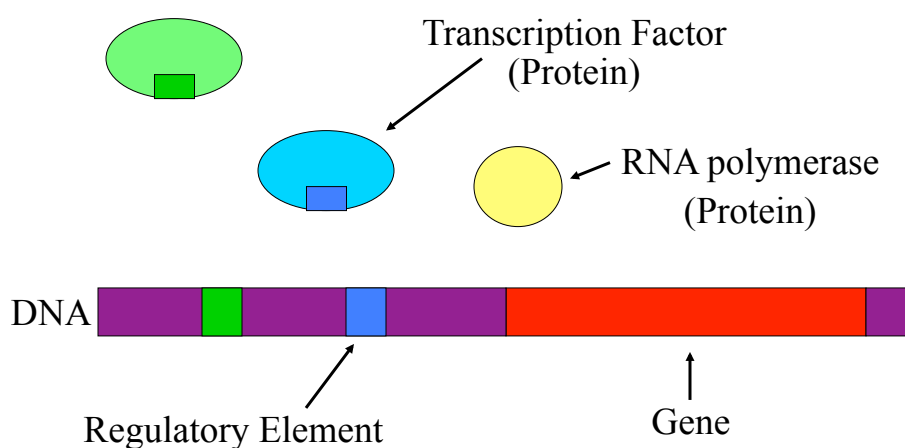


# ONE SHOULD NEVER MISTAKE PATTERN FOR MEANING

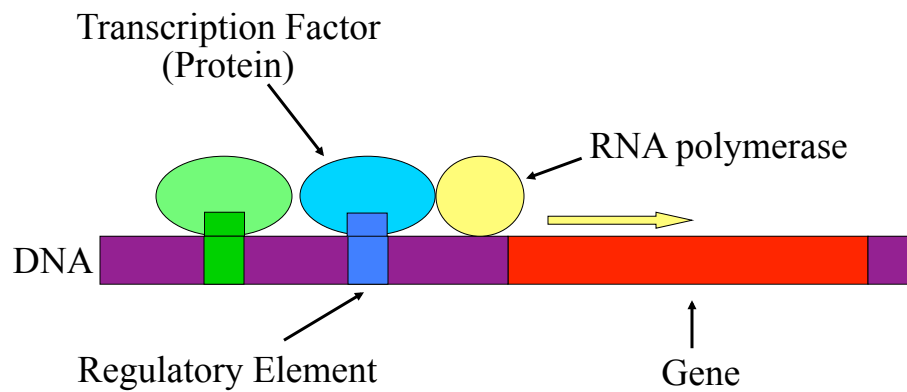
IAIN BANKS

## Regulation of Genes



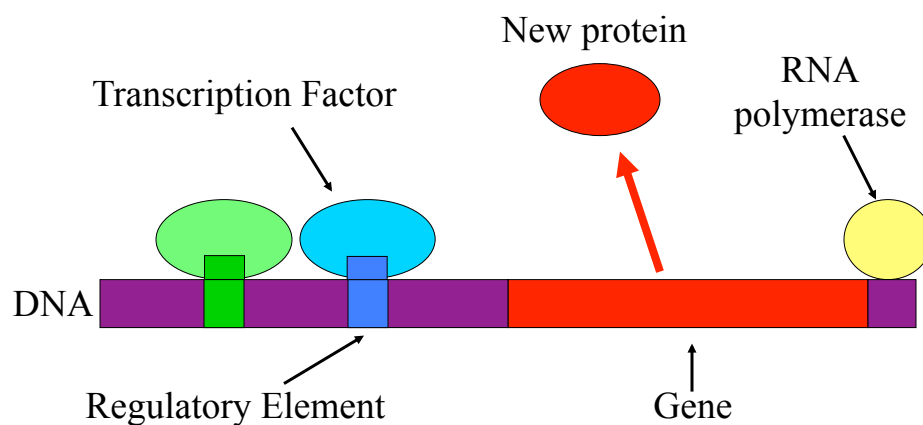
source: [M. Tompa](#), U. of Washington

## Regulation of Genes



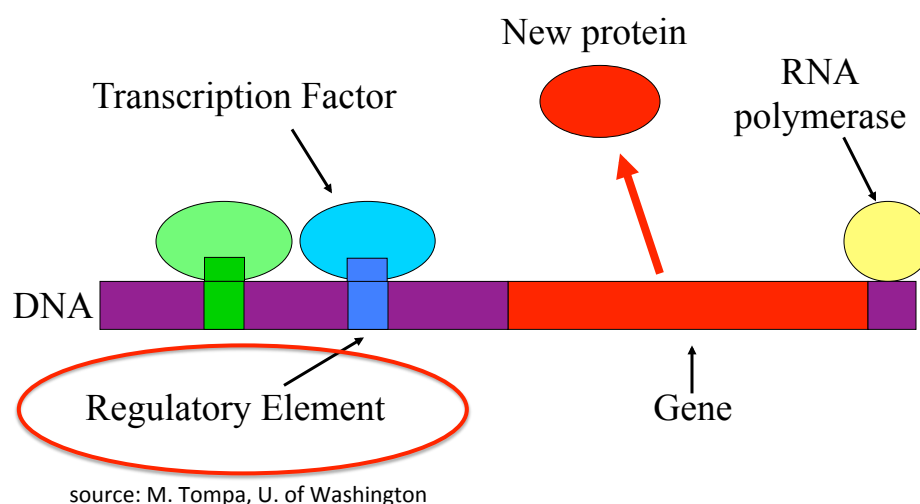
source: [M. Tompa](#), U. of Washington

## Regulation of Genes



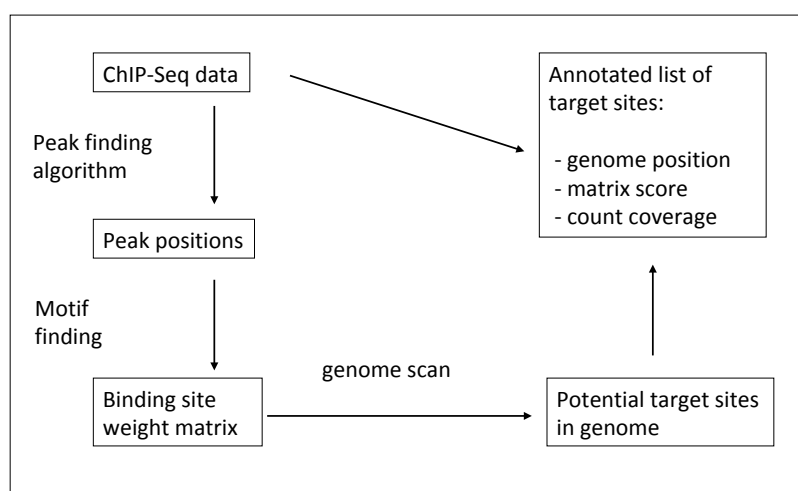
source: [M. Tompa](#), U. of Washington

## Regulation of Genes



## Selective occupancy of genomic TF target sites

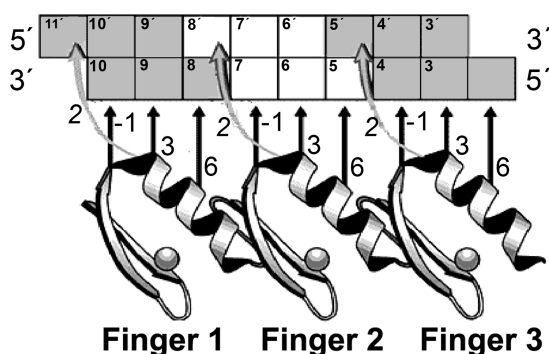
General data processing pipeline:



## What is a motif?

- A subsequence that occurs in multiple sequences with a biological importance (e.g. at sites of protein binding).
- Motifs can be totally constant (substring) or have variable elements (pattern).
- Protein Motifs often result from structural features.
- DNA Motifs (regulatory elements)
  - Binding sites for proteins
  - Short sequences (5-25 bps)
  - A handful to millions of binding sites in the genome

### Cys<sub>2</sub>His<sub>2</sub> Zinc Finger: Canonical DNA binding model

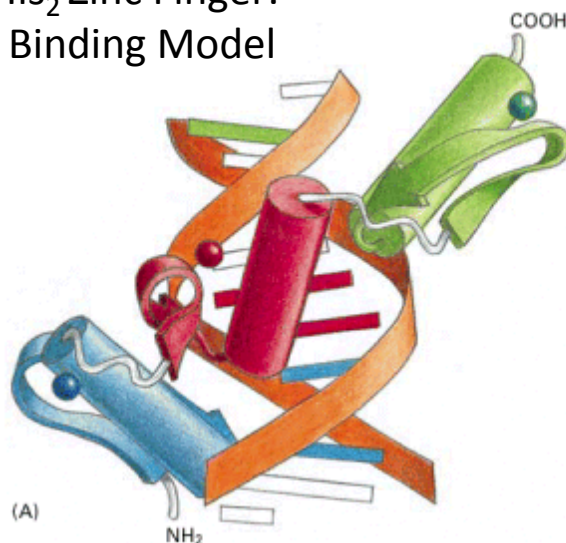


**Finger 1   Finger 2   Finger 3**

Residues at positions 6, 3, 2, and -1 (relative to the beginning of the  $\alpha$ -helix) at each finger interact with adjacent nucleotides in the DNA molecule (interactions shown with arrows).

Kaplan. *et al.*, PLoS Comput Biol, 2005

## Cys<sub>2</sub>His<sub>2</sub> Zinc Finger: DNA Binding Model



source: Molecular Biology of the Cell (4<sup>th</sup> ed.), A. Johnson, et al.

9

## Sequence motif – a pattern of nucleotide or amino acid sequences

### DNA motif:



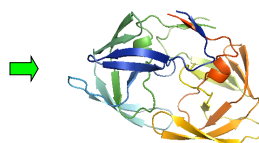
### Transcription Factor Binding Sites (TFBS)

### Protein motif:

DREDDPafly : NSASAT **YRPFKVRHFCIFPIAMAGSNLLIHLDTIDQNDLIYVERDMNFAQEVGLG** FL-LYC **DDHSDATYILQKLLAMTRSDFPQSDLLIKFAK**

DREDDPbfly : NSASAT **YRPFKVRHFCIFPIAMAGSNLLIHLDTIDQNDLIYVERDMNFAQEVGLG** FL-LYC **DDHSDATYILQKLLAMTRSDFPQSDLLIKFAK**

DREDDPcfly : NSASAT **YRPFKVRHFCIFPIAMAGSNLLIHLDTIDQNDLIYVERDMNFAQEVGLG** FL-LYC **DDHSDATYILQKLLAMTRSDFPQSDLLIKFAK**



## Motif Representations

- Consensus sequence: a single string with the most likely sequence

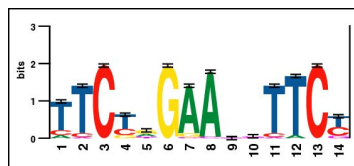
TTCTGGAACCTTCT

- Regular expression: a string with wildcards, constrained selection (+/- wildcards & ambiguity)

YTCYXGAAXTTCY

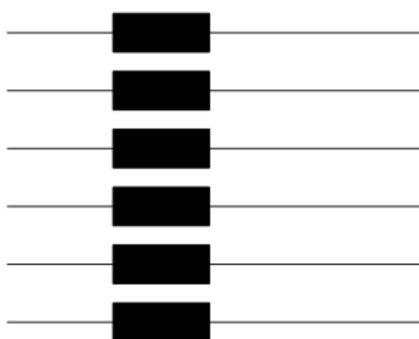
- Profile: a list of the letter frequencies at each position

- Sequence Logo:
  - graphical depiction of a profile
  - conservation of elements in a motif.

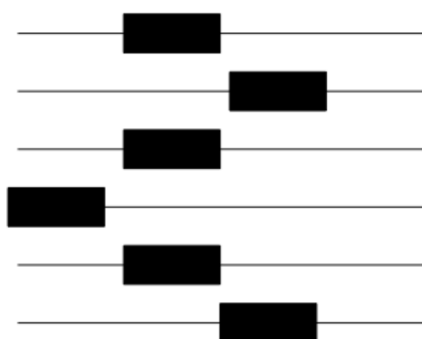


## How do we describe a motif?

YES!



NO



Given a multiple sequence alignment, it's trivial

C C T C C T A A T C C C T C										Majority
10										
G	C	C	C	T	A	A	T	C	C	eve2 01.SEQ
C	C	A	T	C	T	A	A	T	C	hbp2 07.SEQ
T	T	G	G	C	T	A	A	T	C	eve2 02.SEQ
G	C	C	A	C	T	A	A	T	C	btd 06.SEQ
C	A	A	C	G	T	A	A	T	C	hbp2 01.SEQ
A	A	T	T	A	T	A	A	T	C	sal 05.SEQ
T	G	T	C	C	T	A	A	T	C	hbp2 05.SEQ
T	C	C	T	T	A	A	A	T	C	kr 04.SEQ
G	C	T	G	C	T	A	A	G	C	hbp2 02.SEQ
T	G	C	G	G	T	A	A	T	C	btd 02.SEQ
C	C	T	C	G	T	A	A	T	C	btd 01.SEQ
A	T	G	C	A	T	A	A	T	C	btd 03.SEQ
C	G	C	A	T	T	A	A	T	C	btd 04.SEQ
C	G	G	G	G	T	A	A	T	C	btd 05.SEQ
G	A	C	T	A	T	A	A	T	C	eve2 03.SEQ
A	C	T	A	A	T	A	A	T	C	eve2 04.SEQ
C	G	T	G	T	T	A	A	T	C	eve2 05.SEQ
T	C	C	G	C	T	A	A	G	C	hbp2 03.SEQ
C	A	T	C	C	A	A	A	T	C	hbp2 04.SEQ
A	T	C	C	G	T	G	A	T	C	hbp2 06.SEQ
A	A	A	T	T	T	A	A	T	C	kr 01.SEQ
A	C	A	A	A	T	A	A	T	C	kr 02.SEQ
A	A	G	C	T	T	A	A	T	C	kr 03.SEQ
T	C	T	G	T	T	A	A	T	C	kr 05.SEQ
G	A	A	C	T	A	A	A	T	C	kr 06.SEQ
G	C	G	G	C	T	A	A	T	C	runt 01.SEQ
C	T	G	C	T	T	A	A	T	C	runt 02.SEQ
A	A	T	C	T	T	A	A	T	C	runt 03.SEQ
T	T	C	G	A	T	A	A	G	C	sal 01.SEQ
C	G	G	A	C	A	A	A	T	C	sal 02.SEQ
G	C	T	G	C	A	A	A	T	C	sal 03.SEQ
T	A	T	G	C	A	A	A	T	C	sal 04.SEQ
C	T	A	T	T	T	A	A	C	C	tll 01.SEQ
C	G	T	C	T	T	A	A	C	C	tll 02.SEQ

T A A T C C C										Motif
G	C	C	C	T	A	A	T	C	C	eve2 01.SEQ
C	C	A	T	C	T	A	A	T	C	hbp2 07.SEQ
T	T	G	G	C	T	A	A	T	C	eve2 02.SEQ
G	C	C	A	C	T	A	A	T	C	btd 06.SEQ
C	A	A	C	G	T	A	A	T	C	hbp2 01.SEQ
A	A	T	T	A	T	A	A	T	C	sal 05.SEQ
T	G	T	C	C	T	A	A	T	C	hbp2 05.SEQ
T	C	C	T	T	A	A	A	T	C	kr 04.SEQ
G	C	T	G	C	T	A	A	G	C	hbp2 02.SEQ
T	G	C	G	G	T	A	A	T	C	btd 02.SEQ
C	C	T	C	G	T	A	A	T	C	btd 01.SEQ
A	T	G	C	A	T	A	A	T	C	btd 03.SEQ
C	G	C	A	T	T	A	A	T	C	btd 04.SEQ
C	G	G	G	G	T	A	A	T	C	btd 05.SEQ
G	A	C	T	A	T	A	A	T	C	eve2 03.SEQ
A	C	T	A	A	T	A	A	T	C	eve2 04.SEQ
C	G	T	G	T	T	A	A	T	C	eve2 05.SEQ
T	C	C	G	C	T	A	A	G	C	hbp2 03.SEQ
C	A	T	C	C	A	A	A	T	C	hbp2 04.SEQ
A	T	C	C	G	T	G	A	T	C	hbp2 06.SEQ
A	A	A	T	T	T	A	A	T	C	kr 01.SEQ
A	C	A	A	A	T	A	A	T	C	kr 02.SEQ
A	A	G	C	T	T	A	A	T	C	kr 03.SEQ
T	C	T	G	T	T	A	A	T	C	kr 05.SEQ
G	A	A	C	T	A	A	A	T	C	kr 06.SEQ
G	C	G	G	C	T	A	A	T	C	runt 01.SEQ
C	T	G	C	T	T	A	A	T	C	runt 02.SEQ
A	A	T	C	T	T	A	A	T	C	runt 03.SEQ
T	T	C	G	A	T	A	A	G	C	sal 01.SEQ
C	G	G	A	C	A	A	A	T	C	sal 02.SEQ
G	C	T	G	C	A	A	A	T	C	sal 03.SEQ
T	A	T	G	C	A	A	A	T	C	sal 04.SEQ
C	T	A	T	T	T	A	A	C	C	tll 01.SEQ
C	G	T	C	T	T	A	A	C	C	tll 02.SEQ

("Consensus String")

W A A T C C N												← Motif (Regular Expression)	
W = T or A													
N = A,C,G,T													
	G	C	C	C	T	A	A	T	C	C	C	T	eve2 01.SEQ
	C	C	A	T	C	T	A	A	T	C	C	C	hbp2 07.SEQ
	T	T	G	G	C	T	A	A	T	C	C	C	eve2 02.SEQ
	G	C	C	A	C	T	A	A	T	C	C	C	btd 06.SEQ
	C	A	A	C	G	T	A	A	T	C	C	C	hbp2 01.SEQ
	A	A	T	T	A	T	A	A	T	C	C	C	sal 05.SEQ
	T	G	T	C	C	T	A	A	T	C	C	A	hbp2 05.SEQ
	T	C	C	T	T	A	A	A	T	C	C	C	kr 04.SEQ
	G	C	T	G	C	T	A	A	G	C	T	G	hbp2 02.SEQ
	T	G	C	G	G	T	A	A	T	C	C	G	btd 02.SEQ
	C	C	T	C	G	T	A	A	T	C	C	T	btd 01.SEQ
	A	T	G	C	A	T	A	A	T	C	C	A	btd 03.SEQ
	C	G	C	A	T	T	A	A	T	C	C	G	btd 04.SEQ
	C	G	G	G	G	T	A	A	T	C	C	T	btd 05.SEQ
	G	A	C	T	A	T	A	A	T	C	G	C	eve2 03.SEQ
	A	C	T	A	A	T	A	A	T	C	T	C	eve2 04.SEQ
	C	G	T	G	T	T	A	A	T	C	C	G	eve2 05.SEQ
	T	C	C	G	C	T	A	A	G	C	T	C	hbp2 03.SEQ
	C	A	T	C	C	A	A	A	T	C	C	A	hbp2 04.SEQ
	A	T	C	C	G	T	G	A	T	C	C	T	hbp2 06.SEQ
	A	A	A	T	T	T	A	A	T	C	C	G	kr 01.SEQ
	A	C	A	A	A	T	A	A	T	C	C	A	kr 02.SEQ
	A	A	G	C	T	T	A	A	T	C	A	C	kr 03.SEQ
	T	C	T	G	T	T	A	A	T	C	T	C	kr 05.SEQ
	G	A	A	C	T	A	A	A	T	C	C	G	kr 06.SEQ
	G	C	G	G	C	T	A	A	T	C	G	C	runt 01.SEQ
	C	T	G	C	T	T	A	A	T	C	C	G	runt 02.SEQ
	A	A	T	C	T	T	A	A	T	C	C	T	runt 03.SEQ
	T	T	C	G	A	T	A	A	G	C	C	G	sal 01.SEQ
	C	G	G	A	C	A	A	A	T	C	C	T	sal 02.SEQ
	G	C	T	G	C	A	A	A	T	C	C	G	sal 03.SEQ
	T	A	T	G	C	A	A	A	T	C	C	G	sal 04.SEQ
	C	T	A	T	T	T	A	A	C	C	C	T	t11 01.SEQ
	C	G	T	C	T	T	A	A	C	C	C	T	t11 02.SEQ

### Alternative way to represent motif

G	C	C	C	C	T	A	A	T	C	C	C	T	eve2 01.SEQ
C	C	A	T	C	T	A	A	T	C	C	C	T	hbp2 07.SEQ
T	T	G	G	C	T	A	A	T	C	C	C	A	eve2 02.SEQ
G	C	C	A	C	T	A	A	T	C	C	C	G	btd 06.SEQ
C	A	A	C	G	T	A	A	T	C	C	C	A	hbp2 01.SEQ
A	A	T	T	A	T	A	A	T	C	C	C	T	sal 05.SEQ
T	G	T	C	C	T	A	A	T	C	C	A	G	hbp2 05.SEQ
T	C	C	T	T	A	A	A	T	C	C	C	T	kr 04.SEQ
G	C	T	G	C	T	A	A	G	C	T	G	G	hbp2 02.SEQ

1	1	9	9	0	0	0	1	A
6	0	0	0	0	9	8	7	C
1	0	0	0	1	0	0	1	G
1	8	0	0	8	0	1	0	T

Position weight matrix (PWM)  
 Or simply, "weight matrix"  
 Sometimes also referred to as Position Specific Scoring Matrix (PSSM)  
 (in this instance it's a frequency matrix!)



## Consensus Model

Alignment

```

a G g t a c T t
C c A t a c g t
a c g t T A g t
a c g t C c A t
C c g t a c g G

```

- Line up the patterns by their start indexes

$$s = (s_1, s_2, \dots, s_t)$$

Profile

A	3	0	1	0	3	1	1	0
C	2	4	0	0	1	4	0	0
G	0	1	4	0	0	0	3	1
T	0	0	0	5	1	0	1	4

- Construct matrix profile with frequencies of each nucleotide in columns

Consensus

A C G T A C G T

- Consensus nucleotide in each position has the highest score in column

## PWM Model

Alignment

```

a G g t a c T t
C c A t a c g t
a c g t T A g t
a c g t C c A t
C c g t a c g G

```

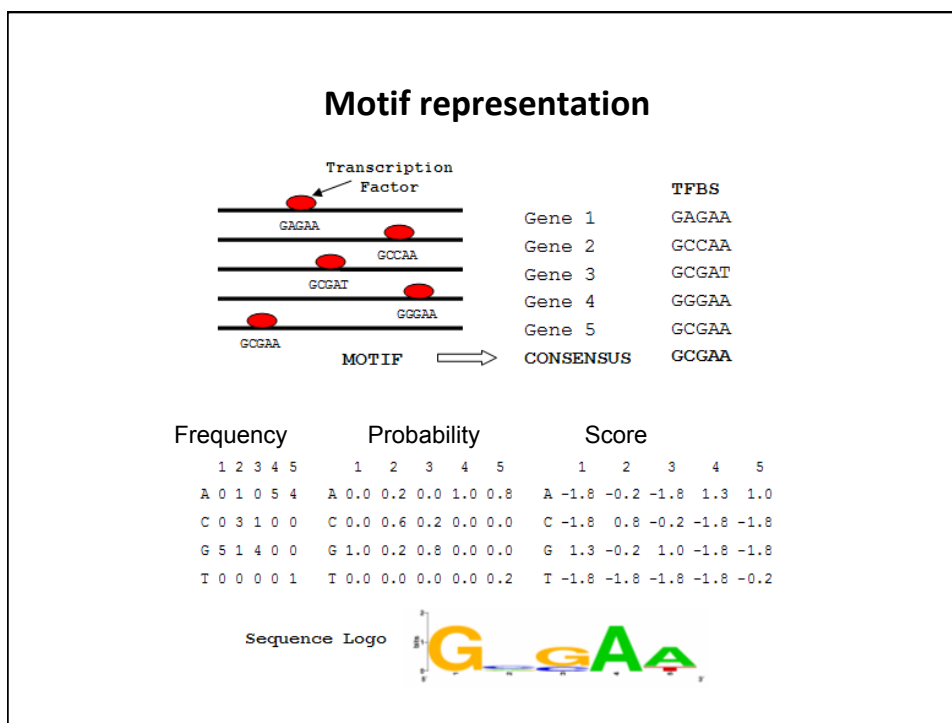
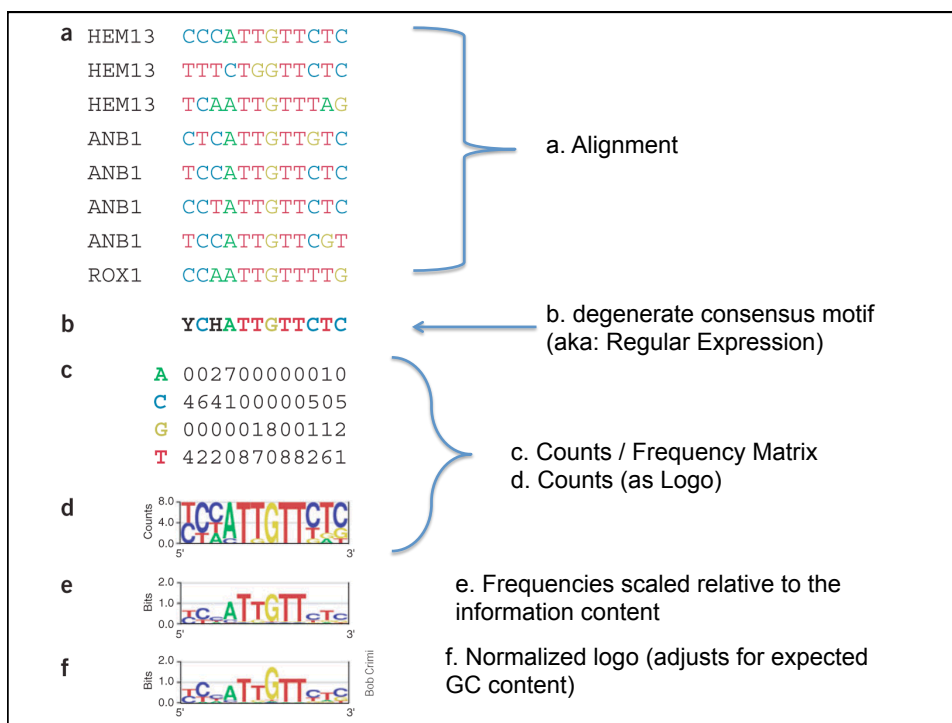
- Line up the patterns by their start indexes

$$s = (s_1, s_2, \dots, s_t)$$

Profile

A	3	0	1	0	3	1	1	0
C	2	4	0	0	1	4	0	0
G	0	1	4	0	0	0	3	1
T	0	0	0	5	1	0	1	4

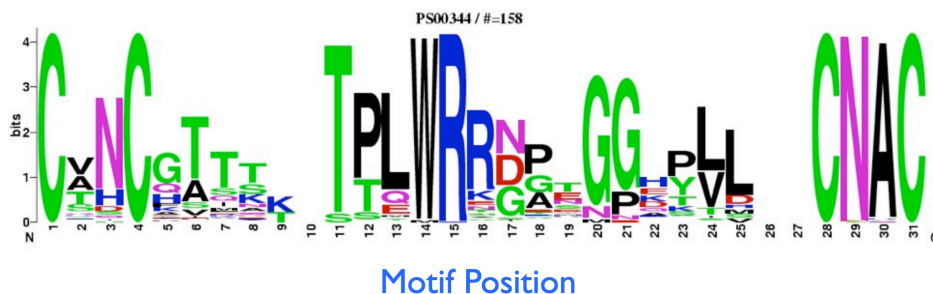
- Construct matrix profile with frequencies of each nucleotide in columns
- Convert the frequency matrix to scores – but how?





Height of letter  $\approx$  fraction  
of time that letter is  
observed at that position.

(Height of all the letters in  
a column  $\approx$  to how  
conserved the column is)



## What is a “bit” in this context?

Each position scaled with the  
information content of the base  
frequencies at that position:

$$I_i = 2 + \sum_b f_{b,i} \log_2 f_{b,i}$$

Positions are calculated using log likelihoods, the score of  
a sequence can be calculated by adding (rather than  
multiplying as is necessary with probabilities).

Perfect conservation = 2 bits  
Every base equal = 0 bits

Assumes all four bases occur equally likely.

## What is a “bit” in this context?

Better measure adjust for background bias:

$$I_{seq}(i) = -\sum_b f_{b,i} \log_2 \frac{f_{b,i}}{p_b}$$

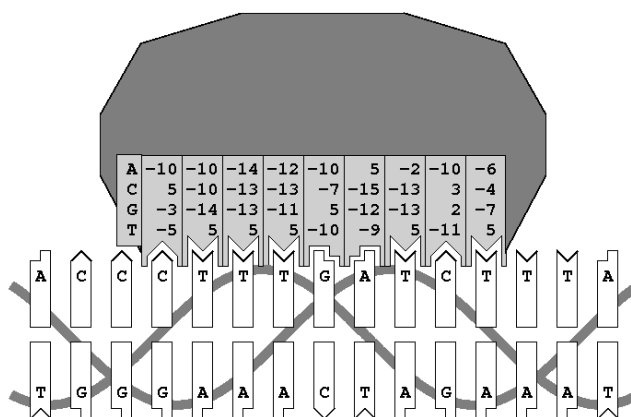
Aka. “relative entropy”

Notice its equivalent to a log-likelihood ratio!

Play with Logos:

Steven Brenner’s WebLogo (<http://weblogo.berkeley.edu/>)  
 enoLOGOS3 (<http://biodev.hgen.pitt.edu/enologos>)

## Physical interpretation of an weight matrix



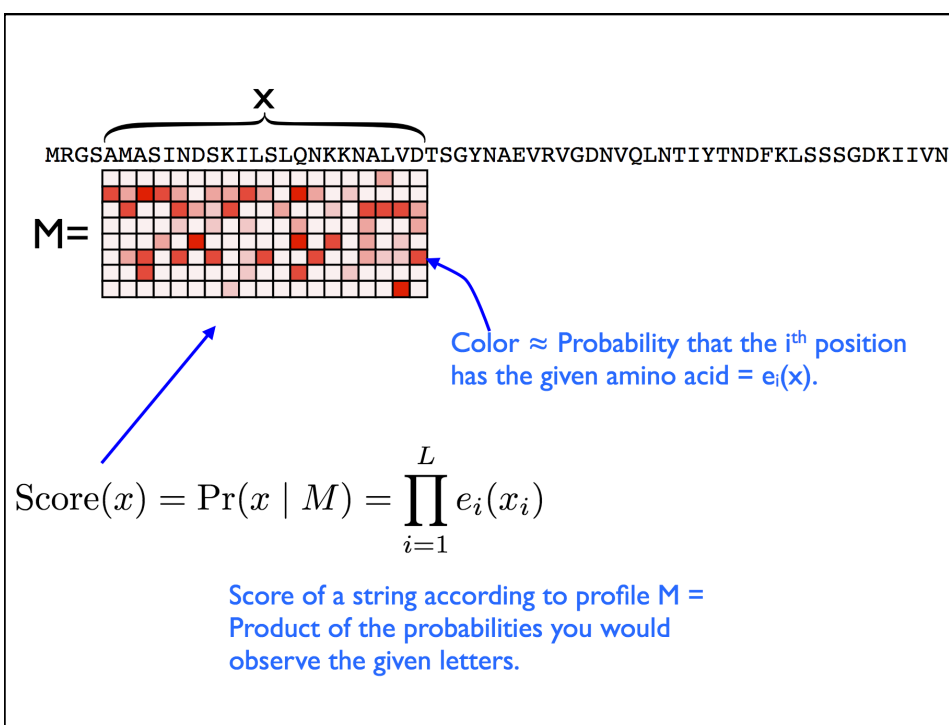
Weight matrix elements represent relative binding energies between DNA base-pairs and protein surface areas (base-pair acceptor sites).

A weight matrix column describes the base preferences of a base-pair acceptor site.

Represents sequence of one strand, though physical interactions may be with other strand.

## Motif scanning

- Given a motif (e.g., consensus string, regular expression, or weight matrix), find the binding sites in an input sequence (e.g. a genome)
- For consensus string, problem is trivial
  - For each position in input sequence, check if substring starting at position  $i$  matches the motif.
- For weight matrix, not quite so trivial



For example ...

- Given a string  $s$  of length = 7

- $x = x_1 x_2 \dots x_l$

- $\Pr(x \mid M) = \prod_{i=1}^L e_i(x_i)$

- Example:

$$\Pr(\text{CTAATCCG}) = 0.67 \times 0.89 \times 1 \times 1 \times 0.89 \times 1 \times 0.89 \times 0.11$$

.11	.11	1	1	0	0	0	.11	<b>A</b>
.67	0	0	0	0	1	.89	.78	<b>C</b>
.11	0	0	0	.11	0	0	.11	<b>G</b>
.11	.89	0	0	.89	0	.11	0	<b>T</b>

Probability of each base  
in each column

$W_{\beta k}$  = probability of base  $\beta$  in column  $k$

Here we are looking at probabilities, so we must multiply!

With normalized “scores,”  
the positions are additive.

	1	2	3	4	5	6	7	8	9
<b>A</b>	-10	-10	-14	-12	-10	5	-2	-10	-6
<b>C</b>	5	-10	-13	-13	-7	-15	-13	3	-4
<b>G</b>	-3	-14	-13	-11	5	-12	-13	2	-7
<b>T</b>	-5	5	5	5	-10	-9	5	-11	5

Strong Binding site    C    T    T    T    G    A    T    C    T  
                              5   + 5   + 5   + 5   + 5   + 5   + 5   + 3   + 5 = 43

Random Sequence    A    C    G    T    A    C    G    T    A  
                              -10   -10   -13   + 5   -10   -15   -13   -11   - 6 = -83

(This is similar to scoring matrices for sequence alignment!)

## Binding sites from a weight matrix motif

- Given sequence  $S$  (e.g., 1000 base-pairs long)
- For each substring  $x$  of  $S$ ,
  - Compute  $\Pr(x|M)$
  - If  $\Pr(x|M) > \text{some threshold}$ , call that a binding site
- Look at  $S$ , *as well as its reverse complement*

## But what threshold?

- In reality, every protein binds to every sequence with at least **some** affinity. But poorer matches to matrix are weaker (i.e. less frequent and transient) binding.
- Therefore, cutoff is related to how “strong” a site you are looking for ... depends on protein (TF), its concentration and the question at hand.
- However, empirically 60% of max is often used as a cutoff. This is another arbitrary but commonly used threshold (like 0.05 as a p-value cutoff).

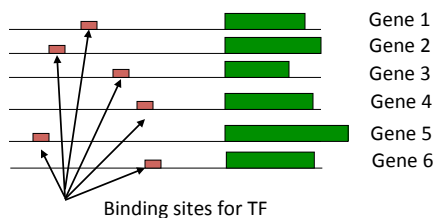


## A couple things to ponder ...

- Note how the scoring scheme used for motifs resembles the scoring schemes we discussed for alignment, only now they are position dependent.
- Also notice that a position specific scoring matrix is essentially a very simple HMM.

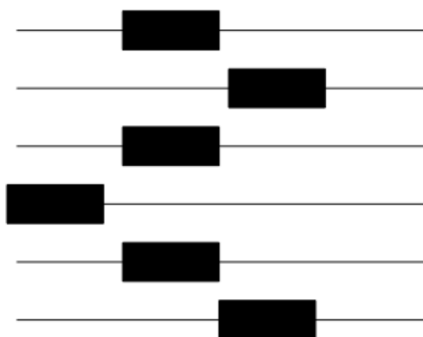
## We will leverage “patterns” to identify transcription factor motifs.

- Say a transcription factor (TF) controls six different genes
- Each of the six genes will have binding site(s) for the TF in their promoter region



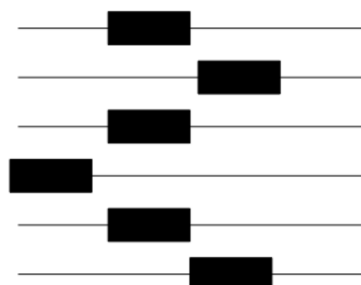
## The motif finding problem

- Now suppose we are given the regions of six genes G1, G2, ... G6 identified by ChIP as bound by TF.
- Can we find the binding sites of TF, without knowing about them *a priori* ?
  - Binding sites are similar to each other, but not necessarily identical (e.g. we expect a PWM)

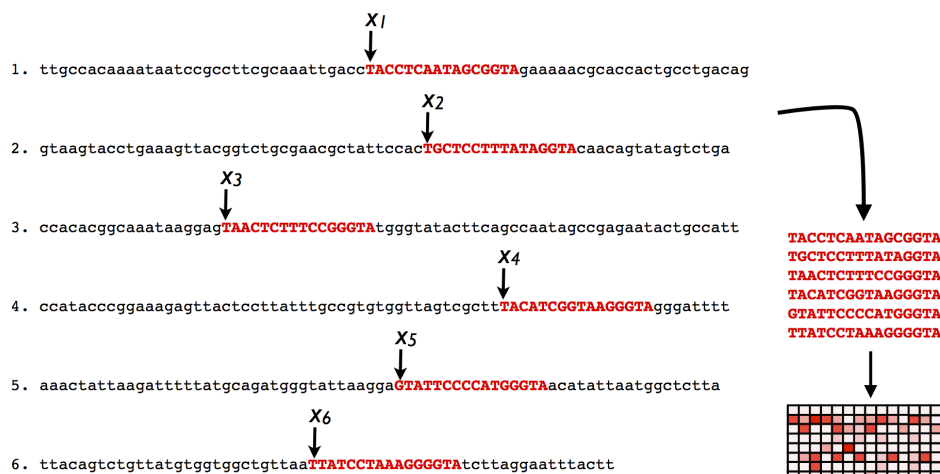


## Identifying Motifs: Complications

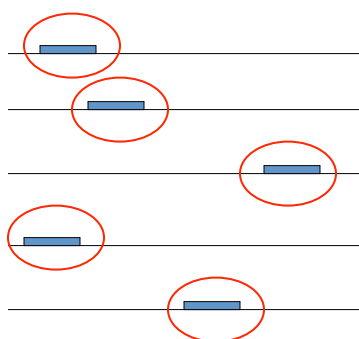
- We do not know the motif sequence
- We do not know where it is located within the sequence fragment
- Motifs can differ slightly from one gene to another
- How to discern it from “random” sequence?



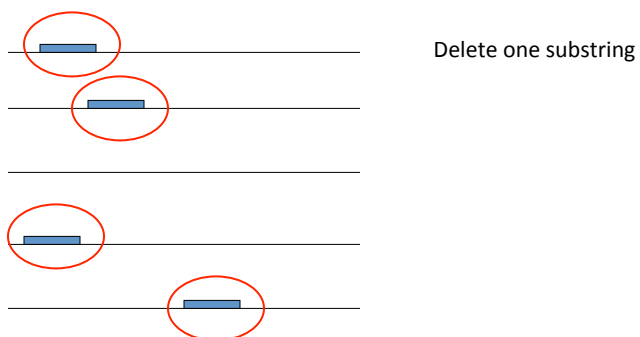
If we knew the starting point of the motif in each sequence, we could construct a Sequence Profile (PSSM) for the motif:



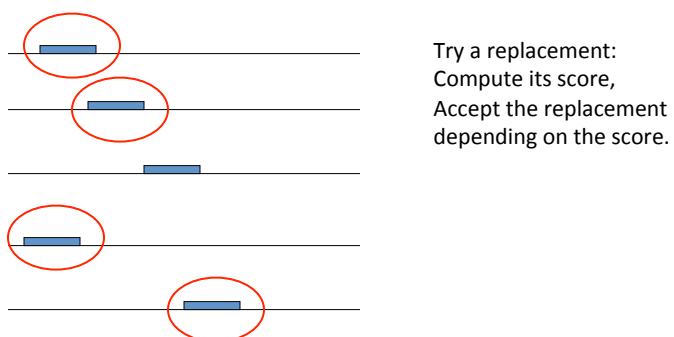
## Gibbs sampling: basic idea



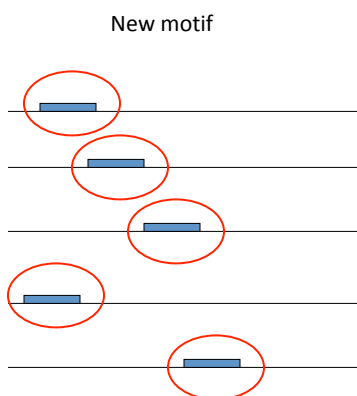
## Gibbs sampling: basic idea



## Gibbs sampling: basic idea



## Gibbs sampling: basic idea



Not guaranteed to find  
“best motif”.

Works well in practice.

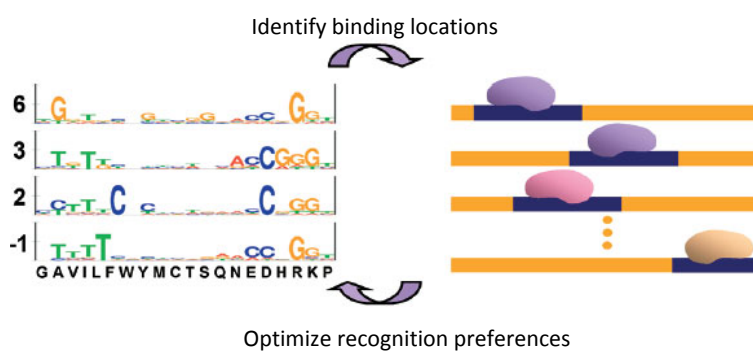
Best if you “restart”  
process many times  
(resample).

## Expectation Maximization

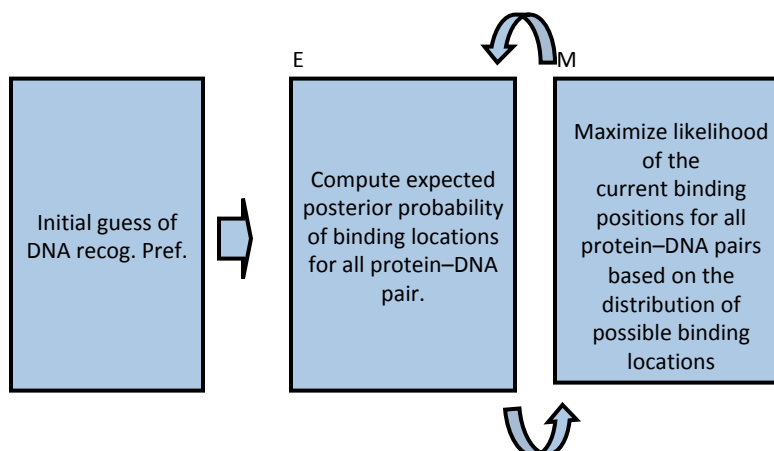
- Popular algorithm for motif discovery
- Motif model: Position Weight Matrix
- Local search algorithm
  - Move from current choice of motif to a new similar motif, so as to improve the score
  - Keep doing this until no more improvement is obtained : Convergence to local optima

## Estimating DNA Recognition Preferences


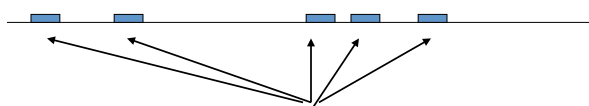
- Apply [Expectation Maximization](#)

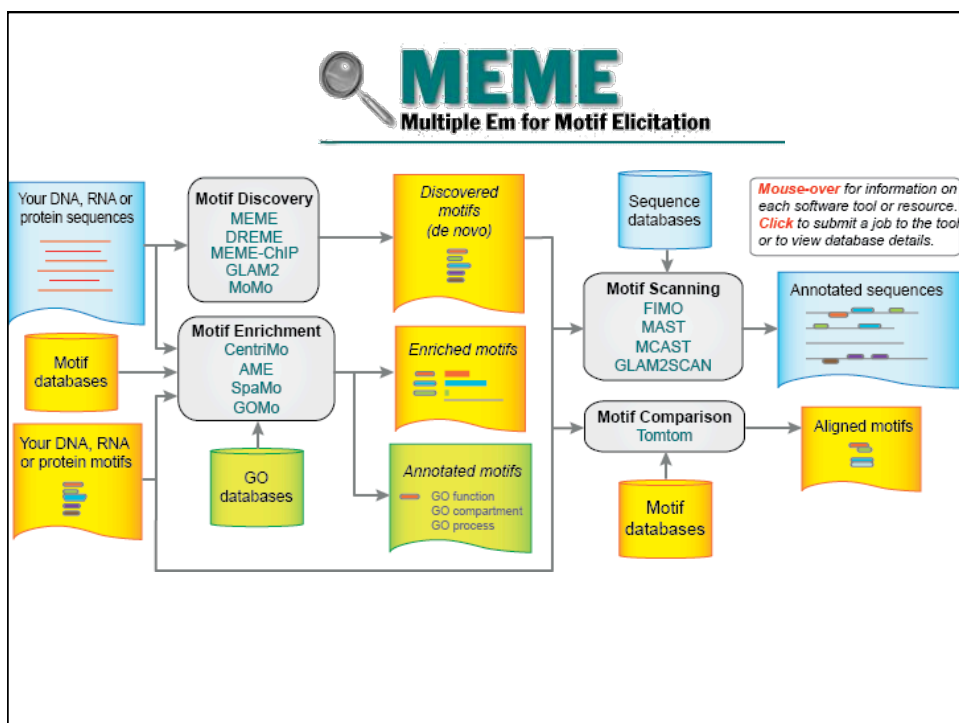


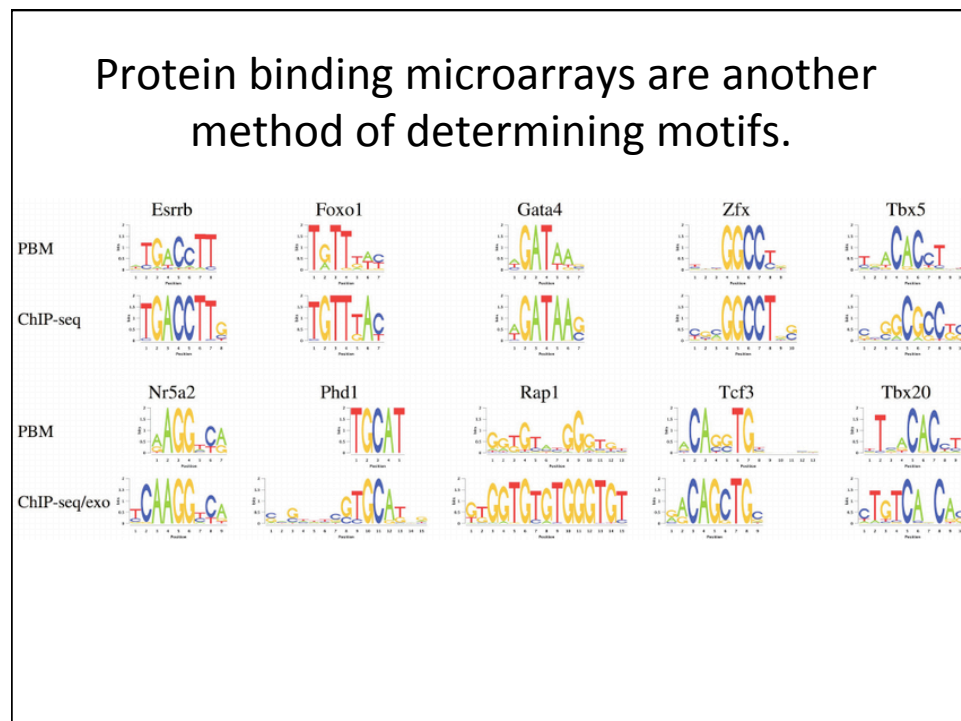
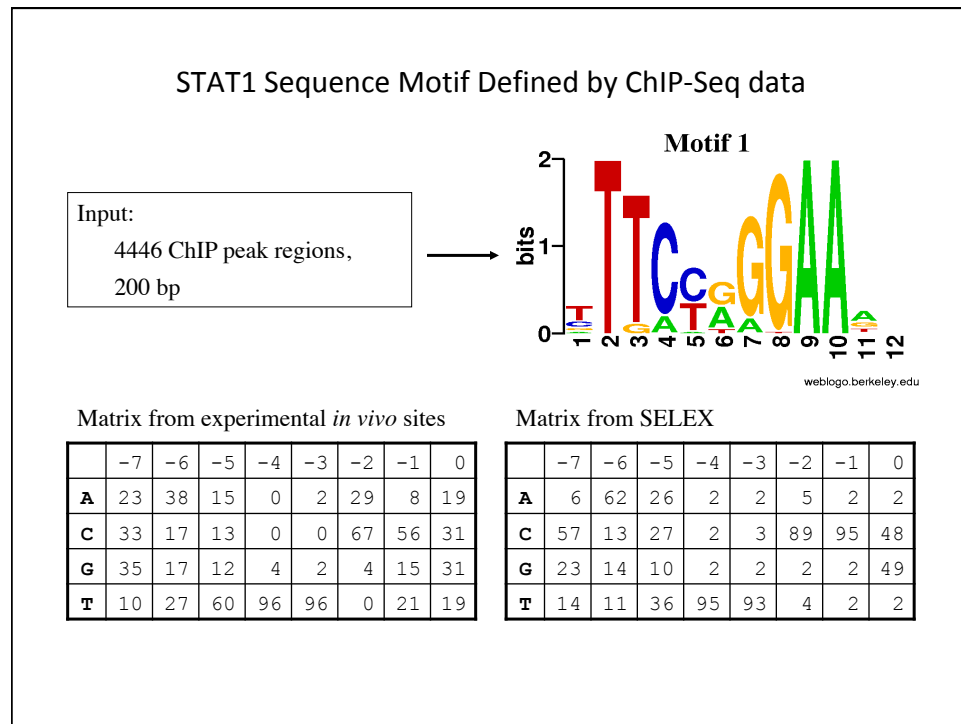
## Expectation Maximization algorithm



## Basic idea of iteration

1. **PWM**  
 ← Current motif
2. Scan sequence(s) for good matches to the current motif.  

3. Build a new PWM out of these matches, and make it the new motif





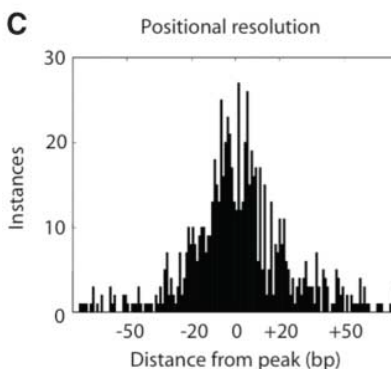


## Motif Databases

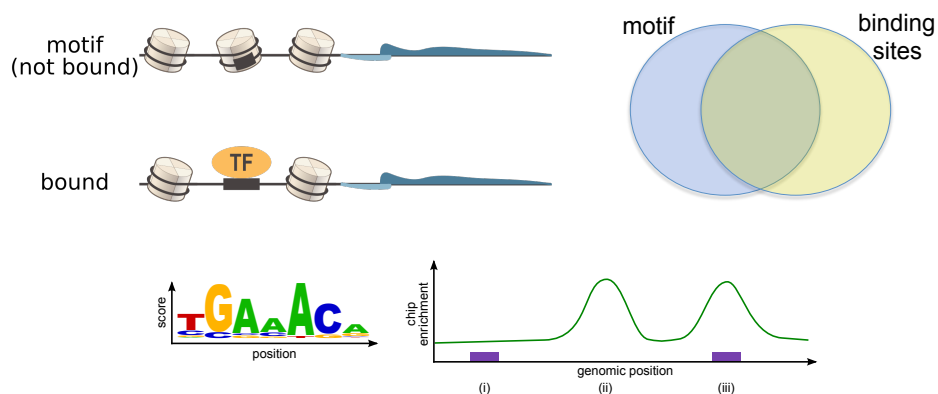


## Precision of ChIPSeq

- Evaluated against the center of high-scoring canonical motifs.
- 94% of these strong motifs fall *within 50bp* of the called experimental peak.



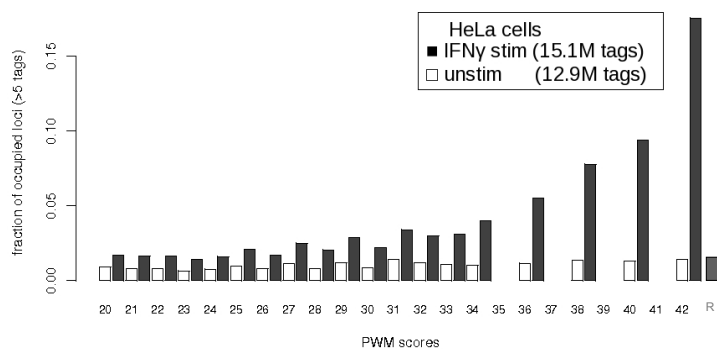
But .. only a fraction of motifs are bound



Dowell (2010)

Binding influenced by nucleosomes ...  
e.g. competition!

But higher quality motif sites (better matches to PSSM)  
are *more likely* to be bound.



STAT1 matrix

<b>A:</b>	0	-12	-12	-1	-6	0	0	-8	5	5	2
<b>C:</b>	0	-9	-10	5	4	0	-13	-12	-4	-13	-2
<b>G:</b>	-2	-13	-4	-12	-13	0	4	5	-10	-9	0
<b>T:</b>	2	5	5	-8	0	0	-6	-1	-12	-12	0