SSIE 660: Stochastic Systems

Homework assignment 5: Hint

1. Solve Chapter 3. Problem 8.

   (a) $E[X] = E[X|\text{first roll is 6}]P[\text{first roll is 6}] + E[X|\text{first roll not is 6}]P[\text{first roll is not 6}]$

   (b) $E[X|Y=1]$: This is the case where the first outcome is a five.

   (c) $E[X|Y=5]$: This implies that the first four outcomes are not a five and the fifth outcome is a five. Given this assumption, the probability that the first outcome is a six is $1/5$, because we know that the first outcome is not a five (there are only 5 possibilities left).

2. Solve Chapter 3. Problem 15.

   Find $f_{X|Y=y}(x|y)$ first. Then, $E[X^2|Y=1] = \int x^2 f_{X|Y=y}(x|y)dx$.

3. Solve Chapter 3. Problem 20.

   (a) This problem asks you to find $f(x|\text{disease})$. Hint: $P(\text{disease}) = \int P(\text{disease}|x)f(x)dx = \int P(x)f(x)dx$

   (b) This problem asks you to find $f(x|\text{no disease})$

   (c) This problem asks you to explain $\frac{f(x|\text{disease})}{f(x|\text{no disease})}$.

4. Solve Chapter 3. Problem 30.

   It's a geometric distribution. Therefore, $E[N|X_0 = j] = \frac{1}{p(j)}$. Now, find $E[N]$.

5. Solve Chapter 3. Problem 36.

   $$E[X] = E[X|X \neq 0](1 - p_0) + E[X|X = 0]p_0$$

   Think about what $E[X|X = 0]$ is.

6. Solve Chapter 3. Problem 40.

   Let X denote the number of the door chosen, and let N be the total number of days spent in jail.

(a) This is the problem we already solved in class.

(b) Let N_i denote the number of additional days the prisoner spends after having initially chosen cell i.

$E[N] = \frac{1}{3}(2 + E[N_1]) + \frac{1}{3}(3 + E[N_2]) + \frac{1}{3}(0)$. Then, think about how to find $E[N_1]$ and $E[N_2]$.