Lin Mod M1 : 5B

MD

$$Y'(I-M)Y = \sum_{i=p+1}^{n} y_*^2$$

This is the sum of squares of last $(n-p)$ values of Y_*

As before :

$$\underset{n\times 1}{Y_*} = \begin{bmatrix} U'ULV'\beta + U'\varepsilon \\ U_{rest}'ULV'\beta + U_{rest}'\varepsilon \end{bmatrix} = \begin{bmatrix} \underset{p\times 1}{\square} \\ \underset{n-p\times 1}{\square} \end{bmatrix}$$

Last $(n-p)$ elements of Y_* : $U_{rest}'ULV'\beta + U_{rest}'\varepsilon = Y_{*rest}$

$$\begin{aligned}
\sum_{i=p+1}^{n} y_*^2 &= [Y_{*rest}]'[Y_{*rest}] \\
&= [U_{rest}'ULV'\beta + U_{rest}'\varepsilon]'[U_{rest}'ULV'\beta + U_{rest}'\varepsilon] \\
&= [U_{rest}'X\beta + U_{rest}'\varepsilon]'[U_{rest}'X\beta + U_{rest}'\varepsilon] \\
&= [U_{rest}'\varepsilon]'[U_{rest}'\varepsilon] \\
&= \varepsilon' U_{rest} U_{rest}'\varepsilon \\
&= \varepsilon'\varepsilon \\
&= [(I-M)Y]'[(I-M)Y] \\
&= Y'(I-M)'(I-M)Y \\
&= Y'(I-M)Y \quad \blacksquare
\end{aligned}$$

$X = ULV'$

$$\left.\begin{aligned} &U' \in C(X), \\ &U_{rest}' \in C(X)^\perp \\ &X\beta \in C(X) \end{aligned}\right\} (*)$$

$(*)$ implies $U_{rest}' \perp X\beta$

and $U_{rest}'X\beta = 0$

$\varepsilon \in C(X)^\perp$

$U_{rest}' \in C(X)^\perp$

$(I-M)$ is p.p.m onto $C(X)^\perp$

$\varepsilon = (I-M)Y$

U_{rest} are orthonormal,

So $U_{rest}U_{rest}' = I$