SSIE 660: Applied Stochastic Processes

Homework assignment 9 – Hint

Nov. 30, 2016
Due: Dec 7, 2016 Before class starts

1. Solve Chapter 6. Problem 4.

   Let $N(t)$ denote the number of customers in the station at time t. Then $\{N(t)\}$ is a birth and death process. The entering rate is the arrival rate multiplied by the probability of entering the system.

2. Solve Chapter 6. Problem 8.

   The number of failed machines is a birth and death process with

   $$\lambda_0 = 2\lambda, \lambda_1 = \lambda,$$
   $$\lambda_n = 0, n > 1$$
   $$\mu_1 = \mu_2 = \mu,$$
   $$\mu_n = 0, n \neq 1, 2$$

   Find more parameters such as v_0, q_{01}, etc. Then write the Kolmogorov backward equations using such parameters.

3. Solve Chapter 6. Problem 13.

   With the number of customers in the shop as the state, we get a birth and death process with

   $$\lambda_0 = \lambda_1 = 3, \mu_1 = \mu_2 = 4$$

   Therefore

   $$P_1 = \frac{3}{4}P_0, P_2 = \frac{3}{4}P_1 = \left(\frac{3}{4}\right)^2 P_0$$

   And since $\sum_0^2 P_i = 1$, we get

   $$P_0 = \left(1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2\right)^{-1} = \frac{16}{37}$$

4. Solve Chapter 6. Problem 14.

Letting the number of cars in the station be the states, we have a birth and death process with

$$\lambda_0 = \lambda_1 = \lambda_2 = 20$$
$$\lambda_i = 0, i > 2$$
$$\mu_1 = \mu_2 = 12 \text{ (12 cars per hour)}$$

Hence,

$$P_1 = \frac{5}{3}P_0$$
$$P_2 = \frac{5}{3}P_1 = \left(\frac{5}{3}\right)^2 P_0$$
$$P_3 = \frac{5}{3}P_2 = \left(\frac{5}{3}\right)^3 P_0$$

5. Solve Chapter 6. Problem 15.
With the number of customers in the system as the state, we get a birth and death process with

$$\lambda_0 = \lambda_1 = \lambda_2 = 3$$
$$\lambda_i = 0, i > 2$$
$$\mu_1 = 2, \mu_2 = \mu_3 = 4$$

Therefore, the balance equations reduce to

$$P_1 = \frac{3}{2}P_0, P_2 = \frac{3}{4}P_1 = \frac{9}{8}P_0, P_3 = \frac{3}{4}P_2 = \frac{27}{32}P_0$$

And therefore,

$$P_0 = \left(1 + \frac{3}{2} + \frac{9}{8} + \frac{27}{32}\right)^{-1} = \frac{32}{143}$$

6. Solve Chapter 6. Problem 19.

There are four states. Let state 0 mean that no machines are down, state 1 that machine 1 is down and 2 is up, state 2 that machine 1 is up and 2 is down, and 3 that both machines are down (machine 1 is in service).