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Lin Mod M2 : 1

1A (i) Under M1, $S(Y) = \dfrac{Y'(M-M_1)Y/r(M-M_1)}{Y'(I-M)Y/r(I-M)}$

$$\sim F\left(r(M-M_1),\ r(I-M),\ \beta'X'(M-M_1)X\beta/2\sigma^2\right)$$

according to Theorem 3.2.1.

⊛ Here, $r(M-M_1) = (p+q)-p = q$, and $r(I-M) = n-(p+q) = n-p-q$.

So $S(Y) = \dfrac{Y'(M-M_1)Y/q}{Y'(I-M)Y/(n-p-q)} \sim F\left(q,\ n-p-q,\ \beta'X'(M-M_1)X\beta/2\sigma^2\right)$

(ii) Under M∅, ⊛ still applies, and by the same theorem,

$$S(Y) \sim F\left(r(M-M_1),\ r(I-M),\ \emptyset\right) = F\left(q,\ n-p-q,\ \emptyset\right).$$

$\boxed{1B}$ Show that the LRT $\equiv$ a test that rejects $M\emptyset$ for large values of $S(Y)$.

LRT rejects $M\emptyset$ when $L(\hat{\theta}_0)$ is not sufficiently bigger than $L(\hat{\theta})$. Using c, the LRT states that it will reject $M\emptyset$ if the likelihood under $M\emptyset$ is lower than the likelihood under M1.

$$\text{LRT: Reject if } \frac{L(\hat{\theta}_0)}{L(\hat{\theta})} < c$$

Lower $L(\hat{\theta}_0)$ means that $M\emptyset$ didn't describe the data as well as the other model, and has larger errors than the other model.

Larger errors under $M\emptyset$ $\Rightarrow$ Larger $Y'(M-M_1)Y$ $\Rightarrow$ Larger $S(Y)$

$\Rightarrow$ More likely to reject $M\emptyset$