Mean-centered

$y_i = \beta_0 + \beta_1 x_{i1} + \dots \beta_{p-1} x_{ip-1} + e_i$

$X = [J \;\vdots\; X_*]$, X_* is $n\times(p-1)$

$\beta = \begin{bmatrix} \beta_0 \\ | \\ \beta_* \\ | \end{bmatrix}$, β_* is $(p-1)\times 1$

$y_i = \gamma_0 + \gamma_1(x_{i1} - \bar{x}_{\cdot 1}) + \dots + \gamma_{p-1}(x_{ip-1} - \bar{x}_{\cdot p-1}) + e_i$

$X = [J \;\vdots\; (I-M_J)Z]$, $(I-M_J)Z$ is $n\times(p-1)$

$\gamma = \begin{bmatrix} \gamma_0 \\ | \\ \gamma_* \\ | \end{bmatrix}$, γ_* is $(p-1)\times 1$

$$\beta_0 + X_*\beta_* = E[Y] = \gamma_0 + [(I-M_J)Z]\gamma_* \quad \boxed{\star}$$

Average over observations:

$$\beta_0 + \beta_1\bar{x}_1 + \dots + \beta_{p-1}\bar{x}_{p-1} = \gamma_0 + \underbrace{\gamma_1(\bar{x}_1 - \bar{x}_1) + \dots + \gamma_{p-1}(\bar{x}_{\cdot p-1} - \bar{x}_{\cdot p-1})}_{\emptyset}$$

Let $\underset{(p-1)\times 1}{\bar{X}_*} = [\bar{x}_{\cdot 1}, \dots, \bar{x}_{\cdot p-1}]'$

$$\beta_0 + \bar{X}_*'\beta_* = \gamma_0$$

$$\beta_0 = \gamma_0 - \bar{X}_*'\beta_*$$

Substitute β_0 into $\boxed{\star}$: $\cancel{\gamma_0} - \bar{X}_*'\beta_* + X_*\beta_* = \cancel{\gamma_0} + [(I-M_J)Z]\gamma_*$

$$[\underset{n\times(p-1)}{X_*} - \underset{n\times(p-1)}{1_n\bar{X}_*'}]\beta_* = [(I-M_J)Z]\gamma_*$$

For each i, $[(x_{i1} - \bar{x}_{\cdot 1}), \dots, (x_{ip-1} - \bar{x}_{\cdot p-1})]\beta_* \equiv [(I-M_J)Z]\beta_*$

$$\boxed{\text{Therefore } \beta_* = \gamma_*, \text{ and } \beta_0 = \gamma_0 - \bar{X}_*'\gamma_*}$$

Book denotes $\bar{X}_*'$ as $(\frac{1}{n})J_1^n Z$, so $\beta_0 = \gamma_0 - (\frac{1}{n})J_1^n Z\gamma_*$

From above, in the mean-centered version, $C((I-M_J)Z) = C(J)^{\perp}_{C(X)} = C(M-M_J)$

Normal equation $X'X\gamma = X'Y$: $X'X = \underset{p\times n}{\begin{bmatrix} - J - \\ -(I-M_J)Z- \end{bmatrix}} \underset{n\times p}{\begin{bmatrix} | & | \\ J & (I-M_J)Z \\ | & | \end{bmatrix}} = \underset{p\times p}{\begin{bmatrix} n & 0 \\ 0 & [(I-M_J)Z]'[(I-M_J)Z] \end{bmatrix}}$,

$[X'X]^{-1} = \underset{p\times p}{\begin{bmatrix} \frac{1}{n} & 0 \\ 0 & \frac{1}{[(I-M_J)Z]'[(I-M_J)Z]} \end{bmatrix}}$ is invertible because X is full rank, $X'Y = \underset{p\times 1}{\begin{bmatrix} \Sigma y_i \\ [(I-M_J)Z]'Y \end{bmatrix}}$

The L.S. estimate of γ is $\hat{\gamma} = (X'X)^{-1}X'Y = \begin{bmatrix} \frac{\sum y_i}{n} \\ \frac{[(I-M_J)z]'Y}{[(I-M_J)z]'[(I-M_J)z]} \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \frac{[(I-M_J)z]'Y}{[(I-M_J)z]'[(I-M_J)z]} \end{bmatrix} = \begin{bmatrix} \hat{\gamma}_0 \\ \hat{\gamma}_* \end{bmatrix}$

From previous result, know that $\gamma_* = \beta_*$, so $\hat{\gamma}_* = \hat{\beta}_*$. This is useful in that we can relate Y under the reduced model, with $X = [J \,\vdots\, (I-M_J)z]$, to the Least Squares estimate $\hat{\beta}_*$.

For L.S., generally $MY = X\hat{\beta}$. Here $M_{(I-M_J)z}Y = X\hat{\beta}_*$. $M_{(I-M_J)z}$ is defined below.

$SSReg = Y'(M-M_J)Y$. Since $C(M-M_J) = C((I-M_J)z)$, can write ppm onto $(I-M_J)z$ as

$$(I-M_J)z\left[z'(I-M_J)'(I-M_J)z\right]^{-}z'(I-M_J)' = M_{(I-M_J)z}$$

$= Y'(I-M_J)z\left[z'(I-M_J)'(I-M_J)z\right]^{-}z'(I-M_J)'Y$	Replacing $M-M_J$ with $M_{(I-M_J)z}$
$= \hat{\beta}_*'z'(I-M_J)z\left[z'(I-M_J)'(I-M_J)z\right]^{-}z'(I-M_J)'z\hat{\beta}_*$	Use normal equations to get $\hat{\beta}_*$, then $E[Y] = z\hat{\beta}_*$
$= \hat{\beta}_*'z'(I-M_J)z\left[z'(I-M_J)z\right]^{-}z'(I-M_J)'z\hat{\beta}_*$	Collapse ppm's, since they are idempotent.
$= \hat{\beta}_*'z'(I-M_J)z\hat{\beta}_*$ ∎	Allow generalized inverse to cancel out.