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THERMAL DEFORMATION IN FRICTIONALLY HEATED CONTACT*

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Summary

When sliding occurs with significant frictional heating thermoelastic deformation may lead to a transition from smoothly distributed asperity contact to a condition where the surfaces are supported by a few thermal asperities. This circumstance may be associated with a transition to a condition of severe wear because of the elevated contact pressure and temperature, and also because of production of tensile stresses. This second stress component may lead to heat checking whereupon the rough checked surface acts to abrade the mating material.

The factors influencing transition are discussed, including wear, cooling and hydrodynamic lubrication. The transition state is also discussed with regard to the stress distribution, rate of movement of the contact patches and temperatures.

1. Introduction

Modern contact theory has drawn upon the powerful ideas of Holm [1], Bowden and Tabor [2] and Merchant [3] to provide a model of surfaces which are rough on the microscale and form localized highly stressed contact junctions. This model replaced the earlier asperity engagement models of Belidor [4] and Coulomb [5]. The idea of a high temperature in a frictionally heated contact was introduced by Blok [6] and applied by him to explain the failure of lubricated Hertzian contacts as encountered in gearing. Holm [7], Archard [8] and others have drawn upon similar concepts to provide an estimate of localized high temperatures in frictional sliding.

As experimental findings accumulate and as analytical modeling is extended and improved, it has become clear that the asperity model must be supplemented by the concept of thermal deformation and the creation of

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"thermal asperities". This concept helps to explain many instances of high wear, anomalous partitioning of heat [9] and a chain of events sometimes leading to catastrophic failure.

The idea of thermal asperities is supplemental to that of roughness asperities and in no way antagonistic to it. On the one hand a process of instability can produce thermal asperities on surfaces approaching absolute smoothness. On the other hand a thermal asperity may itself correspond to one of the original roughness peaks, or it may give rise to a contact patch wherein a large number of roughness asperities engage those of a second body just as they would in a large area of undeformed flat surface contact.

The process of transition from nominally flat to highly deformed surfaces can be termed *thermoelastic transition.* It sometimes occurs in a sequence of stable continuously related states as operating conditions are changed. At other times, however, the stably evolving behavior of the sliding system crosses a threshold whereupon a sudden change of contact conditions occurs as the result of an instability. This involves a feedback loop which comprises localized elevation of frictional heating, resultant localized thermal bulging, localized pressure increase as the result of the bulging and further elevation of frictional heating as the result of the pressure increase. When this process leads to an accelerated change of contact stress distribution it is known as *thermoelastic instability* (TEI).

The ultimate result of the growth of the thermal disturbance is the separation of the surfaces in some portions of the nominal contact area with the gap height being several times the roughness asperity height. As a consequence of the reduction of the nominal contact area the remaining contact patches acquire elevated stress.

Contact patch formation can occur in lubricated as well as dry contact and is influenced by wear, cooling, materials properties and macroscopic constraints on the contacting bodies. The physics of thermoelastic transition, thermoelastic instability and contact patch behavior have received serious interest only in the last few years and the explanation of many important effects is not generally well known. For this reason space must be devoted here to a brief introduction to the phenomena involved before discussion can proceed to problem areas needing further research.

2. Historical background

Prior to the introduction of the concepts of thermoelastic transition Ling and Mow [10] developed an influence function for surface displacement for high Peclet number* sliding of bodies in plane strain and outlined the procedure for treating a moving contact patch on the surface of a slab,

*The Peclet number is a dimensionless measure of the speed of movement of a heat source and is of the form cl/k where c is the speed of movement, l is a characteristic length (patch length) and k is the thermal diffusivity.

also for high Peclet number. Mow and Cheng [11] have examined the companion problem of thermal stress in plane elastohydrodynamic contact. Early investigations of thermoelastic effects on lubricated sliding were reported by Korovchinsky [12] for the change of the contact stress between a ball and a frictionally heated contact and by Nica [13] for the change in the radius of journal bearings with a fixed external radius. The bearing work has been broadened [14, 15] to include cooling effects and more realistic boundary conditions for fluid-film bearings, and also to show that similar phenomena exist for rolling contact bearings [16]. The most interesting result obtained is the role of thermal expansion in the catastrophic chain of events leading to seizure of the bearing.

Sibley and Allen [17] carried out a series of experiments on seal materials and developed a criterion for thermal checking and showed photographic evidence of systematically moving hot patches in the contact zone.

Interest in contact instabilities and patch formation was accelerated by the work of Barber [18 - 20] who has demonstrated the phenomenon experimentally and has provided analyses which partially explain his observations as well as making fundamental contributions to the field of thermoelasticity. His explanations draw upon the modification of asperity contact by frictional heating and wear. His interest was initially in explaining hot-spot effects in railroad brakes.

Dow [21, 22] addressed the problem of a scraper sliding perpendicular to its edge on a conductive slab and showed that instabilities would be predicted in the absence of wear and would be modified by wear which would (a) raise the instability threshold and (b) sometimes give rise to oscillating contact pressure. He carried out a numerical simulation [23] which predicts the formation of contact patches and their translation along the edge of the scraper as the result of wear. More recently he has carried out experiments which vividly display patch formation [24]. The onset of such patches is close to his theoretical critical sliding speed for instability for dry contact, but although qualitatively similar it is only poorly predicted for wick-lubricated contact.

Kennedy and Ling [25] attacked the problem of severely loaded aircraft brakes using first numerical analysis and later experiments [26]. They postulated a wear model which suggests a transition from mild wear to severe wear, assuming that no wear occurs until a critical shear stress is reached in the material. For sandwiches of alternating moving and stationary disks they found contact reducing to a band at the outer radius and then moving inward, only to repeat the sweep again several times in a typical "stop". The confinement of the disk brake causes the material properties and operating parameters to interact differently than in the case of the "floating" scraper. Furthermore the appearance of patch contact in the brake may be a transient phenomenon rather than an instability in that there is some evidence that the sweep would ultimately die out as the brake wore in. This cannot be tested since the overall temperature rises severely even in the brief runs reported and thus limits the time of operation.

Nerlikar [27] has addressed the problem of ring contact in an idealized face seal sliding tangentially and has shown that the relative conductivities of the two bodies strongly influence instability. The least stable is a thermal conductor sliding on an insulator and the most stable is a material sliding on itself. This has been extended [28] to show that extremely thin contaminant, solid lubricant or oxide films can play a major role in determining instability. Nerlikar has also explored the influence of roughness on instability and has calculated the gap width and contact temperatures for seals in the contact patch configuration.

Lebeck [30] has improved the model for sealing ring contact, allowing for thin beam bending and showing how to account for cooling from the sides of the rings.

Kilaparti has addressed the problem of patch contact in the absence of wear [31] and with wear [32, 33], and has developed an improved influence function similar to that of Ling and Mow [34] and discovered that there is a critical wear coefficient above which instability will not occur.

Banerjee [35] treated the idealized seal with liquid lubricant in the hydrodynamic short bearing regime. He predicted a range of instability which is suppressed at the thin film extreme by elastic deformation under contact pressure and suppressed at the thick film extreme by convection of heat in the film. They have carried out experiments (see Figs. 1 and 2) which show the instability to occur where expected [36]. More recent work [37] has modified his model but does not alter the basic conclusions.

Closely related to this work is that of Hahn and Kettleborough [38], Ettles [39] and Tanaguchi [40] on thrust washers where only thermoelastic deformation can explain the load support of notched rings in lubricated wide bearing contact. Why these rings (and for that matter sliding systems such as

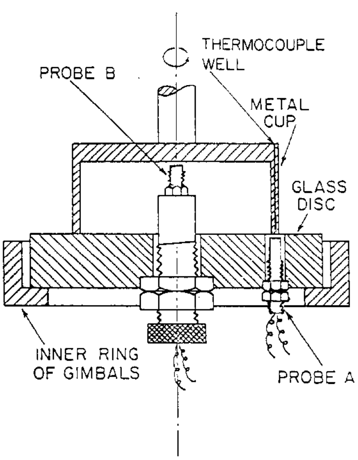


Fig. 1. Cross section of the apparatus where ring contact occurs between an inverted cup and a glass plate. Probe A reads the surface profile of the cup and probe B indicates the overall rising or bouncing.

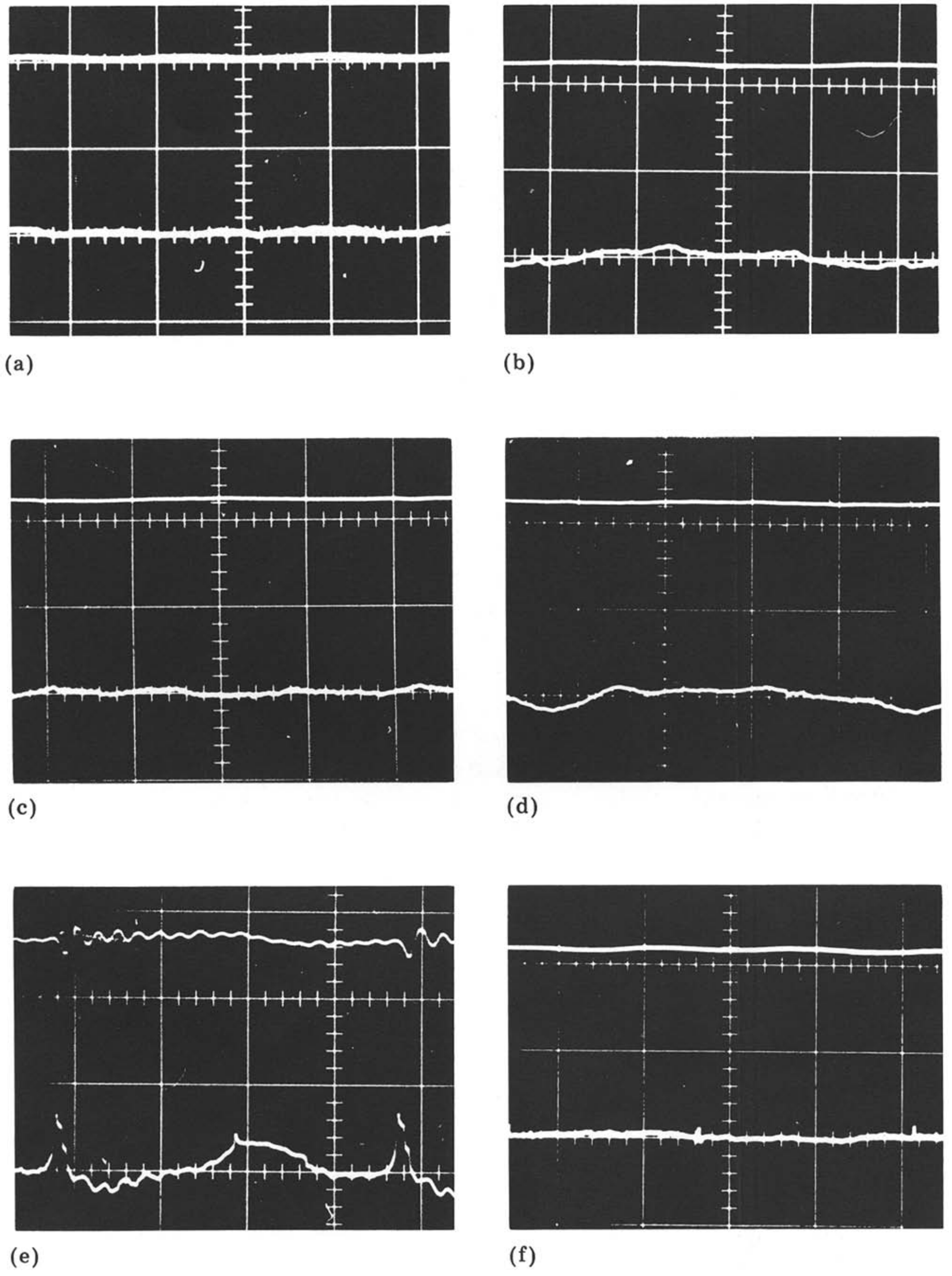


Fig. 2. In these oscilloscope traces of probe outputs the upper trace is probe B and the lower is probe A. As speed is increased moving waves grow in the cup face and at 13 m s^{-1} there is a sudden formation of a thermal asperity. Slight bouncing is indicated as this goes over the probe indentation. Upon lowering the speed (f) the thermal asperity disappears except for a tiny wear scar. (a) U = 4.6 m s^{-1}; (b) U = 10.7 m s^{-1}; (c) U = 10.7 m s^{-1}; (d) U = 10.7 m s^{-1}; (e) U = 13 m s^{-1}; (f) U = 4.6 m s^{-1}.

engine pistons) do not show thermoelastic instability is a question that must yet be answered. We can only note here that Banerjee predicted a stabilizing effect from increased face width.

Heckmann [41] has reassessed the ring and scraper instability problems from the point of view of controlling dimensionless groups. These groups are w/w^* which is a wear measure and H and ζ which are two groups for cooling. He has also extended the seal and scraper problem to line contact on a slab [42], thus bringing three dimensionality in to replace the earlier two-dimensional models. An axially symmetric contact patch on a slab has also been treated elsewhere [43]. Heckmann's studies show that except for quantitative differences a cylinder sliding in line contact with a slab is substantially the same as Dow's scraper. Indeed, Dow's experimental pieces [24] simulating Wankel seals lie somewhere between scrapers and cylinders.

Wangkrajong [44] has carried out high speed sliding experiments for carbon graphite on mild steel where the evidence at 400 ft s^{-1} and higher suggests extremely small contact area. Work in progress on high speed turbine blade and labyrinth seal rubs not only suggests patch contact but calls for analyses to treat bouncing and vibratory compliance of the contacting members [45]. Although this 1000 - 1500 ft s^{-1} contact represents a spectacular application where thermoelastic instability is inevitable, one should remember that the phenomenon is also found at speeds as low as 10 ft s^{-1} in some geometries (see also ref. 46).

3. Thermoelastic transition of nominally flat sliding contact

Face seals represent one of the best definable examples of thermoelastic transition. Such seals are ordinarily in the form of concentric rings meeting at a plane perpendicular to their axis with contacting surfaces lapped to quarter-light-wave smoothness and supported by bellows or O-ring arrangements to make them self-aligning. At times they may operate with no lubricant, or they may incorporate solid lubricants, boundary lubricants or even liquid lubricants which provide hydrodynamic support.

An experiment [36] which simulates the frictional contact of a face seal is illustrated in Fig. 1. A cylindrical cup of metal is inverted and supported by an axial stem in the chuck of a drill press (Fig. 1). The circular edge of the cup is pressed against a flat glass plate which is supported in gimbals to permit a self-aligning almost uniform contact. Whether the surfaces are lubricated or dry, as the cup turns about its axis one will find a temperature rise which is determined by speed and load but is typically only about 100 °C. As the speed is increased the interface can be observed from below looking through the glass plate, and at a characteristic somewhat reproducible speed a red light flashing from the contact is observed.

If the glass and cup are inverted so that the glass turns and the cup is stationary, the event of light evolution will be found to correspond to the appearance of two or three discrete spots on the cup which glow orange-red and twinkle slightly. When operation is continued the spots move slowly from their original position at a rate of about 10° min^{-1}. Thus even though the glass may be turning at 2000 rev min^{-1} the hot spots will take roughly 30 min to make a circuit around the circumference of the cup. Typically they will remain as two spots 180° from one another, although for short times more spots may appear and later vanish.

Examination of the metal cup immediately after the formation of a hot spot shows evidence of some plastic flow and a darkened or "burned" patch on the metal. If examination follows a longer period of operation it will show tracks on the surface over which the spots have traversed.

Although the spots are red hot the general temperature of the cup remains moderate. Damage is not catastrophic because only a shallow layer of metal is affected. Nevertheless there is evidence that prolonged operation in this regime may significantly affect the wear rate and can produce surface cracking.

The first explanation that comes to mind is that the contact is at the peaks of roughness asperities. However, when one measures the contour of the metal surface in the deformed state one finds that the hot spots can be associated with pimples on the surface which are as high as 10^{-3} cm while the roughness is about 2.5×10^{-5} cm and the initial waviness is 10^{-4} cm or less (see Fig. 2).

More interesting is the fact that if the speed of sliding is reduced the pimples shrink back into the surface and in some cases are replaced by slight depressions. To explain this we may postulate a thermal expansion which produces the pimples. Then because there is concentrated contact on their peaks frictional heating is concentrated there so as to maintain the local thermal expansion. Since frictional heating is speed dependent it would be reduced upon reduction of speed and would ultimately be insufficient to maintain the height of the pimples which therefore collapse back to flat surface contact.

3.1. Conditions for self-sustaining patch contact

To show the factors which permit the existence of patch contact let us assume that the edge of a plate rubs against a rigid thermal insulator and that the contact patch has a width $2l$ on the edge of the plate. Since we do not know the contact pressure distribution at this point we assume it to be a uniform value $\bar{p}$; thus if the sliding speed is V and the friction coefficient is μ the heat input per unit of area is

$$q = \mu \bar{p} V \tag{1}$$

It has been shown that the curvature of the edge of a plate is related to heat input such that [48]

$$\frac{d^2 v}{dx^2} = \frac{1}{R} = \frac{\alpha q}{K} \tag{2}$$

where R is the radius of curvature, v the normal displacement and K the thermal conductivity. At this point we may ask what contact pressure would be required to flatten this curvature across the contact zone, and turn to the well known relation for Hertzian contact:

$$l/R = 2.3\bar{p}/E^* \tag{3}$$

Eliminating R between eqns. (2) and (3) and substituting for q from eqn. (1)

$$l = 2.3K/E^*V\mu\alpha \tag{4}$$

More exact studies bear out the essential correctness of this finding which shows two interesting features: (1) the sliding speed required to sustain a patch of a particular l is independent of the loading; (2) there is nothing to prevent there being one, three or any number of contact patches on a given sliding edge which thus poses a problem of uniqueness. With regard to the first feature we note that a load influence could appear in the physical problem if μ, E, α and K depend on temperature or stress level. With regard to the second feature there is a problem of uniqueness which disappears when a second consideration is introduced, this being the stability of the solution under small disturbances. A test for this can easily be introduced at the present level of argument.

Nerlikar's work [29] showed that if the contact load on a patch is increased the surfaces will move further apart in the region between patches. This paradoxical effect causes leakage to increase with increased face load on a seal. It also gives rise to an instability which causes the number of contact patches to reduce to the minimum number consistent with mechanical equilibrium. For a self-aligning seal the minimum is *two* patches; for a seal where tilting is constrained *one* patch is the minimum.

To visualize this we note that the seal can be developed into an infinite plate where contact patches are cyclically repeated. If alternate patches are reduced in load the others must experience load increase. These load shifts give rise to displacements which further increment the load changes until ultimately the patches with decreasing load break contact.

3.2. *Thermoelastic instability*

The question arises now as to how the transition from smooth contact to the patch configuration occurs. This may be posed as a stability problem where the conditions may be found for runaway growth of a small departure from uniform pressure. If the edge of the cup is allowed to have a circumferential waviness and if the waviness is pressed flat by axial loading the contact pressure can be represented by

$$p = \bar{p} + \hat{p}\cos\theta = \bar{p} + \hat{p}\cos\kappa x \tag{5}$$

where θ is a measure of angular position and x is a curvilinear coordinate running around the edge of the cup. The pressure $\hat{p}$ is known as the pressure perturbation. Frictional heating will occur according to eqn. (1) but the uniform portion produced by $\mu\bar{p}V$ will not affect waviness but only cause the cup to flare out by a small amount. The perturbation will give rise to a distribution $\hat{q}\cos\kappa x$. This can be inserted into eqn. (2) and integrated to produce a waviness

$$v = \frac{\alpha}{K}\frac{q}{\kappa^2}\cos\kappa x \tag{6}$$

From elastic theory it is known that the pressure required to press flat a sine wave of displacement v is

$$p = E\kappa v/2 \tag{7}$$

Hence if the displacement increment from the heating is held flat it will supplement the initial perturbation by an amount

$$p_{\text{th}} = \frac{E\alpha}{2K\kappa}\hat{q}\cos\kappa x' \tag{8}$$

However, the heating perturbation will now be determined by

$$\hat{q} = \mu V(\hat{p} + p_{\text{th}}) \tag{9}$$

Inserting this into eqn. (8) yields

$$p_{\text{th}} = \frac{E\alpha\mu V}{2K\kappa}(\hat{p} + p_{\text{th}}) \tag{10}$$

Let us use the convenient grouping of variables V^* to simplify eqn. (10) where $V^* = 2K\kappa/E\alpha\mu$:

$$p_{\text{th}} = \frac{V\hat{p}}{V - V^*} \tag{11}$$

It is now apparent that if $V = V^*$, $p_{\text{th}} \to \infty$ irrespective of the size of $\hat{p}$. This suggests that V^* is the critical sliding speed for transition, since ultimately the negative portion of the p_{th} wave would exceed $\bar{p}$ and the surfaces would part thus initiating patch contact.

Table 1 is a list of thermal and elastic properties of some typical materials. When the material aluminum for example is selected for the cup, one finds that for $\mu = 0.1$ and $\kappa = 2$ rad cm^{-1}

$$V^* = 0.64 \text{ m s}^{-1} \tag{12}$$

This corresponds to about 150 rev min^{-1} for a cup of diameter 75 mm, yet no major transition has actually been observed up to about 2500 rev min^{-1}. Clearly the analysis is incomplete. Indeed, wear and other factors will be shown to have dominant effects upon the growth of waviness.

Although this analysis predicts a critical speed at which waviness is greatly multiplied in amplitude it also predicts that at higher speeds the amplitude will drop and extremely high speeds would be associated with improved smoothness. This would at first appear to be something like the resonance phenomenon of synchronous whirl where machines regularly operate above the critical speed, but more careful analysis will show an important difference.

TABLE 1

Properties of representative materials

	Aluminum	*Cast iron*	*SiC*	*Graphite*	*Glass*
E (MPa)	6.8×10^4	1.2×10^5	8×10^4	0.68×10^3	8×10^4
α (°C^{-1})	17×10^{-6}	11×10^{-6}	4.7×10^{-6}	4.7×10^{-6}	5.4×10^{-6}
K (N s^{-1} °C^{-1})	227	50.4	18	13.5	0.9
k (cm^2 s^{-1})	83×10^{-2}	11×10^{-2}	6×10^{-2}	7.9×10^{-2}	0.31×10^{-2}

3.3. Conditions for exponential growth of surface waviness

The existence of steady state solutions as discussed in Section 3.2 does not ensure that the system will be bound to these if they are themselves unstable to small disturbances. A test for such an instability can be made in several well established ways, one of which is to hypothesize an exponentially growing waviness and to test for the circumstances under which this can exist. The argument behind this would be that if (1) the system were disturbed and (2) the disturbance contained a time-dependent component corresponding to exponential growth, then such growth would continue no matter how small the initial disturbance. Thus a grain of dirt or a temperature fluctuation could trigger the growth process.

Before proceeding with such a test let us review the initial analysis, noting that sinusoidal waves of contact pressure were accompanied by sinusoidal waves of frictional heating and thermal expansion. Further examination of the equation for heat transfer in the tube would suggest that there is also a corresponding sinusoidal temperature perturbation proportional to $\hat{p}$ and having its peak where $\hat{p}$ is a maximum [22, 35]. Let us therefore postulate a temperature perturbation of the type

$$T = \hat{T} \exp(-by) \exp(i\beta t) \cos \kappa x = T_s \exp(-by) \tag{13}$$

Then contact pressure will be given by

$$p = E\alpha T_s \frac{\kappa}{\kappa + b} \tag{14}$$

Here it is again assumed that the axial load is sufficient to ensure full surface contact. Returning to eqn. (13) we find that the heat flux through the surface will be

$$q = -K\left(\frac{\partial T}{\partial y}\right)_{y=0} = KbT_s \tag{15}$$

Let us now require that for a self-sustained wave to exist the frictional heating μVp must correspond to the heat passing through the surface as given in eqn. (11). It follows that

$$KbT_s = \mu VE\alpha T_s(\kappa/b + \kappa) \tag{16}$$

Drawing upon the definition of V^* this reduces to

$$\frac{(1 + b/\kappa)b}{\kappa} = \frac{2V}{V^*} \tag{17}$$

If the temperature distribution in eqn. (13) is to satisfy the Fourier heat flow equation

$$\nabla^2 T = \frac{1}{\kappa}\frac{\partial T}{\partial t} \tag{18}$$

the following relation must prevail when b must be positive to satisfy distant boundary conditions [22]:

$$b = (\kappa^2 + \mathrm{i}\beta/k)^{1/2} \tag{19}$$

Substituting this into eqn. (17) and simplifying we find

$$\left(1 + \frac{\mathrm{i}\beta}{k^2}\right)^{1/2} = -\frac{1}{2} \pm \left(\frac{1}{4} + 2\frac{V}{V^*}\right)^{1/2} \tag{20}$$

Examination of this equation shows that $\mathrm{i}\beta$ can be replaced by a real positive number when

$$V/V^* \geqslant 1$$

Hence the condition for a self-sustained exponentially growing perturbation is that the speed of sliding exceeds V^*.

Again we note that the rationale behind this test is that if such a condition can exist it may be triggered by fluctuations in operating conditions during extended periods of nominally steady operation. Obviously if such a pressure perturbation continues to grow it will reach a condition where the negative lobes exceed the initial contact pressure provided by the axial loading. This will lead to regions of parting between the surfaces. As has been shown this condition ultimately stabilizes the contact in a new configuration where contact is in patches spaced around the edges of the cup.

Returning to the definition of V^* it is seen that the transition speed is dependent upon the wavenumber κ which is π/λ where λ is half the wavelength of the perturbation. Since small κ (or large wavelength) has the lowest critical speed, it follows that upon increasing the speed from zero instability will first be reached for the longest permissible wavelength. If the glass and cup are stiffly mounted to prevent tilting the critical wavelength will correspond to one cycle around the cup. However, if the glass is held in gimbals it will tilt so as to make the existence of $p = \hat{p}\cos\theta$ impossible. The next longest wavelength would be that for $p = \hat{p}\cos 2\theta$ and would have two pressure maxima 180° apart. In experiments such a disturbance tends to dominate in that hot spots (or contact patches) 180° apart are frequently observed.

As a word of caution, these derivations apply to the cup described or to relatively stiff rings. Lebeck [30] has pointed out that slender rings which obey beam theory are more compliant and provide a stabilization of long wavelength disturbances. They may also give rise to more contact patches than the minimum mentioned above.

Returning to eqn. (11) the equation for critical speed can be rewritten as

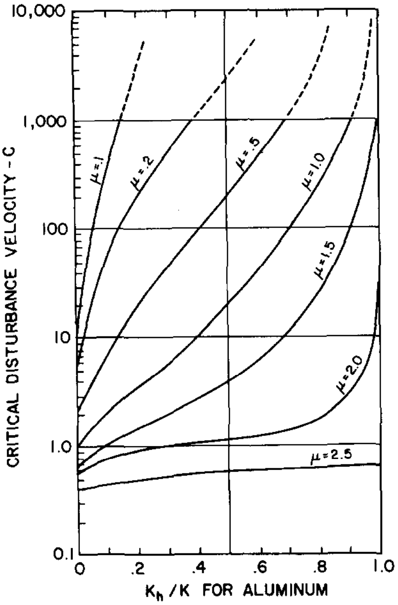
$$V^* = 4\pi\kappa/E\alpha\mu\lambda$$

where λ is the wavelength. For very thin bearings the longest wavelength permissible is $3.7h$ where h is the axial length of the ring. As the speed is raised longer wavelengths or spacings between contact patches will be tolerated, the length rising as $V^{1/3}$, until the number of patches reaches the minimum permissible number given earlier.

3.4. Effects of heat conduction and wear on instability

Because uniform heat flow down the axis of a ring or tube does not lead to waviness in the axial direction, flat contact instability is independent of this factor. Patch contact is, however, influenced by the overall heat flow. In both cases the relative conductivities of the contacting bodies are of major importance. For the tube-on-tube geometry Nerlikar [27] has carried out a theoretical study allowing both bodies to have the properties of aluminum with one having a hypothetically altered conductivity K_h (the effect of this on diffusivity is also accounted for). Results for critical sliding speed are as shown in Fig. 3 as a function of the ratio of conductivities and the friction coefficient. Except for high friction the curves turn upward at $K_h/K = 1$, and the critical sliding speed goes to infinity for material sliding on itself. Another effect of differing conductivities is that the perturbation wave is not stationary in either body but moves more slowly relative to the more conductive body. Even when both bodies are of the same material the asymmetry necessary for instability can arise from thin insulating films on one or other of the two bodies. Such a film will offer much less resistance to heat flow for low speed disturbance movement than for high speed movement. Consequently a condition can arise where a disturbance is nearly stationary in one body and has a high relative speed on the second body, and the apparent resistances of the bodies can be such as to lead to instability at low sliding speeds. Calculations for aluminum on aluminum with a glassy film of thickness λ show that for infinite film thickness on one body the critical speed approaches that of a conductor on an insulator whereas a film of reduced thickness leads to an increased critical speed (Fig. 4). There is, however, no critical speed for zero film thickness or zero friction coefficient.

Figure 5 is based on calculations where frictional heating is assumed to take place at the interface. However, it is possible that some of the heating may be due to plastic deformation below the surface and thus the film would not impede the flow of this heat component into the material. This question requires further investigation. However, it does not obscure the fact that even if only half the generated heat is affected by the film instability would be called for, yet it would be absent without the film. When wear is present [42, 22] it can be treated in terms of the modified form of Archard's wear law [47]



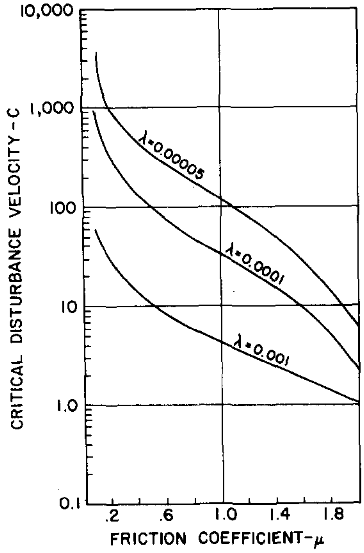


Fig. 3. Critical disturbance velocity in the less conductive body for a range of friction coefficients and for aluminum with a hypothetical reduced conductivity sliding on aluminum.

Fig. 4. Effect of thin glass films on the critical disturbance velocity in aluminum.

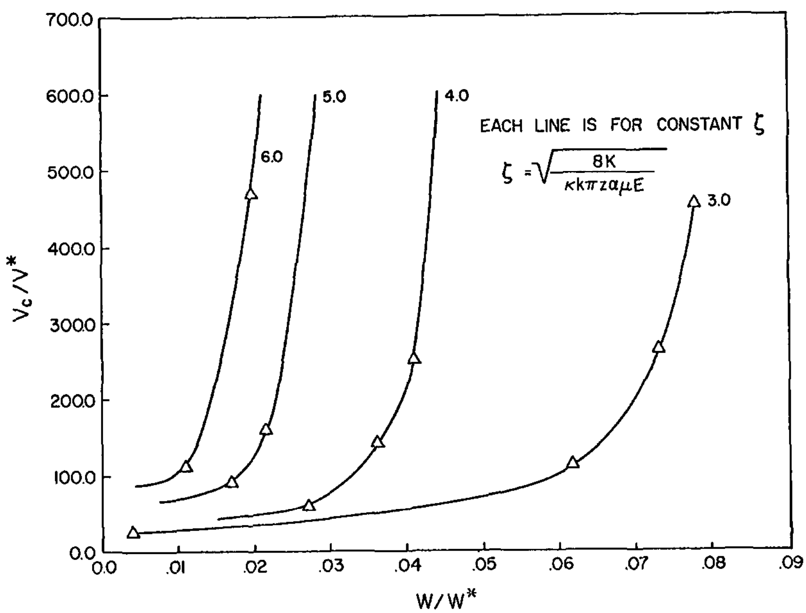


Fig. 5. Effect of wear and heat transfer on the stability of a scraper; as ζ increases the heat transfer to the surface against which the scraper slides increases.

$$dv/dt = wpV$$

where p is the contact pressure and V is the sliding speed. The quantity w is an empirical wear coefficient which is influenced by material properties, temperature and other environmental factors. The wear coefficient can be put into dimensionless form by dividing by w^*, a group of variables known as the critical wear coefficient:

$$w^* \equiv 2\alpha\mu k/K \tag{21}$$

For a conductor sliding on insulator, when $w/w^* \geqslant 1$ in the conductor, instability will not occur for a finite sliding speed. The critical sliding speed V_w^* in the presence of wear is approximately

$$V_w^* = V^*\left(\frac{1}{1-1.2w/w^*}\right)\exp\left(\frac{w}{w^*}\right) \tag{22}$$

Dow [22] has treated the case of a scraper running against a conductive drum in the presence of wear. Heckmann [41] has generalized these results in terms of dimensionless quantities as shown in Fig. 5. The critical sliding speed is set in ratio to V^* for the scraper for several values of the conduction parameter ζ. The physical quantities incorporated into ζ are those of the scraper except for the diffusivity k of the drum. Cooling from the sides of the scraper (and also circular seals) was also investigated and the effects were found to be small for reasonable values of convection heat transfer coefficient.

3.5. *Instability in a hydrodynamically lubricated seal*

The instability shown in Fig. 2 took place in a seal with sufficient oil present to ensure hydrodynamic short-bearing support for the faces which were self-aligning and supported a fixed axial load. Banerjee [35] has investigated the stability of seals with constant mean film thickness and found the critical sliding speed to be

$$U_{\text{crit}} = (K_m/\alpha_m\eta)^{1/2}\bar{h}\kappa \tag{23}$$

where the metal runs against a rigid insulator as in the prior illustrations. Strikingly good experimental confirmation of this result was obtained as shown in ref. 36. An improved model, where the constant $\bar{h}$ condition is relaxed, has been investigated and has led to the paradoxical conclusion that quasi-equilibrium states of the system approach but do not cross the stability threshold. Because of the approach to the conditions of eqn. (23) it is not surprising that the equation is satisfied. Recent experiments confirm that with great care steady operation can be carried out without transition.

Turning now to Dow's results for a scraper running on a drum [22], Table 2 compares the measured wear coefficient w and that required to predict the observed transition speed w_c. It should be noted that for combinations run dry agreement is good except for steel on steel which may have been influenced by the overall heat transfer through the contact *after*

TABLE 2

Drum	*Blade*	*Friction coefficient* μ	V^* (m s^{-1})	*Measured* w	*Calculated* w_c
Al_2O_3	Aluminum	0.38	7.11	14×10^{-10}	13×10^{-10}
Al_2O_3	Brass	0.16	8.13	9×10^{-10}	1.7×10^{-10}
Al_2O_3	Steel	0.26	5.08	2.5×10^{-10}	1.1×10^{-10}
Steel	Steel	0.25	12.20	13×10^{-10}	0.5×10^{-10}
		Lubricated			
Steel	Aluminum graphite	0.026	32	0.26×10^{-10}	<0
Steel	Alloy steel	0.051	23.9	0.54×10^{-10}	1×10^{-13}

the transition. For the lubricated runs prediction is poor but the transition was observed to be qualitatively the same as for the dry contact. Actually it was the deformed patch contact state that was observed as evidence of transition. In view of this it is possible that the friction and wear coefficients on the hot patches would be nearer to those for dry contact than for the smooth sliding of lubricated material. If that is so, then the similarity to dry contact behavior can easily be explained.

4. Patch contact with heat transfer and wear

The experiments of Dow [22] and Banerjee [35] suggest that if the deformed state *can* exist it *may* exist. In the earlier studies of conductors on insulators the condition for patch contact to exist was shown to be the same as that for thermoelastic instability. However, when more realistic models are considered, the formation of patches is found to be strongly influenced by overall heat transfer from one body to another, whereas the idealized flat surface instability is not, being dependent only on the growth of zero-average waves — at least for the case of the axisymmetric ring configuration. This can be demonstrated by referring back to eqn. (2) and adapting it to the case of heat flow from one body to another. If both are of the same material the one losing heat will curve inward and the one receiving heat will bulge outward by the same amount. If they are different there will be a relative curvature change (see also ref. 49)

$$\left(\frac{d^2V}{dx^2}\right)_{rel} = \left(\frac{\alpha_1}{K_1} - \frac{\alpha_2}{K_2}\right)q \qquad (24)$$

Depending upon the sign of the heat flow and the materials, this influence may greatly alter the patch size. It may also give rise to patch contact for normally flat surfaces even in the absence of sliding. For these reasons patch contact observations require a knowledge of heat partitioning for their full interpretation.

Although study of the stationary patch with or without wear has provided considerable physical insight it can easily be shown that the patches must be moving relative to both contacting surfaces when wear is present and each body is a thermal conductor. Even at low Peclet number the nature of the stress distribution is strongly affected by this movement. Kilaparti [32] has examined the case of the uniformly heated patch (q is constant) and has shown that as the patch moves wear causes a reduction of the outward displacement v from the leading edge to the trailing edge. At the same time absorption of heat causes a rise in v over the same distance. Hence if the patch is in contact with a rigid body these two effects must cancel one another. It follows that the speed of traversal goes up with wear rate and must exceed infinity for a wear coefficient exceeding W^*. Even in cases of so-called severe wear W/W^* lies below unity for several abradable materials. A more detailed study [50] has shown that the pressure distribution on the contact becomes nearly triangular as the wear rate is increased; however, the study was limited to a modestly high Peclet number because the numerical scheme required an excessively fine grid for higher values. A third study directed towards a higher Peclet number [33] has reviewed the work of Ling and Mow [10] and suggested a simplified relation for surface displacement:

$$\int_x^{\infty} \frac{2\alpha k}{cK}\, q(\xi)\, \mathrm{d}\xi + \int_{-\infty}^{x} \frac{2}{\pi^{1/2}}\left(\frac{k}{c}\right)^{3/2} \frac{\alpha}{K} \frac{q(\xi)\, \mathrm{d}\xi}{(x-\zeta)^{1/2}} = \delta(x) \tag{25}$$

Derivations based on this function have supported the earlier prediction of nearly triangular contact pressure distribution.

4.1. *Contact stress*

Recent calculations show that a tensile stress given by

$$\sigma_{x\,\mathrm{th}} = \frac{E\alpha Q}{K(1-\nu)\epsilon}\,(0.8\mathrm{Pe}_l) \tag{26}$$

where $\mathrm{Pe}_l = cl/k\pi$ may appear at the edge of a moving contact patch with uniformly distributed heat input. In one example where the contact patch was virtually stationary in the carbon of a carbon/cast iron contact the rate of movement c relative to the iron would be approximately the sliding speed. For both bodies close to ambient temperature in the bulk almost all of the frictional heat would be expected to pass into the iron; hence $Q \approx \mu PV$. Under these circumstances eqn. (26) becomes for $\nu = 0.3$ and $l/\epsilon = 1$

$$\sigma_{x\,\mathrm{th}} = \frac{1.14E\alpha P}{Kk}\left(\frac{l}{\epsilon}\right)V^2 \tag{27}$$

and for V in metres per second and $\sigma_{x\,\mathrm{th}}$ in bars one finds upon substituting the properties of cast iron

$$\sigma_{x\,\mathrm{th}} = 26\,394\ V^2$$

Obviously this can lead to large stresses.

On the one hand this gives an easy explanation of surface heat checking where patch contact passes over the surface. On the other hand it raises the question as to why heat checking is not observed more often. One possible explanation may be that the depth affected is shallow. It is also possible that the numerical magnitude may be reduced by substituting a different distribution of heat input on the patch. Whatever the modifying factors may be they offer an attractive problem for further investigation.

An illustration of such heat checking is shown in Fig. 6 which is an enlarged photograph of the band of contact of a large seal. The light colored band is over 2 mm wide and lies on the face of a hard brittle alloy which has been run against a carbon ring with a raised nose. The tiny hairline cracks run across the contact band and are believed to be the result of tangential tension. Evidence suggests that the thermal asperity forms and moves slowly on the carbon, and that the tension described earlier arises in the metal as it passes through the contact patch. There is no evidence of gross heating, and such heating is not necessary to produce the stresses necessary for cracking.

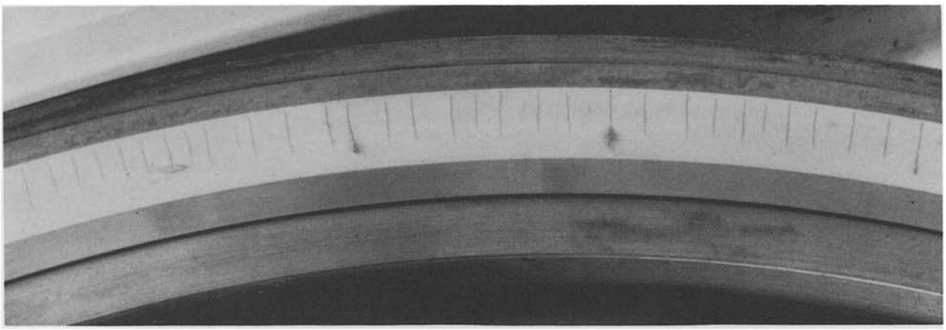


Fig. 6. Radial hairline cracks across the contact band (light area) on the metal face of a seal after running against a carbon ring at a high peripheral speed.

5. Directions for further investigation

One of the most troublesome problems left unsolved in the work reported in the preceding sections involves delineating the path from stable operation to patch contact. When this is understood we shall be in a better position to avoid the formation of patches with their severe effects. Undoubtedly along the way we shall learn a great deal more about the mechanisms of lubrication and wear in general.

This is of particular interest in the case of lubricated contact where predictability is the poorest. We should find out when flat sliding surfaces (such as the sides of pistons) develop thermal asperities or else discover the mechanisms which prevent this.

Although the radial clearance loss due to thermal deformation has been investigated for journal bearings, we know of no investigation of ellipticity or multilobar deformation patterns which might be thermally excited and could lead to catastrophic failure. In other bearing configurations such as tilted pads one can visualize a thermal asperity forming at high speed. Again it should be determined whether or not this actually happens in practice.

It is our opinion that even the minimum film thickness criterion of bearing failure should be reinvestigated with a thermoelastic analysis of the thin film region to determine the possibility of a load-concentrating temperature rise and thermal breakdown of the film.

It is clear that thermoelastic effects distort gas bearings, but as far as we can determine no stability analysis has been made of such systems. Furthermore, although ball bearings have been shown to be possibly subject to thermoelastic seizure processes, the study of thermoelastic effects in catastrophic failures is in the most primitive stage of development and should be extended.

Thermoelastic deformation of supposedly non-contacting jet engine shaft seals appears on initial investigation to have a strong likelihood of occurring and could lead to thermal asperities on the seal face or concentration of load on two or more pads with attendant increased leakage and possibly failure. The mating rings may also deform significantly.

The high speed rubs of turbomachines in the form of blade tip contact and labyrinth seal contact are already under investigation. Further investigation of thermoelastic deformations brought about by such contact may help to provide guides to improve designs.

Granting that thermal asperities do appear in important applications such as seals, it is essential to understand the thermoplastic behavior of the patches and to estimate the stresses, metal working and fatigue mechanisms in such contacts. In conventional face seals, where the effects are most easily demonstrated, the complex interactions of tilt, nutation and other motions with those associated with thermal deformation can only be conjectured yet experiments which have been made suggest that these interactions are complex and closely intercoupled. Stress analysis for the thermoelastic regime for hard materials should be undertaken and the conditions for stress cracking (heat checking) should be defined.

Turning now to physically different problems, Marshall [51] has reported contact patch formation in high current electrical brushes. It appears that current as well as friction can produce the deformed state, and the combined effects of these should be studied to determine their influence upon wear as well as the limits of operation of brushes.

Although no publications have appeared on the subject, we have been told that thermoelastic instability involving roller deformation sets the limit for the speed of rolling of thin sheet. Rollers may be undercut to compensate for "barreling out" as a result of heating. However, at some limiting speed this is no longer effective in ensuring uniform sheet.

Recently Quinn [52] has given evidence for the thermoelastic oscillation of asperities in a concentrated rider-on-drum contact, and has provided estimates of the number of asperities in contact at one time. On the basis of earlier theories thermal augmentation of asperities would not have been expected in the small dimensions involved, but the evidence is compelling.

In summary it is our opinion that thermal deformation and the formation of the thermal asperity are not isolated phenomena to be encountered in special circumstances. They appear and sometimes become predominant in quite ordinary applications. Indeed it is believed that neglect of accounting for them has needlessly complicated the interpretation of many friction and wear observations. Increased understanding has led to improved tools to use in studying these effects. To apply these to some of the obvious problem areas listed above should provide in some cases improved physical understanding, in others design guides and in a few a clearing of the air by showing that such effects are under control.

Nomenclature

b	parameter in the heat equation
c	speed of movement of the contact patch along the surface
E	Young's modulus
E^*	$\dfrac{1}{(1/E_1 + 1/E_2)}$, modified modulus for two-body contact
h	axial length of the sealing ring
$\bar{h}$	film thickness in the seal
k	thermal diffusivity
K	thermal conductivity
l	half-length of the contact patch
p	pressure
q	heat flow through a unit area of surface
Q	total heat flow through the contact patch
R	radius of curvature of the surface
t	time
T	temperature
V	sliding speed
V^*	critical sliding speed
w	wear coefficient
w^*	critical wear coefficient
x	coordinate along the direction of sliding
y	coordinate normal to the surface
Pe	Peclet number
Pe_l	Peclet number of the contact patch
α	coefficient of thermal expansion
ζ	heat transfer parameter
η	viscosity
κ	wavenumber
λ	wavelength
μ	friction coefficient
ξ	dummy variable

The circumflex denotes the amplitude of the wave and the bar denotes the mean value.

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