MCMC M1: C Conditional Posterior Distributions

MD

Given the joint, which is proportional (to a constant) to the posterior, each conditional posterior can be found by regarding all other variables as constant.

$p(z_i|\ldots)$: Probability terms for π_j, r_j, q_j, and z_j are all functions of z.

$$P(z|\ldots) \propto P(\underline{\pi}|z,\pi^*,q,r)\cdot P(\underline{r}|z,a_0)\cdot P(q|z,a_1)\cdot P(z|\rho)$$

SEE BELOW *

$$\propto \left[\prod_{j\in A_1} q_j\right](\pi^*)^{M_1}\cdot\left[\prod_{j\in A_0} r_j\right]\cdot(1-\pi^*)^{M_0}\cdot\left[\prod_{j\in A_0} r_j^{a_0-1}\right]\cdot\left[\prod_{j\in A_1} q_j^{a_1-1}\right]\cdot\rho^{M_1}(1-\rho)^{M_0}$$

$$\propto \left[\prod_{j\in A_1} q_j^{a_1}\right]\cdot\left[\prod_{j\in A_0} r_j^{a_0}\right]\cdot(\pi^*)^{M_1}(1-\pi^*)^{M_0}\cdot\rho^{M_1}(1-\rho)^{M_0} \sim Dir(M_1; a_1+1,\ldots,a_1+1)\cdot Dir(M_0; a_0+1,\ldots,a_0+1)\cdot Beta(M_1+1, M_0+1)\cdot Beta(M_1+1, M_0+1)$$

$P(\pi^*|\ldots)$: Only the π_j and π^* probability distributions contain π^*.

$$P(\pi^*|\ldots) \propto \prod_{j=1}^{N}\left[\delta(z_j=1)\cdot\pi^* q_j + \delta(z_j=0)\cdot(1-\pi^*)\cdot r_j\right]\cdot(\pi^*)^{a^*-1}(1-\pi^*)^{b^*-1}$$

$$\propto \left[\prod_{j\in A_1}\pi^* q_j\right]\cdot\left[\prod_{j\in A_0}(1-\pi^*)\cdot r_j\right]\cdot(\pi^*)^{a^*-1}(1-\pi^*)^{b^*-1}$$

Recall $|A_1| = M_1$, $|A_0| = M_0$

$$\propto \left[\prod_{j\in A_1} q_j\right]\left[\prod_{j\in A_0} r_j\right]\cdot(\pi^*)^{M_1+a^*-1}\cdot(1-\pi^*)^{M_0+b^*-1}$$

$$\sim Beta(M_1+a^*, M_0+b^*)$$

$$* \; P(z_i|\ldots) \propto I(z_i=1)\left[q_i^{a_1}\cdot\pi^*\cdot\rho\right] + I(z_i=0)\cdot\left[r_i^{a_0}\cdot(1-\pi^*)(1-\rho)\right]$$

$P(q|\ldots)$: Only the π_j an $q_1, \ldots, q_N$ terms contain q.

$$P(q|\ldots) \propto \left[\prod_{j \in A_1} q_j\right] \cdot \left[\prod_{j \in A_1} \frac{q_j^{a_1 - 1}}{B(a_1)}\right]$$

$$\propto \prod_{j \in A_1} q_j \cdot q_j^{a_1 - 1} = \prod_{j \in A_1} q_j^{a_1} \sim Dir\left(M_1; a_1+1, \ldots, a_1+1\right)$$

$P(r|\ldots)$: Only the π_j and $r_1, \ldots, r_N$ terms contain r.

$$P(r|\ldots) \propto \left[\prod_{j \in A_0} r_j\right] \left[\prod_{j \in A_0} \frac{r_j^{a_0 - 1}}{B(a_0)}\right]$$

$$\propto \prod_{j \in A_0} r_j \cdot r_j^{a_0 - 1} = \prod_{j \in A_0} r_j^{a_0} \sim Dir\left(M_0; a_0+1, \ldots, a_0+1\right)$$