Lin Mod M2 : 2

MB

For M1 : $Y = X\beta + \varepsilon$, $\varepsilon \sim N(0, \sigma^2 I)$

Consider constraint $\Lambda'\beta = 0$ with $\Lambda' = P'X \Rightarrow P'X\beta = \emptyset$.

Show that $\beta'X' M_{MP} X\beta = \emptyset$ iff $\Lambda'\beta = \emptyset$.

" $\Leftarrow$ " $\Lambda'\beta = P'X\beta = P'MX\beta = \emptyset$.

$\Rightarrow X\beta \in C(MP)^{\perp} \Rightarrow \beta'X' M_{MP} X\beta = \beta'X' \cdot \emptyset = \emptyset$

More details

Given U s.t. $C(U) = C(\Lambda)^{\perp}$, $\beta \in C(U)$ and $Y = X\beta + \varepsilon = XU\gamma + \varepsilon$

for some γ. XU can be re-written as X_1 , where $X = [X_1 \; X_2]$

and $C(X_1) \subset C(X)$. $X\beta = XU\gamma \in C(MP)^{\perp}$.

Therefore, $C(MP) = C(XU)^{\perp}_{C(X)} = C(X_1)^{\perp}_{C(X)} = C(M - M_1)$.

$X\beta \in C(MP)^{\perp} \equiv X\beta \in C(M-M_1)^{\perp} \Rightarrow \forall g \in C(M-M_1), \; X\beta \cdot g = \emptyset$

$\beta'X' M_{MP} X\beta = \underbrace{\beta'X'(M-M_1)X\beta}_{\emptyset} = \emptyset$ ∎

" $\Rightarrow$ " $\beta'X' M_{MP} X\beta = \emptyset$

M_{MP} is projection matrix onto $C(MP)$, so $\beta'X'(M_{MP})' M_{MP} X\beta = \emptyset$

$\Rightarrow (M_{MP}X\beta)' M_{MP} X\beta = \emptyset \Rightarrow M_{MP} X\beta = \emptyset \Rightarrow MP(P'MP)^{-} P'\underbrace{MX\beta}_{X\beta} = \emptyset$

$\Rightarrow MP(P'MP)^{-} P'X\beta = \emptyset \Rightarrow MP(P'MP)^{-} \Lambda'\beta = \emptyset$

$\Rightarrow \Lambda'\beta = \emptyset$ ∎

3