

SSIE 660: Stochastic Systems

Homework assignment 7 - Key

1. Solve Chapter 4. Problem 14.

- (a) $\{0, 1, 2\}$ Recurrent
- (b) $\{0, 1, 2, 3\}$ Recurrent
- (c) $\{0, 2\}$ Recurrent, $\{1\}$ Transient, $\{3, 4\}$ Recurrent,
- (d) $\{0, 1\}$ Recurrent, $\{2\}$ Recurrent, $\{3\}$ Transient, $\{4\}$ Transient

2. Solve Chapter 4. Problem 30.

Letting X_n be 0 if the n^{th} vehicle is a car and letting it be 1 if the vehicle is a truck gives rise to a two-state Markov chain with transition probabilities.

$$P_{00} = 4/5, P_{01} = 1/5, P_{10} = 3/4, P_{11} = 1/4$$

The long-run proportions are the solutions of

$$r_0 = \frac{4}{5}r_0 + \frac{3}{4}r_1$$

$$r_1 = \frac{1}{5}r_0 + \frac{1}{4}r_1$$

$$r_0 + r_1 = 1$$

Solving these gives the result

$$r_0 = \frac{15}{19}, r_1 = \frac{4}{19}$$

That is, 4 out of every 19 cars is a truck.

3. Solve Chapter 4. Problem 42.

- (a) This is the long-run proportion of transitions that go from a state in A to one in A^C .
- (b) This is the long-run proportion of transitions that go from a state in A^C to one in A .
- (c) Between any two transitions from A to A^C there must be one from A^C to A . Similarly between any two transitions from A^C to A there must be one from A to A^C . Therefore, the long-run proportion of transitions that are from A to A^C must be equal to the long-run proportion of transitions that are from A^C to A .

4. Solve Chapter 4. Problem 52.

Let the state be the successive zonal pickup locations. Then, $P_{A,A} = .6, P_{B,A} = .3$. The long-run proportions of pickups that are from each zone are

$$\pi_A = .6\pi_A + .3\pi_B = .6\pi_A + .3(1 - \pi_A)$$

Therefore, $\pi_A = 3/7, \pi_B = 4/7$. Let X denote the profit in a trip. Conditioning on the location of the pickup gives

$$E[X] = \frac{3}{7}E[X|A] + \frac{4}{7}E[X|B] = \frac{3}{7} [.6 * 6 + .4 * 12] + \frac{4}{7} [.3 * 12 + .7 * 8] = 62/7$$