Linear Models Midterm 2

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1 Question 3C

1.1 Strategy

Find $P(\beta_i|\tau,\sigma^2)$, still assuming $\tau = \sigma^2 = 1$. To do this, first find joint distribution $P(\beta_i, \lambda_i|\tau, \sigma)$, then marginalize out λ_i.

1.2 Joint Distribution of β_i, λ_i

$$\begin{aligned}
P(\beta_i, \lambda_i, \tau, \sigma^2) &= P(\beta_i|\lambda_i)P(\lambda_i|\tau)P(\tau|\sigma)P(\sigma^2) \\
&\text{Note that the assumptions on } \tau, \sigma^2 \text{ imply } P(\tau|\sigma) = P(\sigma^2) \propto 1 \\
P(\beta_i, \lambda_i|\tau, \sigma^2) &\propto N(\beta_i|0, \lambda_i^2)C^+(\lambda_i|0, \tau) \\
&\propto \left((\lambda_i^2)^{-1/2} exp\Big(-\frac{1}{2}\frac{\beta_i^2}{\lambda_i^2}\Big)\right)\left(\frac{1}{1+\lambda_i^2}\right) \\
&\propto (\lambda_i^2)^{-1/2}(1+\lambda_i^2)^{-1} exp\Big(-\frac{1}{2}\frac{\beta_i^2}{\lambda_i^2}\Big)
\end{aligned}$$

1.3 Marginalize out λ_i

$$\begin{aligned}
P(\beta_i|\tau, \sigma^2) &= \int P(\beta_i, \lambda_i|\tau, \sigma^2) d\lambda_i \\
&\propto \int (\lambda_i^2)^{-1/2}(1+\lambda_i^2)^{-1} exp\Big(-\frac{1}{2}\frac{\beta_i^2}{\lambda_i^2}\Big) \\
&\propto \int \frac{1}{\lambda_i + \lambda_i^3} exp\Big(-\frac{1}{2}\frac{\beta_i^2}{\lambda_i^2}\Big)
\end{aligned}$$

1.4 Numerical Integration

```r
# Create grid of Betas.
betas <- seq(-3, 3, length=1000)

# Define function to integrate. Integrate function for each Beta_i.
INTEGRAND <- function(beta.i) {
  integrand <- function(x) {
    (1/(x+x^3))*(exp((-1/2)*beta.i^2/x^2))
  }
  return (integrand)
}

# Do integration on Lambda from 0 to Inf, and plot results.
len <- length(betas)
results <- matrix(0, nrow=len, ncol=1)
for (i in 1:len) {
  results[i,] <- integrate(INTEGRAND(betas[i]), lower=0, upper=Inf)$value
}

plot(betas,results, xlab="Betas",
     main=expression(paste("Numerical Evaluation of P(", beta[i], "|",
                          tau, ",", sigma^2, ")")),
     ylab=expression(paste("P(", beta[i], "|", tau, ",", sigma^2, ")")))
```