SSIE-523
Take-Home Midterm Exam

Due: 12:00pm (noon) Wednesday October 19th, 2016

Solve the problems given in the following page. Write your answers in a single PDF document and submit it through the TurnItIn Assignment page on the Blackboard by the deadline given above. No late submissions will be accepted.

NOTE: You MUST carefully read, understand, then include in the cover page, the following honor statement:

> ***I hereby declare that I have prepared the answers provided in this report all by myself. I did not seek any help or hint from any other person, either offline or online. I fully understand that if any substantial part of my answers are found to be too similar to answers of other students in class, or to be plagiarized from any other offline/online materials, then I will receive a zero point for this midterm exam and I will also be subject to further investigation and disciplinary actions according to the Watson School's and the University's code of academic honesty and other regulations.***

Any submission that does not have this statement will be rejected.

You can use your computer and Python. You can look at the textbook, the course slides, and any other offline/online materials to develop your answers. However, you must provide full reference information of such materials if you used them.

If you need clarifications about these problems, email the instructor (sayama@binghamton.edu). No response will be given to questions that directly or indirectly seek answers to the exam problems.

Q1.

Choose two topics from the following list:

a) Phase space visualization
b) Linear stability analysis
c) Eigenvalues and eigenvectors
d) Variable rescaling
e) Bifurcation diagram

For each topic, describe ***what it is <u>in your own words</u>***, and discuss ***how it can be relevant to your own domain of study***. Use about 1/2 page (single spaced) for each topic. (10x2 = 20 points)

Q2.

Consider the following continuous-time model of the dynamics of an epidemic system made of a growing susceptible population (x) and an infected population (y), with $x \geq 0$, $y \geq 0$, $a > 0$, $b > 0$, and $c > 0$. The carrying capacity of the environment is 1 (i.e., $0 \leq x + y \leq 1$).

$$\frac{dx}{dt} = ax(1-(x+y)) - bxy + cy$$

$$\frac{dy}{dt} = bxy - cy$$

It is assumed that (i) the susceptible population grows until the total population reaches the carrying capacity of the environment, (ii) the infection of the disease changes susceptible individuals into infected ones, and (iii) the infected individuals recover and become susceptible again at a certain rate. Answer the following.

1. Explain how each of the above three assumptions were represented in the equations. (3x3 = 9 points)
2. Find the equilibrium points. There are three such points. (5x3 = 15 points)
3. Conduct linear stability analysis for each equilibrium point and discuss the conditions under which they are stable/unstable. (5x3 = 15 points)
4. Identify the critical condition of parameter values at which a bifurcation occurs. (6 points)
5. Draw the phase spaces of this model using Python for several different parameter values to confirm the prediction of bifurcation derived in the previous question. Discuss how the system's behavior is affected by the parameter values. (15 points)

Q3.

Demonstrate a different form of bifurcation (and/or chaos, if possible) of any kind by modifying the model discussed above in Q2 in any way you want. Explain what you did, and present and discuss the result you obtained. (20 points)