StatMod 2 - Hierarchical-Shrinkage POLLS MODEL

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Given:

State: $i = 1, \dots, n$

person: $j = 1, \dots, n_i$

$$P(y_{ij} = 1) = \Phi(z_{ij}), \quad z_{ij} = \mu_i + x_{ij}'\beta$$

Add 1 to design matrix
for person-level intercept

$$\pi(\beta) = 1 \quad ; \quad \pi(\tau^2) = 1$$

$$\mu_i \overset{ind}{\sim} N(0, \tau^2)$$

Joint Posterior:

$$P(z_{ij}, \beta, \tau^2, \mu_i | y) \propto \prod_{i=1}^{n} \prod_{j=1}^{n_i} \left[\mathbb{1}(y_{ij}=1)\, \mathbb{1}(z_{ij}>0) + \mathbb{1}(y_{ij}=0)\, \mathbb{1}(z_{ij} \leq 0) \right] \exp\left\{ -\frac{1}{2} \left[z_{ij} - \mu_i - x_{ij}'\beta \right]^2 \right\}$$

$$\times \prod_{i=1}^{n} \left(\frac{1}{\tau^2} \right)^{\frac{1}{2}} \exp\left\{ -\frac{1}{2} \frac{\mu_i^2}{\tau^2} \right\}$$

Full Conditionals:

$$P(z_{ij} | \dots) \propto N(z_{ij} | \mu_i + x_{ij}'\beta, 1) \begin{cases} \text{trunc } + & \text{if } y = 1 \\ \text{trunc } - & \text{if } y = 0 \end{cases}$$

$$P(\mu_i | \dots) \propto N\left(\left(\frac{n_i}{1} + \frac{1}{\tau^2} \right)^{-1} \left(\sum_{j=1}^{n_i} (z_{ij} - x_{ij}'\beta) + \phi \right), \left(\frac{n_i}{1} + \frac{1}{\tau^2} \right)^{-1} \right)$$

$$P\left(\frac{1}{\tau^2} \Big| \dots \right) \propto \left(\frac{1}{\tau^2} \right)^{\frac{n}{2}} \exp\left\{ -\frac{1}{2} \left(\frac{1}{\tau^2} \right) \sum_{i=1}^{n} \mu_i^2 \right\}$$

$$\propto Ga\left(\frac{n}{2} + 1, + \frac{\sum_{i=1}^{n} \mu_i^2}{2} \right)$$

$$P(\beta | \dots) \propto \exp\left\{ -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n_i} \left[x_{ij}'\beta - (z_{ij} + \mu_i) \right]^2 \right\}$$

$$\propto \exp\left\{ -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n_i} \left[(x_{ij}'\beta)(x_{ij}'\beta) - 2 x_{ij}'\beta z_{ij} + 2 x_{ij}'\beta \mu_i + \dots \right] \right\}$$

$$\propto \exp\left\{ -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n_i} \left[\beta' x_{ij} x_{ij}'\beta - 2\beta x_{ij}'(z_{ij} - \mu_i) + \dots \right] \right\}$$