Observe $y_i = f(x_i) + \varepsilon_i$, $\quad \varepsilon_i \sim N(0, \sigma^2)$ for unknown f.

$P(f) \sim GP(0, C)$. $\quad \hookrightarrow y_i \sim N(f_i, \sigma^2)$

So: Likelihood: $P(y|f) = N(f, \sigma^2 I) \quad \exp\left\{-\frac{1}{2}\left(\frac{(y-f)'(y-f)}{\sigma^2}\right)\right\}$

$\quad$ Prior: $P(f) \sim GP(0, C) \quad \exp\left\{-\frac{1}{2}\left(f'C^{-1}f\right)\right\}$

Posterior: $P(f|y) \propto P(y|f)\,P(f) = \exp\left\{-\frac{1}{2}\left[f'\left(\frac{I}{\sigma^2}+C^{-1}\right)f' - f'\left(\frac{y}{\sigma^2}\right) + \ldots\right]\right\}$

$$\sim N\left(\left(\frac{I}{\sigma^2}+C^{-1}\right)^{-1}\left(\frac{y}{\sigma^2}\right), \left(\frac{I}{\sigma^2}+C^{-1}\right)^{-1}\right)$$