StatMod2 - Hierarchical Models and Shrinkage - Exercises 4

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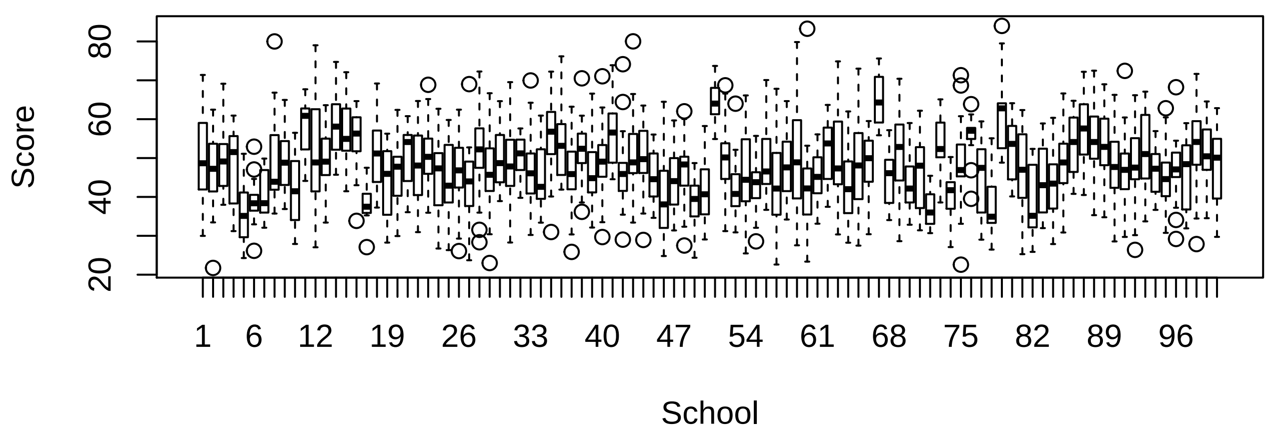
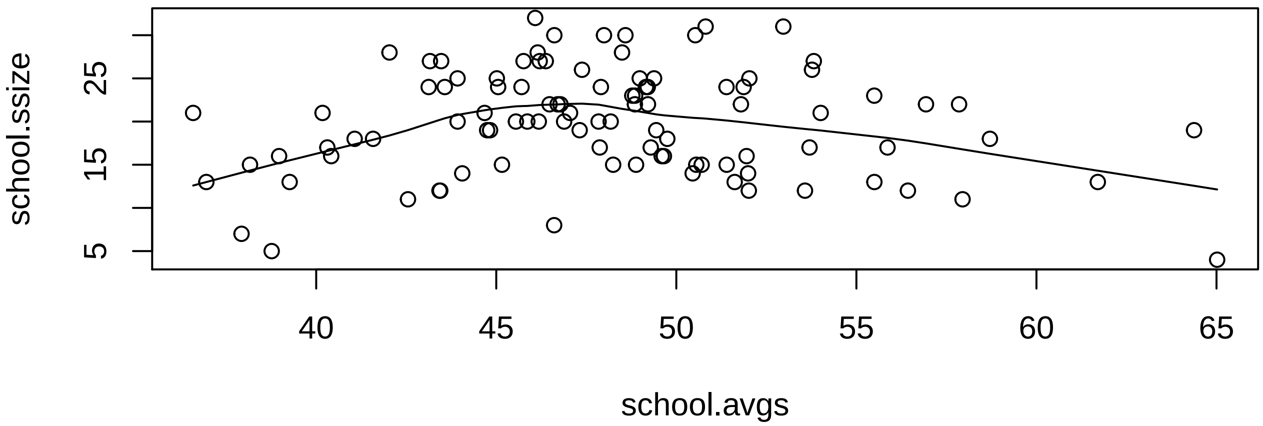
1 School Averages and Sample Size

Larger samples tend to "smooth" out extreme scores, so small samples are more likely to be extreme.

```
attach(data)
boxplot(mathscore ~ school, xlab="School", ylab="Score")
```




```
school.avgs <- aggregate(data, list(school=school), mean)[,3]
school.ssize <- aggregate(data, list(school=school), length)[,3]
plot(cbind(school.avgs, school.ssize))
fit <- lowess(cbind(school.avgs, school.ssize))
lines(fit)
```



1.1 Normal Hierarchical Model with Gibbs Sampling

1.1.1 Model

For school $i = 1, \cdots, p$; student $j = 1, \cdots, n_i$; and $\sum n_i = n$; let $a = b = c = d = 1$, and note that for this data, $p = 100$ and $n = 1993$:

$$\begin{aligned} y_{ij} &\sim \; N(\theta_i, \sigma^2) \\ \theta_i &\sim \; N(\mu, \tau^2) \\ \mu &\sim \; N(m, v) \\ \sigma^2 &\sim \; InvGa(a, b) \\ \tau^2 &\sim \; InvGa(c, d) \end{aligned}$$

1.1.2 Joint and Posterior Distributions

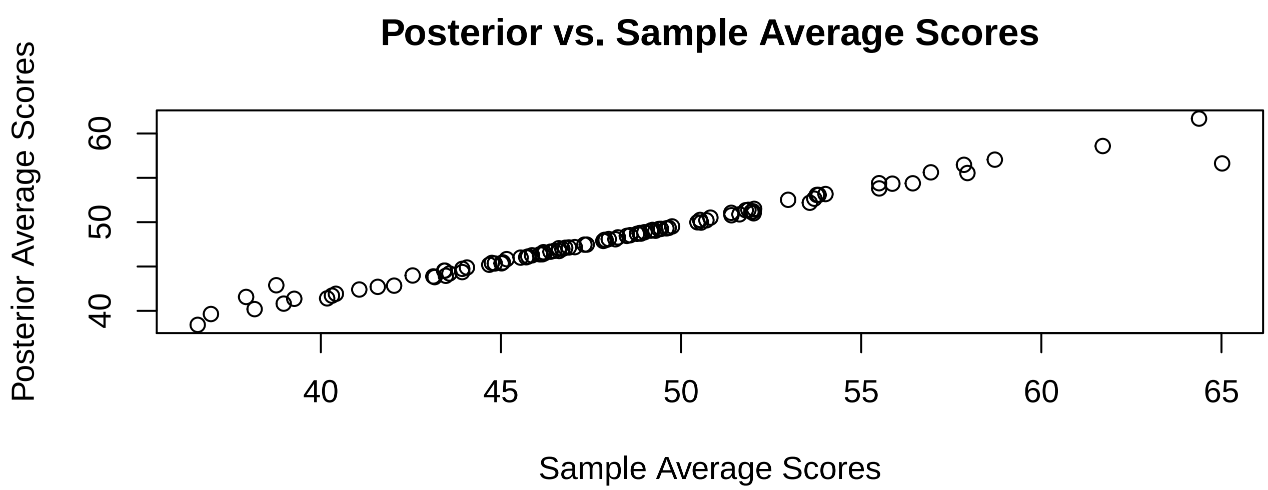
See attached sheets.

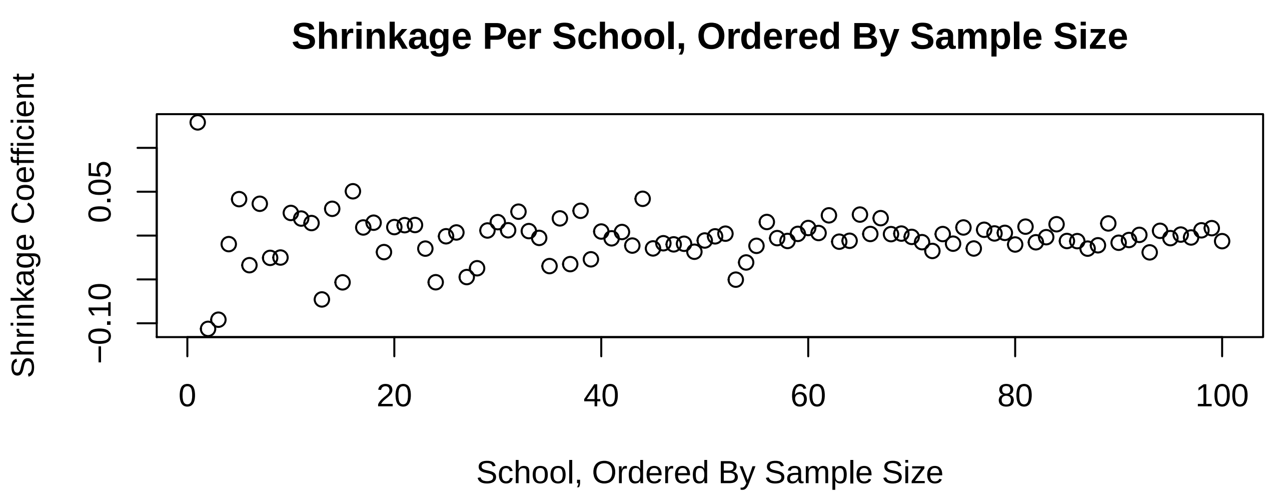
1.1.3 Gibbs Sampler

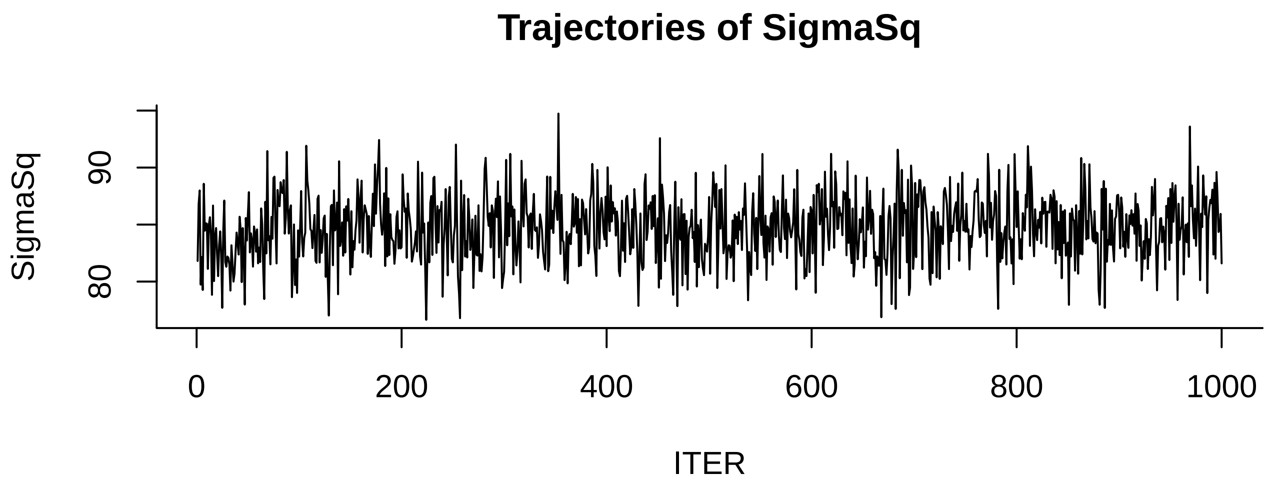
The Gibbs Sampler produces the following results. Immediately below are the posterior estimates for θ.

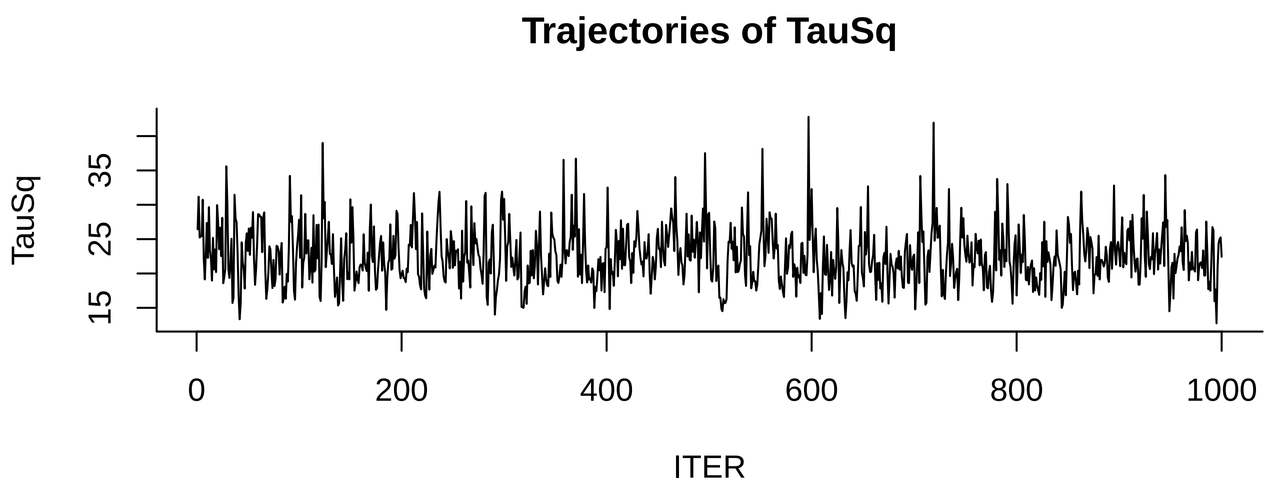
```
##   [1] 50.51003 46.76601 48.68838 47.46504 38.42135 40.81311 41.92385
##   [8] 48.75465 49.04415 42.40514 55.54075 50.25178 49.24118 57.04885
##  [15] 54.38310 54.39402 41.56460 49.98703 44.52868 46.26326 50.21306
##  [22] 47.97774 51.22848 46.00253 45.33894 46.74912 44.53177 51.34644
##  [29] 46.61217 49.30503 49.06949 49.95425 47.46797 46.04563 54.34171
##  [36] 52.19013 46.37511 51.15531 46.49257 48.86489 55.62025 46.38389
##  [43] 50.77899 49.03702 45.82999 41.40744 44.73532 47.10874 40.19376
##  [50] 42.70538 61.67738 49.22951 43.98558 46.66859 43.88010 49.12179
```

```
## [57] 44.37257 48.45245 49.51989 42.84086 45.46777 50.88362 49.35530
## [64] 43.80215 48.00024 48.28518 56.62764 45.40195 51.38905 43.95266
## [71] 46.98120 39.64060 53.07232 41.71179 48.77520 53.80514 45.19835
## [78] 41.35697 58.58670 51.00293 47.14796 42.88651 44.88587 44.21317
## [85] 48.70792 52.64602 56.46694 53.17146 53.07175 48.53131 47.88498
## [92] 48.07271 51.49883 47.06121 45.36017 47.18530 46.13284 52.52240
## [99] 51.05314 48.09516
```









1.2 Shrinkage

In general, the smaller the sample size, the more extreme the sample mean, and the larger the shrinkage coefficient.