MD

Lin Mod M1 : 5c

$$\hat{\beta}'\hat{\beta} = \sum_{i=1}^{p} \frac{y_{*i}^2}{\lambda_i^2}$$

Recall: First p elements of $Y_* = U'ULV'\beta = Y_{*p}$

Recall: $\sum_{i=1}^{p} y_{*i}^2 = [U'ULV'\beta]'[U'ULV'\beta] = Y_{*p}'Y_{*p}$

$$= \begin{bmatrix} y_{*1} & \cdots & y_{*p} \end{bmatrix} \begin{bmatrix} y_{*1} \\ \vdots \\ y_{*p} \end{bmatrix}$$

Want same method but include $\frac{1}{\lambda_i}$ in each element.

Try $$\begin{bmatrix} y_{*1} & \cdots & y_{*p} \end{bmatrix} \begin{bmatrix} \frac{1}{\lambda_1} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{\lambda_p} \end{bmatrix} = \begin{bmatrix} \frac{y_{*1}}{\lambda_1} & \cdots & \frac{y_{*p}}{\lambda_p} \end{bmatrix}$$

$$\underset{1 \times p}{Y_{*p}'} \cdot \underset{p \times p}{L^{-1}} = \underset{1 \times p}{Y_{*p}' L^{-1}}$$

Now we have $\frac{1}{\lambda_i}$ in each element:

$$\sum_{i=1}^{p} \frac{y_{*i}^2}{\lambda_i^2} = \begin{bmatrix} \frac{y_{*1}}{\lambda_1} & \cdots & \frac{y_{*p}}{\lambda_p} \end{bmatrix} \begin{bmatrix} \frac{y_{*1}}{\lambda_1} \\ \vdots \\ \frac{y_{*p}}{\lambda_p} \end{bmatrix}$$

$$= \left[Y_{*p}' L^{-1}\right]\left[Y_{*p}' L^{-1}\right]'$$

$$= \left[(U'ULV'\beta)' L^{-1}\right]\left[(U'ULV'\beta)' L^{-1}\right]'$$

$$= \beta' V L' \underbrace{U'U}_{I} L^{-1} (L^{-1})' \underbrace{U'U}_{I} L V' \beta$$

$$= \beta' V L L^{-1} L^{-1} L V' \beta$$

$$= \beta' V V' \beta$$

$$= \beta'\beta \quad \blacksquare$$

$Y_{*p} = U'ULV'\beta$

$U'U = I$ orthonormal columns of U

$L' = L$, $L^{-1} = (L^{-1})'$

since each is square, diagonal

$V'V = I$ orthonormal columns of V

$\Rightarrow V' = V^{-1}$, so $VV' = VV^{-1} = I$