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Modeling the price dynamics of CO_2 emission allowances

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ABSTRACT

In this paper we analyze the short-term spot price behavior of carbon dioxide (CO_2) emission allowances of the new EU-wide CO_2 emissions trading system (EU ETS). After reviewing the stylized facts of this new class of assets we investigate several approaches for modeling the returns of emission allowances. Due to different phases of price and volatility behavior in the returns, we suggest the use of Markov switching and AR–GARCH models for stochastic modeling. We examine the approaches by conducting an in-sample and out-of-sample forecasting analysis and by comparing the results to alternative approaches. Our findings strongly support the adequacy of the models capturing characteristics like skewness, excess kurtosis and in particular different phases of volatility behavior in the returns.

1. Introduction

In January 2005 the EU-wide CO_2 greenhouse gas emissions trading system (EU ETS) has formally entered into operation.[1] The new system represents a shift in paradigms, since environmental policy has historically been a command-and-control type regulation where companies had to strictly comply with emission standards or implement particular technologies. The EU ETS requires a cap-and-trade program whereby the right to emit a particular amount of CO_2 becomes a tradable commodity.

By forcing the participating companies to hold an adequate stock of allowances that corresponds to their CO_2 output, the carbon market provides new business development opportunities for market intermediaries and service providers. Risk management consultants, brokers and traders buy and sell emission allowances and their derivatives. Especially for these groups, the price behavior and dynamics of this new asset class—CO_2 emission allowances—is of major importance. According to IETA (2005) and PointCarbon (2005) previous carbon trading activities have been mostly conducted by OTC activities and brokers.

Since allowance trading has primarily been applied in the US, the majority of publications about price behavior of tradable emission allowances assesses the market for SO_2 emissions under the *Acid Rain Program* of the US Environmental Protection Agency (EPA). By using industrial organization models they account for changes in parameters of technology (Rezek, 1999) and electricity demand (Schennach, 2000) and their impact on the optimal equilibrium price path for SO_2 permits. There is also a number of empirical investigations on ex-post market price analysis, among them Ellerman and Montero (1998), Burtraw (1996) and Carlson et al. (2000). For CO_2 market price simulation studies with respect to changes in market design parameters see e.g. Burtraw et al. (2002), Böhringer and Lange (2005), Kosobud et al. (2005) or Schleich et al. (2006). Kosobud et al. (2005) analyze monthly returns of SO_2 allowances with respect to other financial assets and find no statistically significant correlation between spot prices in the US and returns from various financial investments.

However, literature examining the CO_2 allowance prices from an econometric or risk management angle is rather sparse. Exceptions include Daskalakis et al. (2005), Paolella and Taschini (2006), Seifert et al. (2008) and Uhrig-Homburg and Wagner (2006). While Uhrig-Homburg and Wagner (2006) investigate the success chances and optimal design of derivatives on emission allowances, Seifert et al. (2008) develop a stochastic equilibrium model reflecting in a stylized way the most important features of the EU ETS and analyze the resulting CO_2 spot price dynamics. Their main findings are that an adequate CO_2 process does not necessarily have to follow any seasonal patterns. It should possess the martingale property and exhibit a time- and price-dependent volatility structure. Paolella and Taschini (2006) provide an econometric analysis addressing the unconditional tail behavior and the heteroskedastic dynamics in the returns on CO_2 and

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[1] The agreement on a common position was reached in December 2002 and passed the EU-parliament's second reading in the summer of 2003 (European Union, 2003). The Commision of the European Communities (2001) had already published a proposal for a Directive in October 2001.

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SO_2 allowances. They find that models based on the analysis of fundamentals or on the future-spot parity of CO_2 yield implausible results due to the complexity of the market and advocate the use of a new GARCH-type structure. Finally, examining emission allowance prices and derivatives, Daskalakis et al. (2005) find some evidence that market participants adopt standard no-arbitrage pricing.

We differ from the analysis of the mentioned papers by also concentrating on the out-of-sample performance of the models with respect to forecasting. In particular, we evaluate price, volatility and density forecasts for the different approaches what can be considered as a substantial issue in managing price risk. With an increasing range of new instruments (e.g. spot, forwards, futures, etc.) the carbon market is steadily gaining in complexity. Risk managers and traders constantly have to hedge their positions against irregular and unexpected carbon price fluctuation. Hence, they are not only interested in the long-term perspective of emission allowance prices but also in short-term price dynamics of the assets. Having a reliable pricing and forecast model will allow companies, investors and traders to realize efficient trading strategies, risk management and investment decisions in the carbon market.

The aim of this paper is to provide an analysis of the short-term spot price behavior of CO_2 emission allowances focusing on the price dynamics and changes in the volatility of the underlying stochastic price process. Since CO_2 emission allowances are a new trading good in the European commodity market, there is not much historical data available. By studying the new market mechanism and analyzing first empirical data we consider the appropriateness of several stochastic price processes. The suggested econometric models can be used in particular for short-term forecasting and Value-at-Risk (VaR) calculation. Thus, they could be especially helpful for risk managers or traders in the market, but might also enable companies to monitor the costs of CO_2 emissions in their production process.

2. CO_2 emission allowances—market mechanism and instruments

2.1. The EU ETS and classification of emission allowances

The EU ETS will result in the world's largest greenhouse gas (GHG) emissions trading system. Under the Kyoto Protocol the EU has committed to reduce GHG emissions by 8% compared to the 1990 level by the years 2008–2012. The system regulates an annual allocation of the allowances.[2] Surplus allowances can be transferred for use during the following year (banking).[3] Borrowing is principally prohibited between 2007 and 2008 (Commitment Period I), as well as between all future commitment periods. Failure to submit a sufficient amount of allowances results in sanction payments.[4] In addition, companies will have to surrender the missing allowances in the following year.

Generally, a company's stock of emission allowances determines the degree of allowed plant utilization. Thus, a lack of allowances requires from the company either some plant-specific or process improvements, a cut- or shutdown of the emission producing plant or the purchase of additional allowances and emission credits. With the latter two alternatives CO_2 becomes a new member of the European commodity trading market. There is, however, a fundamental difference between trading in CO_2 and more traditional commodities. What is actually sold is a lack or absence of the gas in question. Sellers are expected to produce fewer emissions than they are allowed to, so they may sell the unused allowances to someone who emits more than her allocated amount. Therefore, the emissions become either an asset or a liability for the obligation to deliver allowances to cover those emissions (PointCarbon, 2004).

Benz and Trück (2006) point out the differences between emission allowances and classical stocks. While the demand and the value of a stock is based on profit expectations of the underlying firm, the CO_2 allowance price is determined directly by the expected market scarcity induced by the current demand and supply at the carbon market. Notably, firms by themselves are able to control market scarcity and hence the market price by their CO_2 abatement decisions. It is important to note that the annual quantity of allocated emission allowances is limited and already specified by the EU-Directive for all trading periods. Additionally, in case of an intertemporal ban in banking of CO_2 emission allowances, the certificates have a limited duration of validity. The value of an individual allowance expires after each commitment period. Allowing for an intertemporal transfer, the allowances only lose their value once used for covering CO_2 emissions.

A more appropriate approach in specifying CO_2 emission allowances is their consideration as a factor of production (Fichtner, 2005). The shortage of emission allowances by reducing the emissions cap for the commitment periods classifies the assets as 'normal' factors of production. They can be 'exhausted' for the production of CO_2 and after their redemption or at the end of the commitment period when they expire, they are removed from the market. Additionally, if there is an intertemporal ban on banking between the commitment periods—as it was the case from the pilot phase to the CP I—all allowances become worthless at the end of the periods and thus are non-storable. On the other side, if banking is allowed the validity of allowances is renewed for the upcoming commitment period. Accordingly, it seems more adequate to compare the right to emit CO_2 with other operating materials or commodities than with a traditional equity share and hence to adopt rather commodity than stock pricing models (see Section 3).

2.2. Price determinants of CO_2 emission allowances

Having gained knowledge about the particularities of the new assets, it is essential for carbon market players to learn about their price dynamics in order to realize trading strategies, risk strategies and investment decisions. In this section, we identify the key price determinants of the CO_2 emission allowances, which an appropriate commodity pricing model should be able to display. According to the investigation of SO_2 permit prices by Burtraw (1996), we categorize the principle driving factors of CO_2 allowance prices into (i) policy and regulatory issues and (ii) market fundamentals that directly concern the production of CO_2 and thus demand and supply of CO_2 allowances.

Monitoring price sources from part (i), it is reasonable to assume that they have a long-term impact on prices. However, for our model we are only interested in those policy issues, which additionally have a rather low probability for an exact forecast. Changes in policy directives or regulations may have substantial consequences on actual demand and supply and thus on short-term price behavior of emission allowances. This is comparable with the effect that some good or bad news published on an individual company may have on its share price. In the carbon market these could be decisions and announcements concerning the National Allocation Plans (NAPs) that set the rules and reduction targets (e.g. NAP revisions or cut of national emission caps). Hence, the consequences of changes in such regulatory or policy issues may be sudden price jumps, spikes or phases of extreme volatility in allowance prices.

Note that aspects concerning the regulatory framework like explicit trading rules (e.g. intertemporal trading), the linkage of the EU ETS with the market of project-based mechanisms and/or with the Kyoto Market in the future have an important impact on prices, too. However, they are the result of a long discussion process whose consequences have to be studied extensively in advance, see e.g. Anger (2008), Schleich et al. (2006) and Seifert et al. (2008). Hence, market

[2] Allowances may either be allocated free of charge, auctioned off or sold at a fixed price. Hybrid systems are also possible.

[3] Banking from the pilot period (2005–2007) into the first Kyoto-commitment (2008–2012) period is left up to the individual member states to decide. Only France and Poland allow for restricted banking from 2007 to 2008.

[4] In the pilot period sanction payments are 40 Euro per missing ton of CO_2-allowances, and 100 Euro in the commitment periods.

participants might be able to hedge themselves against these foreseen 'price risks' in the long term. They are not incorporated in our econometric models focusing on short-term price behavior.

Incorporating part (ii), allowance prices may also show phases of specific price behavior due to fluctuations in production levels. In general, CO_2 production depends on a number of factors, such as weather data (temperature, rain fall and wind speed), fuel prices and economic growth. Especially unexpected (environmental) events[5] and changes in fuel spreads will shock the demand and supply side of CO_2 allowances and consequently market prices. Cold weather increases energy consumption and hence CO_2 emissions through power and heat generation; rainfall and wind speed affect the share of non-CO_2 power generating sources and thus emission levels. A short-term measure for the power and heat sector to invest in CO_2 abatement projects are the relative costs of coal and cleaner fossil fuels such as oil and natural gas. Europe's cheapest path is to switch from coal-fired to gas-fired power generations, which need less than half of the allowances required by their coal-fired counterparts to produce the same amount of electricity.[6] Therefore, this source of price uncertainty may have a rather short or medium-term impact on market liquidity of the allowances that possibly increases volatility of the allowance prices.

Overall, we assume that allowance prices and returns will exhibit different periods of price behavior including price jumps or spikes as well as phases of high volatility and heteroscedasticity in returns. It is the challenge of an appropriate stochastic model to capture such a price pattern.

3. Modeling the price dynamics of CO_2 emission allowances

In this section we incorporate the aforementioned characteristics of CO_2 allowances and their price determinants, in particular the different phases of volatility behavior and the dependence of the variability of the time series on its own past in an adequate stochastic model. Hence, we suggest models allowing for heteroscedasticity like ARCH, GARCH or regime-switching models. While the former two suggest a unique stochastic process but conditional variance, the latter divides the observed stochastic behavior of a time series into several separate phases with different underlying stochastic processes.

3.1. GARCH models

While the traditional linear ARMA-type models assume homoscedasticity, i.e. a constant variance and covariance function, the autoregressive conditional heteroskedastic (ARCH(p)) time series model of Engle (1982) was the first formal model which successfully addressed the problem of heteroskedasticity. In this model the conditional variance of the time series $(y_t)_{t\geq 0}$ is represented by an autoregressive process (AR), namely a weighted sum of squared preceding observations:

$$y_t = \varepsilon_t \sigma_t, \quad \text{with} \quad \sigma_t^2 = a_0 + \sum_{i=1}^{q} a_i y_{t-i}^2, \tag{1}$$

where ε_t are i.i.d. with zero mean and finite variance (typically it is assumed that $\varepsilon_t \overset{\text{iid}}{\sim} N(0,1)$).

In practical applications to financial time series data it turns out that the order q of the calibrated model is rather large (Pagan, 1996). However, if we let the conditional variance depend not only on the past values of the time series but also on a moving average of past conditional variances the resulting model allows for a more parsimonious representation of the data. This model, the generalized autoregressive conditional heteroskedastic model (GARCH(p,q)) put forward by Bollerslev (1986) and Taylor (1986) is defined as

$$y_t = \varepsilon_t \sigma_t, \quad \text{with} \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i y_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{2}$$

where ε_t are as before. The coefficients have to satisfy $\sum\alpha_i + \sum\beta_j < 1$ and $\alpha_i, \beta_j \geq 0$, $\alpha_0 > 0$ to ensure stationarity and a conditional variance that is strictly positive. Identification and estimation of GARCH models is performed by maximum likelihood estimation, e.g. documented by Brooks et al. (2001).

Obviously, the GARCH model is especially designed to model the conditional volatility of a time series. However, the variance equation can be coupled for example with an AR(r) process for the mean of the time series

$$y_t = c + \sum_{k=1}^{r} \phi_k y_{t-1} + \varepsilon_t, \tag{3}$$

where $\phi_k < 1$ and c denote real constants. Then the model provides a promising approach to model both the mean and the variance of the considered time series—the AR–GARCH model. The literature on GARCH or AR–GARCH models for analyzing financial time series is extensive. Applications to models for commodities include Garcia et al. (2005), Morana (2001), Mugele et al. (2005), Ramirez and Fadiga (2003).

3.2. Regime-switching models

The second class of pricing models that we suggest are the so-called regime-switching models. Hereby, we follow the idea of Goldfeld and Quandt (1973), Hamilton (1989, 1990) who introduced regime-switching models and successfully suggested their use for financial time series. There are also a number of recent publications where the models are used to describe asset returns in financial markets (Kanas, 2003; Kim and Nelson, 1999; Kim et al., 2004; Schaller and van Norden, 1997). In the last decade the models also became especially popular for modeling electricity spot prices (Bierbrauer et al., 2004; Ethier and Mount, 1998; Huisman and Mahieu, 2001; Weron et al., 2004). Due to their promising features of modeling different regimes of price and volatility behavior we suggest the approach also for modeling CO_2 emission allowances' logreturns.

In general, regime-switching models divide the time series into several phases that are called *regimes*. For each regime one can define separate and independent underlying price processes. The literature distinguishes between two main classes of regime-switching models (Franses and van Dijk, 2000). In the first one, the regime can be determined by an observable variable. Consequently, the regimes that have occurred in the past and present are known with certainty. In the second class the regime is determined by an unobservable, latent variable. In this case we can never be certain that a particular regime has occurred at a particular point in time, but we can only assign or estimate probabilities of their occurrences. In the following we will suggest to use the second class of models that is often referred to as Markov regime-switching models. We argue that it is rather questionable to assume that the regime-switching mechanism is simply governed by a fundamental variable or the price process itself. As described in Section 2.2, spot prices or returns of CO_2 emission allowances are the outcome of a vast number of variables including fundamentals (like weather or macroeconomic variables) but also the unquantifiable regulatory, policy and sociological factors that can cause an unexpected and irrational buyout or lead to price jumps and periods of extreme volatility.

Hence we assume that the switching mechanism between the states is governed by an unobserved random variable R_t. For example, a model with two regimes follows a Markov chain with two possible states, R_t={1,2}. Hereby, the spot price or return may be assumed to

[5] E.g. power plant breakdowns (nuclear-, coal-fired- or hydroelectric power plants) where more emission intensive power stations have to be set up or unexpected environmental disasters (forest fire, earthquakes, etc.) shock the demand and supply side of CO_2 allowances.

[6] Depending on the capacity, the turning-on of gas turbines only takes several minutes (BMWT, 2006).

display either low or very high volatility at each point in time t, depending on the regime $R_t=1$ or $R_t=2$. Consequently, we have a probability law that governs the transition from one state to another, while the processes y_{t,R_t} for each of the two regimes are supposed to be independent from each other. Further, a transition matrix **Q** contains the probabilities q_{ij} of switching from regime i at time t to regime j at time $t+1$, for $i, j=\{1,2\}$:

$$\mathbf{Q} = (q_{ij}) = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} = \begin{pmatrix} q_{11} & 1-q_{11} \\ 1-q_{22} & q_{22} \end{pmatrix}. \tag{4}$$

Due to a property of Markov chains the current state R_t only depends on the past through the most recent value R_{t-1}:

$$P\{R_t = j | R_{t-1} = i, R_{t-2} = k, \dots\} = P\{R_t = j | R_{t-1} = i\} = p_{ij} \tag{5}$$

Consequently the probability of being in state j at time $t+m$ starting from state i at time t is given by

$$(P(R_{t+m} = j | R_t = i))_{i,j=1,2} = \left(\mathbf{Q}'\right)^m \cdot e_i, \tag{6}$$

where $\mathbf{Q}'$ denotes the transpose of **Q** and e_i denotes the ith column of the 2×2 identity matrix. The variation of regime-switching models is due to both the possibility of choosing the number of regimes and different stochastic processes assigned to each regime. In the literature, often a mean-reverting process with Gaussian innovations is used for the various regimes reverting process with Gaussian innovations is used for the various regimes (Bierbrauer et al., 2004; Huisman and Mahieu, 2001) while other model specifications are possible and straightforward. Hamilton (1989) for example suggests an autoregressive process of higher order for both regimes, while for return modeling a white-noise process for either regime may be adequate (Kim and Nelson, 1999; Schaller and van Norden, 1997).

Given the stated assumptions about the price behavior of CO_2 emission allowances, applying regime-switching models may be a promising approach. It reflects the concept of having a systematic change between stable and unstable states which results from fluctuations in demand and supply on markets as assumed for the CO_2 allowance market in the previous section. Furthermore, the model allows for several consecutive price jumps or extreme returns that are important when talking about risk management and pricing of derivative instruments.

Unfortunately, parameter estimation of the two underlying processes is not straightforward since the regime is latent and hence not directly observable. Hamilton (1990) introduced an application of the EM algorithm by Dempster et al. (1977) for the estimation procedure. The regime R_t is modeled as the outcome of an unobserved two-state Markov chain with $R_t=\{1, 2\}$. Additionally, the estimation process needs a stochastic process for each regime $y_{t,R_t}, R_t=\{1, 2\}$, $t=1, \dots, T$ and a transition matrix **Q**. The EM algorithm uses an iterative procedure to collect and estimate the parameter set θ based on an initial parameter estimate $\hat{\theta}^{(0)}$. Then each iteration of the EM algorithm generates new estimates $\hat{\theta}^{(n+1)}$ of the unknown parameter set based on the previously calculated vector set $\hat{\theta}^{(n)}$. The algorithm stops as soon as the change in the log-likelihood function (LLF) is small enough, i.e. when the process has converged. Hamilton (1990) shows, that each iteration cycle of the sample increases the LLF and the limit of this sequence of estimates reaches a (local) maximum of the LLF. For a detailed technical specification refer to Dempster et al. (1977) and Hamilton (1994).

4. Empirical results

4.1. The data

In this section we investigate the appropriateness of the suggested time series models for logreturns of daily EU CO_2 allowance (EUA) prices. The considered time period is from January 3, 2005–December 29, 2006. Hereby, the data from period January 3, 2005–December 30, 2005 is used for the calibration of the models, while the period January 3, 2006–December 29, 2006 is used for out-of-sample testing.

Fig. 1 shows a plot of daily EUA prices for the period August 27, 2003–December 29, 2006. The data are provided by Spectron one of the major brokers in the energy trading industry and stems from OTC transactions (Spectron, 2005).

The operational trade with EUAs already began in 2003, before the official agreement on the EU ETS. In the “pre-2005” period, the traded volume was quite low, at some days even zero as the highest bidder price was smaller than the lowest seller price. The market price then was just determined by the mean of the two figures. Bid-ask spreads were quite large, often exceeding 4 Euro, indicating that prices could not be considered as stemming from real trading activities. One should note, that prices before 2005 are forward prices on a not yet traded underlying. Hence they have to be considered with care when they are compared to spot prices starting 2005. Ulreich (2005) points out that the pre-2005 period was more useful for setting up the infrastructure for the official start of the EU ETS in 2005 than getting important market price signals.

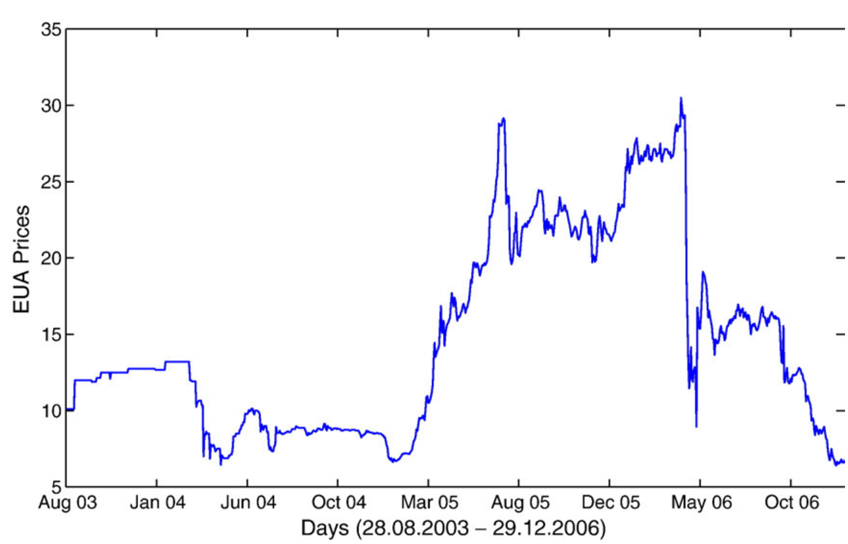

Fig. 1. Daily EUA Prices from August 28, 2003–December 29, 2006.

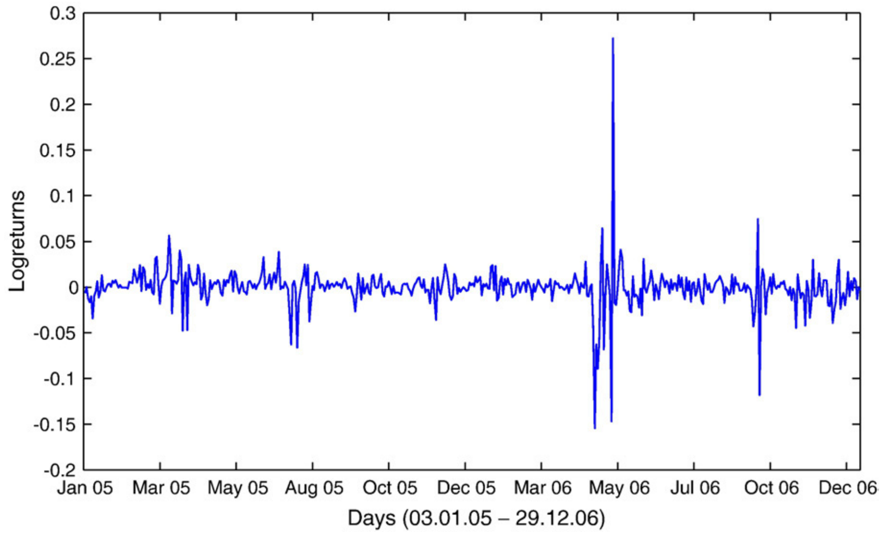

Fig. 2. Daily EUA logreturns for both calibration and test period from January 3, 2005–December 29, 2006.

Having a closer look at the run of the curve in Fig. 1, our model for EUAs' key price drivers from Section 2.2 can be verified.[7] Before the EU-parliament agreed on the introduction of the EU ETS in July 2003 and before the first suggestions for National Allocation Plans (NAPs) were published at the end of 2003, prices were quite stable.[8] Both announcements led to an increase in prices. Because of the initially generous allocation of allowances to the countries prices calmed down again between February and March 2004. Reviewing and accepting the NAPs in the second half of the year, prices increased and settled down around 9 Euro. As the main framework of the trading scheme has been defined, the price determinants became more fundamental after January 2005 (Ulreich, 2005).

The market began to respond increasingly to changes in the underlying energy markets and the weather. We find that prices initially fell due to a quite mild climate and high supply of wind energy from Scandinavia and North Germany. At the end of January an extreme cold snap and constant high UK gas and oil prices, compared to relatively low coal prices, led to a drastically price increase (PointCarbon, 2005). This effect was boosted by an extremely dry summer in July 2005 in the southwest of Europe. The absence of necessary rainfall prevented full utilization of hydraulic plants, especially in Spain. Additionally, the lack of cooling water for nuclear plants led to higher emission-intensive power plant utilization and therefore increased the demand for CO_2 permits. By mid of July 2005 prices peaked at 29.15 Euro. Since then, during the last four months of 2005 prices fell and stabilized around 22 Euro. However, in the beginning of 2006, a renewed increase in the price level could be observed to approximately 27 Euros by the end of March. Reasons for that, once again, may be the extremely long and cold winter in 2005/2006.

May 2006 saw the completion of the first full compliance cycle of the EU ETS with the publication of the 2005 verified emissions data. But already in April 2006, it became clear that corporate participants had been granted around 10% more allowances than they actual needed to cover their 2005 emissions. Consequently, surplus EUAs flooded the market, prices crashed 60% within 1 week, from a high of around 30 Euros per ton of CO_2 to 11 Euros. Traders began to express the fear that the emissions price would drop to zero. With so many allowances being given out, even factors such as the fluctuations in the use of fossil fuel associated with yearly variations in weather are now playing havoc with demand, putting prices in doubt. Then, prices stayed volatile, especially since no European government wanted to be the first to reduce radically the number of allowances granted to the industry. In June and July 2006 the EUA market recovered as industrial companies started selling EUAs to utilities and financial players and the hot, dry July in Europe led to higher demand for electricity even as hydro resources were low and nuclear resources were off-line pushing the spot price of EUAs higher to around 16 Euros. In September, EUA spot prices declined sharply following the collapse of spot prices of natural gas in Europe. Over the next months, the EU Commission began to review the proposed allocation plans by Member States for Phase 2 of the EU ETS (NAPs II).

Overall, we can conclude that spot price behavior in the CO_2 emission allowance market confirm our motivation for looking at a price model that deals with volatile price processes induced by short-term factors like the spread between fuel prices, precipitation, summer and winter temperature and the setup of a trading environment.

4.2. Analyzing logreturns of CO_2 allowances

Fig. 2 shows a plot of the EUA logreturns $y_t=\log(S_t)-\log(S_{t-1})$ for the whole considered period while a summary statistics for the EUA prices S_t and logreturns y_t in the calibration and forecasting period is presented in Table 1. Obviously, the data show heteroskedasticity and volatility clustering. In 2005, the calibration period, in March as well as during the very dry summer in July the logreturns exhibit a clearly increased volatility. As a consequence, both maximum positive and negative logreturns could be observed during this period. The former was 0.1298 on July 4, 2005 while the latter could be observed on July 14, 2005 and was −0.1528. For the logreturns we get a skewness parameter of $s=-0.83$ and a kurtosis of $k=8.57$ in the calibration period and $s=2.06$ and $k=43.13$ in the out-of-sample period. We conclude that in both periods the logreturns exhibit skewness and excess kurtosis. However, logreturns are left-skewed during the

Table 1
Summary statistics for the EUA logreturns y_t for the in-sample period January 3, 2005–December 30, 2005 and out-of-sample period January 3, 2006–December 29, 2006

Series	N	Mean	Median	Min	Max	Standard deviation	Skew	Kurt
In-sample	256	0.0037	0.0046	−0.1528	0.1298	0.0319	−0.83	8.57
Out-of-sample	253	−0.0047	−0.0017	−0.3551	0.6267	0.0653	2.06	43.13

[7] Price data have not been available for July 2003. Consequently, the price path in Fig. 1 starts in August 2003.

[8] For each commitment period, the member state have to develop a NAP that sets the reduction targets for the covered sectors and how it is divided among the covered installations.

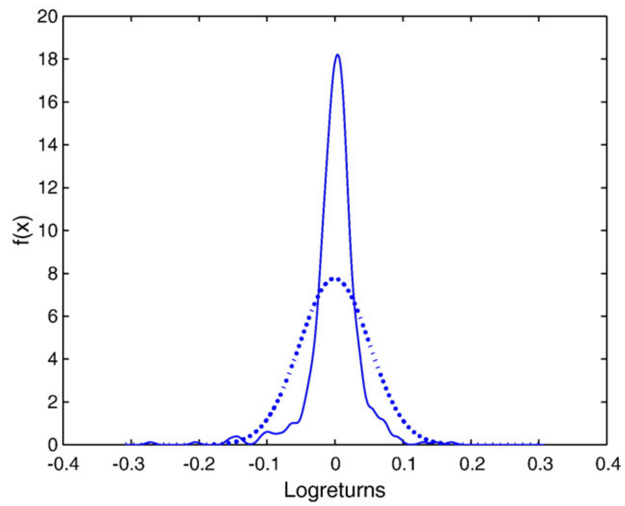

Fig. 3. Empirical distribution obtained by kernel estimator (solid) and Gaussian (dashed) fit to daily EUA logreturns from January 3, 2005–December 29, 2006.

calibration period and right-skewed during the out-of-sample period. Fig. 3 provides the empirical distribution of the logreturns for the whole period from January 3, 2005–December 29, 2006, including a fit of the normal distribution to the data. Due to asymmetry, excess kurtosis and heavy tails, the normal distribution doesn't fit the data very well. Hence, alternative models allowing for changes in the volatility structure, asymmetry and excess kurtosis should provide a better fit to the time series.

4.3. Time series models

After examining daily logreturns of EUA, in a second step we investigate the adequacy of the suggested AR–GARCH and regime-switching models for the time series. To benchmark our estimation results, we also compare them to the results of a simple normal distribution for the logreturns as well as to an AR(r). For the AR–GARCH models, we have to specify both the mean and variance equation. For the mean equation we chose the same AR(r) process as in Eq. (3). However, taking non-constant variance in the residuals into account, the noise terms are not just i.i.d. $(0, \sigma^2)$ but are given by a GARCH(p,q) process. All models are estimated by using maximum likelihood estimation.

For analyzing returns of financial time series with regime-switching models, Kim and Nelson (1999) suggest a white-noise processes for both regimes, while Schaller and van Norden (1997) investigate whether stock market logreturns are drawn from Gaussian distributions with the same or different means and variances. The structure and parameter estimation of such models have been described in the previous section. Hence, the remaining task consists of specifying the two stochastic processes $y_{t,1}$ and $y_{t,2}$. Following the literature we suggest either a white-noise process with different mean and variance for both regimes, R_t={1, 2} (Kim and Nelson, 1999; Schaller and van Norden, 1997) or a mean-reverting process for the 'base regime' (R_t=1) (Ethier and Mount, 1998; Bierbrauer et al., 2004) while the 'spike regime' (R_t=2) is modeled by independent and identically distributed realizations of a Gaussian distribution (Huisman and Mahieu, 2001). In summary, we are considering the following stochastic processes for the regimes:

For the base regime we either use

$$y_{t,1} \overset{\text{iid}}{\sim} N(\mu_1, \sigma_1^2), \quad t \in \mathbb{N} \tag{7}$$

in the model labeled 'Gaussian' or

$$y_{t,1} = \phi y_{t-1,1} + c + \varepsilon_t, \quad t \in \mathbb{N} \tag{8}$$

in the model labeled 'Mean Reversion' (MR), while for the spike regime we suggest

$$y_{t,2} \overset{\text{iid}}{\sim} N(\mu_2, \sigma_2^2), \quad t \in \mathbb{N}. \tag{9}$$

for both model specifications. The innovations ε_t in Eq. (8) are assumed to be i.i.d. centered normal ($\varepsilon_t \overset{\text{iid}}{\sim} N(0, \sigma^2)$), ϕ<1, c denote real constants and N displays the normal distribution with parameters μ_i and σ_i^2, i={1, 2}. Process (8) is the discrete version of a standard Vasiček model. We choose to specify the stochastic processes in discrete-time, simplifying the estimation procedure which is based on a discrete-time sample.

4.4. In-sample results

In the following we discuss the estimation and in-sample results for the suggested models. We first consider the results from fitting a Gaussian distribution and an AR(r) process to the logreturns. For the simplest model—fitting a normal distribution to the data—we obtain the parameters μ=0.0037 and σ=0.0319. As indicated by Schwartz (1997), many commodity prices are in general regarded to be mean-reverting. In discretized form, a mean-reverting process is then equivalent to a Gaussian AR(1) process:

$$r_t = c + \phi r_{t-1} + \varepsilon_t, \tag{10}$$

where ϕ<1 and c denote real constants and the innovations ε_t are assumed to be i.i.d. normal centered ($\varepsilon_t \overset{\text{iid}}{\sim} N(0, \sigma^2)$). The parameter estimates for the AR(1) process are c=0.0033, ϕ=0.2122 and σ_ε=0.0313. Note that also AR processes of higher order were tested, but according to the Schwarz Bayesian Criterion the AR(1) specification was considered as optimal. Obviously, both model specifications provide almost the same estimate for the variance. Further, the added explanatory power by the AR(1) process is rather limited what is also indicated by the only slightly increasing LLF in Table 2.

The residuals obtained by the fitted AR(1) process seemed to exhibit non-constant variance. Testing with the Lagrange multiplier ARCH test statistics (Engle, 1982) the heteroskedastic effects are highly significant. To capture this behavior also a GARCH(p, q) model was calibrated to the data. Hereby, for the mean equation we chose an AR (1) process, while for the variance equation we test different GARCH specifications. It turns out that parameter estimates for higher orders of p or q are not significant. Thus, we obtain the simple setup of an AR (1)–GARCH(1,1) model and the following variance equation:

$$\varepsilon_t = u_t \sigma_t, \quad \text{with} \quad \sigma_t^2 = k + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{11}$$

where u_t is i.i.d. with zero mean and finite variance and k, α, β are real constants.

Note that to simplify the notation, in the following we will refer to the AR–GARCH model as GARCH model. Estimation results of the GARCH model are provided in Table 3, all estimated coefficients of the model are significant. Fig. 4 displays graphs of the innovation, conditional standard deviation and the logreturn time series for the estimated model. As expected, during times of extreme positive or negative returns, the estimated conditional variance increases substantially. While the estimates of the conditional standard deviation

Table 2
Number of parameters k, log-likelihood, Akaike information criterion (AIC), and Bayesian information criterion (BIC) for the estimated models

	k	LLF	AIC	BIC
i.i.d. Normal	2	519.45	−1034.90	−1027.82
AR(1)	3	525.78	−1045.56	−1034.94
GARCH(1,1)	5	575.72	−1141.44	−1123.73
'Gaussian'	6	574.55	−1137.10	−1115.85
'MR'	7	578.96	−1143.92	−1119.13

Table 3
Parameter estimates of the AR(1)–GARCH(1,1) model for the in-sample period January 3, 2005–December 30, 2005

	Coefficient	Standard error	*t*-statistic
Mean equation			
c	0.00243	0.00129	1.881
ϕ	0.29713	0.06652	4.467
Variance equation			
k	3.7309e−005	1.0445e−005	3.5720
α	0.53253	0.036733	14.4973
β	0.36747	0.055515	6.6193

Table 4
Estimation results for logreturns with the two-state regime-switching model with (a) a simple normal distribution in both regimes (model 'Gaussian') and (b) a mean-reversion process in the base regime and a Gaussian distribution in the spike regime (model 'MR')

Panel (a): Model 'Gaussian'

	Parameter estimates			Statistics	
Regime	μ_i	σ_i	p_{ii}	$P(R_t=i)$	$E(y_{t,i})$
Base (i=1)	0.0029	0.0122	0.8768	0.5814	0.0029
Spike (i=2)	0.0050	0.0476	0.8289	0.4186	0.0050

Panel (b): Model 'MR'

	Parameter estimates					Statistics	
Regime	ϕ	c	μ_i	σ_i	p_{ii}	$P(R_t=i)$	$E(y_{t,i})$
Base (i=1)	0.2661	0.0029	–	0.0137	0.8834	0.6598	0.0040
Spike (i=2)	–	–	0.0042	0.0513	0.7738	0.3402	0.0042

are clearly below 0.05 during rather quiet periods, they increase up to 0.1 during volatile periods like in March/April or July/August 2005. Obviously, the GARCH model describes the data better than a simple normal distribution or an AR process and seems more appropriate for the price dynamics of EUA logreturns.

Finally, we estimate the two different regime-switching specifications for the in-sample period. Parameter estimates are displayed in Table 4. We first compare the estimated standard deviation of the 'Gaussian' model—a mixture of two normal distributions—to the simple model of a single normal distribution for the logreturns. We find that the standard deviation of the 'naive' fit lies between the two estimated standard deviations $\sigma_1=0.0122$ for the base regime and $\sigma_2=0.0476$ for the spike regime. The same is true for μ being higher than the expected logreturn in the base regime $\mu_1=0.0029$ but clearly lower than $\mu_2=0.0050$ in the spike regime. In terms of the variance of the regimes, very similar results are obtained also for the model 'MR'. However, here the expected value with $\mu_1=0.0040$ for the base regime and $\mu_2=0.0042$ for the spike regime do not differ that much. Overall, both regime-switching models seem to distinguish between two phases of logreturns: one phase with clearly higher variance for the volatile periods, and one for the less volatile period yielding a lower mean and variance in the returns.

Moreover, the estimated volatility in the two regimes is of special interest, since in the empirical data we observe periods of very low volatility being followed by phases of much higher volatility. In both specifications the estimates for σ_2 are approximately four times higher than σ_1. This results in a variance about 16 times higher for the spike regime than for the base regime. In both models the probability of being in the base regime is higher, approximately 58% for the model 'Gaussian' and 66% for the model 'MR', while the spike regime has the probability of approximately 42% and 34%. Consequently, the probability for remaining in the same regime p_{ii} is higher for the base regime: we have approximately $p_{11}=88\%$ for both model specifications. This indicates that a change in the regimes does not occur frequently. We conclude, that for the considered data the estimated parameters are meaningful and can be interpreted in terms of an adequate distinction between the different phases of volatility behavior. Furthermore, we find that both regime-switching specifications lead to similar results.

Another decisive question is whether the models are able to significantly distinguish between the regimes in terms of the assigned probabilities to either one of the two regimes. Fig. 5 provides a graph that shows the original logreturn series and the corresponding estimated probability of being in the spike regime for the model 'Gaussian'. Note, as the results for the model 'MR' are quite similar we forbear from showing these graphs. Higher volatility, price jumps and therefore, changes in the regime are often related to the aforementioned political and fundamental factors. Most notably, for the periods March/April as well as for June to August and November/December the model assigns many observations to the spike regime. This behavior can be explained by means of the two principle driving factors introduced in Section 2.2. First, at beginning of March the cold weather forced Spanish and French firms to enter the market and thus increased the demand side. Additionally, the sudden increase in spot prices of UK natural gas and oil in March and April increased the spot price for CO_2. Besides, within these two months important political decisions were made with respect to NAPs: by mid March is was announced to cut the Polish and Czech NAP and at the same time to possibly increase allocation for UK and Italy. Further, by early April the decision of the European Council, to reduce emissions by 15–30% until 2020 and up to 60–80% by 2050, was published. Fig. 6 provides a closer look at the allocation to the regimes for the period June 1, 2005 to August 31, 2005. In this phase prices were mainly driven by fundamentals, most notably the dry summer period in July, which boasted emissions in particular in Spain and France. Besides, over the whole period, again high oil and gas price (relative to coal) drove the price for CO_2 and also lead to an increased price volatility. Finally, at the end of 2005 the market reacted again to three important market announcements, the World Bank's forecast for CER supply, the drastic cut of the Italian NAP by 10 mt of CO_2 and the agreement on a cap-and-trade program by seven US states, the Reginal Greenhouse Gas Initiative (RGGI). To sum up, from the probabilities in Figs. 5 and 6 it becomes obvious that with high probability the model assigns most of the logreturns to either one of the two regimes. This indicates the model's ability to distinguish between the two regimes of different volatility.

For model evaluation, we also examined the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) for the

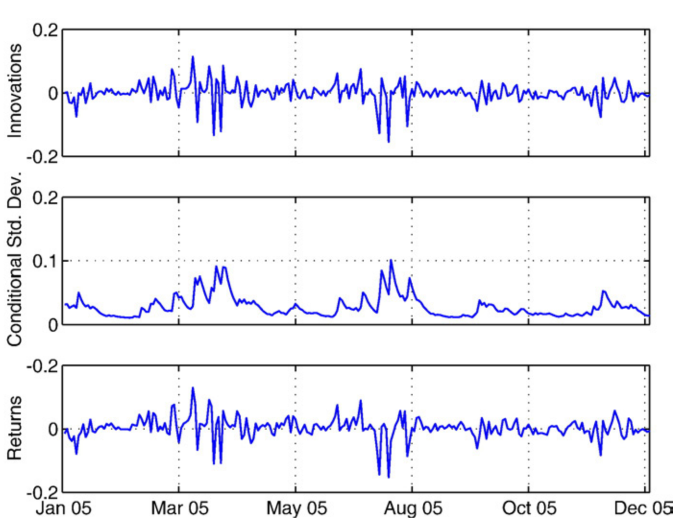

Fig. 4. Innovations, conditional standard deviations and logreturns of the estimated AR(1)–GARCH(1,1) model for the in-sample period January 3, 2005–December 30, 2005.

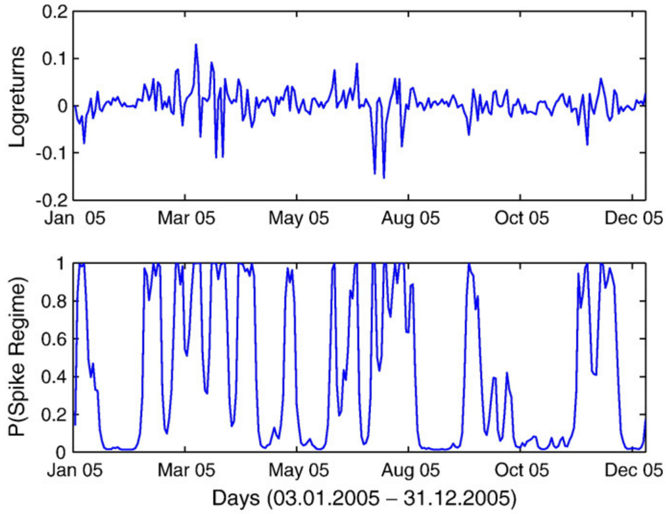

Fig. 5. *Top panel:* Logreturns of EUA prices from January 3, 2005 to December 30, 2005. *Bottom panel:* Probability of being in the spike regime for the defined two regimes 'Gaussian' model for the same period.

estimated models. We find that according to the chosen parsimony model criteria our results are confirmed: the GARCH and regime-switching models clearly outperform the approach of fitting a normal distribution or an AR(1). The results for a the GARCH model and the regime-switching models 'Gaussian' and 'MR' are quite similar. While for the AIC the best results are obtained for the regime-switching model with an AR process for the base regime ('MR'), for the BIC the GARCH model gives the best results. However, similar to the results for the log-likelihood function, the differences between the two regime-switching and the GARCH model are quite small. We conclude that as far as in-sample results are concerned, GARCH and regime-switching models are adequate approaches for modeling EUA logreturns.

4.5. Forecasting results

We conduct an out-of-sample analysis of the models by comparing one-day-ahead point and density forecasts for EUA logreturns for the period January 3, 2006 to December 29, 2006. Hereby, both a static approach using the estimated models for the whole out-of-sample period as well as a recursive and rolling window technique with reestimation of the parameters after each day were examined. For the recursive window approach the initial estimation date is fixed and additional observations are added one at a time to the estimation period. For a rolling window, on the other hand, the length of the in-sample period is fixed. In this case the start date and end date successively increase by one observation.

Overall, reestimating the parameters on a daily basis improved the forecasting ability of the model. The results for a recursive and rolling window technique were similar, when the length of the rolling window was chosen to be at least 9 months or longer. For shorter windows, the parameter estimates for the GARCH and in particular for the regime-switching models showed some instability. In the following only the results for the recursive window approach are provided. However, the results for the static and rolling window approach are available upon request to the authors.

For point forecasts we measure the average prediction errors by computing the mean absolute error (MAE) and mean-squared error (MSE) of the one-day-ahead forecasts. The results for the different models can be found in Table 5. We observe the smallest MAE for the 'AR' model despite the superior in-sample fit of the GARCH or regime-switching model. On the other hand, the smallest MSE can be observed for the regime-switching model 'MR' with an autoregressive term in the base regime. We also find that for both criteria, the GARCH model yields the worst results. However, the differences between the results for all models are rather small, since the values for MAE range from 0.0306 to 0.0310 and for MSE from 0.0042 for the regime-switching model 'MR' to 0.0049 for the GARCH model. Overall, we conclude that for point forecasts the results are mixed while there are no substantial differences between the models.

In a second step we investigate the ability of the models to provide accurate forecasts of the whole density function or intervals. Especially for risk management purposes such forecasts are highly relevant, since traders and brokers are more interested in predicting intervals or densities for future price movements than in simple point estimates. The literature suggests different approaches to evaluate interval or density forecasts, see e.g. Christoffersen (1998), Christoffersen and Diebold (2000), Crnkovic and Drachman (1996), Diebold et al. (1998). One approach (Christoffersen, 1998) is to evaluate the quality of confidence interval forecasts by comparing the nominal coverage of the models to the true coverage in the out-of-sample period. However, tests being based on confidence intervals may be unstable in the sense that they are sensitive to the choice of the confidence level α. We overcome these deficiencies by applying a test

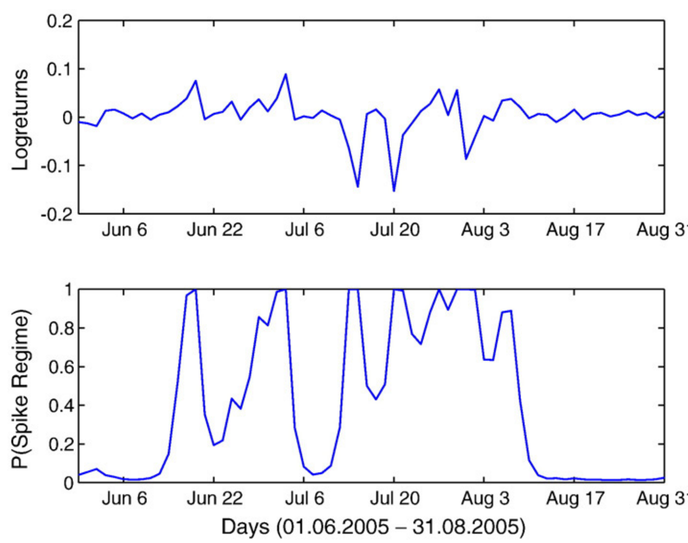

Fig. 6. *Top panel:* Logreturns of EUA prices from June 1, 2005 to August 31, 2005. *Bottom panel:* Probability of being in the spike regime for the defined two-regime 'Gaussian' model for the same period.

Table 5
Results for the mean absolute error (MAE) and mean-squared error (MSE) for the point forecasts of the considered models

	MAE	MSE
i.i.d. Normal	0.0307	0.0043
AR(1)	0.0306	0.0047
GARCH(1,1)	0.0310	0.0049
'Gaussian'	0.0308	0.0044
'MR'	0.0308	0.0042

that investigates the complete distribution forecast, instead of a number of quantiles only. Evaluating the accuracy of the density forecasts we perform a distributional test following Crnkovic and Drachman (1996) and Diebold et al. (1998). We are interested in the distribution of the logreturn y_{t+1}, $t>0$, which is forecasted at time t. Further, let $f(y_{t+1})$ be the probability density and $F(y_{t+1})=\int_{-\infty}^{y_{t+1}} f(x)\mathrm{d}x$ be the associated distribution function of y_{t+1}. To conduct the test, we determine $\hat{F}(y_{t+1})$ by using the parameter estimates from the in-sample period and the observations y_s, $s=0,..,t$. Rosenblatt (1952) shows that if $\hat{F}$ is the correct loss distribution, the transformation of y_t, namely

$$u_{t+1}=\int_{-\infty}^{y_{t+1}} \hat{f}(x)\mathrm{d}x=\hat{F}(y_{t+1}), \tag{12}$$

is i.i.d. uniformly on [0, 1]. The method can be applied to test for violations of either independence or uniformity.

Fig. 7 presents the corresponding probability integral transforms of the one-day ahead forecasts based on the 'naive' model of a simple normal distribution, the AR(1) model, the GARCH model and the regime-switching model specification 'MR'. It turns out that the observations for u_t of the models with a simple normal distribution and the AR(1) process for the logreturns are far from being uniformly distributed. A very high fraction of the probability integral transforms lies in the two central quartiles between 0.25 and 0.75, indicating that using a simple normal distribution or AR(1) model, very often the forecasted confidence intervals for the next day are too wide. This is also confirmed by Fig. 8, displaying the observed logreturns and predicted 95%-confidence intervals for the different models from July 3, 2006 to December 29, 2006. Similar to the in-sample period, the changes in regimes and volatility can be related to the principle driving factors of emission allowances: in mid and late September, there was an increase in volatility and prices dropped from approximately 16 to 12 Euro. Reasons for this were on the one hand the relatively mild temperatures in September and similar forecasts of warm weather for the upcoming weeks. On the other hand there were news on September 19 that Poland discarded the plans to restrict the sales of huge over-allocations of emission allowances. During the period late October to early December when the price volatility was increased for another time, the Stern report on the economics of climate change appeared and was discussed heavily. Further, the UN climate change conference in Nairobi started, stating another time the oversupply of emission allowances for the pilot period. Further, also this period was characterized by comparably mild weather that lead to a lower energy consumption and hence CO_2 emissions through power and heat generation than expected. The phases of higher volatility are clearly mirrored in the widened confidence intervals for the one-day ahead forecasts of the models with conditional variance. As a consequence, for density forecasts the GARCH and regime-switching models provide significantly better results. The corresponding probability integral transforms are closer to a uniform distribution. As Fig. 8 indicates, the width of the confidence intervals varies with the conditional variance of the density forecast, such that during periods of higher volatility the intervals become wider. However, both for the

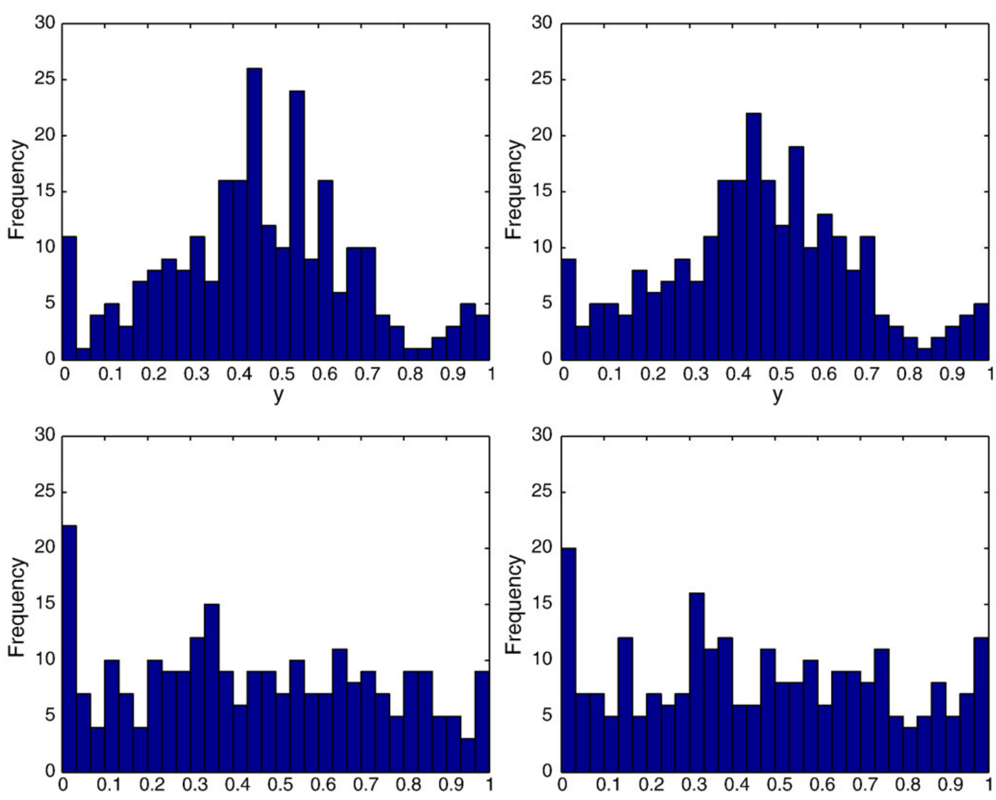

Fig. 7. Histogram of the probability integral transforms of the one-day ahead forecasts for logreturns of CO_2 emission allowances for January 1 2006–May 31, 2006. Results for the 'naive' model of a simple normal distribution (*Upper left panel*), AR(1) model (*Upper right panel*), the GARCH(1,1) model (*Lower left panel*) and the 'MR' regime-switching model (*Lower right panel*).

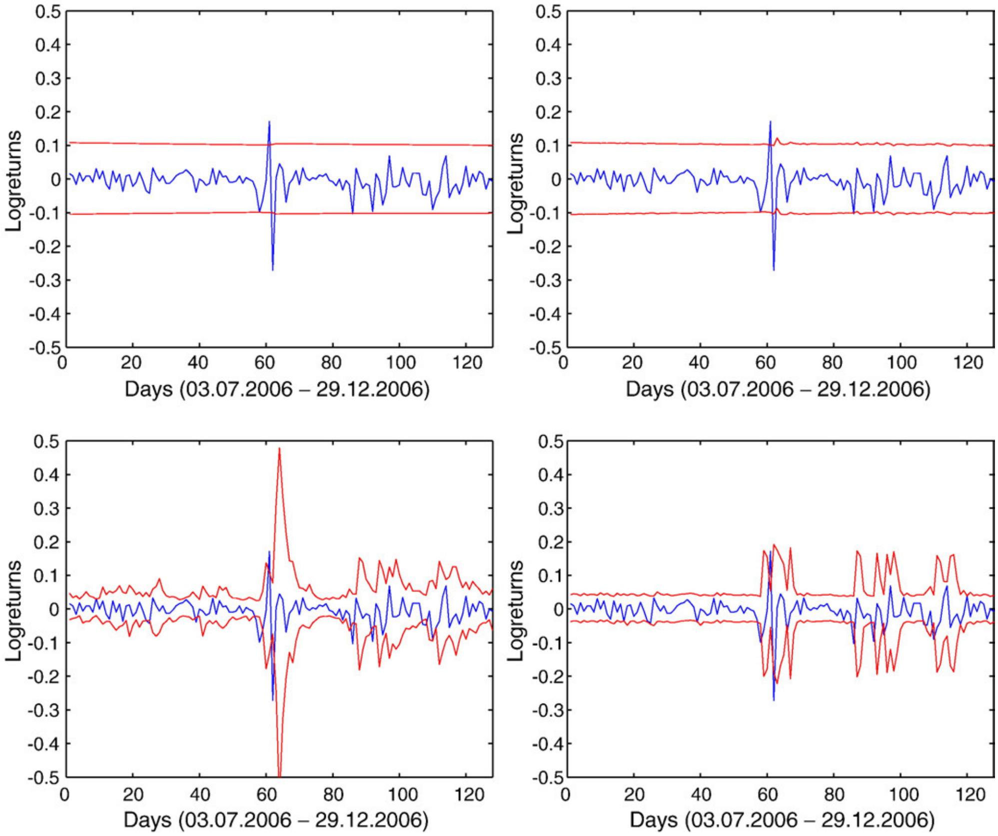

Fig. 8. Logreturns and predicted 95%-confidence intervals for the different models from July 3, 2006 to December 29, 2006. Results for the 'naive' model of a simple normal distribution (*Upper left panel*), AR(1) model (*Upper right panel*), the GARCH(1,1) model (*Lower left panel*) and the 'MR' regime-switching model (*Lower right panel*).

GARCH and regime-switching models there is a high number of observations with probability integral transforms close to zero. This may be due to the fact that substantial price shocks like in April 2006 are rather difficult to predict with an econometric model. As a consequence, several of the large negative returns could not be captured despite the use of models with conditional variance and resulting wider confidence intervals for these periods.

Testing for uniformity, Crnkovic and Drachman (1996) suggest to use a test that is based on the distance between the empirical and the theoretical cumulative distribution function of the uniform distribution. This may be done using e.g. the Kolmogorov–Smirnov (KS) or Kuiper statistic. The former is usually denoted by $D_{KS}=\max\{D^+, D^-\}$ while the latter is $D_{\text{Kuiper}}=D^+ + D^-$ with $D^+=\sup\{F_n(u)-\hat{F}(u)\}$ and $D^-=\sup\{\hat{F}(u)-F_n(u)\}$. Hereby $F_n(u)$ denotes the empirical distribution function for the probability integral transforms of the one-day ahead forecasts and $F(u)$ is the *cdf* of the uniform distribution.

Table 6 presents the test results for the models. We find, that the 'naive' model of a simple normal distribution for the logreturns gives the worst results. Probability integral transforms of the one-day ahead forecasts are non-uniformly distributed. Both tests reject the hypothesis of a uniform distribution even at the 1% level. Similar results are obtained for the AR model. The KS and Kuiper test statistics also significantly reject the assumption of uniformity at the 1% level. The results obtained for the GARCH and regime-switching models are clearly superior. As indicated by Fig. 7 the probability integral transforms are much closer to uniformity in comparison to the normal distribution and the AR model. For all three models, the assumption of uniformity cannot be rejected even at the 10% significance level. The best results for the KS test are obtained for the regime-switching 'MR' model. However, the results for the other regime-switching and GARCH model are only slightly worse. For the Kuiper test, the GARCH model outperforms all its competitors, but the distance for the two regime-switching models is in a similar range. So despite the fact that the GARCH and regime-switching models have some difficulties in forecasting a number of extreme negative price shocks, the density forecasts using these models seem to be adequate.

Overall, in terms of density forecasting, the GARCH and regime-switching models significantly outperform the models with constant variance. This suggests the models as particularly useful for risk management purposes and short-term forecasting of future price ranges for emission allowances.

4.6. Comparison with results from other papers

Daskalakis et al. (2005) suggests that CO_2 emission allowance price levels are non-stationary and exhibit abrupt discontinuous shifts. For logarithmic returns they find that the distribution is clearly non-normal and characterized by heavy tails. They further find that the best model fit for allowance prices in terms of likelihood and parsimony is obtained by a geometric Brownian motion with an additional jump-diffusion component. This model is also able to produce the discontinuous shifts in the underlying diffusion that are

Table 6
Results for Kolmogorov–Smirnov and Kuiper statistics

	KS	Kuiper
i.i.d. Normal	0.1760***	0.2620***
AR(1)	0.1750***	0.2591***
GARCH	0.0748	**0.0828**
'Gaussian'	0.0712	0.0893
'MR'	**0.0709**	0.0885

Best results are highlighted in bold. The asterisk further denote rejection of the model at the 1% ***, 5% ** or 10% * level, for $n=253$ observations.

observed in the CO_2 emission allowances prices. Although our approach differs from their analysis, we find a superior performance of the models with non-constant variance like GARCH or regime-switching confirming the non-normality and heavy tails in the logreturns. The models not only provide the best in-sample fit but also outperform alternative approaches with constant variance in density and volatility forecasting. Hence, similar to Daskalakis et al. (2005), we find that issues like shifts in prices, non-normality or short periods of extreme volatility have to be incorporated into adequate pricing or forecasting models for CO_2 allowances or returns.

Paolella and Taschini (2006) examine the performance of different GARCH models for CO_2 and SO_2 certificates. Similar to our results, they observe heteroskedasticity in the returns and obtain an adequate fit for models with conditional variance. They conclude that for sound risk management, hedging or purchasing strategies the choice of an adequate statistical model is a crucial task. Finally, Seifert et al. (2008) develop a stochastic equilibrium model in order to analyze the dynamic behavior of CO_2 emission allowances spot prices for the European emissions market. According to their analysis, spot prices must always be positive and bounded by the penalty cost plus the cost of having to deliver any lacking allowances. As far as volatility is concerned, they argued that a steep increase will occur when the end of the trading period is approaching. This also recommends the use of models with conditional variance to capture the fact whether the market is in period of higher or lower volatility.

5. Summary and conclusion

In this paper we examine the spot price dynamics of CO_2 emission allowances in the EU ETS. Short-term dynamics of the new asset are of particular interest for market participants like risk managers or traders, but also for CO_2 emitting companies, as they must model the behavior of their production costs. We find that the logreturns exhibit skewness, excess kurtosis and different phases of volatility behavior coming from fluctuations in demand for CO_2 allowances. The best in-sample fit to the data is provided by a regime-switching model with an autoregressive process in the base regime and a normal distribution for the spike regime. Results for the GARCH and normal mixture regime-switching models are only slightly worse while the fit of the models with constant variance like a Gaussian distribution and an AR (1) process is clearly inferior. We also provide an out-of-sample forecasting analysis for the CO_2 allowance logreturns. In terms of point forecasts we observe only very small differences between the models for the evaluated MAE and MSE measures. We also conduct an analysis on density and interval forecasts which is more relevant for risk managers than simple point forecasts. For one-day ahead density forecasts, the AR–GARCH and regime-switching models clearly outperform the models with constant variance. The adequacy of a simple normal distribution or AR process is significantly rejected.

The superior performance of the models with conditional variance can be explained to a high extend by the relationship between allowance prices, regulatory factors and fundamental variables. In particular, we motivated the impact of political issues like National Allocation Plans, which decide on the CO_2 emissions level of the trading countries. Also the uncertainty about future emission caps of the participating nations play a major role: consequences were a substantial drop or increase in prices and following phases of high volatility that are captured much better by GARCH and regime-switching models. Also periods of unexpected weather like cold snaps or extremely hot and dry summer months lead to phases of price behavior that favours the more flexible models with conditional volatility. The suggested models can be used in particular for Value-at-Risk purposes. Modeling the short-term price behavior of emission allowances will be especially helpful for risk managers, brokers or traders in the market, but might also enable companies to monitor the costs of CO_2 emissions in their production process. Our results strongly support the use of AR–GARCH or regime-switching models for modeling the returns of CO_2 emission allowances. The models may also be used for the pricing of relates derivates on emission allowances. For further references on option and derivative pricing with GARCH and regime-switching models we refer e.g. to Huisman and De Jong (2003) and Duan (1995). Overall, we suggest further investigating the use of these models for this new class of assets in future work when more empirical data is available.

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