In [5]:
```
pip install tracywidom
```

```
Collecting tracywidom
  Downloading TracyWidom-0.3.0.tar.gz (12 kB)
Requirement already satisfied: numpy>=1.7.0 in /usr/local/lib/python
3.7/dist-packages (from tracywidom) (1.19.5)
Requirement already satisfied: scipy>=0.13.0 in /usr/local/lib/python
3.7/dist-packages (from tracywidom) (1.4.1)
Building wheels for collected packages: tracywidom
  Building wheel for tracywidom (setup.py) ... done
  Created wheel for tracywidom: filename=TracyWidom-0.3.0-py3-none-an
y.whl size=11748 sha256=7050c0a84dcc06fc6139a79036f6a335257401f70f183
65009f6be60f832fe7a
  Stored in directory: /root/.cache/pip/wheels/8a/b7/de/25c063ec4c03e
c8de200c68295c4bd7c9a063477d90f734fec
Successfully built tracywidom
Installing collected packages: tracywidom
Successfully installed tracywidom-0.3.0
```

Tarea 5

Temas Selectos de Estadística

Diego Ramírez Araque

Ejercicio 1

Calculamos los parámetros de centrado y escalamiento para calcular θ_α dados por:

$$\mu(p,m,n) = 2\log\tan(\frac{\phi+\gamma}{2}) \qquad \sigma^3(p,m,n) = \frac{16}{(m+n-1)^2}\frac{1}{\sin^2(\phi+\gamma)\sin\phi\sin\gamma}$$

en donde γ, ϕ se definen por:

$$\gamma = 2\arcsin\sqrt{\frac{\min(p,n)-1/2}{m+n-1}}$$

$$\phi = 2\arcsin\sqrt{\frac{\max(p,n)-1/2}{m+n-1}}$$

Y tomamos $f_\alpha = 0.9793$

In [6]:
```
import numpy as np
from TracyWidom import TracyWidom

x = np.linspace(-10, 10, 101)
tw1 = TracyWidom(beta=1)  # allowed beta values are 1, 2, and 4
pdf = tw1.pdf(x)
cdf = tw1.cdf(x)

p = 4
n = 5
m = 42
phi = 2*np.arcsin(np.sqrt((min([p,n])-1/2)/(m+n-1)))
gamma = 2*np.arcsin(np.sqrt((max([p,n])-1/2)/(m+n-1)))
mu = 2*np.log(np.tan((phi+gamma)/2))
aux = pow(np.sin(gamma+phi),2)*np.sin(phi)*np.sin(gamma)
sigma3 = (16/pow(m+n-1,2))*(1/aux)
sigma = pow(sigma3,1/3)
theta_a = (np.exp(mu)+0.9793*sigma)/(1+np.exp(mu)+0.9793*sigma)
```

In [7]:
```
print('mu(p,m,n): \n',mu)
print('sigma(p,m,n): \n',sigma)
print('theta_a: \n',theta_a)
```

```
mu(p,m,n):
 -0.7696258935699679
sigma(p,m,n):
 0.3026877995501765
theta_a:
 0.4316917587377988
```

En este caso obtenemos que $\theta_\alpha = \frac{e^{\mu+f_\alpha\sigma}}{1+e^{\mu+f_\alpha\sigma}}$ = 0.4316917587377988.

A comparación de $\theta_{obs} = 0.652$, θ_α es una cota superior de θ_{obs} o al menos se aproximan de manera exacta y la diferencia entre estas dos es significativa. De esta forma rechazamos la Hipótesis nula por que no hay evidencia de igualdad.

In [9]:
```
tw1.cdf(0.9793)
```

Out[9]:
```
0.9500003304691891
```

Ejercicio 3

In [10]:
```
import numpy as np
import matplotlib.pyplot as plt
from numpy import linalg as LA
import seaborn as sns
```

- Ley de Semicírculo GUE y Tracy-Widom de orden 2.

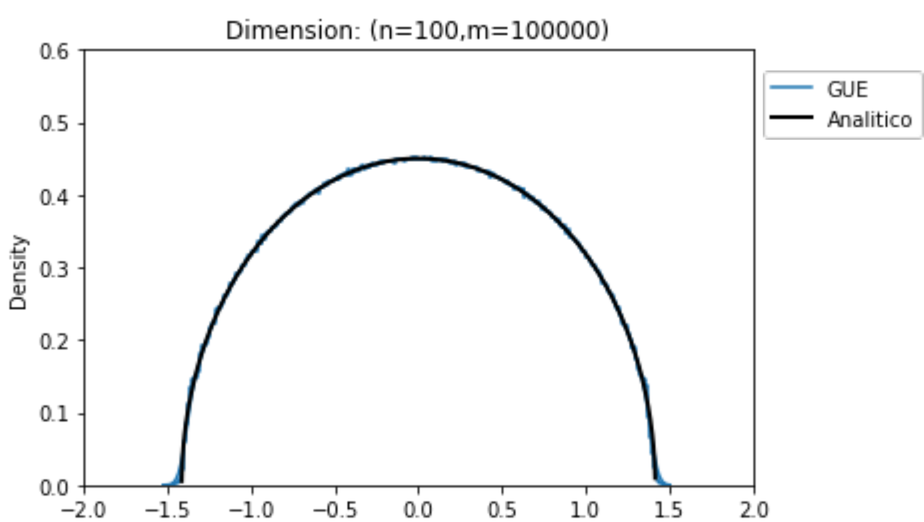
Calculamos con ayuda del código de sesiones pasadas la Ley de Semicírculo y Tracy Widom

In [11]:
```
### Theoretical
def f(x):
    return 1/np.pi * np.sqrt(2-x**2)
```

In [ ]:
```
### Numerical: GUE
n = 100
m = 100000
eig_GUE = []
k = 0
s = np.zeros([(n-1)*m])
for i in range(m):
    H_GUE = (np.random.normal(size=(n, n))+np.random.normal(size=(n,
n))*1j)
    Hsy_GUE = (H_GUE + np.transpose(H_GUE).conjugate())/2.
    w_GUE,v_GUE = LA.eigh(Hsy_GUE)
    eig_GUE.append(w_GUE)
```

In [22]:
```
GUE = np.sort(np.sqrt(1/(2*n))*np.array(eig_GUE).flatten())
sns.histplot(data=GUE,stat="density", element="step", fill=False,kde=Tr
ue, label='GUE')
eps = 0.0001
t = np.arange(-np.sqrt(2)+eps, np.sqrt(2)-eps, 0.001)
plt.plot(t, f(t), 'k',linewidth=2,label=r'Analitico')
plt.title('Dimension: (n='+str(n)+',m='+str(m)+')')
plt.xlim([-2,2])
plt.ylim([0,0.6])
plt.legend(loc='upper left',bbox_to_anchor = (1, .975))
```

Out[22]: <matplotlib.legend.Legend at 0x7f1af373af10>



In [78]:
```
GUE = np.array(eig_GUE).flatten()
```

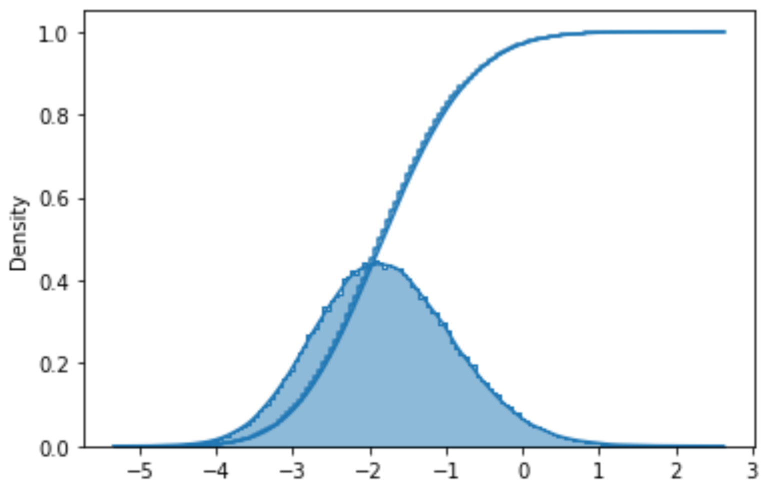
In [79]:
```
eig_GUE_n = []
for k in range(len(eig_GUE)):
  eig_GUE_n.append((n**(1/6))*(eig_GUE[k]-2*np.sqrt(n)))
```

In [80]:
```
tracy_widom = [max(eig_GUE_n[i]) for i in range(len(eig_GUE))]
```

- Tracy Widom con eigenvalores y acumulativa

In [81]:
```
sns.histplot(data=tracy_widom,stat="density", element="step", fill=True,kde=True, label='GUE')
sns.histplot(data=tracy_widom,stat="density", element="step", fill=False,kde=True, label='GUE',cumulative=True)
#plt.xlim([-6,4])
#plt.ylim([0,1])
```

Out[81]: <matplotlib.axes._subplots.AxesSubplot at 0x7f1ae8825610>



- Estadístico $\hat{R}$ para $h_1 - h_0 = 1, 2, \ldots, 8$

Ordenamos los eigenvalores obtenidos, centramos y tomamos los primeros 10.

In [83]:
```
[sorted(n**(2/3)*(eig_GUE_n[i]-2),reverse = True)[:11] for i in range(len(eig_GUE))][0]
```

Out[83]:
```
[-82.30709280932105,
 -115.17623148945904,
 -145.5505615660613,
 -173.89629288830804,
 -204.4271002802564,
 -244.1269976988969,
 -260.0865608165396,
 -293.48143510447375,
 -312.12329551285075,
 -335.095591136391,
 -370.1826958634954]
```

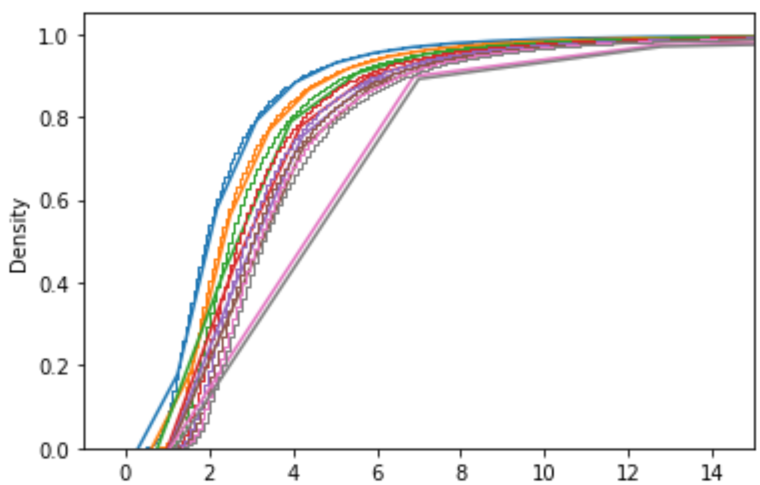
In [90]:
```
#values = [sorted(n**(2/3)*(eig_GUE_n[i]-2),reverse = True)[:11] for i
in range(len(eig_GUE))]
values = [sorted(eig_GUE_n[i],reverse = True)[:11] for i in range(len(e
ig_GUE))]
```

In [91]:
```
R = {}
for i in range(9):
  R[i] = [(values[k][i]-values[k][i+1])/(values[k][i+1]-values[k][i+2])
for k in range(len(eig_GUE))]
```

In [92]:
```
R1 = [ max([R[i][k] for i in range(2)]) for k in range(len(eig_GUE))]
R2 = [ max([R[i][k] for i in range(3)]) for k in range(len(eig_GUE))]
R3 = [ max([R[i][k] for i in range(4)]) for k in range(len(eig_GUE))]
R4 = [ max([R[i][k] for i in range(5)]) for k in range(len(eig_GUE))]
R5 = [ max([R[i][k] for i in range(6)]) for k in range(len(eig_GUE))]
R6 = [ max([R[i][k] for i in range(7)]) for k in range(len(eig_GUE))]
R7 = [ max([R[i][k] for i in range(8)]) for k in range(len(eig_GUE))]
R8 = [ max([R[i][k] for i in range(9)]) for k in range(len(eig_GUE))]
```

In [93]:
```
## Distribución acumulada R
sns.histplot(data=R1,stat="density", element="step", fill=False,kde=Tru
e, label='1',cumulative=True,linewidth=1)
sns.histplot(data=R2,stat="density", element="step", fill=False,kde=Tru
e, label='2',cumulative=True,linewidth=1)
sns.histplot(data=R3,stat="density", element="step", fill=False,kde=Tru
e, label='3',cumulative=True,linewidth=1)
sns.histplot(data=R4,stat="density", element="step", fill=False,kde=Tru
e, label='4',cumulative=True,linewidth=1)
sns.histplot(data=R5,stat="density", element="step", fill=False,kde=Tru
e, label='5',cumulative=True,linewidth=1)
sns.histplot(data=R6,stat="density", element="step", fill=False,kde=Tru
e, label='6',cumulative=True,linewidth=1)
sns.histplot(data=R7,stat="density", element="step", fill=False,kde=Tru
e, label='7',cumulative=True,linewidth=1)
sns.histplot(data=R8,stat="density", element="step", fill=False,kde=Tru
e, label='8',cumulative=True,linewidth=1)
#plt.legend(['1','2','3','4','5','6','7','8'])
plt.xlim([-1,15])
#plt.ylim([0,0.6])
```

Out[93]: (-1.0, 15.0)



- Tabla de valores críticos del estadístico R

In [94]:
```
significancia = [1-.15,1-.10,1-.09,1-.08,1-.07,1-0.06,1-0.05,1-0.04,1-
0.03,1-0.02,1-0.01]
tabla_valores = {}
tabla_valores[1] = np.quantile(R1, significancia)
tabla_valores[2] = np.quantile(R2, significancia)
tabla_valores[3] = np.quantile(R3, significancia)
tabla_valores[4] = np.quantile(R4, significancia)
tabla_valores[5] = np.quantile(R5, significancia)
tabla_valores[6] = np.quantile(R6, significancia)
tabla_valores[7] = np.quantile(R7, significancia)
tabla_valores[8] = np.quantile(R8, significancia)
```

In [95]:
```
len(significancia)
```

Out[95]: 11

In [96]:
```
import pandas as pd
pd.DataFrame.from_dict(tabla_valores)
```

Out[96]:

	1	2	3	4	5	6	7	8
0	3.597543	4.122725	4.525492	4.857600	5.136769	5.415943	5.649198	5.862339
1	4.279583	4.887033	5.333539	5.718398	6.050890	6.337374	6.605421	6.836232
2	4.471406	5.084102	5.568441	5.958258	6.287924	6.610439	6.874673	7.111639
3	4.699016	5.328292	5.834215	6.222879	6.576084	6.910282	7.178024	7.419989
4	4.954462	5.615743	6.137214	6.540706	6.914724	7.247601	7.548529	7.814829
5	5.258174	5.962233	6.492183	6.942778	7.307242	7.689682	8.018960	8.272237
6	5.650884	6.375424	6.962814	7.415501	7.839224	8.230171	8.547435	8.840000
7	6.150531	6.956938	7.545524	8.088534	8.517002	8.911481	9.258050	9.581509
8	6.868381	7.738833	8.425157	8.980571	9.437701	9.893134	10.275562	10.623630
9	8.000601	8.993003	9.763884	10.357151	10.935532	11.426233	11.856356	12.269652
10	10.275209	11.531287	12.367150	13.210050	14.017074	14.662887	15.282976	15.741200

In [ ]:
```
## Empezando con 15 y acabando con 1
```

- Tracy Widom numérica

In [20]:
```
import numpy as np
from TracyWidom import TracyWidom

x = np.linspace(-10, 10, 101)
tw1 = TracyWidom(beta=2)  # allowed beta values are 1, 2, and 4
pdf = tw1.pdf(x)
cdf = tw1.cdf(x)
plt.plot(x, pdf,label='f2')
plt.plot(x, cdf,label='F2')
plt.legend(['f2', 'F2'])
plt.suptitle('F2 y f2')
```

Out[20]:
```
Text(0.5, 0.98, 'F2 y f2')
```