StatMod2 - Backfitting - Exercises 4

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1 Model

Constraining $\hat{\alpha}$ to be the sample average of y, removes issues of unidentifiability.

Initialize $\hat{\alpha} = 1/N \sum_{i=1}^{N} y_i, \hat{f}_j \equiv 0, \forall j$

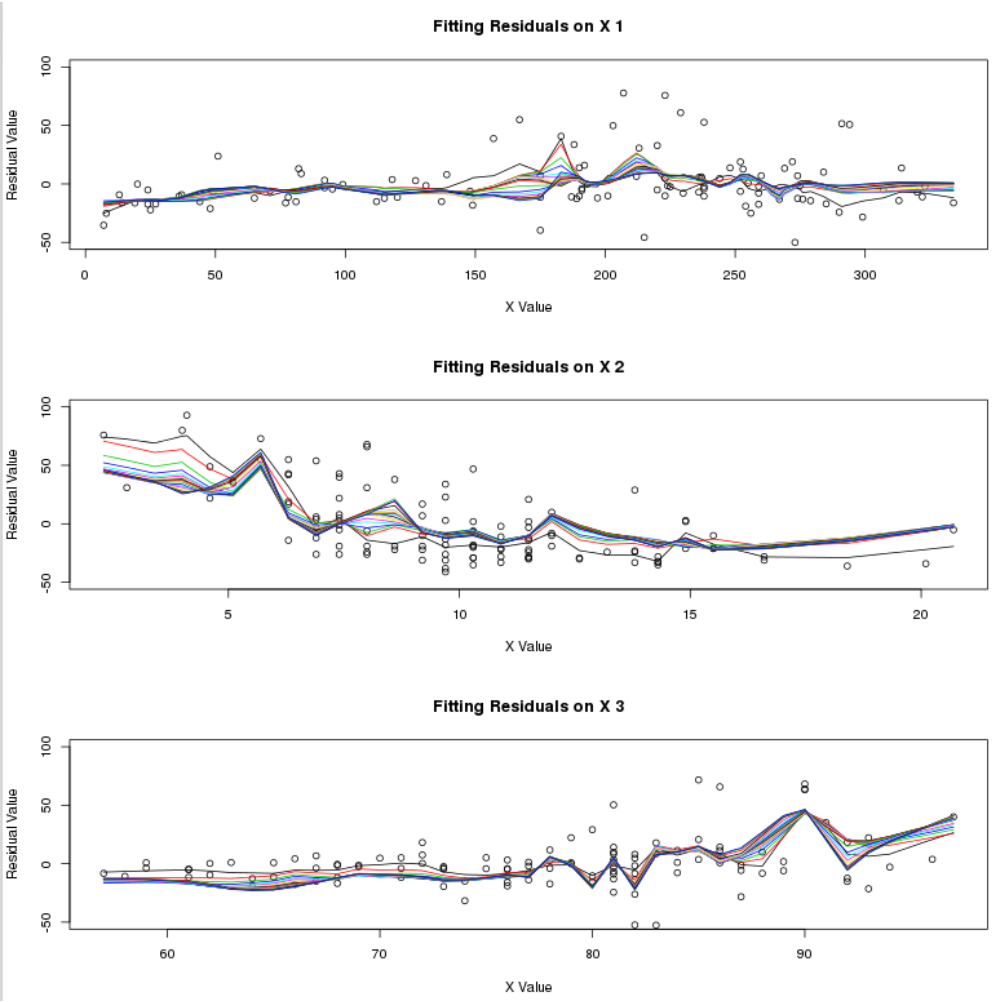
Do until $\hat{f}_j$ converge:

  For each predictor j:

    (a) $\hat{f}_j \leftarrow \text{Smooth}[\{y_i - \hat{\alpha} - \sum_{k \neq j} \hat{f}_k(x_{ik})\}_1^N]$ (backfitting step)

    (b) $\hat{f}_j \leftarrow \hat{f}_j - 1/N \sum_{i=1}^{N} \hat{f}_j(x_{ij})$ (mean centering of estimated function)

2 Results



```
> # StatMod2 - Backfitting HW - Exercices 4
>
> library(stats)
> #setwd("~/Google Drive/2. SPRING 2015/STAT MOD 2 - Prof Scott/backfitting")
> setwd("~/Documents/latex")
> data <- read.csv("air.csv")
> data <- data[order(data[, 1]), ]
> y <- as.matrix(data$Ozone)
> x <- data[,c("Solar.R","Wind","Temp")]
> n <- length(data[,1])
> p <- dim(x)[2]
> times <- 20
> # Initialize alpha, f, and mses.
> a <- mean(y)
> f <- array(0, dim=c(p, n))
> mses <- NULL
> Backfit <- function(q) {
+
+   # Do t iterations.
+   for (t in 1:times) {
+     # Rather than convergence, I record all MSEs for review, below.
+
+     # Do for each partial residual (kth predictor).
+     #for (k in c(1, 2, 3)) {
+     #for (k in c(3, 2, 1)) {
+     for (k in c(2, 1, 3)) {
+
+       # Remove one column, to make p-1 by 1 matrix.
+       f.minus.k <- as.matrix(f[-k,])
+
+       # Prepare value to smooth; the Yi - a - sum(...).
+       to.smooth <- rep(0, n)
+       for (i in 1:n) {
+         to.smooth[i] <- y[i] - a - sum(f.minus.k[,i])
+       }
+
+       # Fit kth residual against kth column of x.
+       partial.data <- cbind(y, x[k],to.smooth)
+       partial.data <- partial.data[order(partial.data[,2]),]
+       predicted <- rep(0, n)
+       predicted <- lowess(partial.data[,c(2,3)], f=.1)$y
+
+       # Plot stuff.
+       if (k==q) {
+         if (t==1) {
+           plot(t(x[k]), to.smooth, main=paste("Fitting Residuals on X", k),
+                ylim=c(-50,100), xlab="X Value", ylab="Residual Value")
+         }
+         lines(partial.data[,2], predicted, col=t)
+       }
```

```
+
+       # Attach predicted values to a resorted (by y) matrix.
+       to.sort <- cbind(partial.data, predicted)
+       resorted <- to.sort[order(to.sort[,1]),]
+       # Record K-th predictor for t-th iteration.
+       f[k,] <- resorted$predicted - mean(resorted$predicted)
+     }
+
+     mses[t] <- (1/n)*sum((y - a - colSums(f))^2)
+   }
+   return (mses)
+ }
> par(mfrow=c(3,1))
> for (q in 1:p) {
+   mses <- Backfit(q)
+ }
> summary(mses)

   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  207.9   210.9   229.0   240.1   257.6   366.0
```