SSIE 660: Stochastic Systems

Homework assignment 8 - Hint

1. Solve Chapter 5. Problem 4.

   B and C must finish service before A.
   (b) Service time for A: 3, Service time for B and C: 1, respectively.

   (c) (The probability that B finishes the service before A) multiplied by (The probability that C finishes the service before A)

2. Solve Chapter 5. Problem 6.

   Condition on which server initially finishes first. Now,

   $$P[\text{Smith is not last}|\text{server 1 finishes first}] = P[\text{server 1 finishes before server 2}]^2$$

   Repeat for the case where server 2 finishes first.

3. Solve Chapter 5. Problem 15.

   Let T_i denote the time between $(i-1)^{th}$ and the i^{th} failure. Then the T_i are independent with T_i being exponential with rate $(101-i)/200$.

4. Solve Chapter 5. Problem 16.

   (a) Suppose i and j are initially begun, with k waiting for one of them to be completed. Then,
   $$E[T_i] + E[T_j] + E[T_k] = \frac{1}{\mu_i} + \frac{1}{\mu_j} + \frac{1}{\mu_i + \mu_j} + \frac{1}{\mu_k} = \sum_{i=1}^{3} \frac{1}{\mu_i} + \frac{1}{\mu_i + \mu_j}$$
   Think about how to minimize this.

   (b) Letting X_i be the processing time of job i. Then, $\sum_{i=1}^{3} X_i$ = Time for which two processors working + Time for which one processor working.

   (c) $E[S]$ = (expected time for job 1 or 2 * prob. that the last one is job 1 or 2) + (expected time for job 3 * prob. that the last one is job 3)

   (d) $P_{1,2}(\mu) = \dfrac{\lambda}{\mu + \lambda}$. What is $P_{1,3}(\mu)$?

5. Solve Chapter 5. Problem 37.

   This is the summation of 1) expected time of failure and 2) expected time until the machine is found to be failed, once it fails. Consider the memoryless effect.

6. Solve Chapter 5. Problem 42.

   (a) $E[S_4] =$ Expected time of T_1 + Expected time of T_2 + Expected time of T_3 + + Expected time of T_4

   (b) $E[S_4|N(1) = 2] = 1 + E[\text{time for 2 more events}]$