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Correction of highly noisy strong motion records using a modified wavelet de-noising method

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ABSTRACT

During the past earthquakes, many valuable acceleration time histories were recorded by analog and digital accelerometers. These records are important sources of information in the field of earthquake engineering and engineering seismology. However, a large number of these records are contaminated by noise and it is necessary to correct them for practical applications. On the other hand, only a few records can be corrected using conventional filtering because of mathematical limitations of the method. However, advances in the field of time–frequency analysis and wavelet transform theory provide useful non-linear and adaptive de-noising methods for removing of non-stationary and high-energy noise from the recorded signals. In this paper, the characteristics and capabilities of the modified non-linear adaptive wavelet de-noising method are examined for correction of highly noisy strong motion records. In the frequency domain, it is shown that this method can attenuate the noise in the whole frequency range of engineering interest while in the time domain it can detect and remove non-stationary noise. In addition, the displacement response spectra of these wavelet de-noised records are more stable than conventional filtered records with respect to different correction functions. It is found that a large number of noisy acceleration records that are usually discarded from sets of records used for estimating the ground motions can be corrected using this new method.

1. Introduction

Every earthquake is an important source of information for the progress in the fields of earthquake engineering and engineering seismology. Actually, every engineering model of ground shaking is constructed based on the information obtained through analyses of strong motion records. Accordingly, it is an important issue to gather valid information about the characteristics of ground shaking. On the other hand, every measurement of ground shaking is contaminated by ambient or instrumental noise. In this regard, implementing a correction method is an attempt to suppress the noise and preserve recorded earthquake information. From mathematical point of view, any correction scheme is an estimation which is calculated by an operator that attenuates the noise while preserving the signal.

During the past decades, many strong motion records of major earthquakes are recorded by analog instruments. Among these earthquakes, the records obtained from some important events like San Fernando 1978, Tabas Iran 1978, Landers 1992, Northridge 1994, Kobe 1995 and Izmit Turkey 1999 had great impacts on the progress of earthquake engineering and engineering seismology. In Iran and among 24 major earthquakes examined by Zafarani et al. [1], all acceleration time histories from large events with $M_w > 7.0$ were obtained by analog instruments of the Building and Housing Research Centre of Iran (BHRC). However, major proportion of analog strong motion records is felt to be too poor to be included in practical engineering applications. As an example, using the conventional filtering method, only 3 out of 27 records of Tabas 1978, Mw 7.4 [2] and 15 out of 68 records of Manjil 1990, Mw 7.1 [3] earthquakes have been corrected and other important records of these events are remained useless. In the case of digital instruments, in spite of improvement of dynamic range, sampling period and analog-to-digital (A/D) process, a large number of time histories recorded by these instruments cannot be corrected using the conventional method. As an example, in the case of 2003 Bam, Iran earthquake, 24 SSA2 stations have been recorded this event [4]. However, only 5 records with high signal-to-noise ratio (SNR) in the frequency range of interest have been identified as suitable for different seismic studies [5].

Douglas [6] studied strong motion records from analog instruments that are missing their initial part due to late triggering of the instrument and also strong motion records from digital instruments with low A/D convertor resolution to examine

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whether these records can be used for response spectral analysis. It has been shown that accurate response spectral ordinates can often be obtained from records affected by such problems only in the period range of 0.2–2 s. It has also been mentioned that a significant proportion of strong motion records within the Imperial Collage European strong motion databank are affected by such problems and they are usually not included in strong motion studies because they are thought to be of too poor quality.

The conventional method of record correction mainly consists of three steps: (1) non-linear baseline correction, (2) linear or non-linear instrument correction and (3) linear filtering of noise [7]. In the case of digital accelerograms, because the natural frequency of transducer is more than the frequency range of interest and damping ratio is properly selected, there is always no need for instrument correction [8].

In both analog and digital accelerograms, the long period components of the records are often plagued by the baseline offsets [8–13]. Direct integration of these records results in non-physical values for ground velocity and displacement. As a result, a major step in the correction procedure is the adjustment of baseline.

The most important step in the correction of acceleration records is suppressing the noise from the signal. In the conventional methods of correction, the contaminating noise is removed through band-pass filtering of the noisy signal. The applicability of linear filtering method is dependent on the following conditions: (1) possibility to introduce a representative model of contaminating noise for obtaining the characteristics of the noise in the time and frequency domains, (2) the noise is stationary, hence, it is possible to filter out the noise in the frequency or time domain through convolution of the signal and filter function and (3) the SNR is relatively high in the frequency range of interest so it is possible to cancel out the noise only outside this frequency range. However, if a complex signal is highly contaminated by noise, it is not really possible to introduce a reasonable model of noise. Indeed, the noise can be non-stationary and dependent to the characteristics of the measured signal [8].

For the case of analog acceleration signals, not only the assumptions of stationary noise and high SNR in the frequency range of interest is questionable but also in many cases, because of the lack of pre-event, it is not possible to introduce a representative model of noise even in the case of low noisy signals. Hence, it is not possible to implement the traditional filtering methods because without having prior information of noise, it is not possible to determine the cut-off frequencies of the filtering. Actually, for correction of analog records, the cut-off frequencies of band-pass filtering are mainly determined based on the subjective experience and judgment of the individuals.

Another characteristic of traditional filtering is the strong dependence of results on the parameters of the filters like type of filter, order of filter function, magnitude of pass-band or stop-band ripples and the phase behavior of the filter function. In practice, it is desirable to have a correction procedure with a minimum number of parameters which its results have less sensitivity to these parameters. This issue is of great importance especially in non-linear analysis of structures [14 and references therein].

Thresholding estimators in the wavelet domain are proven to be useful tools for suppressing additive noise and restoring the signals [15–18]. This method is used by Galiana-Merino et al. [19] for de-noising of seismograms. To et al. [20] have compared Fourier-based and wavelet-based de-noising techniques applied to both synthetic and real experimental geophysical data.

Since the wavelet de-noising method is an adaptive algorithm and represents a self-organized model of the noise for each time–frequency component of the signal, it seems to be a suitable tool for correction and modification of highly noisy analog and digital strong motion records. However, direct implementation of wavelet de-noising method to acceleration records does not remove the long period noise effectively. To solve this problem, Ansari et al. [21] proposed a modified two step method and had a comprehensive study on the applicability of this method for the correction of acceleration records contaminated with different stationary and non-stationary noise. It was also shown, through examining synthetic signals, that it is possible to effectively remove the noise from highly noisy records using the proposed method.

In this paper, the application of proposed wavelet de-noising method of Ansari et al. [21] is examined in details for the correction of highly noisy real acceleration signals. Aiming to this subject, first a brief introduction to the principles of wavelet de-noising procedure is represented. In succession, a through comparison is made between the results of wavelet de-noising and conventional method of correction in order to reveal the capabilities of the proposed method in modification of high noisy signals. Moreover, to better examine the capability of the proposed method in correction of highly noisy signal, an analysis is performed on the synthetic signal.

Using the modified wavelet de-noising method, it is possible to retrieve a large number of analog acceleration records which was not possible to correct using conventional methods of correction.

2. A brief introduction to wavelet analysis and wavelet de-noising

2.1. Wavelet transform

The wavelet transform of $f(u)$ is defined as [22]

$$Wf(\lambda,\tau)=\int_{-\infty}^{+\infty} f(u)\frac{1}{\sqrt{\lambda}}\psi\left(\frac{u-\tau}{\lambda}\right)du \tag{1}$$

where λ and τ are the scaling and shifting parameters, respectively. The function $\psi_{\lambda,\tau}(u)$ is called the wavelet function. If parameters λ,τ in Eq. (1) take discrete values, the transform is called discrete wavelet transform (DWT), otherwise it is called continuous wavelet transform (CWT).

Having a more subtle view on Eq. (1), it is found that the kernel function $\psi((u-\tau)/\lambda)$, also known as window function, is described by two independent parameters τ, λ rather than only one in the case of Fourier transform. The parameter τ is the shifting parameter representing the movement of the window function and λ is the scaling parameter controlling the resolution of the analysis and has a relationship with frequency. As the scaling parameter λ increases, the time resolution decreases and the frequency resolution increases.

For a constant value of scaling parameter λ, the time and frequency resolutions will be constant and the wavelet transform of signal f is only a function of time. This time signal is an approximation of the original signal. By changing the value of scaling parameter λ, a set of signal approximations with different time and frequency resolutions will be obtained. In practice, λ is usually taken as a power of 2, say, $\lambda=2^i$ and the corresponding approximation is the ith approximation. The $i+1$th approximation is obtained using $\lambda=2^{i+1}$. This process is called "multi-resolution analysis". The detail at level i is the difference between two successive approximations $i+1$ and i. By adding the detail at the resolution level i to the approximation at the level i, a better approximation at the level $i+1$ will be obtained, providing that the wavelet function is orthogonal with respect to the scaling parameter λ.

In practice, there is always a question about suitable level of multi-resolution analysis. From one point of view, the minimum frequency is controlled by the length of the signal. In other words, at least one period of the signal has to be included in the window of wavelet function. This condition determines the lower limit of the meaningful frequency of the analysis [23]. However, in many cases the lower limit of the frequency range of interest is more than this limit. In practical applications, the most direct way of determining the suitable level of analysis is obtained by plotting the Fourier amplitude spectrum of the details at each level. The decomposition of the signal has to be performed up to the level whose frequency content is of interest.

2.2. Wavelet de-noising

The estimation of signal from noisy record is optimized by finding a representation that discriminates signal from noise. An estimation is calculated by an operator that attenuates the noise while preserving the signal. Linear operators have long predominated because of their simplicity, despite their limited performance. However, it is possible to keep the simplicity while improving the performance with non-linearities in the sparse representation [24].

The unique properties of DWT led Donoho [15–18] to propose a non-parametric adaptive de-noising method which is called "wavelet shrinkage de-noising". Wavelet shrinkage de-noising does involve shrinking in the wavelet transform domain, and consists of three steps: (1) a linear forward wavelet transform, (2) a non-linear shrinkage de-noising and (3) a linear inverse wavelet transform. The non-linear shrinking of coefficients in the transform domain distinguishes this procedure from linear de-noising methods like conventional linear filtering. Furthermore, wavelet shrinkage de-noising is considered as a non-parametric method and as a result, it is distinct from parametric methods, in which it is necessary to estimate parameters for a particular model that must be assumed a priori. Actually, in the wavelet de-noising method the correction is performed entirely based on the specific characteristics of the signal in different frequency bands. In the remaining of this section, the principles of wavelet de-noising are introduced through a very brief review of the mathematical basis of conventional filtering method [24].

Consider a signal $f[n]$ of size N which is contaminated by noise. This noise is modeled as the realization of a random process $W[n]$. The measured data are

$$X[n]=f[n]+W[n] \tag{2}$$

The signal f is estimated by transforming the noisy data X with a decision operator D. The resulting estimator is

$$\tilde{F}=DX \tag{3}$$

The goal is to minimize the error of the estimation, which is measured by a loss function. The risk of the estimator $\tilde{F}$ of f is the average loss, calculated with respect to the probability distribution of the noise $W[n]$:

$$r(D,f)=E[\left|\left|f-DX\right|\right|^2] \tag{4}$$

In real applications, the signal f is unknown and it is not possible to compute $r(D,f)$. In order to compute the risk r and the estimate $\tilde{F}$, it is necessary to make some assumptions about f. The Bayes principle, which is the basis of all linear filtering methods, supposes that signal f is realization of a random vector F whose probability distribution is known a priori. Moreover, it is assumed that $W[n]$ and $F[n]$ are zero-mean, stationary and independent random variables. Under these conditions, it is proved in Wiener theorem that an estimation of the signal f can be obtained by frequency filtering of noisy signals. In real applications, there is always a hidden assumption that both $W[n]$ and $F[n]$ are Gaussian processes.

Although we may have some prior information about signal f, it is rare that we know the probability distribution of complex signals like acceleration records of earthquakes. Moreover, as mentioned by Boore and Bommer [8], in many acceleration records, the noise is also generated by the peaks of the signal and is dependent on it. They named this noise as "signal generated noise". It is obvious that such signal generated noise is not stationary. As a result, since the principle assumptions of Bayes and Wiener theorems are no longer hold in these cases, it is not possible to obtain a reasonable estimation of signal f through conventional frequency filtering of the observed signal X.

The Minimax theorem provides a mathematical framework to obtain an estimation of signal f which is contaminated by non-stationary white or colored noise. The wavelet de-noising method which is introduced by Donoho [15–18] is actually the practical result of implementing the Minimax theorem for correction of noisy signals.

Using the linearity of wavelet transform an equivalent formulation of (2) in the wavelet domain can be written as

$$g_i=c_i+\varepsilon_i \tag{5}$$

where g_i, c_i and ε_i are wavelet transforms of $X[n]$, $f[n]$ and $W[n]$, respectively. The main idea of signal de-noising using wavelet is to modify g_i to obtain an estimate $\hat{c}_i$ of c_i which minimizes the risk $r(\hat{c},c)$. This is equivalent to find $\hat{f}$ in the original domain. The most common criterion of modification involves choosing a threshold level and modifying the amplitude of the signal where it is higher than this threshold. Donoho [15–18] proposed two modification functions $T(D,\tau)$ as thresholding functions. They depend on a single positive parameter τ, called threshold as

$$\text{Soft thresholding}: T_S(D,\tau)=\begin{cases} D-\tau, & D\geq\tau \\ 0, & |D|<\tau \\ D+\tau, & D\leq-\tau \end{cases}$$

$$\text{Hard thresholding}: T_H(D,\tau)=\begin{cases} 0, & |D|<\tau \\ D, & |D|\geq\tau \end{cases} \tag{6}$$

As it is clear, both T_H and T_S operators are non-linear and as a result, this method is categorized as non-linear. The question of how the noisy wavelet coefficients should be modified then reduces to the numerical choice of τ.

The threshold T must be chosen so that there is a high probability that it is just above the maximum level of the noise. If the real signal is almost zero, then the wavelet coefficients of noise must have a high probability of being below T. However if the real function is not zero then T must not be too large, avoiding setting to zero too many wavelet coefficients of the real function which are larger than the noise variance.

Generally, it is possible to classify thresholds into "single thresholds" and "level-dependent thresholds". In the single threshold methods, a single value is selected based on the characteristics of the signal and thresholding of different wavelet coefficients are performed using this single value. On the other hand, it is possible to determine the threshold values based on the characteristics of different components of the signal and have level-dependent thresholds.

The most common method for determination of single level threshold is called *VisuShrink* which is based on choosing τ to minimize a constant term in the upper bound for the minimax risk of estimating a function using the shrinkage estimator [16]. This leads to

$$\tau=\hat{\sigma}.\sqrt{2\log n} \tag{7}$$

where $\hat{\sigma}$ is an estimate of the noise level σ obtained from the finest or most detailed component of wavelet transform and n is length of the signal. The *VisuShrink* method is more suitable for cancellation of white noise. In the case of colored noise, the threshold value is dependent on the details of the different levels.

SureShrink is a method for determination of level-dependent threshold. It is based on Stein's Unbiased Risk Estimation. *SureShrink* is smoothness adaptive: if the unknown function f contains jumps, the reconstruction $\hat{f}$ essentially does also; if the unknown function has a smooth piece, then the reconstruction is as smooth as the mother wavelet function will allow [17].

Taswell [25] made comparisons between different methods for selection of threshold value and concluded that *SureShrink* has some advantages against other methods. Donoho and Johnson [17], represented many sample noisy functions to show the capabilities of this method. As it is mentioned before, the method is adaptive and level-dependent and as a result it is very flexible for dealing with different functions and different noise conditions.

Ansari et al. [21] proposed a two step procedure which implements the wavelet de-noising method for correction of acceleration signals. In the first step, the wavelet decomposition of the signal is performed in N levels. It is important that the Nth detail of the signal be in the frequency range more than the desired low-cut frequency. The acceleration details then are thresholded by the values determined for each level. The corrected signal of the first stage will be achieved by reconstruction of the modified details and approximation.

In the second stage, the wavelet decomposition will be applied to the velocity record in M levels with $M \geq N+2$. The velocity record is obtained by integration of the first stage acceleration. The threshold values are obtained using *SureShrink* procedure. The modified velocity record will be reconstructed by only adding the details of the signal. This is identical to the high-pass filtering of the velocity record using a specified mother wavelet function.

In the proposed method, in the first stage, since the correction is applied to acceleration time history, it is more effective for modification of high frequency components of the motion. In the second stage, the moderate and low-frequency components of the motion are amplified due to integration. Thus wavelet de-noising method in this stage is more effective in removing the noise of these components. In the second stage, the analysis is performed for at least two additional low-frequency decompositions of the signal ($M \geq N+2$) for correction of more low-frequency components.

3. Comparison between filtering and modified wavelet de-noising algorithm

In this section, a comprehensive comparison is made between the results of conventional filtering method and modified wavelet de-noising procedure proposed by Ansari et al. [21]. The comparison is made in the time and frequency domains. Without limiting the generality of comparison and discussion, the comparison is mainly made for correction of two representative analog acceleration time histories of September 16, 1978 Tabas Iran (Mw 7.4) earthquake. These records are obtained from two distant stations and are highly contaminated by noise. These records are shown in Fig. 1 and their characteristics are presented in Table 1 [4]. Moreover, to examine the capability of the proposed method in correction of highly noisy signal, an analysis is performed on a synthetic signal.

3.1. Comparison between filtering and wavelet de-noising in the time domain

In this section, a comparison is made between strong motion parameters including peak ground acceleration (PGA), velocity (PGV), displacement (PGD), Arias intensity (I_A) and significant duration of motion (T_d). Moreover, the effect of different correction procedures on the general condition of the records and displacement spectra is also presented.

As it is clear from Fig. 1, there is neither pre-event nor fixed trace in these records and it is not possible to identify any portion of these time histories as representative of the noise. These characteristics are common in many analog and distant records which are highly contaminated by noise. Hence, in these cases it is not possible to obtain cut-off frequencies of the band-pass filtering. However for the purpose of discussion and comparison

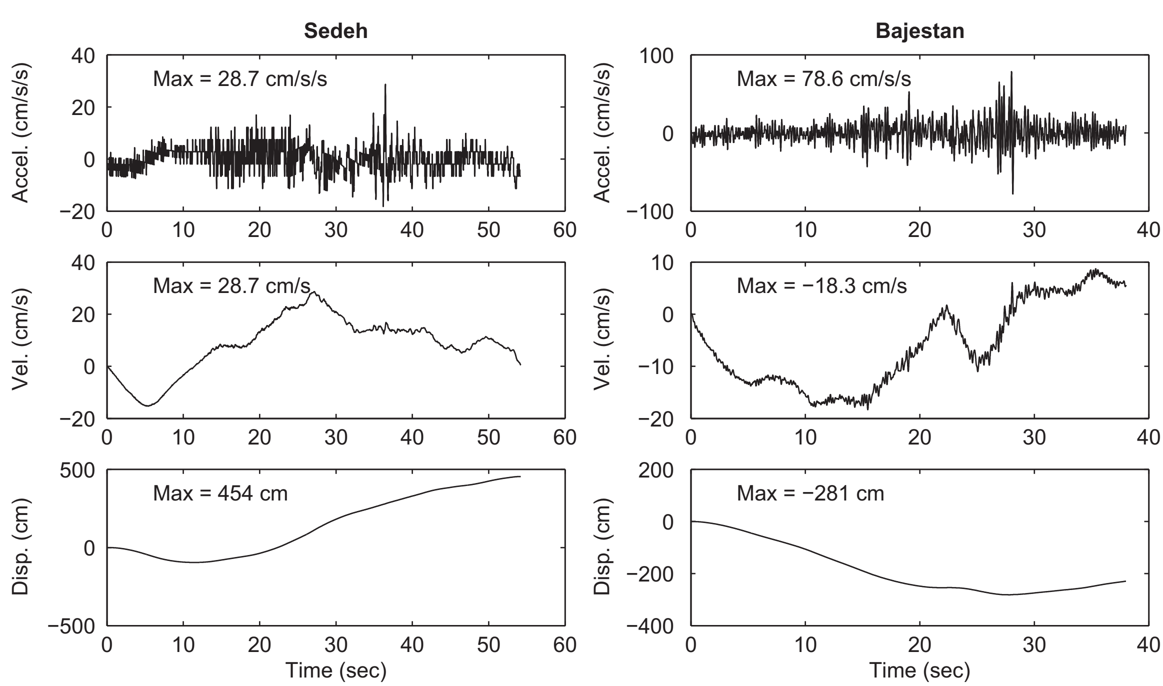

Fig. 1. Uncorrected acceleration, velocity and displacement time histories of Sedeh and Bajestan records of 1978 Tabas, Iran earthquake.

Table 1
Characteristics of records used in this study.

No.	Record no.	Station name	Station coord.		Ep. Dist. (km)	Comp.	PGA (cm/s/s)	Type	Instrument	
			Lat.	Long					Damp	Period
1	1085/02	Sedeh	33.3	59.2	167	L	28.67	SMA-1	0.57	0.04
2	1086	Bajestan	34.5	58.2	145	L	78.61	SMA-1	0.54	0.04

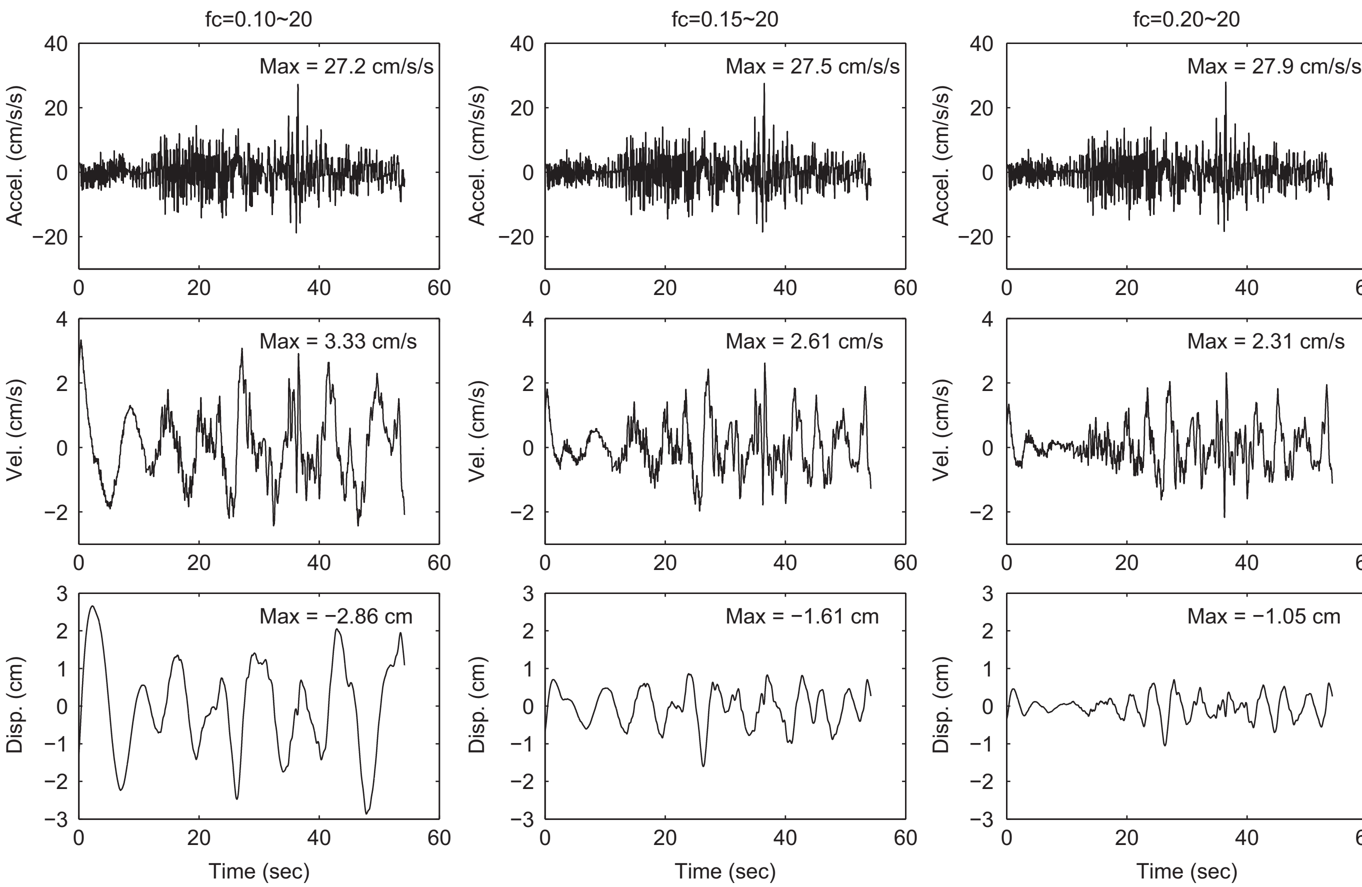

Fig. 2. Acceleration, velocity and displacement time histories obtained from band-pass filtering of Sedeh record with different high-pass cut-off frequencies of 0.1 Hz (left column), 0.15 Hz (middle column) and 0.2 Hz (right column). Filtering was done using linear phase Butterworth filter of order 4.

with the results of wavelet de-noising, we filter these records with Butterworth filter of order four using different cut-off frequencies. To eliminate the spurious low-frequency noise of filtering and phase distortion effect, zero pads with sufficient length are added at the beginning and end of the signals [26] and all filtering functions are applied in forward and backward directions. The baseline correction is also performed before filtering by fitting a quadratic function to velocity record.

In Fig. 2, the acceleration, velocity and displacement time histories of filtered signals of Sedeh station are shown for different high-pass cut-off frequencies. Similarly in Fig. 3, the corresponding time histories are shown by changing the low-pass cut-off frequency for the Bajestan record.

Visual inspection of these figures turns out that filtering does not improve the general condition of these time histories. The reason is that the energy of the noise is high in the frequency range of band-pass filtering. In Table 2, the Arias intensity of the uncorrected and filtered signals is compared. Arias intensity is actually a measure of the energy of the signal. As it is clear from this table, the energies of the signals before and after filtering are not changed considerably. As a result, for such signals, not only it is not possible to determine the cut-off frequencies of the band-pass filtering but also it is not possible to improve the general condition of the signals by blind filtering. In other words, as it was indicated in the previous section, the linear filtering method has some intrinsic limitations in dealing with highly noisy signals.

In Fig. 4, the corrected signals of Sedeh and Bajestan stations are shown which are modified by the proposed wavelet de-noising method of Ansari et al. [21]. Fig. 4 shows that a significant improvement is achieved by correcting the signals using wavelet de-noising method. It can be inferred clearly from the corrected time histories of Fig. 4 that the first 10 s of both Sedeh and Bajestan records is dominated by noise.

We have investigated the influence of using different cut-off frequencies of filtering and different wavelet functions on important strong motion parameters (PGA, PGV, PGD, I_A, T_d) for both Sedeh and Bajestan records. The corresponding results are presented in Table 2. The results show that PGA is more stable for filtered signals, however, the stability of PGV and PGD are much more for wavelet de-noised records. In almost all filtered signals,

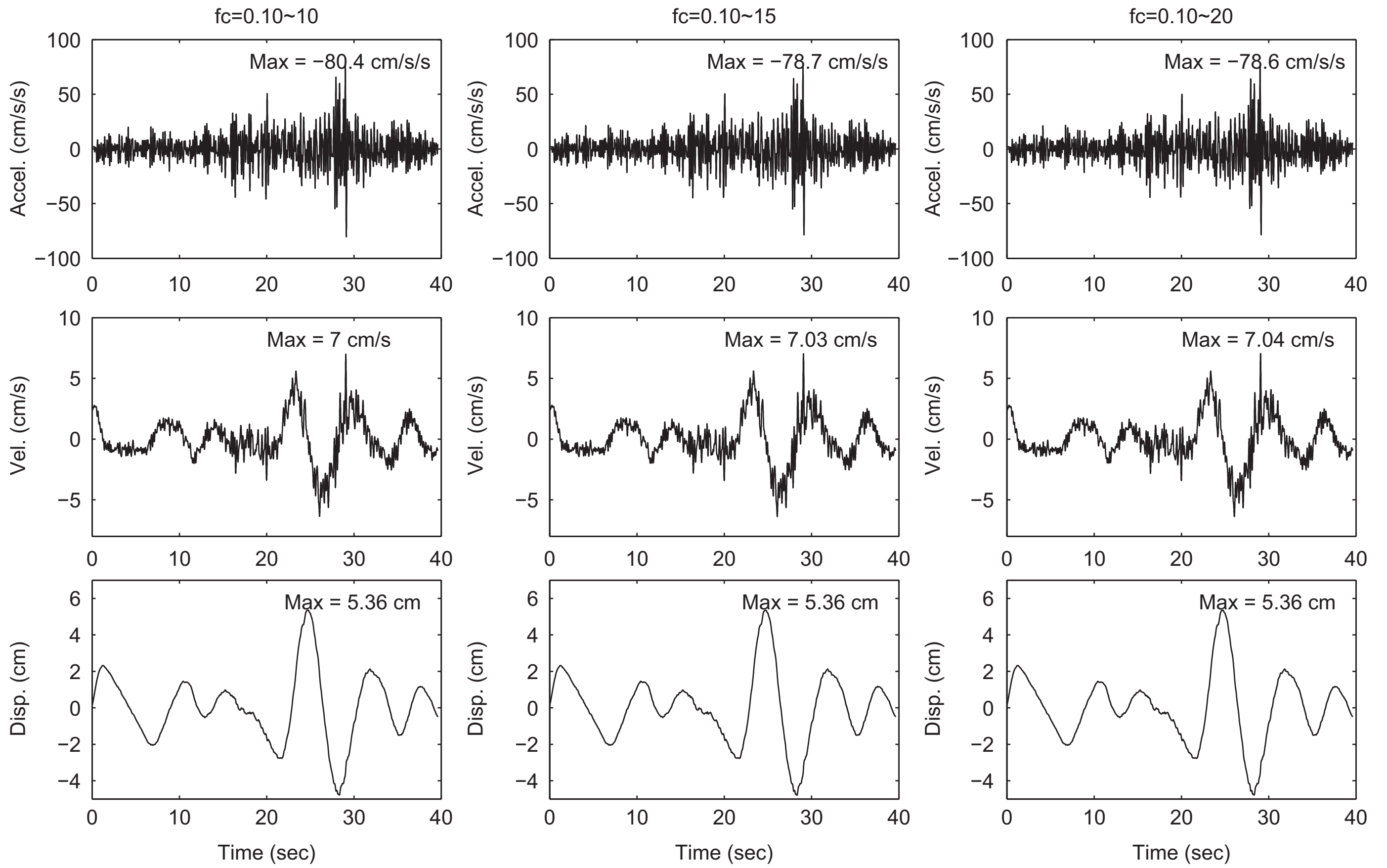

Fig. 3. Acceleration, velocity and displacement time histories obtained from band-pass filtering of Bajestan record with different low-pass cut-off frequencies of 10 Hz (left column), 15 Hz (middle column) and 20 Hz (right column). Filtering was done using linear phase Butterworth filter of order 4.

Table 2
Strong ground motion parameters: comparison between filtering and wavelet de-noising methods.

Record	Filter/ wavelet	PGA (cm/s^2)	PGV (cm/s)	PGD (cm)	I_A (cm/s)	T_d (s)
Bajestan	Uncorrected	78.6	–	–	12.5	29.0
	f_c=0.10–20 Hz	78.6	7.0	5.4	12.0	27.3
	f_c=0.15–20 Hz	80.0	6.2	2.1	12.0	28.3
	f_c=0.20–20 Hz	80.4	5.2	−0.8	12.0	28.3
	sym8	70.03	3.4	0.9	3.5	14.7
	sym6	67.35	3.5	1.1	3.5	14.2
	coif4	62.94	3.3	1.1	3.7	14.7
Sedeh	Uncorrected	28.7	–	–	2.1	46.5
	f_c=0.10–20 Hz	27.2	3.3	−2.9	1.7	41.1
	f_c=0.15–20 Hz	27.5	2.6	−1.6	1.7	36.8
	f_c=0.20–20 Hz	27.9	2.3	−1.1	1.6	36.3
	sym8	18.5	1.8	0.9	0.4	36.6
	sym6	18.7	1.8	0.7	0.4	33.1
	coif4	23.3	1.9	1.0	0.5	34.7

there is not a considerable difference between the Arias intensity (I_A) of corrected and uncorrected signals. On the contrary, the I_A of wavelet de-noised records is approximately 25 percent of the uncorrected records, indicating that the most significant part of the uncorrected signal energy is identified as noise by the wavelet de-noising method. However, the most important result is obtained by comparing the significant duration of different signals. As it is clear from Table 2, there is a considerable reduction in significant duration of wavelet de-noised strong motion records in comparison with uncorrected and filtered records. This issue can also be obtained from Fig. 4 where first and last portions of Sedeh and Bajestan records are identified as noise.

As a way for verification of wavelet de-noising results, it is very tempting to compare wavelet de-noised records of Sedeh and Bajestan stations with those used in Sarkar et al. [27] to determine different sub-events of Tabas earthquake. In Fig. 5, these records are shown together. As it is obvious from this figure, it is difficult to identify the arrival times of S2 and S3 sub-events on the records of Sarkar et al. [27] because of the highly noisy characteristics of these records. However, in the wavelet de-noised records, identification of different sub-events is straightforward. In other words and in these cases, the noise is effectively removed while the main features of the signal are maintained. The case shown in Fig. 5 indicates that wavelet de-noising method is a very capable and efficient tool in seismological inferences.

In Fig. 6, the relative displacement spectra of Sedeh and Bajaestan records are shown. In the left column of this figure, these records are corrected using Butterworth filter of order 4 with different high-pass corner frequencies. For periods more than 6 s, the difference between the spectra corrected with f_c of 0.1 and 0.2 Hz is of order of 10. Thus, in cases where there is not a clear criterion for selection of high-pass corner frequency, it is possible to use only a limited portion of the displacement spectrum in practical applications. Ansari et al. [21] showed that with a constant value of high-pass corner frequency, changing the order and type of filter function causes a difference in response spectra on the order of 3. A similar conclusion is reported by Boore and Bommer [8].

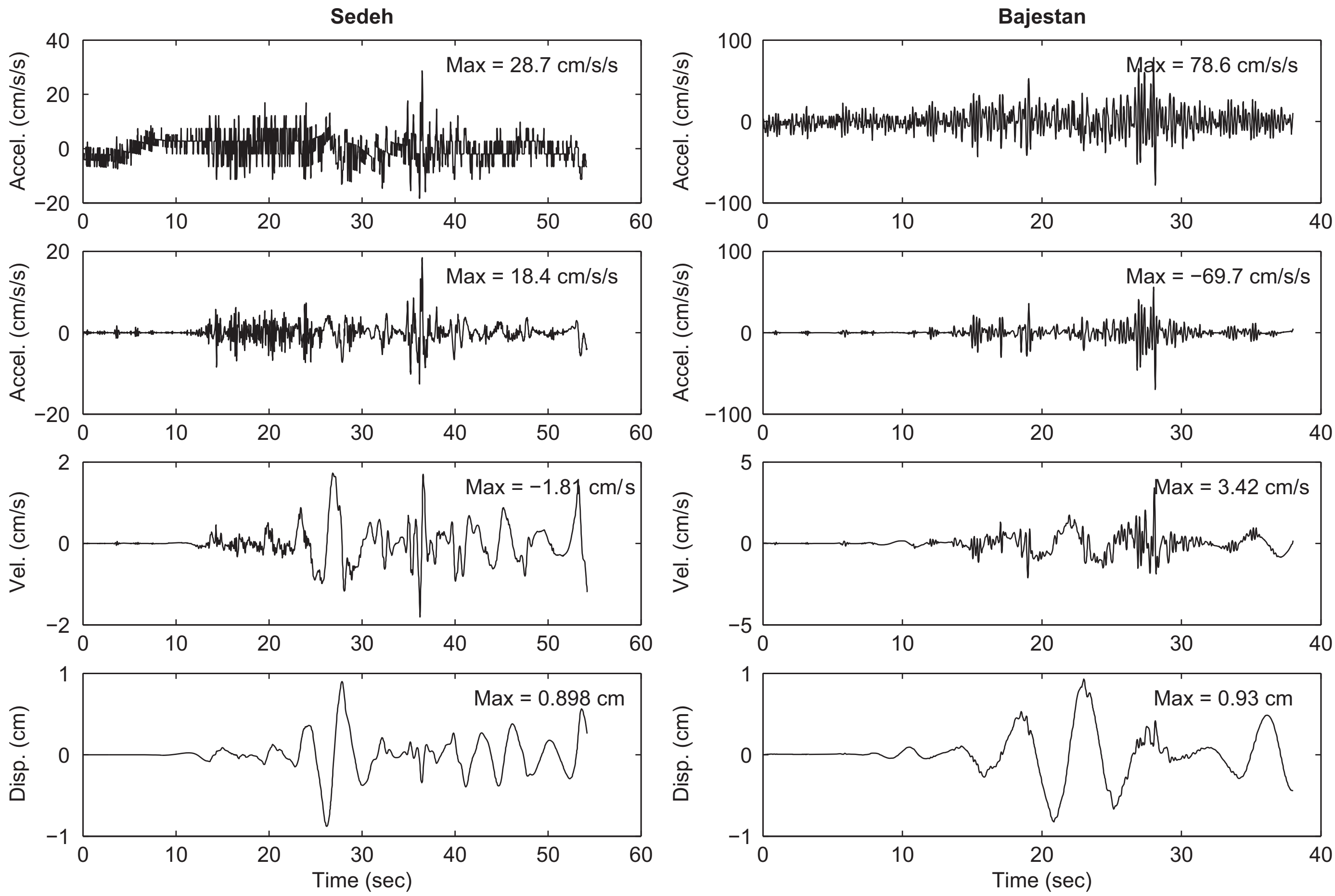

Fig. 4. Uncorrected acceleration, corrected acceleration, velocity and displacement time histories of Sedeh (left column) and Bajestan stations (right column).

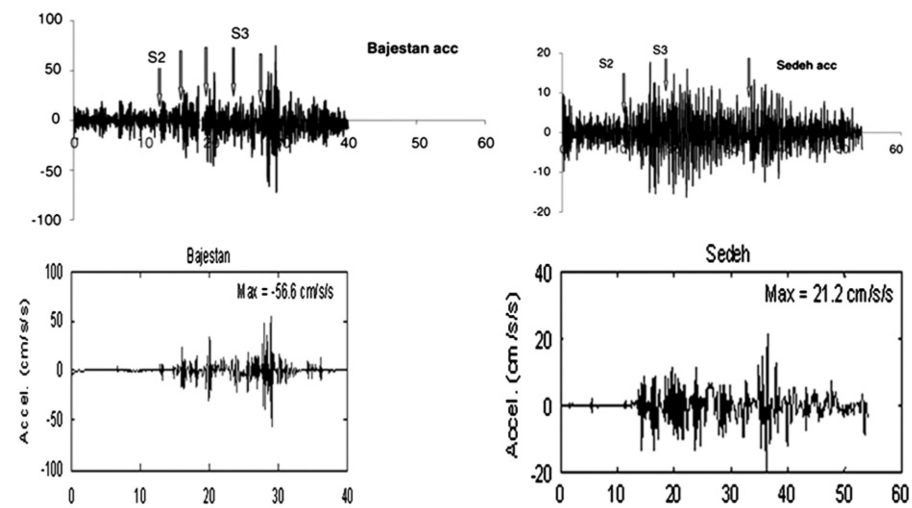

Fig. 5. Comparison between records of Bajestan and Sedeh stations of this study and Sarkar et al. [27].

On the other hand, as it is clear from the right column of Fig. 6, it is possible to obtain stable results where the records are corrected using modified wavelet de-noising method. In these figures, the order of mother wavelet function and its type is changed. For all corrections, the same wavelet functions are used for the first and second stage de-noising. Because of the stable characteristic of relative displacement response spectra, it is possible to use them in a wider period range.

As it was previously mentioned, the basic assumption in the mathematical theory of filtering is the statistical independency of the signal and noise. In other words in Eq. (2), $f[n]$ and $W[n]$ are statistically independent. To test this hypothesis, in Fig. 7, the cross correlations of the noise and corrected velocity signal are presented. As it is clear from this figure, the correlation of the filtered signal and the noise is more than five times the correlation of wavelet de-noised signal in the

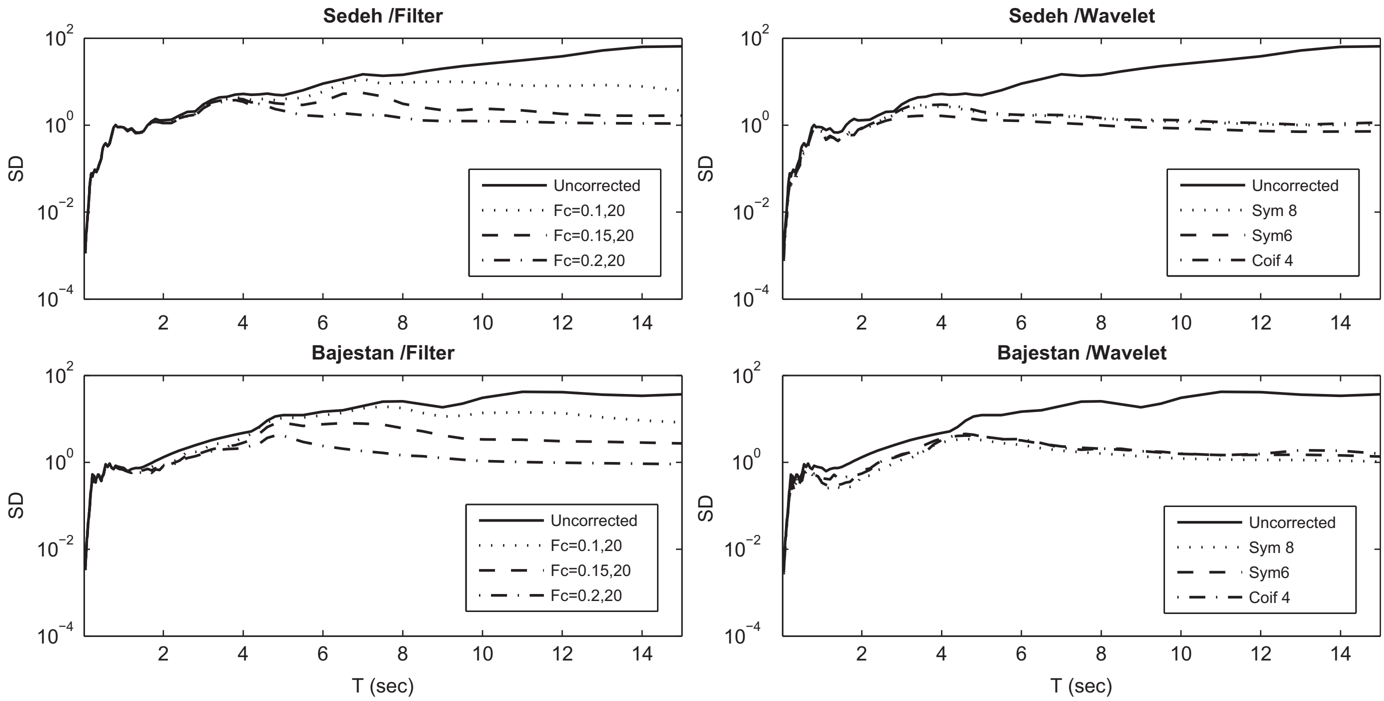

Fig. 6. Displacement spectra of Sedeh and Bajestan records corrected by filtering (left column) and wavelet de-noising methods (right column).

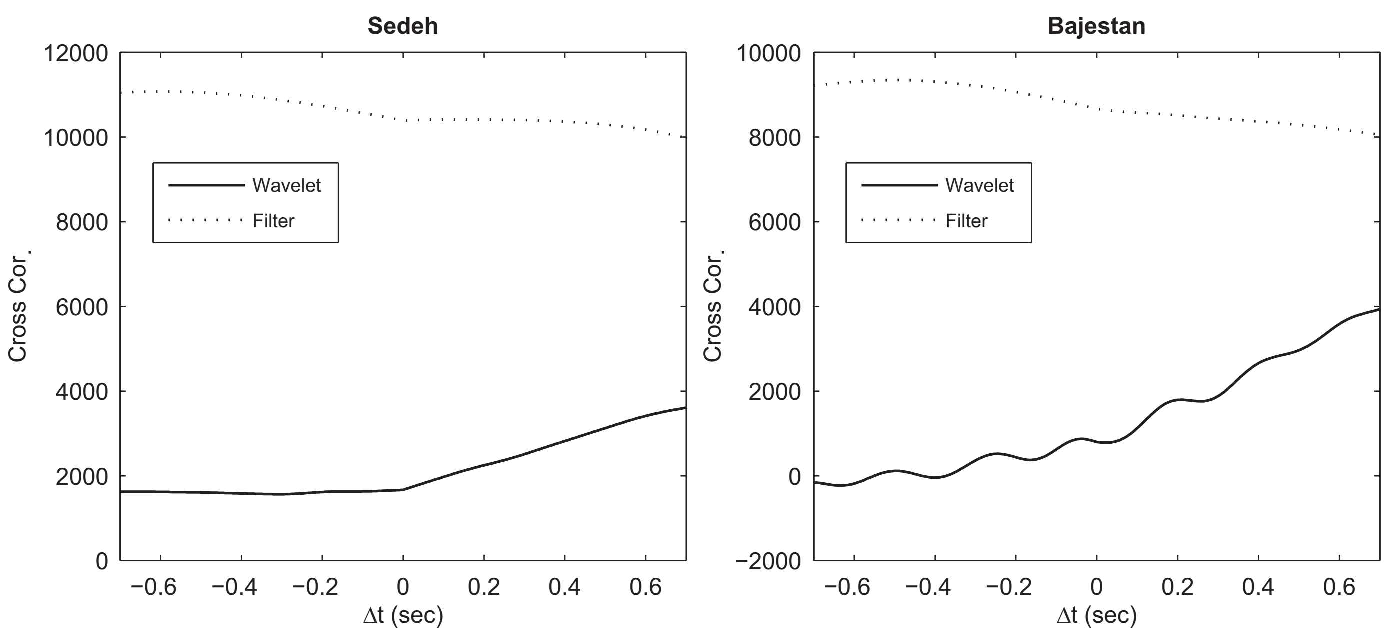

Fig. 7. Cross-correlation of noise with filtered signal and wavelet de-noised signal for Sedeh record (left) and Bajestan record (right).

case of Sedeh record. A similar case can be observed for Bajestan record. It has to be noted that the noise is obtained by subtracting the corrected signal from the original uncorrected record.

3.2. Comparison between filtering and wavelet de-noising in the frequency domain

In Fig. 8, the characteristics of wavelet de-noising method are represented in the frequency domain. In Fig. 8(a), a comparison is made between the noise identified by the wavelet de-noising method and first 10 s of the acceleration records of Sedeh and Bajestan stations. It should be noted that before wavelet de-noising, there is not any evidence that the first 10 s of these records are models of noise, however, for the purpose of comparison and because the results of Fig. 4 indicates that the first 10 s of these records are dominated by noise, such comparison is made in Fig. 8. As it is clear from Fig. 8(a), in the Sedeh record, the spectral characteristics of the first 10 s of the record is different from wavelet noise in the frequency range up to 7 Hz, which is the important range of engineering applications. However, these two models of noise have similar features beyond 7 Hz. In the case of Bajestan record, the wavelet noise and first 10 s of the signal are more coherent. This coherency

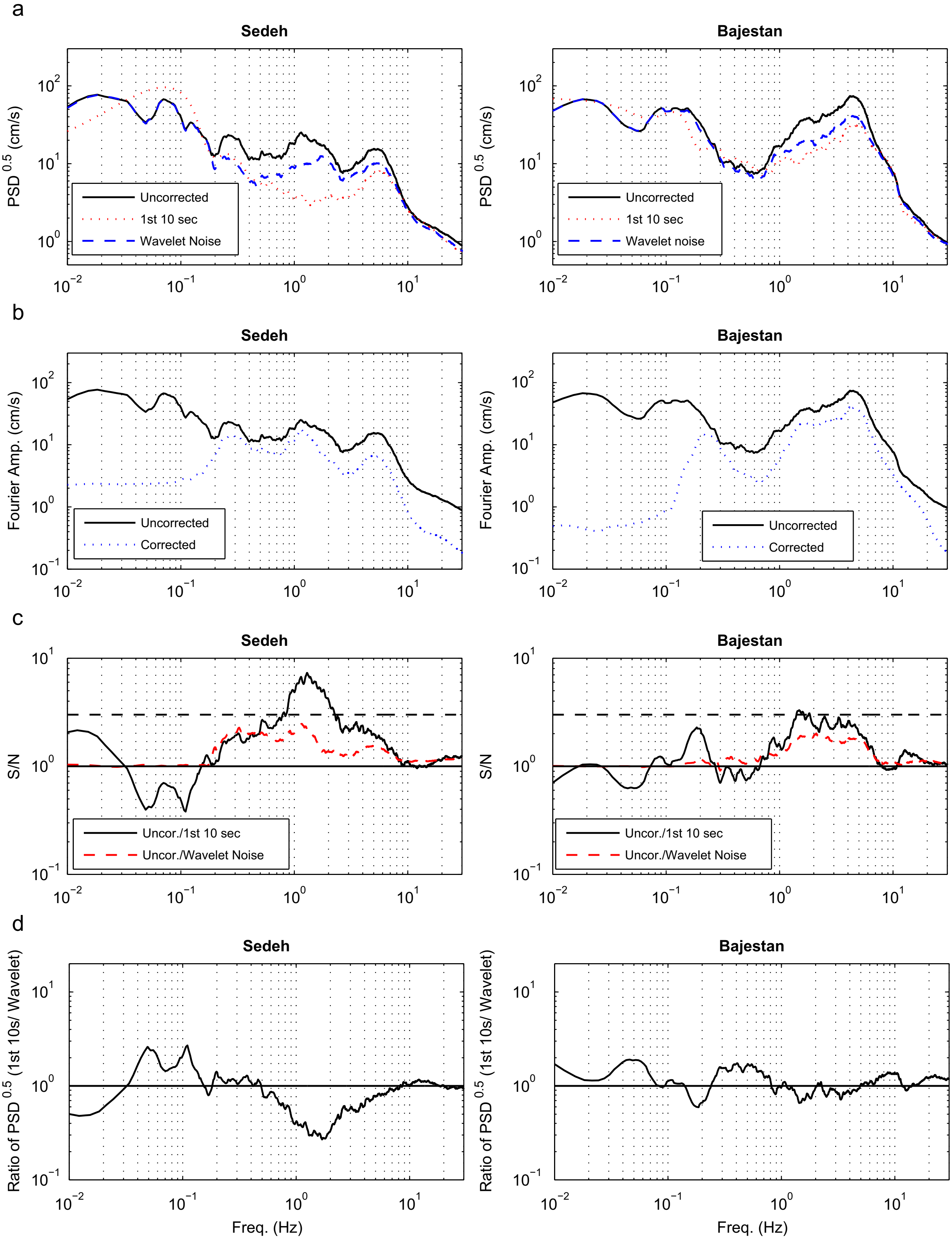

Fig. 8. (a) Fourier amplitude spectra, with correction due to different length of signals, of uncorrected signal (solid black), first 10 s of signal (dashed red), noise identified by wavelet method (dashed blue) of Sedeh and Bajestan records; (b) uncorrected (solid black) and wavelet corrected (dashed blue) signals of Sedeh and Bajestan records; (c) SNR with modeling noise as first 10 s of signals (solid black) or considering noise identified by wavelet de-noising method (dashed red), solid and dashed horizontal black lines correspond to SNR of 1.0 and 3.0 respectively; (d) ratio between wavelet identified noise and the first 10 s of records. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

is also observed with uncorrected signal. In such cases where the signal is over-noisy, it is not possible to discriminate between signal and noise. Thus, every model of noise is coherent with uncorrected signal. In conventional filtering methods, since there is no frequency range where the signal dominates the noise, it is not possible to modify noisy signal. On the contrary, the modification strategy of wavelet de-noising method is quite different. It attenuates all frequency component of the signal up to the level which is determined based on specific characteristics of the noisy signal in that frequency range. This fact can be observed in Fig. 8(b) where in the case of Bajestan record, the Fourier amplitude spectrum of corrected signal is in resemblance to uncorrected signal but with lesser magnitudes in the frequency range more than 0.3 Hz. Fig. 8(b) also shows that the Fourier amplitude spectra of uncorrected and wavelet corrected signals are identical in the frequency range of 0.1–4 Hz for Sedeh record. However, in the case of Bajestan record, the spectra of corrected and uncorrected signals are different in almost all frequency range. This is an indication of the adaptive and flexible characteristic of the wavelet de-noising method.

In Fig. 8(c), the SNR are plotted for two models of noise. Again, the SNR are dissimilar in the Sedeh record and similar in the Bajestan time history. However, in both records, SNRs are below 3.0. The noticeable feature of this figure is those portions of SNR with first 10 s of the signal as the model of noise where this ratio falls below 1.0. It has to be mentioned that SNR is defined as the ratio of the uncorrected signal (sum of original signal and noise) to the noise and thus, this ratio should be always greater than 1.0. As a result, it can be inferred that the first 10 s of the uncorrected signals are not valid models of noise in these cases. In Fig. 8(d), the ratios of two models of noise are shown. It is clear that the first 10 s of the record is underestimated in some frequency ranges and overestimated in other frequency ranges in comparison with wavelet noise.

In Fig. 9, time–frequency decomposition of wavelet noise for Sedeh record is presented. This figure reveals the important advantage of wavelet de-noising method which makes it possible to attenuate the effect of noise in the whole frequency range. As it is clear from this figure, above 10 Hz and below 1 Hz, noise is stationary and spreads the whole duration of the signal. However, between 1 and 10 Hz, non-stationary behavior can be observed. Moreover, the decomposition of the noise at level 5 has the most energy which is concentrated in the frequency range of 3 to 9 Hz. It is clear that this frequency range is the important range from

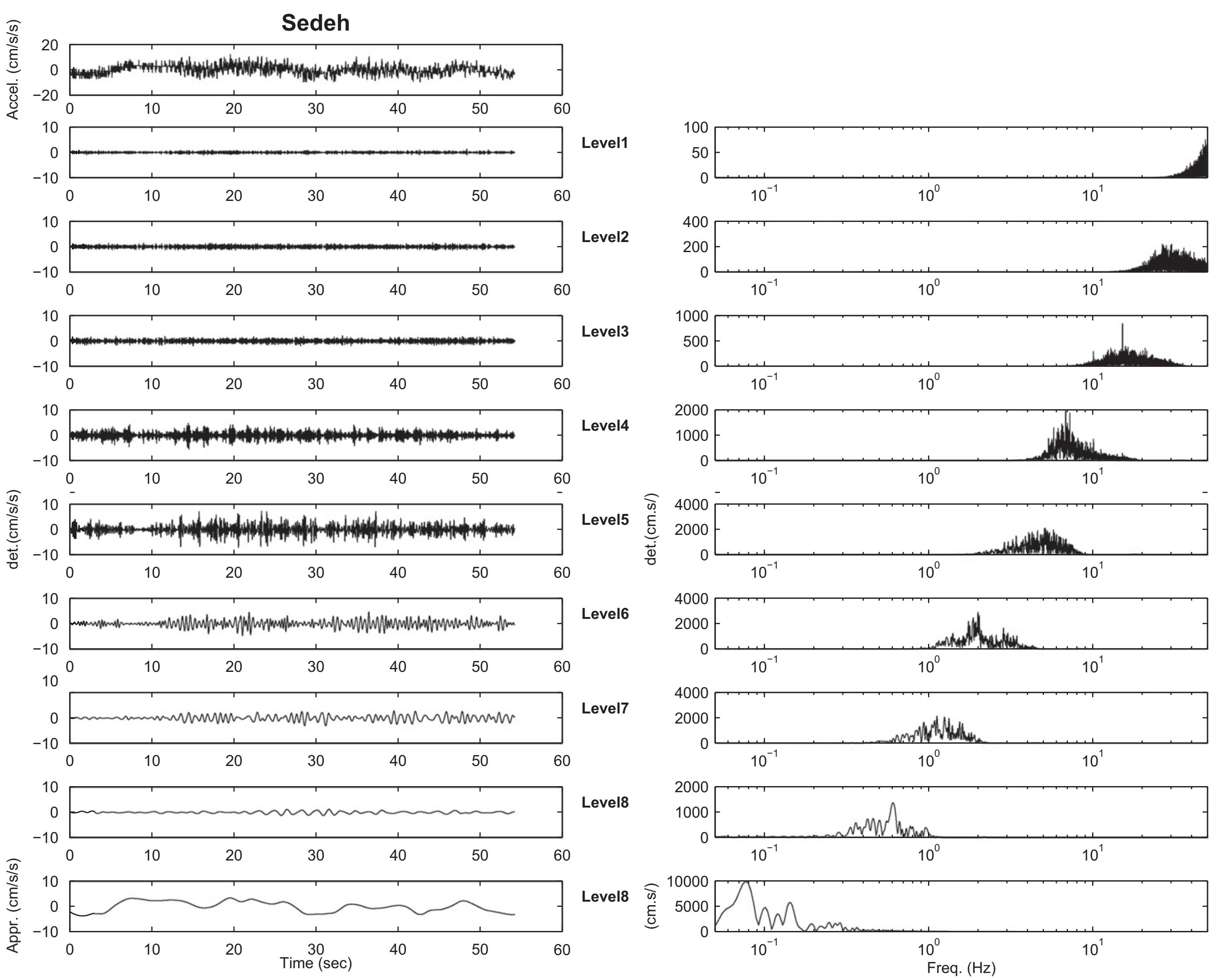

Fig. 9. Wavelet decomposition of noise identified by wavelet de-noising method for Sedeh record (left column) time domain details (right column) corresponding Fourier amplitude spectra.

engineering point of view. Similar conclusions can be made by observing time–frequency decomposition of Bajestan record presented in Fig. 10.

3.3. Correction of highly noisy synthetic signal

To better verify the capability of proposed method, the correction of a highly noisy synthetic signal is considered. In this regard, a corrected acceleration time history is selected as original signal. This acceleration signal is actually the L component of acceleration time history recorded at Mohammad Abad station during 2003 Bam, Iran earthquake after baseline correction. For the baseline correction, a quadratic polynomial was fitted to the signal and subtracted from it. To construct the noisy signal, a Gaussian white noise is generated and band-pass filtered between 0.02 and 30 Hz with Butterworth filter of order 4. These filtering cut-off frequencies are selected such that the frequency bands of the noise are wider than the frequency range of engineering interest. Then this filtered white noise is added to the target signal to construct the noisy acceleration time history. In Fig. 11, the acceleration time histories of the original, filtered Gaussian white noise and noisy signals besides their Fourier amplitude spectra are shown.

In Fig. 12 the S/N is shown for this case with taking the first 10 s of the noisy signal as the representative model of noise. As it is clear, the S/N is less than three in almost all frequencies and the noise contaminates the signal in all frequency range of interest. In such cases the conventional filtering method is not capable of correction of the noisy signal. However, since the wavelet de-noising method detects the noise in all frequencies, it is possible to use it for correction of this noisy signal. In Fig. 13, the results of applying this method are shown beside the result of filtering the noisy signal with conventional Butterworth filter of order 4 in the pass-band of 0.2–20 Hz, which is a common band in real applications. The filtering method is used only for making a better comparison. It is clear from this figure that the proposed method managed to cancel out the noise effectively. The performance of the proposed correction method is more evident in the velocity and displacement time histories where the main features of the signal are retrieved correctly.

4. Summary and conclusion

The wavelet transform theory provides a very efficient tool in many scientific fields. In this paper, the capabilities of the

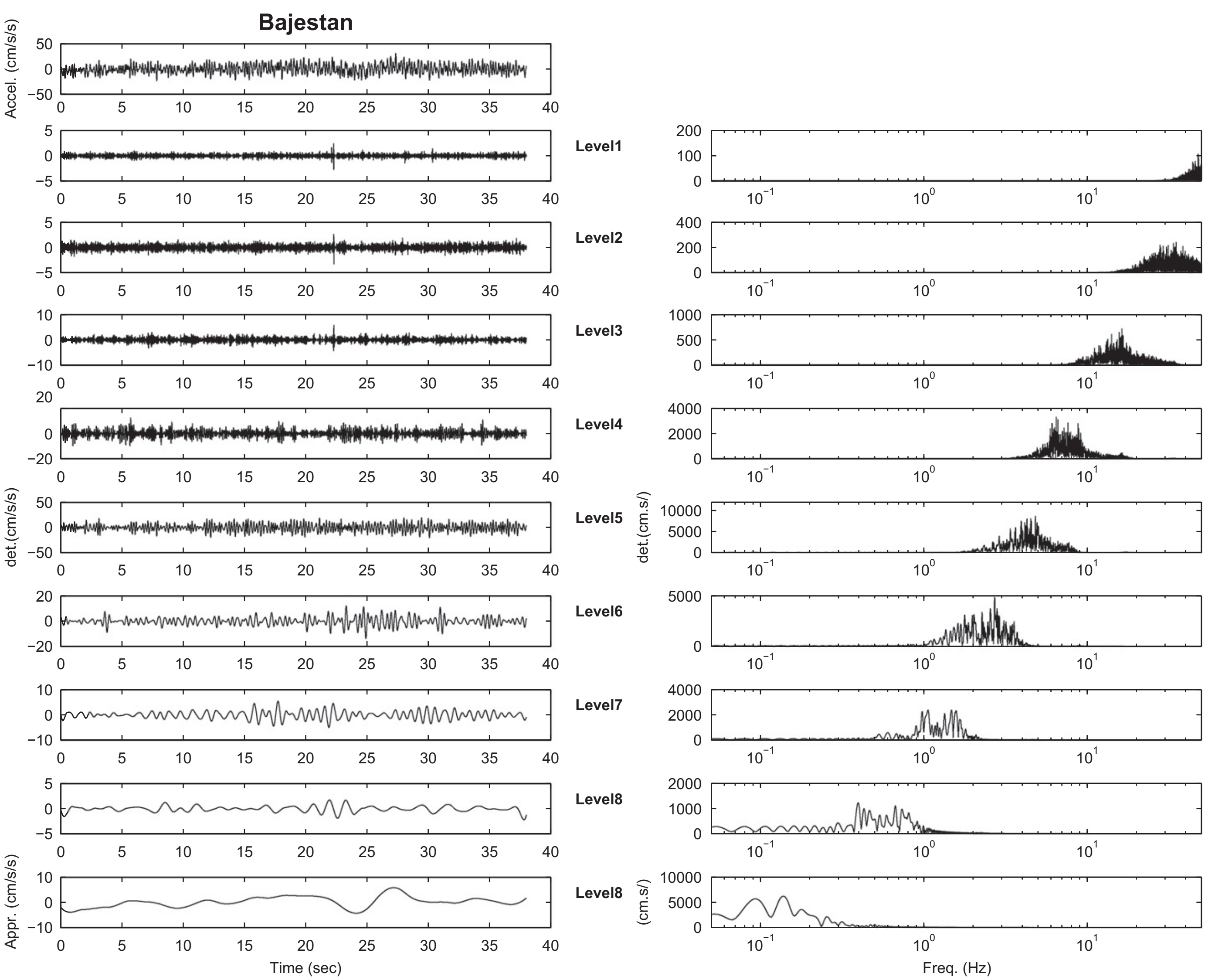

Fig. 10. Wavelet decomposition of noise identified by wavelet de-noising method for Bajestan record (left column) time domain details (right column) corresponding Fourier amplitude spectra.

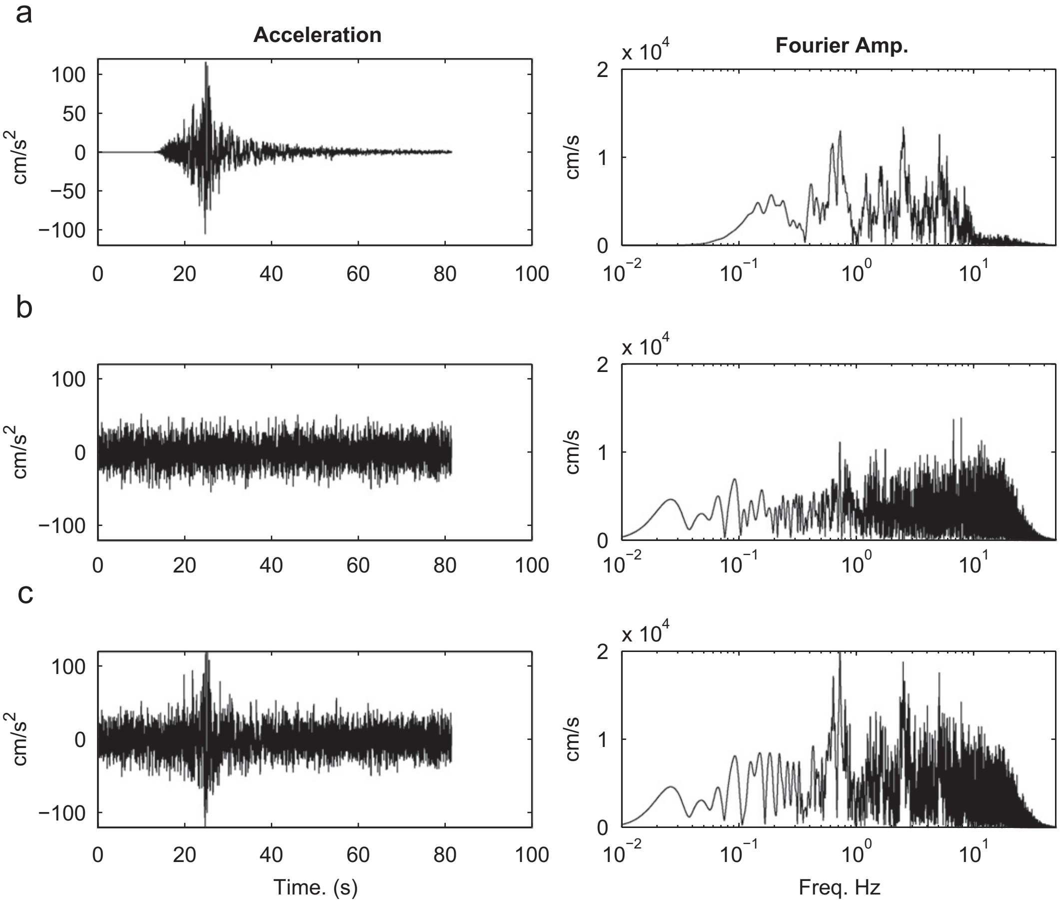

Fig. 11. Acceleration and Fourier amplitude spectrum of (a) original signal, (b) filtered Gaussian white noise and (c) the noisy signal.

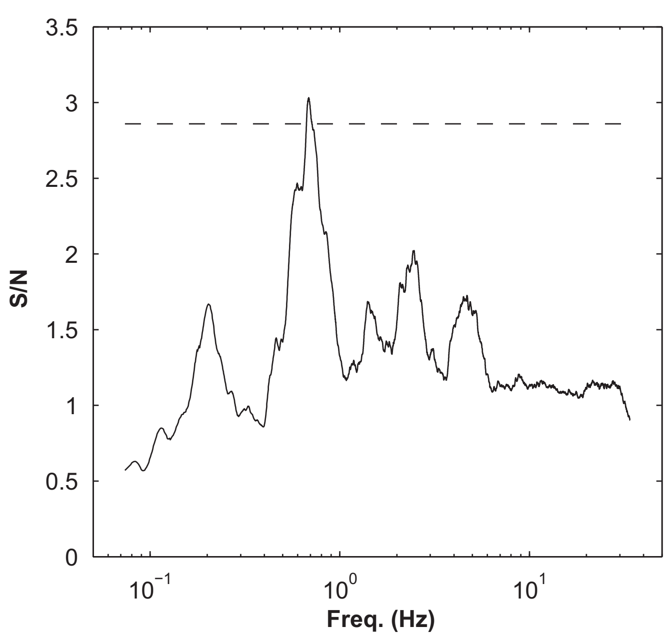

Fig. 12. S/N of the noisy signal.

modified wavelet de-noising method in correction of highly noisy acceleration records were studied and a comprehensive comparison was made between the results of the conventional filtering method and the new wavelet de-noising procedure from different aspects in both time and frequency domains. The capabilities of wavelet de-noising method come from the adaptive nature of this method. In the conventional method, the cut-off frequencies of the filtering are obtained only based on relative energy of signal and noise; but in the wavelet de-noising method, a threshold value is obtained for each time–frequency component of the signal according to the characteristics of that component. As a result, not only no frequency component of the signal is removed, but also each component of the signal will be modified based on the characteristics of the noise in that component. The consequence of applying such correction method is that it is possible to remove the noise from the frequency components of interest. It is worth mentioning that in the conventional method, some low and high frequency components of the motion are removed from the signal and other frequency components of the signal remain unchanged and it is assumed that the energy of the noise is concentrated only in the low and high frequencies. However, in applying the wavelet de-noising method, it is assumed that it is possible to have noise in all frequency components of the motion, just like the case of white noise.

It is shown, by studying two high noisy analog records of Tabas, Iran earthquake, that it is possible to cancel out non-stationary noise from these signals by wavelet de-noising method. It is also shown that the general condition of these signals cannot be improved through conventional filtering even by changing the cut-off frequencies. It is because frequency filtering has some intrinsic mathematical limitations which make it unsuitable for dealing with high noisy signals. The results obtained by wavelet de-noising method also showed more stability with respect to changes of wavelet function, in contrary to the filtering where the

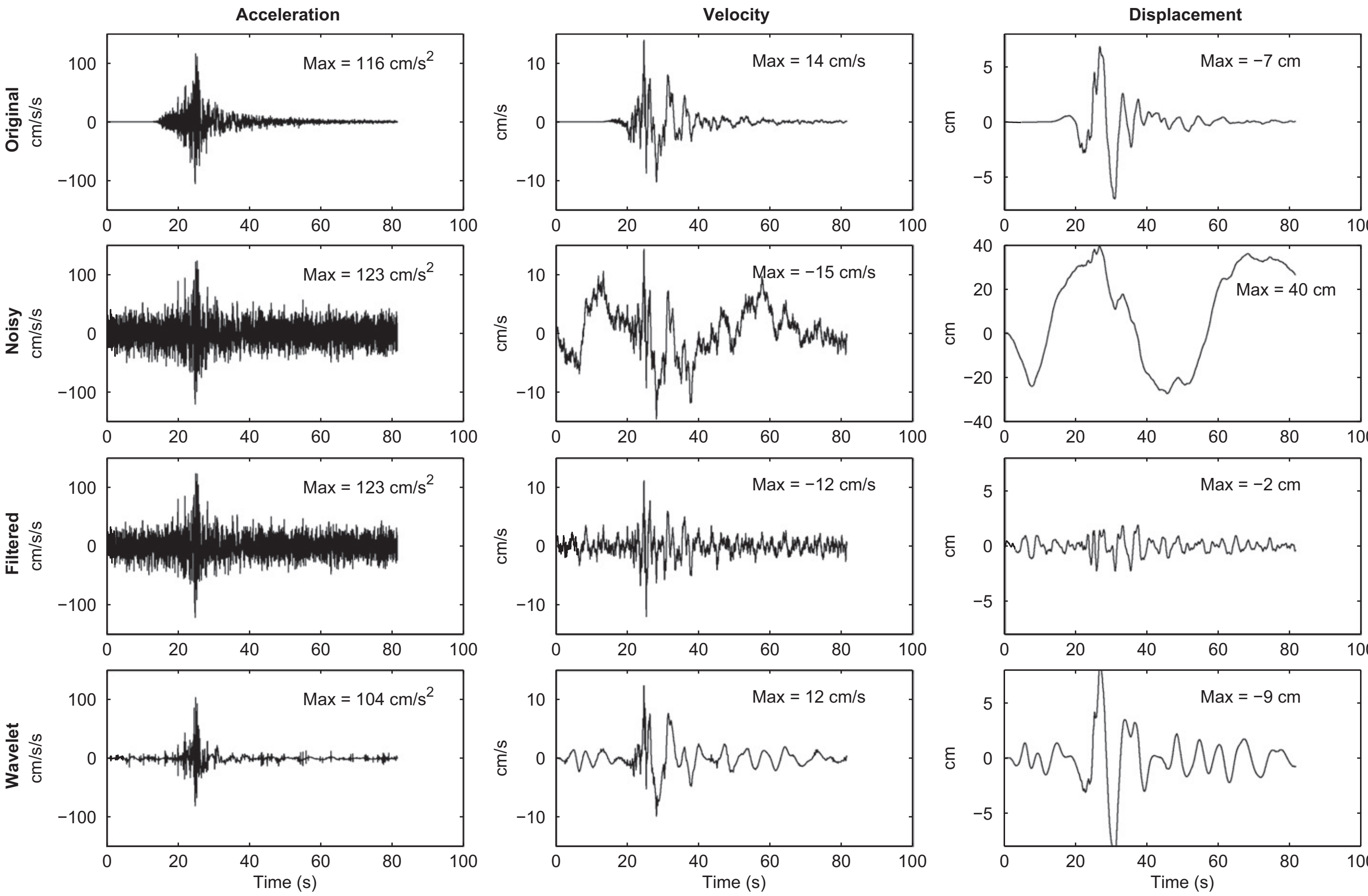

Fig. 13. Acceleration, velocity and displacement time histories of original signal (first row), noisy signal (second row), filtered signal (third row) and wavelet de-noised signal (last row).

results were highly dependent to the function parameters. These results are also verified by correction of highly noisy synthetic signal.

Moreover, because of the success of wavelet de-noising method to effectively remove the noise from highly noisy signals, the resultant corrected signals are more informative. It means that a lot can be inferred from these records with more accuracy. This is a worthy result, because for years, most of the recorded acceleration signals were put aside in different strong motion analyses because there was no way to modify them using conventional correction methods.

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