MCMC M1 : A

MD

Write the joint probability model.

$$p(\pi^*, q_1, \ldots, q_N, r_1, \ldots, r_N, z_1, \ldots, z_N, y_1, \ldots, y_N) = \text{"Joint"}$$

"Joint" distribution:

Prior:

$$Pr(z_j = 1) = p \Rightarrow Pr(z_1, \ldots, z_N) = p^{M_1}(1-p)^{M_0}$$

$$Pr(\pi^* | a^*, b^*) = \frac{1}{B(a^*, b^*)} \cdot (\pi^*)^{a^*-1}(1-\pi^*)^{b^*-1}$$

$$Pr(q_1, \ldots, q_N | z_j, a_1) = \prod_{j=1}^{N} \left[1_{z_j}(1) \cdot \frac{q_j^{a_1-1}}{B(a_1)} + 1_{z_j}(0) \cdot 1 \right]$$

Cases where $z_j = 1$, gets Dirichlet prob

Cases where $z_j = 0$, $q_j = 0$ with prob $= 1$

$$Pr(r_1, \ldots, r_N | z_j, a_0) = \prod_{j=1}^{N} \left[1_{z_j}(1) \cdot 1 + 1_{z_j}(0) \cdot \frac{r_j^{a_0-1}}{B(a_0)} \right]$$

Cases where $z_j = 1$, $r_j = 0$ w/ prob $= 1$

Cases where $z_j = 0$, gets Dirichlet prob.

For each j:

$$Pr(\pi_j | z_j, \pi^*, q_j, r_j) = 1_{z_j}(1) \cdot \pi^* q_j + 1_{z_j}(0) \cdot (1-\pi^*) r_j$$

Likelihood; for each y:

$$P(y | n, \pi_1, \ldots, \pi_N) = \prod_{j=1}^{N} \pi_j^{y_j}$$

"Joint": $P(y | n, \pi_1, \ldots, \pi_N) \cdot \prod_{j=1}^{N} \left[Pr(\pi_j | z_j, \pi^*, q_j, r_j) \right] \cdot Pr(r_1, \ldots, r_N | z_j, a_0) \cdot Pr(q_1, \ldots, q_N | z_j, a_1) \cdot Pr(\pi^* | a^*, b^*) \cdot Pr(z_1, \ldots, z_n)$

$$\text{Joint} = \overbrace{\prod_{j=1}^{N} \pi_j^{y_j}}^{P(y|\pi_{1,\dots,N})} \times \left.\prod_{j=1}^{N} \left[\pi^* \cdot q_j \cdot I(z_j=1) + (1-\pi^*) \cdot r_j \cdot I(z_j=0) \right] \right\} P(\underline{\pi}|\underline{z},\pi^*,\underline{q},\underline{r})$$

$$\times \left.\prod_{j=1}^{N} \left[1 \cdot I(z_j=1) + \frac{r_j^{a_0-1}}{B(a_0)} \cdot I(z_j=0) \right] \right\} P(\underline{r}|\underline{z},a_0)$$

$$\times \left.\prod_{j=1}^{N} \left[\frac{q_j^{a_1-1}}{B(a_1)} \cdot I(z_j=1) + 1 \cdot I(z_j=0) \right] \right\} P(\underline{q}|\underline{z},a_1)$$

$$\times \left.\frac{1}{B(a^*,b^*)} (\pi^*)^{a^*-1} (1-\pi^*)^{b^*-1} \right\} P(\pi^*|a^*,b^*)$$

$$\times \left.\rho^{\sum_{j=1}^{N} I(z_j=1)} \cdot (1-\rho)^{\sum_{j=1}^{N} I(z_j=0)} \right\} P(\underline{z}|\rho)$$

$$= \prod_{j=1}^{N} \pi_j^{y_j} \times \left[\prod_{j\in A_1} q_j \right] \cdot (\pi^*)^{M_1} \cdot \left[\prod_{j\in A_0} r_j \right] \cdot (1-\pi^*)^{M_0}$$

$$\times \left[\prod_{j\in A_0} \frac{r_j^{a_0-1}}{B(a_0)} \right]$$

$$\times \left[\prod_{j\in A_1} \frac{q_j^{a_1-1}}{B(a_1)} \right]$$

$$\times \frac{1}{B(a^*,b^*)} (\pi^*)^{a^*-1} (1-\pi^*)^{b^*-1}$$

$$\times \rho^{M_1} (1-\rho)^{M_0}$$