LinMod M1 : 5A

MD

$$Y'MY = \sum_{i=1}^{p} y_{*i}^2$$

Find first p terms of Y_* : $y_{*1}, \cdots, y_{*p}$

$Y = X\beta + \varepsilon$

Use: $X = ULV'$.

$Y = ULV'\beta + \varepsilon$

Multiply by $U_*' = \begin{bmatrix} U' \\ U'_{rest} \end{bmatrix}$.

$$\underbrace{U_* Y}_{\|} = \underbrace{U_*}_{\|} ULV'\beta + \underbrace{U_*'}_{\|}\varepsilon$$

$$Y_* = \begin{bmatrix} U' \\ U'_{rest} \end{bmatrix} ULV'\beta + \begin{bmatrix} U' \\ U'_{rest} \end{bmatrix}\varepsilon$$

$$\underset{n\times 1}{Y_*} = \begin{bmatrix} \underset{p\times n}{U'}\underset{n\times p}{U}\underset{p\times p}{L}\underset{p\times p}{V'}\underset{p\times 1}{\beta} + \underset{p\times n}{U'}\underset{n\times 1}{\varepsilon} \\ U'_{rest}ULV'\beta + U'_{rest}\varepsilon \end{bmatrix} = \begin{bmatrix} \boxed{}_{p\times 1} \\ \boxed{}_{n-p\times 1} \end{bmatrix}$$

Here, $U'\varepsilon = 0$ since $U' \in C(X)$, $\varepsilon \in C(\varepsilon)$ and $C(X) \perp C(\varepsilon)$.

First p elements of Y_* : $U'ULV'\beta = Y_{*p}$

Sum of first p elements squared : $(Y_{*p})'(Y_{*p})$

$$= [U'ULV'\beta]'[U'ULV'\beta]$$

$$= \beta'VL'U'U\underbrace{U'U}_{I}LV'\beta$$

Note: $U'U = I$, since columns of U are orthonormal,

$$= \beta'VL'U'ULV'\beta$$

$$= (ULV'\beta)'(ULV'\beta)$$

Recall : $X = ULV'$

$$= (X\beta)'(X\beta)$$

$$= (MY)'(MY)$$

$$= Y'M'MY$$

Recall: $M = M'M$ idempotence

$$= Y'MY \quad ∎$$