{\sigma_{X_S}}{\sigma^0_{X_S}} \tag{38}$$

This ratio may be considered as a stochastic index of the structural control effectiveness. It indicates more effectiveness in controlling when its value is smaller than the unit. A value closer to the unit indicates practically negligible effectiveness of the vibration control strategy.

Even if this first criterion represents the most commonly and traditionally used index to measure the effectiveness of the tuned mass dampers, other authors have pointed out that it cannot account for the effects of accumulation of the damage that occurs in the elasto-plastic systems under cyclic loading [6] as, for example, during severe earthquakes. Therefore, the effectiveness of the TMDs cannot be solely judged by the peak response ratio because this does not represent a comprehensive criterion to evaluate the seismic response.

Hence, another objective function (OF_2) is introduced as a supplementary performance index to give a more proper indication of the damage induced in inelastic systems. It concerns the dimensionless ratio of the average value of hysteretic dissipated energies of the protected structure μ_{Eh} to that of the unprotected one, $\mu^0{}_{Eh}$, per given unit of time. The latter has been evaluated by Eq. (41). Therefore, OF_2 can be calculated as follows:

$$OF_2 = \frac{\langle e_h \rangle}{\langle e_h^{(0)} \rangle} = \frac{(1-\alpha_s)\omega_s^2 \gamma_{\dot{x}z} T}{(1-\alpha_s)\omega_s^2 \gamma_{\dot{x}z}^{(0)} T} = \frac{\gamma_{\dot{x}z}}{\gamma_{\dot{x}z}^{(0)}} = \frac{\mu_{E_h}}{\mu^0_{E_h}} \tag{39}$$

In Eq. (39), the terms $\gamma_{\dot{x}z}$ and $\gamma_{\dot{x}z}^{(0)}$ are the cross-covariances of the state variables of the protected and unprotected systems, respectively.

To further examine the usefulness of the TMD in damage reduction of structures subject to earthquake excitations, another OF has been introduced. It consists of a weighted combination of both peak displacement and dissipated energy, as commonly proposed in the literature. Some examples of this are the functional damages formulated by Park and Ang [42] or by Banon and Veneziano [43]. This third index is defined as the norm of the damage variables previously described. Thus the third performance index OF_3 is defined as follows:

$$OF_3 = \sqrt{\beta\, OF_1^2 + (1-\beta) OF_2^2} \tag{40}$$

where the parameter β is a number varying within the range [0,1]. From a stochastic point of view, OF_3 can be considered as a structural damage index.

The TMD optimum design is finally expressed in a completely reliable stochastic approach as the minimization of the displacement ratio (OF_1), or the dissipated energy ratio (OF_2) or the damage index (OF_3).

For the numerical evaluations that are shown in the successive sections, the standard Matlab genetic algorithms (GAs) and [44] have been applied.

5. Numerical results

5.1. Effects of the system nonlinear mechanical parameters

The present analysis is carried out with reference to a simple SDOF Bouc–Wen type system subject to two kinds of ground motions. The first one is representative of a strong motion that occurs on a stiff soil and the second one is a ground motion that occurs on a soft soil. The SDOF system is characterized by a stiffness k, a viscous damping c, and a hysteretic behaviour defined by the internal variable z, as previously described in Section 3.

In order to investigate the TMD effectiveness on the nonlinear structure, the analysis has been conducted by varying the most important parameters that control the nonlinear behaviour.

Among these parameters, the strength reduction factor R has been used. This parameter is defined as the ratio of the maximum elastic force f_{max}^{el} to the yield strength f_y:

$$R = \frac{f_{max}^{el}}{f_y} = \frac{1}{\alpha_f} \tag{41}$$

In the present analysis, the reciprocal of R has been considered to introduce a parameter called α_f, which varies in the range $[0,-1]$. It is computed starting from the maximum elastic displacement X_{max}^{el}, which is estimated to be the peak factor of the random process $X(t)$ determined using the threshold crossing criterion over the observation period T. It is assumed that the maximum displacement is associated with the crossing of the process $X(t)$ over an allowable value X_{adm}^{max}.

Indeed, for systems subjected to random loads, the failure can be generally determined by the first-time crossing of the response parameter X through a given threshold value β. For a generic one degree of freedom system subject to a stationary stochastic process, the reliability is settled as $r(T)=1-P_f(T)$ [45], where P_f is the failure probability:

$$P_f = Pr\left\{X^{el}_{max}(\tau) \geq X^{max}_{adm} | \tau \in [0,T]\right\} \leq \tilde{P}_f \tag{42}$$

In the present work, a value of the admissible failure probability $\tilde{P}_f = 0.01$ was assumed as known. Then the value of P_f was obtained with the threshold crossing theory over the allowable value X^{max}_{adm}. After calculating X^{el}_{max}, the yield strength f_y was determined, assuming a linear elastic law.

As we can see in Eq. (5), the post-yielding stiffness ratio α_s of the main system changes within a fixed range of values. Various mechanical situations are evaluated by varying the main parameters of the system within the following ranges:

- strength ratio α_f: [0.3:0.1:1] and
- stiffness ratio α_s: [0.1–0.5].

It should be underlined that these mechanical system parameters are supposed to be known. Indeed, they can be practically determined for existing structures by means of a widely used nonlinear static procedure such as the *pushover* analysis.

Fig. 3a shows the OF_1 and OF_2 surfaces in the space of the design variables, plotted as functions of the ratio ρ_T of the TMD main frequency to the main system one:

$$\rho_T = \frac{\omega_T}{\omega_s} \tag{43}$$

and the damping coefficient ζ_T of the TMD. Both of the plotted surfaces present a global minimum corresponding to the optimal solution. They are based on displacement (OF_1) and energy (OF_2) criteria. It should be observed that the absolute minimum of OF_2 is lower than the minimum of the OF_1 surface. This means that the energy criterion provides an optimal solution that is smaller than that given by the application of the displacement criterion. This is an expected result since the energy is proportional to the square of the response while the displacement is proportional to the response, so any advantage of a TMD appears greater if considered in terms of energy rather than in terms of displacements.

The optimal points corresponding to each optimization criterion are plotted in Fig. 3b in the design bi-dimensional space. By observing their relative position, it can be seen that the solution obtained by adopting the energy criterion is achieved for very small values of the damping factor and for high values of the frequency ratio. On the contrary, the displacement based optimal solution is given for larger values of damping and for lower values of the frequency ratio.

This finding is of practical significance because it means that TMDs applied on inelastic structures, and whose target performance is the minimization of the dissipated energy, may be designed with low damping level.

Two kinds of soil characteristics are examined to evaluate the effectiveness of the TMD in improving the protection of the buildings. The main filter characteristics are presented in Table 1 and refer to the same white noise power spectral intensity factor S_0=102.268 cm^2/s calculated through Eq. (18) and PGA=0.343g. The structural parameters are fixed for illustrative purposes to ω_s=6.28(rad/s), ξ_s=0.05, and μ=0.025.

Figs. 5–8 show the main trend of the OFs for a fixed value of the strength ratio α_f and for different stiffness ratios α_s on stiff and soft soils. The optimal frequency ratio $\rho_T^{opt} = (\omega_T^{opt}/\omega_s)$ and the optimal damping $\xi_T{}^{opt}$ are also plotted versus the strength ratio α_f. All the results have been evaluated with reference to the displacement (OF_1), energy (OF_2), and mixed criterion (OF_3) for β=0.25. The parameter β is non-negative and represents the effect of cyclic loading on structural damage. This parameter is usually determined experimentally [42].

It is noticeable that all the OF curves decrease monotonically while α_f increases (quite linear in the stiff soil case), reaching the minimum value for α_f=1. This implies that the best performance in vibration reduction always takes place in systems with an elastic behaviour. This can be physically explained as the "instantaneous" stiffness of the system's hysteretic model, and then the frequency of the first vibration mode significantly changes with deformability level, increasing on the plastic branch. In detail, the main frequency of vibration rapidly decreases once the displacement overcomes the elastic yield limit. Thus, once over the yielding force, changes in stiffness due to plastic deformations lead to changes in mechanical properties of the TMD and, consequently, loss of tuning may occur, which could make the TMD less effective in vibration reduction in the nonlinear range due to the detuning effect. It can be also observed that the OFs plotted in the same figures have no null values for α_f=1 (linear elastic case). This is easily understandable for OF_1 due to the intrinsic nature of the hysteretic BW model, which is characterized by a gradual and smoother transition to the plastic range (see Fig. 4). For the same reason, also OF_2 and dissipated energy show similar behaviours. When α_f=1 we suppose that $f_y = f^{el}_{max}$, but this is exactly true only for bilinear models. In the case of the BW model, at α_f=1, the yielding force tends to the maximum elastic limit but does not coincide with it (i.e., $f_y < f^{el}_{max}$ due to the smoother transition to the plastic range). Hence a nonlinear law is recognizable before reaching the conventional elastic limit displacement X_y and therefore a residual irrecoverable amount of energy is dissipated.

Both in stiff and soft soil cases, the frequency optimal ratio $\rho_T{}^{opt}$ shows a convex trend that increases while the strength ratio α_f augments and varies quite rapidly in the lower range of α_f but tends towards a constant value for higher values. The optimal values of $\rho_T{}^{opt}$ tend to increase while the stiffness ratio increases. The optimal damping factor $\xi_T{}^{opt}$ decreases as α_f

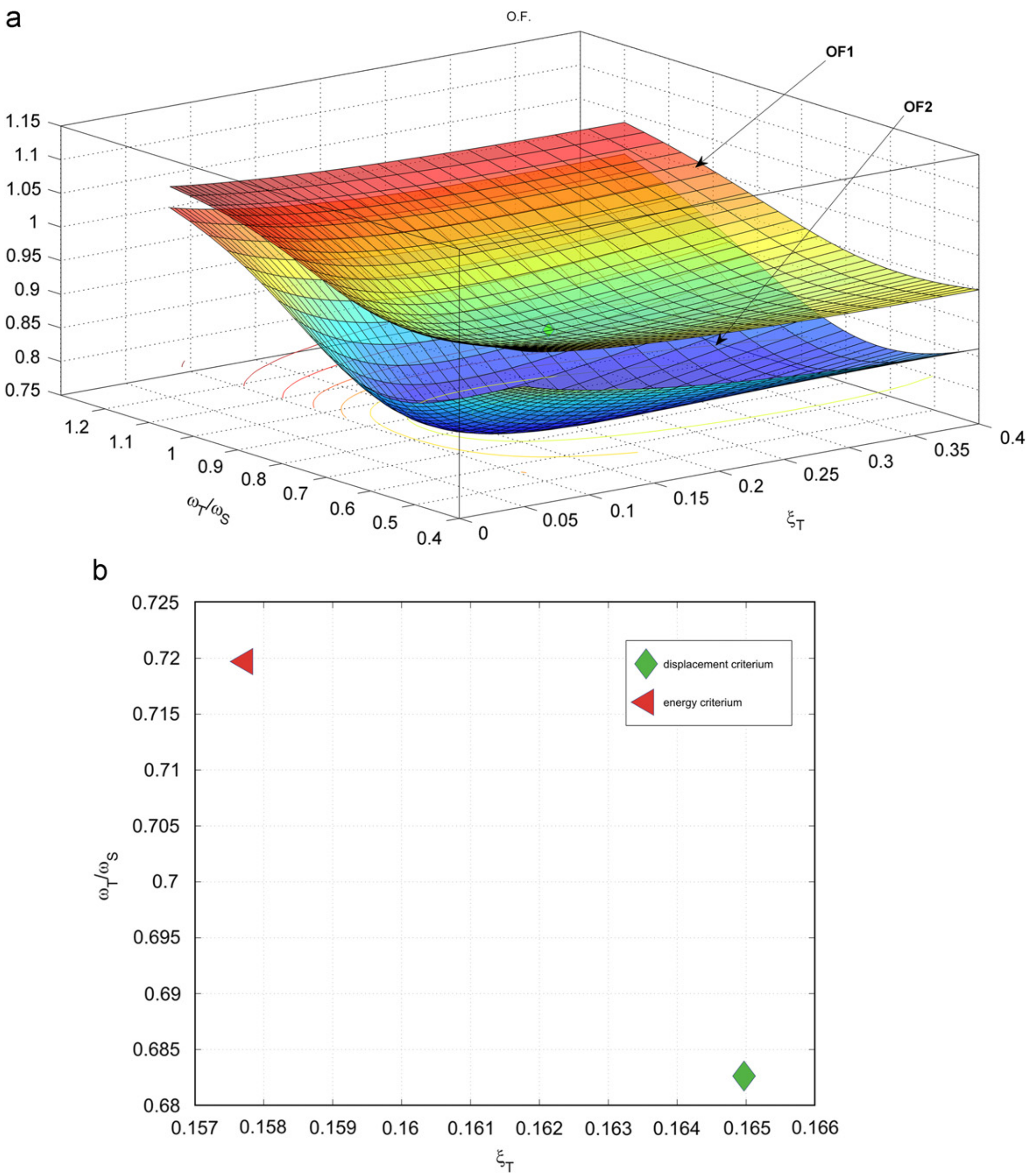

Fig. 3. OF_1 and OF_2 surfaces in the dimensionless design vector space (a) and the optimal solutions for the three optimization criteria for β=0.25(α_f=0.1; α_s=0.1). The mass ratio is fixed at the value μ=0.025.

Table 1
Main filter characteristics.

Soil type	**Filter parameters**	
	ω_f (rad/s)	ξ_f
Stiff	20	0.6
Soft	10	0.2

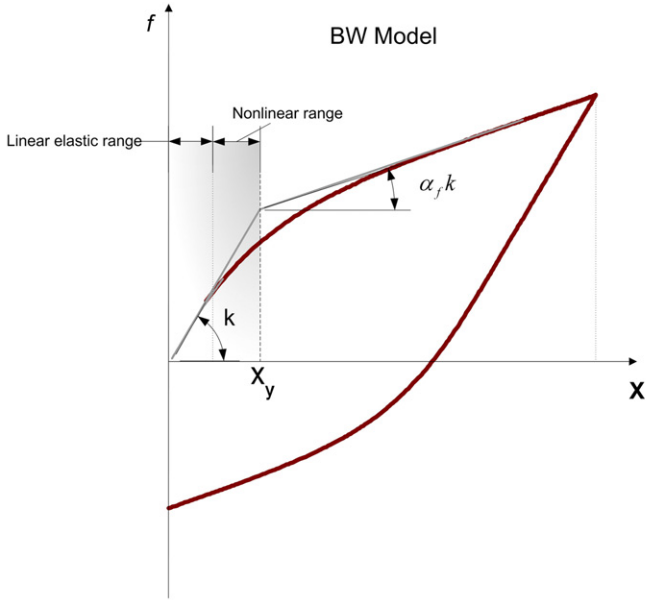

Fig. 4. Bouc–Wen constitutive law and its main mechanical characteristics.

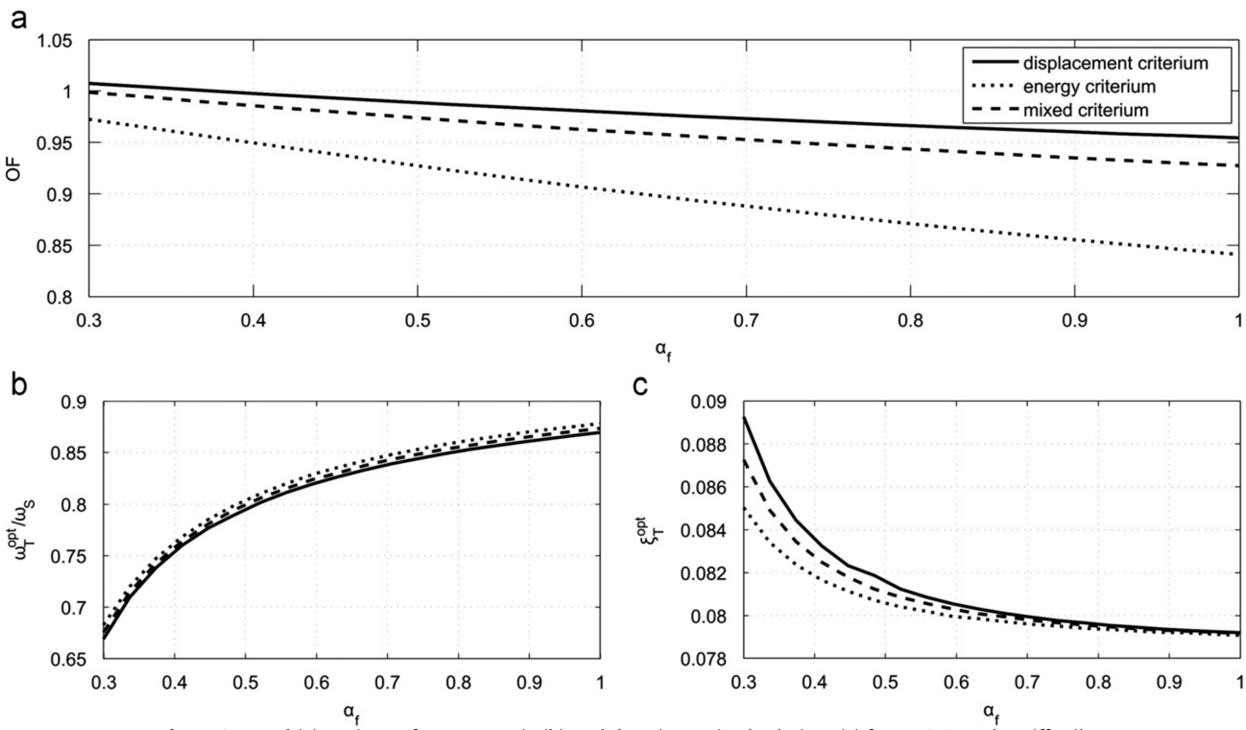

Fig. 5. OF trend (a), optimum frequency ratio (b), and damping optimal solutions (c) for α_s=0.1, on the stiff soil.

increases and assumes larger values on the soft soil in comparison with the stiff soil. It can also be stated that the optimal design parameters of the TMDs, applied on systems with highly nonlinear properties (lower values of α_f and α_s), are characterized by smaller frequency ratio ${\rho_T}^{opt}$ and larger damping ratio ${\xi_T}^{opt}$, compared with the optimal solutions of TMD installed on linear structures. This observation leads to the conclusion that a secondary system linked to a hysteretic one tunes the mass, mainly resulting in an increase of its damping rather than its natural frequency, contrary to the behaviour of a TMD applied on a linear model (Figs. 5–8).

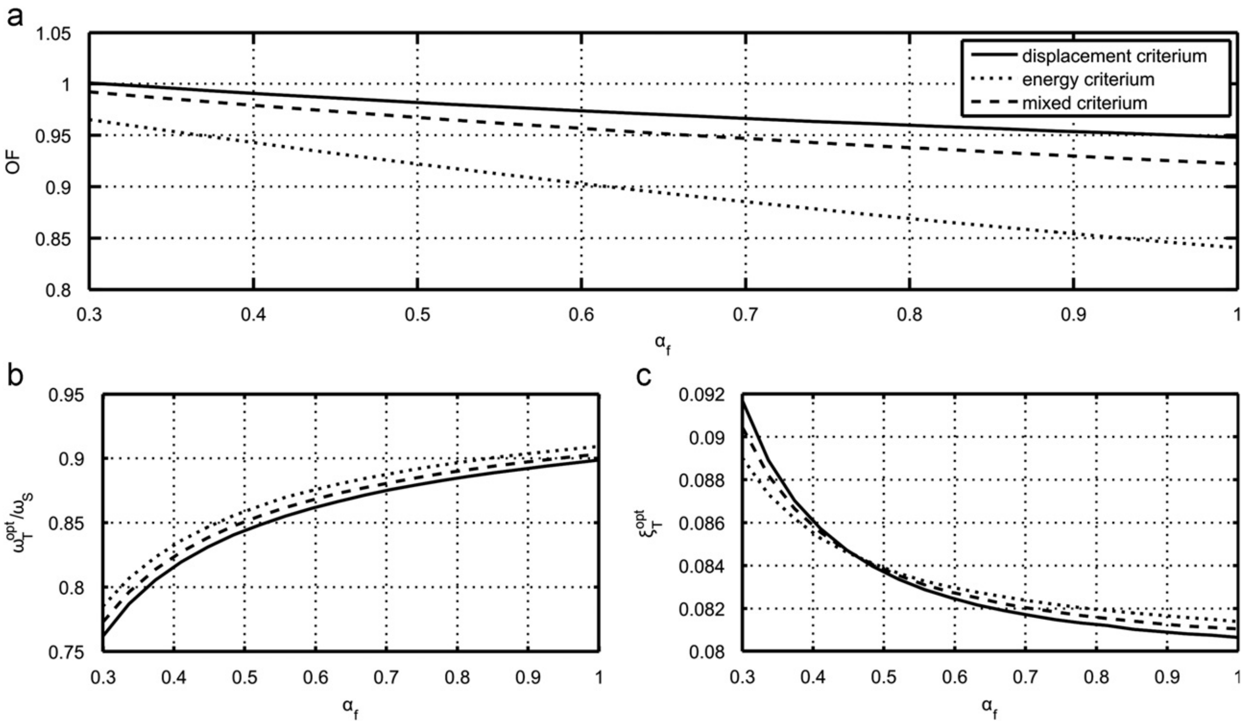

Fig. 6. OF trend (a), optimum frequency ratio (b), and damping optimal solutions (c) for α_s=0.1, on the soft soil.

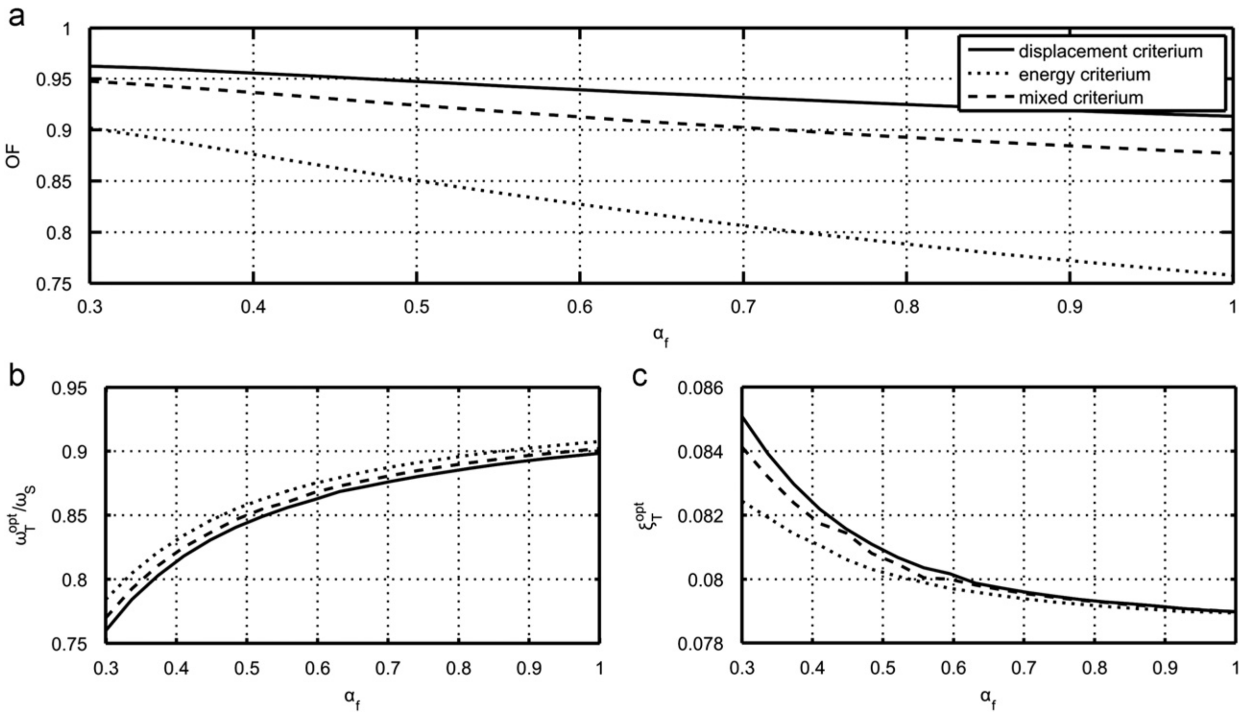

Fig. 7. OF trend (a), optimum frequency ratio (b), and damping optimal solutions (c) for α_s=0.5, on the stiff soil.

5.2. Spectral analysis of the optimal design parameters

A further analysis involves the spectral representation of the OFs with reference to the stiffness ratio α_s=0.1 on stiff and soft soils and for α_f=0.6 and α_f=0.4 (Figs. 9–12, respectively). These curves are also compared to the spectral trend of the optimal solutions of the TMD applied to a linear structure on the basis of displacement criterion (OF_1). As may be expected,

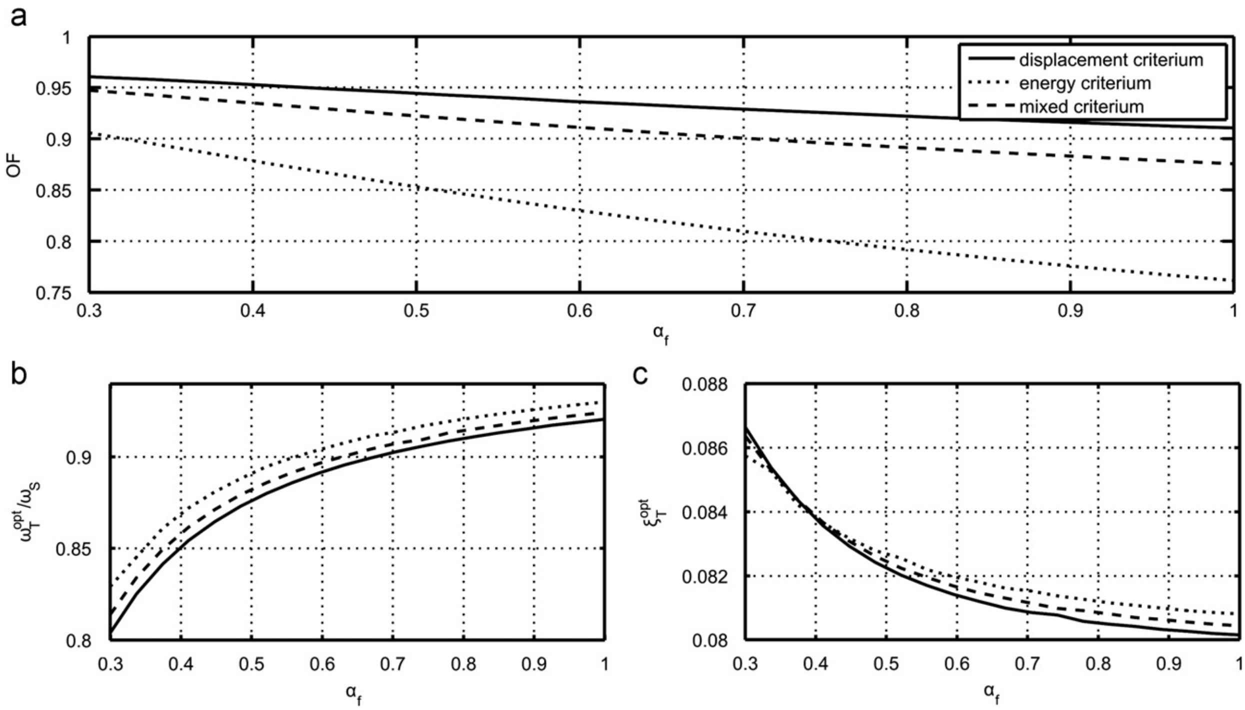

Fig. 8. OF trend (a), optimum frequency ratio (b), and damping optimal solutions (c) for α_s=0.5, on the soft soil.

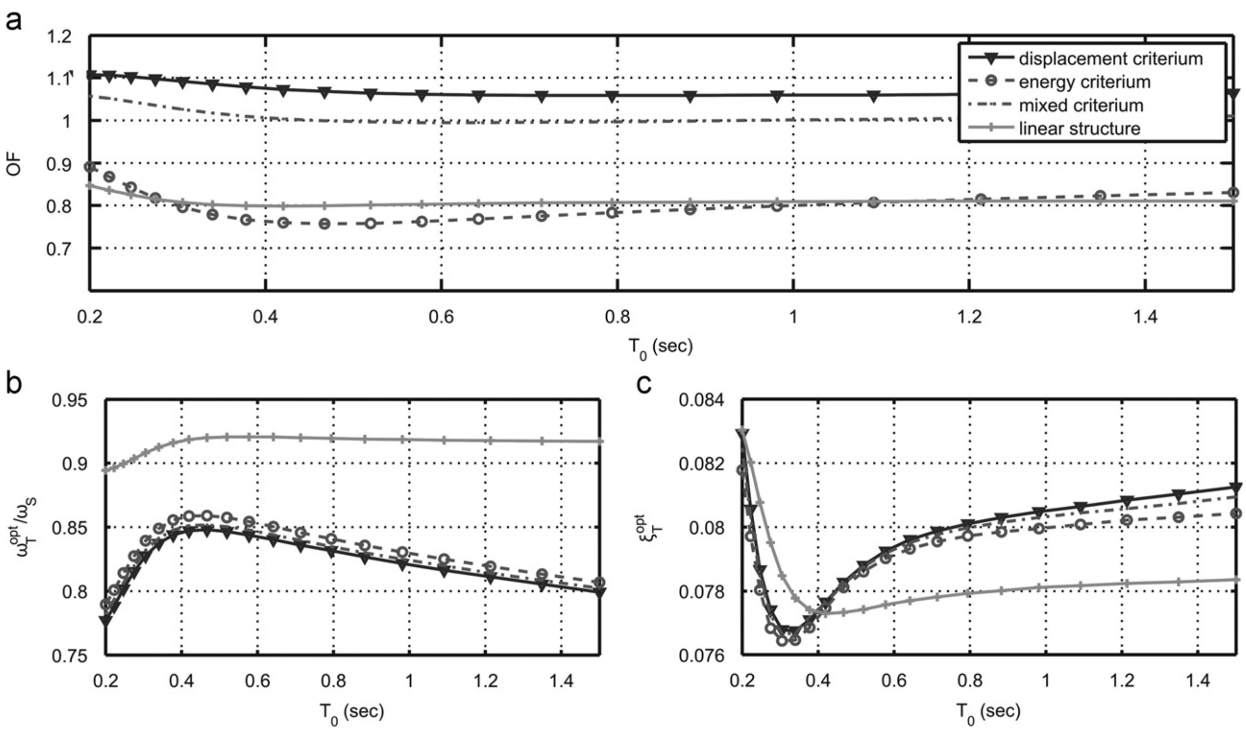

Fig. 9. Spectrum of the OF trend (a), optimum frequency (b), and damping solution (c) for α_s=0.1, α_f=0.6 on the stiff soil.

the OF spectral curves show a global minimum point corresponding to the fundamental period of the ground T_g, which explicates that the TMD provides maximum protection effectiveness. The trend of the optimal parameters, observed both for ${\rho_T}^{opt}$ and for ${\xi_T}^{opt}$, shows a point characterized by a null derivative corresponding to the resonant period. For example, in Fig. 9b, the frequency ratio ${\rho_T}^{opt}$ has a maximum peak at the resonant period ($T_g = 0.314$ s) while the optimal damping

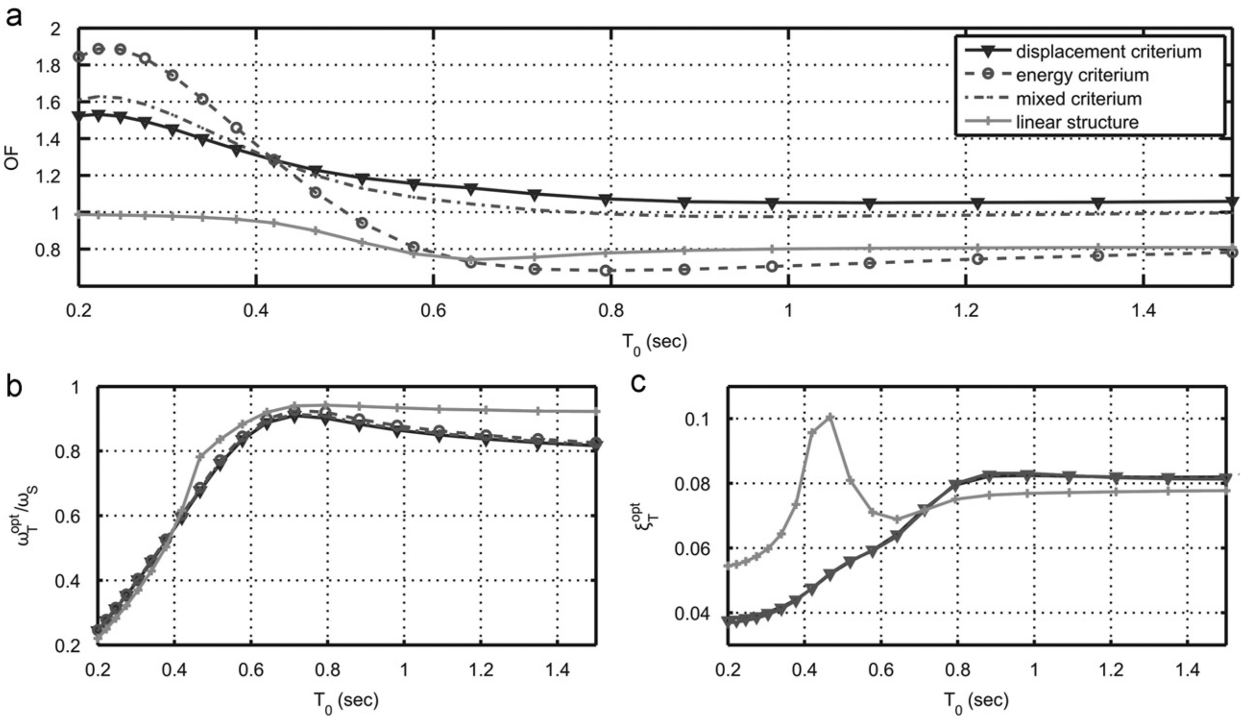

Fig. 10. Spectrum of the O.F trend (a), optimum frequency (b), and damping solution (c) for α_s=0.1, α_f=0.6 on the soft soil.

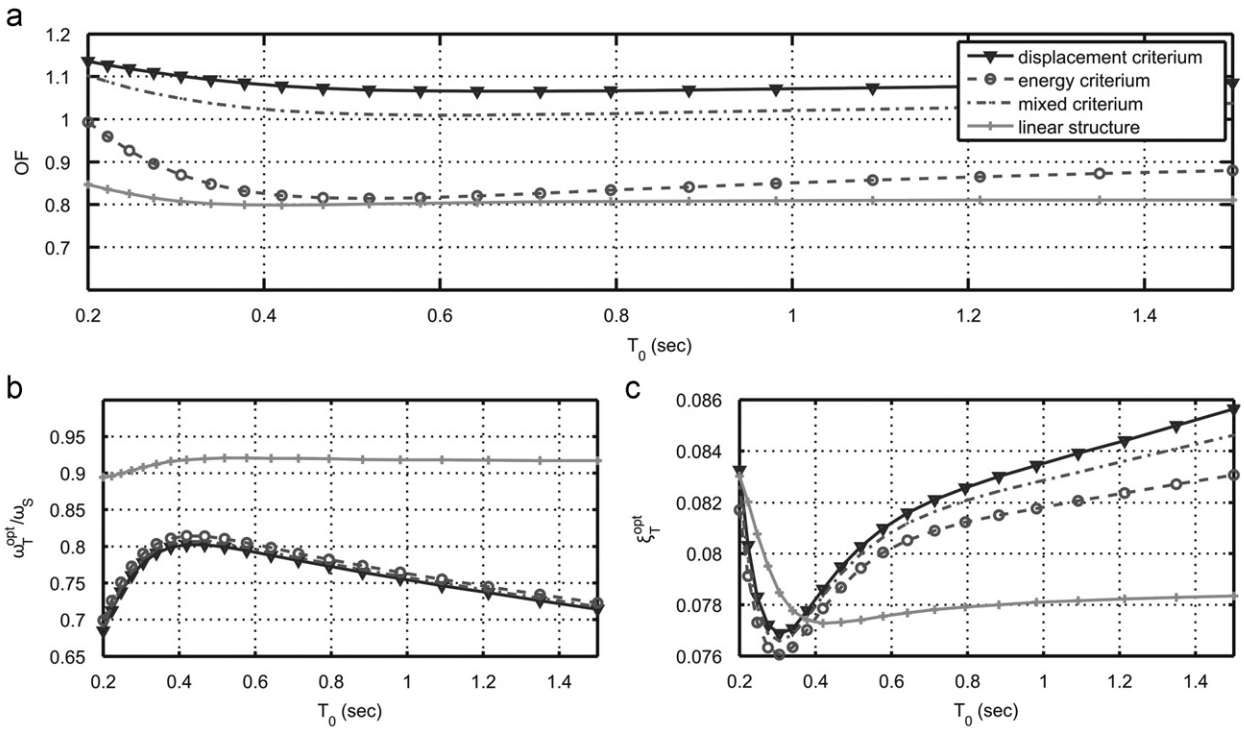

Fig. 11. Spectrum of the OF trend (a), optimum frequency (b), and damping solution (c) for α_s=0.1, α_f=0.4 on the stiff soil.

$\xi_T{}^{opt}$ (Fig. 9c) shows a minimum point. It can be noted that the optimal solutions are minimally sensitive to the optimization criterion adopted. The examined trends can be explained by considering that, at the resonance, the protection system requires a minimum quantity of damping energy to be effective because the TMD functioning is mainly based on

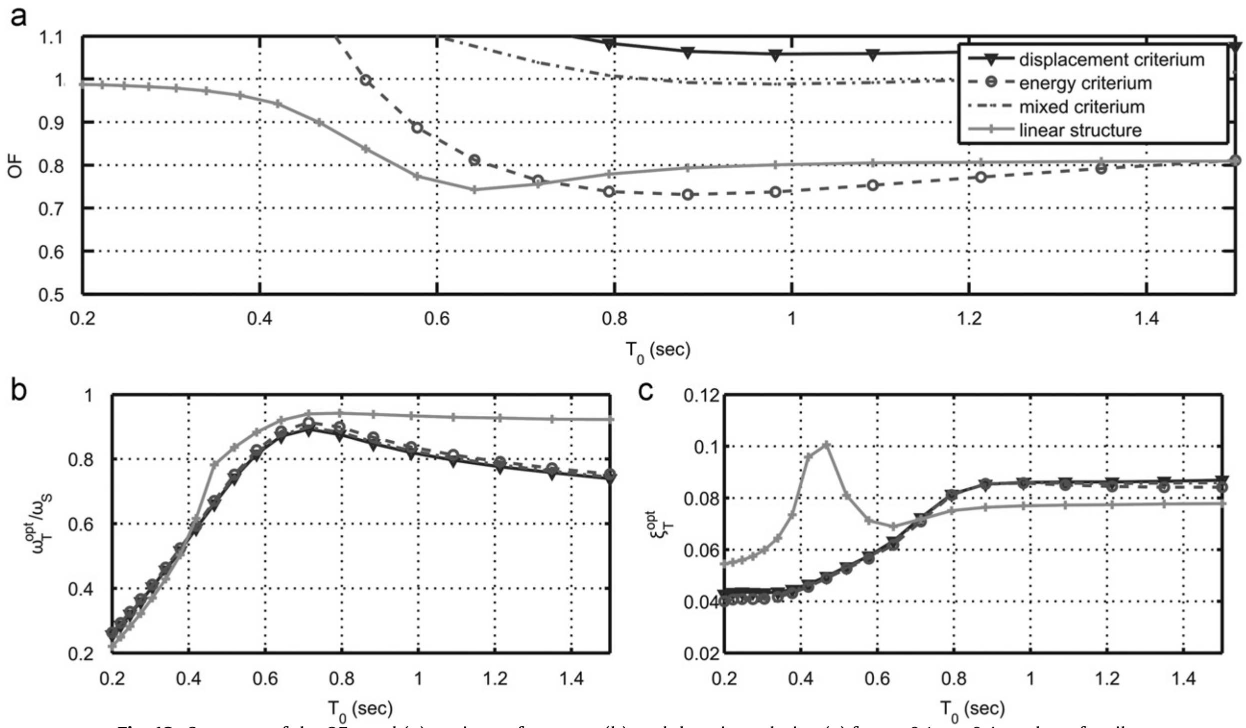

Fig. 12. Spectrum of the OF trend (a), optimum frequency (b), and damping solution (c) for α_s=0.1, α_f=0.4 on the soft soil.

the tuning mechanism. On the soft soil (Fig. 10a) and at the resonance condition ($T_g = 0.628$ s), a maximum value of the frequency ratio (Fig. 10b) can be observed. The optimal damping trend quickly grows while the period increases and tends to settle or rise for periods greater than the resonant one.

It should be noticed that, in all the examined cases, the optimal solutions are not influenced by the optimization criterion adopted.

It can be concluded that, when the natural frequency of the secondary system is less or near the fundamental frequency of the pre-yield primary structure, the optimal solution of the secondary system is characterised by a ''tuning effect'' as the secondary system is tuned with the fundamental mode of the post-yield primary structure. On the contrary, if the system is far from resonance with the soil, a reduction of tuning protection effectiveness occurs and the minimization of the objective function is achieved by augmenting the viscous energy dissipation (''damping effect'').

Besides, in some figures, it can be observed that the curve of OF_1 is above the unit value, while the curve corresponding to OF_2 lies under the unit. This means that the performance of a TMD applied to a nonlinear hysteretic primary system, in comparison with the protection effectiveness of an equivalent TMD on a linear structure, is always unfavourable in terms of displacement reduction. However, the same analysis shows that the mean of the hysteretic energy, dissipated by the primary nonlinear structure, decreases in the presence of a tuned mass.

The spectral curves of the OFs also decrease with an increase in structural period. Thus, the use of TMD, in reducing hysteretic energy response, is more effective for structures with a moderate to long period of vibration than for short period structures, as already found by Wong and Chee [8].

6. Summary and conclusions

In this work, the effectiveness of a tuned mass damper (TMD) device used for the control of buildings with hysteretic behaviour has been investigated under a stationary stochastic model of seismic action. A Bouc–Wen hysteretic model was adopted to describe the structural nonlinear hysteretic behaviour under conditions of severe earthquakes. In order to develop the optimization procedure, different performance indices have been proposed and compared. The traditional stochastic performance ratio of the protected to the unprotected peak displacements has been considered as a first objective function. Further objectives, related to the average value of the hysteretic dissipated energy of the protected structure to the unprotected one and a damage measure including both the displacement and the dissipated energy, have also been taken into account. Two design parameters, (1) the tuning frequency and (2) the damping ratio, have been considered and collected in a single design vector. The results point out that the TMD application on inelastic systems reduces plastic energy dissipation and then the damage in the structure. Hence its use does not appear to be advantageous for displacement control but may be recommended for hysteretic dissipated energy reduction.

In the following, there are some conclusions based on this study:

1. According to previous studies, once over the yielding force, changes in stiffness due to plastic deformations lead to changes in mechanical properties of the TMD and, consequently, loss of tuning may occur. Such loss of tuning could make the TMD less effective in vibration reduction in the nonlinear range due to the detuning effect.
2. The TMD performance reduces as the post-yielding stiffness ratio α_s and the strength ratio α_f increase due to the detuning effect, which occurs in the plastic branch.
3. The optimal solution obtained by adopting the energy criterion is achieved for very small values of the damping factor and for high values of the frequency ratio. On the contrary, the displacement based optimal solution is given for larger values of damping and for lower values of the frequency ratio. However, as a general trend, the optimal solutions are quite insensitive to the adopted optimization criteria.
4. The frequency ratio ${\rho_T}^{opt}$ shows a maximum peak and the optimal damping ${\xi_T}^{opt}$ shows a minimum point near the resonant period. Far from the resonance the trend is opposite. The damping grows and the tuning ratio reduces (this trend is more evident on stiff soils). Thus, when the natural frequency of the secondary system is less or near the fundamental frequency of the pre-yield primary structure, the optimal solution of the secondary system is characterised by a "tuning effect" as the secondary system is tuned with the fundamental mode of the post-yield primary structure. On the contrary, if the system is far from resonance with the soil, a reduction of the tuning protection effectiveness occurs and the minimization of the objective function is achieved by augmenting the viscous energy dissipation ("damping effect").
5. The optimal solutions appear to be larger on soft soils rather than on stiff ones. Nevertheless, this conclusion is based only on two examples of mechanical characteristics of soil type. Therefore, a larger number of examples should be considered in order to give more general conclusions.
6. The use of TMD, in reducing hysteretic energy response, is more effective for structures with a moderate to long period of vibration than for short period structures.

In future developments of this work, a more complete sensitivity analysis will be conducted in order to analyze the various forms of energy in the structure and the influence of peak ground acceleration (PGA) and mechanical properties of the soil, including non-stationary effects.

Appendix 1

Nonlinear Eq. (19) is replaced with a linear function of z and $\dot{x}$ [29], by means of some linearized coefficients:

$$\dot{z} = -K_{eq}(R,t)z - C_{eq}(R,t)\dot{x} \tag{25}$$

where C_{eq} and K_{eq} are coefficients of the linearized equation and $\mathbf{R}$ is the covariance matrix [29].

The linearized coefficients, $C_{eq}(t)$ and $K_{eq}(t)$, are nonlinear functions of the covariance response elements. Atalik and Utku [46] provided these equivalent coefficients. In the most common case of η=1 as applied by many authors, and β=γ and λ=1, in the hypothesis of Gaussian processes z and $\dot{x}$:

$$C_{eq}(t) = \sqrt{\frac{2}{\pi}}\beta\left[\sigma_Z(t) + \frac{\gamma_{\dot{x}z}}{\sigma_{\dot{X}}(t)}\right] - 1 \tag{26}$$

$$K_{eq}(t) = \sqrt{\frac{2}{\pi}}\beta\left[\sigma_{\dot{X}}(t) + \frac{\gamma_{\dot{x}z}}{\sigma_Z(t)}\right] \tag{27}$$

where terms $\sigma_Z(t)$ and $\sigma_{\dot{X}}(t)$ are the standard deviations of the variables Z and $\dot{X}$, respectively, and $\gamma_{\dot{x}z} = \langle \dot{X}(t)Z(t)\rangle$ is the cross-covariance of the mentioned variables.

Appendix 2

The matrix $A^{eq}(R,t)$ can be written as follows:

$$\mathbf{A}^{eq}(\mathbf{R},t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \mathbf{K}_{eq}(\mathbf{R},t) & 0 & \mathbf{C}_{eq}(\mathbf{R},t) & 0 \\ -\omega_T^2 & +\omega_T^2 & +\omega_f^2 & 0 & -2\xi_T\omega_T & +2\xi_T\omega_T & 2\xi_f\omega_f \\ +\mu\omega_T^2 & -(\mu\omega_T^2+\alpha\omega^2) & \omega_f^2 & -(1-\alpha_s)\omega^2 & +2\mu\xi_T\omega_T & -(2\xi\omega+2\mu\xi_T\omega_T) & 2\xi_f\omega_f \\ 0 & 0 & -\omega_f^2 & 0 & 0 & 0 & -2\xi_f\omega_f \end{bmatrix}$$

The matrix equation, whose dimension is 7×7, due to the symmetry of the covariance matrix, can be written as a set of scalar first order differential equations in the 49 unknown elements of the covariance matrix.

References

[1] T.T. Soong, G.F. Dargush, Passive Energy Dissipation Systems in Structural Engineering, John Wiley & Sons, Chichester, ISBN 0-471-96821-8, 1997 (356pp.).
[2] R. Villaverde, LA. Koyoama, Damped resonant appendages to increase inherent damping in buildings, Earthquake Engineering and Structural Dynamics 22 (1993) 491–507.
[3] R. Soto-Brito, S.E. Ruiz, Influence of ground motion intensity on the effectiveness of tuned mass dampers, Earthquake Engineering and Structural Dynamics 28 (1999) 1255–1271.
[4] A.M. Kaynai, D. Veneziano, J.M. Biggs, Seismic effectiveness of tuned mass dampers, Journal of Structural Engineering, ASCE 107 (8) (1981) 1465–1484.
[5] A.H. Chowdhury, M.D. Iwuchukwu, J.J. Garske, Past and future of seismic effectiveness of tuned mass dampers, in: Proceedings of the Second International Symposium on Structural Control, Ontario, Canada, 1985, pp. 105–127.
[6] P. Lukkunaprasit, A. Wanitkorkul, Inelastic buildings with tuned mass dampers under moderate ground motions from distant earthquakes, Earthquake Engineering and Structural Dynamics 30 (4) (2001) 537–551.
[7] T. Pinkaew, T. Lukkunaprasit, P. Chatupote, Seismic effectiveness of tuned mass dampers for damage reduction of structures, Engineering Structures 25 (2003) 39–46.
[8] K.K.F. Wong, Seismic energy dissipation of inelastic structures with tuned mass dampers, Journal of Engineering Mechanics, ASCE 134 (2) (2008).
[9] K.K.F. Wong, Y.L. Chee, Energy dissipation of tuned mass dampers during earthquake excitations, Structural Design of Tall and Special Buildings 13 (2004) 105–121.
[10] J.C. Miranda, On tuned mass dampers for reducing the seismic response of structures, Earthquake Engineering and Structural Dynamics 34 (7) (2005) 847–865.
[11] K.K.F. Wong, J. Johnson, Seismic energy dissipation of inelastic structures with multiple TMD, Journal of Engineering Mechanics 135 (4) (2009).
[12] R. Bouc, Modele Mathematique D'Hysteresis (in French), Acustica (France) 24 (1) (1967) 16–25.
[13] Y.K. Wen, Method for random vibration of hysteretic systems, Journal of the Engineering Mechanics Division, ASCE 102 (1976) 150–154.
[14] W.Q. Zhu, C.D. Huang, T.T. Soong, Response and reliability of secondary systems in yielding structures, Probabilistic Engineering Mechanics 9 (1994) 145–155.
[15] C.D. Huang, W.Q. Zhu, T.T. Soong, Nonlinear stochastic response and reliability of secondary systems, Journal of Engineering Mechanics 120 (1) (1994) 177–196.
[16] T.T. Baber, Y.K. Wen, Random vibration of hysteretic degrading system, Journal of the Engineering Mechanics Division, Proceedings of ASCE 107 (No. EM6) (1981) 1069–1087.
[17] Z. Bažant, Endochronic inelasticity and incremental plasticity, International Journal of Solids and Structures 14 (1978) 691–714.
[18] F. Casciati, 1987. Nonlinear stochastic dynamics of large structural systems by equivalent linearization, in: Proceedings of the Fifth International Conference on Application of Statistics and Probability in Soil and Structural Engineering, ICASP5, Vancouver, B.C., Canada. No. UILU-ENG-99-2002, University Illinois at Urbana-Champaign, Champaign, IL.
[19] R.S. Thyagarajan, W.D. Iwan, Performance characteristics of a widely used hysteretic model in structural dynamics, in: Proceedings of the Fourth US National Conference on Earthquake Engineering, Palm Springs, CA, 1990.
[20] J.E. Hurtado, A.H. Barbat, Improved stochastic linearization method using mixed distributions, Structural Safety 18 (1) (1996) 49–62.
[21] Y.K. Wen, Methods of random vibration for inelastic structures, Applied Mechanics Reviews 42 (2) (1989) 39–52.
[22] G.C. Marano, R. Greco, Damage and ductility demand spectra assessment of hysteretic degrading systems subject to stochastic seismic loads, Journal of Earthquake Engineering 10 (No. 5) (2006) 615.
[23] C.W. Wong, Y.Q. Ni, J.M. KO, Steady-state oscillation of hysteretic differential model. I: response analysis, Journal of Engineering Mechanics 120 (1994) 2271–2298.
[24] R.H. Sues, S.T. Mau, Y.K. Wen, System identification of degrading hysteretic restoring forces, Journal of Engineering Mechanics, ASCE 114 (No. 5) (1988).
[25] H. Tajimi, A statistical method of determining the maximum response of a building during earthquake, in: Proceedings of the Second World Conference on Earthquake Engineering, Tokyo, Japan, 1960.
[26] F.G. Fan, G. Ahmadi, Non stationary Kanai–Tajimi models for El Centro 1940 and Mexico City 1985 earthquakes, Probabilistic Engineering Mechanics 5 (1990) 171–181.
[27] S.H. Crandall, W.D. Mark, Random Vibration in Mechanical Systems, Academic Press, NY, 1963.
[28] N.C. Nigam, Structural optimization in random vibration environment, AIAA (1972) 551–553.
[29] J.B. Roberts, P.D. Spanos, Random Vibration and Statistical Linearization, John Wiley & Sons, Ltd., Chichester, England, 1990.
[30] F. Rudinger, Optimal vibration absorber with nonlinear viscous power law damping and white noise excitation, Journal of Engineering Mechanics, ASCE 132 (1) (2006) 46–53.
[31] J.A. Inaudi, J.M. Kelly, Mass damper using friction-dissipating devices, Journal of Engineering Mechanics 121 (1) (1995).
[32] J.E. Brock, A note on the damped vibration absorber, Journal of Applied Mechanics, ASME 13 (A-284) (1946).
[33] J.P. Den Hartog, Mechanical Vibrations, 4th ed., McGraw-Hill, New York, 1956.
[34] S.H. Crandall, W.D. Mark, Random Vibration in Mechanical Systems, Academic Press, NY, 1973.
[35] G.B. Warburton, Optimum absorbers for simple systems, Earthquake Engineering and Structural Dynamics 8 (1980) 197–217.
[36] R. Rana, T.T. Soong, Parametric study and simplified design of tuned mass dampers, Engineering Structures 20 (1998) 193–204.
[37] G.B. Warburton, E.O. Ayorinde, Optimum absorber parameters for simple systems, Earthquake Engineering and Structural Dynamics 8 (1980) 197–217.
[38] F. Rüdinger, Optimal vibration absorber with nonlinear viscous power law damping and white noise excitation, Journal of Engineering Mechanics—ASCE 132 (No. 1) (2006).
[39] F. Rüdinger, Tuned mass damper with nonlinear viscous damping, Journal of Sound and Vibration, 2006b, doi:10.1016/j.jsv.2006.09.009.
[40] J. Park, D. Reed, Analysis of uniformly and linearly distributed mass dampers under harmonic and earthquake excitation, Engineering Structures 23 (7) (2001) 802–814.
[41] J.-F. Lin, Wang, J.-M. Ueng, Vibration control identification of seismically excited m.d.o.f. structure PTMD systems, Journal of Sound and Vibration 240 (1) (2001) 87–115.
[42] Y.J. Park, A.H.-S. Ang, Mechanistic seismic damage model for reinforced concrete, Journal of the Structural Engineering 111 (4) (1985) 722–739.
[43] H. Banon, D. Veneziano, Seismic safety of reinforced concrete members and structures, Earthquake Engineering and Structural Dynamics 10 (1982) 179–193.
[44] Optimization Toolbox For Use with MATLAB, The MathWorks, Inc., 2006.
[45] G. Muscolino, Dinamica Delle Strutture (in Italian), McGraw Hill, 2002.
[46] S. Atalik, Utku, Stochastic linearization of multi degree of freedom non linear systems, Earthquake Engineering and Structural Dynamics 4 (1976) 411–420.