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LIN MOD M3 : 1

Given 1-way ANOVA model $y_{ij} = \mu + \alpha_i + e_{ij}$, where $i = 1, \dots, t$; $j = 1, \dots, N_i$; and $e_{ij} \sim N(0, \sigma^2)$. Show that $\alpha_1 = \alpha_2 = \dots = \alpha_t$ iff all contrasts $\lambda'\beta = 0$.

First, show implication of $\alpha_1 = \dots = \alpha_t$, i.e. what is actually being tested. Then equate that 'overall property to a property of each individual contrast.

$$Y = \underbrace{\begin{array}{c} \begin{array}{ccccc} \mu & \alpha_1 & \alpha_2 & \cdots & \alpha_t \end{array} \\ \left[\begin{array}{ccccc} 1 & 1 & & & \\ \vdots & \vdots & & & \\ & 1 & & & \\ & & 1 & & \\ & & \vdots & & \\ & & 1 & & \\ & & & \ddots & \\ 1 & & & & \end{array}\right] \end{array}}_{X} \begin{array}{l} \left.\right\} N_1 \\ \\ \left.\right\} N_2 \end{array} \underbrace{\begin{bmatrix} \mu \\ \alpha_1 \\ \vdots \\ \alpha_t \end{bmatrix}}_{\beta} + \underline{e}$$

$\alpha_1 = \dots = \alpha_t$ implies a reduced model:

$$Y = J\mu + \underline{e}$$

<u>Notation:</u>

Consider $X = [\, J \;\vdots\; X_1 \cdots X_t \,]$. Let $X_* = [X_1 \cdots X_t]$, M_* : ppm onto $C(X_*)$, M : ppm onto $C(X)$, and $M - M_*$: ppm onto $C(X - X_*)$.

The estimation space under H_0 is $C(J)$, while the test space is $C(X_*) = C(M_*)$.
(Prop B.32)

With an orthonormal basis $R = [R_1 \cdots R_t]$ of $C(X_*)$, can write $M_* = RR'$ (Th. B.35).

$$\boxed{\begin{array}{l} M_* = RR' = \sum_{i=1}^{t} R_i R_i' = \sum_{i=1}^{t} M_i \\ Y'M_*Y = \sum_{i=1}^{t} Y'M_iY \end{array}}$$

Note: Since R_i's are orthonormal, $M_iM_j = 0$, and functions $Y'M_iY$ and $Y'M_jY$ are independent. (Th. 1.3.7)

A hypothesis tested using $Y'M_*Y$ would test $0 = \beta'X'M_*X\beta$. Since M_* and M_i's are nonnegative definite, for $0 = \beta'X'M_*X\beta = \sum_{i=1}^{t} \beta'X'M_iX\beta$ to hold, $\beta'X'M_iX\beta > 0 \;\; \forall i$.

This implies $\beta'X'[R_i(R_i'R_i)^{-}R_i']X\beta = 0$, or $R_i'X\beta = 0 \;\; \forall i$.

Equivalently, if any $\beta'X'M_iX\beta > 0$, then $\beta'X'M_*X\beta = 0$, and H_0 no longer holds. ∎