MCMC M1: E

Violation of Irreducibility

A Gibbs sampler should be able to reach any configuration of parameters in a finite number of steps, and with positive probability. This is called irreducibility.

Irreducibility is violated if there exists any $\underbrace{\text{configuration of parameters}}_{\equiv \text{ point in the subspace}}$ that cannot be reached ($Pr=0$) in a finite number of steps.

If steps are in the order z_i^{t+1}, $\pi^{*(t+1)}$, q^{t+1}, r^{t+1} suppose that in the current q, $q_i > 0$ and $r_i = 0$, because $z_i = 1$. Now suppose that in the 1st update, that of z_i^{t+1}, z_i^{t+1} becomes $= 0$. The current setting for the second update is:

$$z_i^{(t+1)} = 0, \quad q_i^{(t)} = a : a > 0, \quad r_i^{(t)} = 0$$

The 2nd update will then proceed. $P(\pi^*|\ldots) \propto \prod_{j=1}^{N} \left[\pi^* q_j \cdot I(z_j=1) + (1-\pi^*) r_j \cdot I(z_j=0) \right]$

The i^{th} term of this product will be $\pi^* \cdot q_i\, I(z_i=1) + (1-\pi^*) r_i \cdot I(z_i=0)$.

Since $z_i^{(t+1)} = 0$, the first term is zero because the indicator is zero, and the second term is zero because $r_i = 0$.

This entire product, $P(\pi^*|\ldots)$, equals zero, making some configurations impossible to reach.

This proves that there exist cases where points in the parameter space have zero probability, contradicting irreducibility. ∎