SSIE 660: Stochastic Systems
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In-class Exercise: Chapters 4 and 5

1. Consider the following transition matrix for states 1, 2, 3, and 4.

$$P = \begin{bmatrix} 1-p & 0 & p & 0 \\ q & 0 & q & 0 \\ 0 & 0 & 1-p & p \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

   (a) Find the value of q.

   (b) Classify the states of the chain. You may wish to consider different values of p.

   (c) What is the probability that the chain is in state 4 at the 4^{th} step given that it started in state 2?

2. At the beginning of each day, a machine is inspected to determine its working condition. The equipment can be found in one of four working conditions indexed by 1, 2, 3, and 4. Working condition i is better than $i+1, i = 1, 2, 3$. The equipment deteriorates over time. If the present working condition is i and if no repair is done, then its working condition at the beginning of the next day is j with probability q_{ij} given below.

		j = 1	2	3	4
	1	0.75	0.2	0.05	0
i	2	0	0.5	0.2	0.3
	3	0	0	0.6	0.4

Working condition 4 represents a malfunction which requires a repair that will restore the machine to working condition 1, at the beginning of the next day. If the machine is found to be working condition 3, then a preventive maintenance is done 60% of the time, which will restore the machine to working condition 1, at the beginning of the next day.

(a) Define the states and find the one-step transition probability matrix.

(b) Assume that the machine starts in working condition 1 at the beginning of today ($X_0 = 1$). Then, what is $P[X_3 = 2, X_2 = 1, X_1 = 3]$?

(c) What is $P[X_4 = 1, X_3 = 4, X_2 = 3 | X_1 = 2]$?

(d) What is $P[X_4 = 1, X_3 = 2 | X_1 = 2]$?

3. The demand per day of a certain item follows the probability distribution given below:

Demand	Probability
0	0.2
1	0.5
2	0.3

The ordering policy is as follows: " Order two items, if the inventory at the end of day is 0; otherwise, do not order"

No backorders are allowed. The states of the system are the inventories at the beginning of a day after receipt of ordered quantities.

(a) Obtin all the one-step transition probabilities, using the information given above.

(b) You are given the following. $\pi_1 = 0.385$. The carrying cost per day per unit at the end of a day is \$50.00. and the ordering cost per order is \$100.00. What is the long-run expected cost per day consisting of the expected carrying cost per day and the expected ordering cost per day?

4. In a certain system, a customer must first be served by server 1, and then by server 2. The service times at server i are exponential with rate $\mu_i, i = 1, 2$. An arrival finding server 1 is busy waits in line for that server. Upon completion of service at server 1, a customer either enters service with server 2 if that server is free or else remains with server 1 (blocking any other customer from entering service) until server 2 is free. Customers depart the system after being served by server 2. Suppose that you arrive to find two others in the system, one being served by server 1 and one by server 2.

   What is the expected total time you spend in the system?

5. A subway station has both local and express service, on opposite sides of the same platform. Local and express trains arrive independently according to a Poisson process with rates $1/5$ and $1/15$ per minute, respectively. Both trains stop at your destination, with transit times of 17 minutes for a local train and 11 minutes for an express train.

   (a) What is the waiting time distribution until the next local train arrives?

   (b) What is the waiting time distribution until the next express train arrives?

   (c) What is the waiting time distribution until the next train (either local or express) arrives?

   (d) What is the probability that the next local train arrives before the express?

   (e) If the next train that arrives is a local, should you board that train or wait for an express, assuming that your objective is to minimize your expected travel time?