Model runs once for each store: i = store, j = week

$$\log Q_{ij} \sim N\left(\log\alpha_i + \beta_i \log P_{ij} + \gamma_i D_{ij} + \delta_i(\log P_{ij} \cdot D_{ij}),\ \sigma^2\right)$$

$$\log\alpha_i \sim N(\alpha_0, \tau_1^2)$$

$$\beta_i \sim N(\beta_0, \tau_2^2)$$

$$\gamma_i \sim N(\gamma_0, \tau_3^2)$$

$$\delta_i \sim N(\delta_0, \tau_4^2)$$

Note:

$$\boxed{\sum_j 1 = n_i = \text{number of observations for store } i.}$$

Posterior Full Conditionals:

$$P(\log\alpha_i \mid \ldots) \sim N\left(\left(\frac{1}{\tau_1^2} + \frac{n_i}{\sigma^2}\right)^{-1}\left(\frac{\alpha_0}{\tau_1^2} + \frac{\sum_j(\log Q_{ij} - \beta_i \log P_{ij} - \gamma_i D_{ij} - \delta_i(\log P_{ij} \cdot D_{ij}))}{\sigma^2}\right),\ \left(\frac{1}{\tau_1^2} + \frac{n_i}{\sigma^2}\right)^{-1}\right)$$

$$P(\beta_i \mid \ldots) \sim N\left(\left(\frac{1}{\tau_2^2} + \frac{n_i}{\sigma^2}\right)^{-1}\left(\frac{\beta_0}{\tau_2^2} + \frac{\sum_j(\log Q_{ij} - \log\alpha_i - \gamma_i D_{ij} - \delta_i(\log P_{ij} \cdot D_{ij}))}{\sigma^2}\right),\ \left(\frac{1}{\tau_2^2} + \frac{n_i}{\sigma^2}\right)^{-1}\right)$$

$$P(\gamma_i \mid \ldots) \sim N\left(\left(\frac{1}{\tau_3^2} + \frac{n_i}{\sigma^2}\right)^{-1}\left(\frac{\gamma_0}{\tau_3^2} + \frac{\sum_j(\log Q_{ij} - \log\alpha_i - \beta_i \log P_{ij} - \delta_i(\log P_{ij} \cdot D_{ij}))}{\sigma^2}\right),\ \left(\frac{1}{\tau_3^2} + \frac{n_i}{\sigma^2}\right)^{-1}\right)$$

$$P(\delta_i \mid \ldots) \sim N\left(\left(\frac{1}{\tau_4^2} + \frac{n_i}{\sigma^2}\right)^{-1}\left(\frac{\delta_0}{\tau_4^2} + \frac{\sum_j(\log Q_{ij} - \log\alpha_i - \beta_i \log P_{ij} - \gamma_i D_{ij})}{\sigma^2}\right),\ \left(\frac{1}{\tau_4^2} + \frac{n_i}{\sigma^2}\right)^{-1}\right)$$

For full conditional $P(\log\alpha_i | \ldots)$. i = store
j = week

$$\sum_j \left[\log Q_{ij} - \left(\log\alpha_i + \beta_i \log P_{ij} + \gamma_i D_{ij} \right) \right]^2$$

$$= \sum_j \left[\log Q_{ij}^2 + \left(\underset{a}{\log\alpha_i} + \underset{b}{\beta_i \log P_{ij}} + \underset{c}{\gamma_i D_{ij}} \right)^2 - 2\left(\log Q_{ij}\right)\left(\underset{a}{\log\alpha_i} + \underset{b}{\beta_i \log P_{ij}} + \underset{c}{\gamma_i D_{ij}} \right) \right]$$

$$(a+b+c)(a+b+c) = a(a+b+c) + b(a+b+c) + c(a+b+c)$$

Only terms w/ $\log\alpha_i$:

$$= a^2 + ab + ac + ba + ca = a^2 + 2ab + 2ac$$

$$\propto \sum_j \underset{a^2}{\log\alpha_i^2} + \underset{2ab}{2\log\alpha_i \beta_i \log P_{ij}} + \underset{2ac}{2\log\alpha_i \gamma_i D_{ij}} - 2\log Q_{ij} \cdot \log\alpha_i$$

$$+ 2\log\alpha_i \left(\beta_i \log P_{ij} + \gamma_i D_{ij} \right) - \log\alpha_i \left(2 \log Q_{ij} \right)$$

$$\propto \sum_j \log\alpha_i^2 - 2\log\alpha_i \left(\log Q_{ij} - \beta_i \log P_{ij} - \gamma_i D_{ij} \right)$$

$$P(\log\alpha_i | \ldots) \propto \exp\left\{ -\frac{1}{2} \left[\frac{(\log\alpha_i - \alpha_0)^2}{\tau_i^2} + \frac{\sum_j \left[\log\alpha_i^2 - 2\log\alpha_i \left(\log Q_{ij} - \beta_i \log P_{ij} - \gamma_i D_{ij} \right) \right]}{\sigma^2} \right] \right\}$$

$$\propto \exp\left\{ -\frac{1}{2} \left[\frac{\log\alpha_i^2 - 2\log\alpha_i \cdot \alpha_0}{\tau_i^2} + \frac{n_j \log\alpha_i^2 - 2\log\alpha_i \sum_j \left(\log Q_{ij} - \beta_i \log P_{ij} - \gamma_i D_{ij} \right)}{\sigma^2} \right] \right\}$$

$$\propto \ldots \left(\frac{1}{\tau_i^2} + \frac{n_j}{\sigma^2} \right) \log\alpha_i^2 - 2\log\alpha_i \left(\frac{\alpha_0}{\tau_i^2} + \frac{\sum_j \left(\log Q_{ij} - \beta_i \log P_{ij} - \gamma_i D_{ij} \right)}{\sigma^2} \right)$$

$$\sim N\left(\left(\frac{1}{\tau_i^2} + \frac{n_j}{\sigma^2} \right)^{-1} \left(\frac{\alpha_0}{\tau_i^2} + \frac{\sum_j \left(\log Q_{ij} - \beta_i \log P_{ij} - \gamma_i D_{ij} \right)}{\sigma^2} \right), \left(\frac{1}{\tau_i^2} + \frac{n_j}{\sigma^2} \right)^{-1} \right)$$