CSCI 4830 / 5722
Computer Vision

University of Colorado **Boulder**

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Computer Vision

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Spring 2019
Lecture 7

Reminders

Submissions:

- Homework 2: due Wed 2/13 at 11 pm
- Homework 3: later this week

Readings:

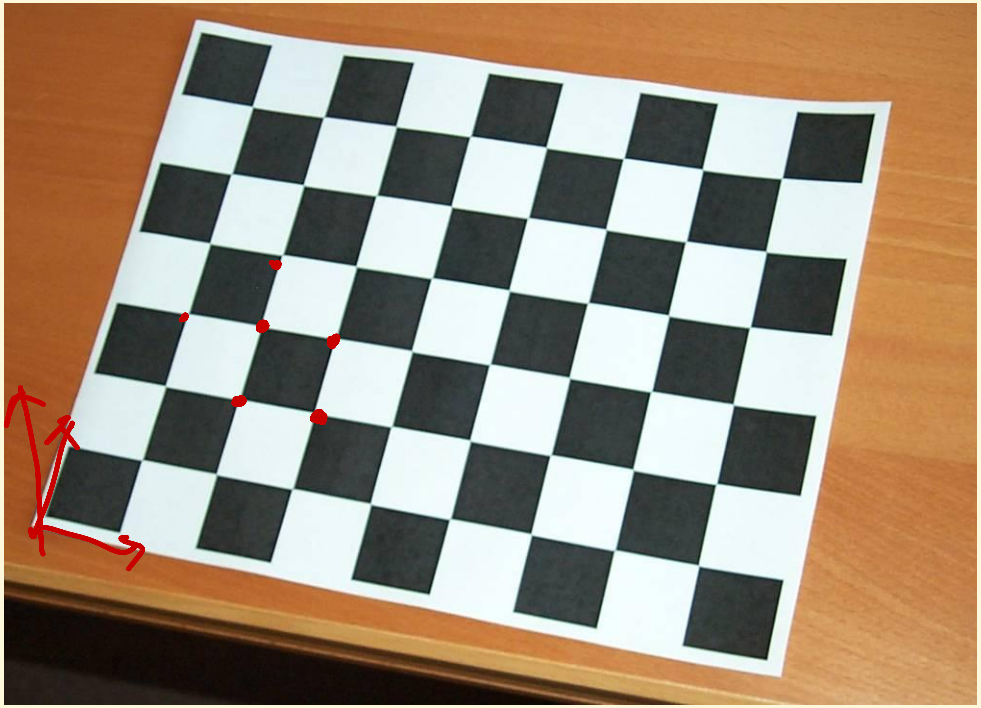
- Szeliski:
  - chapter 3 (filters, changing resolution, Laplacian pyramids, warping)
  - chapter 4.1 (points) and 4.2 (edge detection)
- P&F Ch. 4,5
- Camera Calibration paper

Today

- Camera Calibration paper – cont'
- Estimating homographies
- Homework 2

Camera Calibration: Problem Statement

Given one or more images of a calibration pattern, estimate the camera intrinsic (4 or more) and extrinsic parameters (6) using only observed camera data.



Camera calibration - II

- General strategy:
  - view calibration object; identify image points in image
    - positions are known in some fixed world coordinate system
  - optimization process: *discrepancy between observed image features and their theoretical position is minimized with respect to the camera's intrinsic and extrinsic parameters*
  - obtain camera matrix by minimizing error
  - obtain intrinsic parameters from camera matrix

Camera Calibration

- General strategy:
  - view calibration object
  - identify image points
  - obtain camera matrix by minimizing error
  - obtain intrinsic parameters from camera matrix
- Most modern systems employ the multi-plane method
  - avoids knowing absolute coordinates of calibration points

Methods:

- Photogrammetric Calibration
- Self Calibration
- **Multi-Plane Calibration**

Think of it like of a least squares problem (error minimization):

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix}_{pix} = \begin{pmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} & R & & T \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}_{world}$$

Multi-Plane Calibration

- Hybrid method: Photogrammetric & Self-Calibration.
- Planar pattern imaged multiple times (inexpensive).
- Used widely in practice; many implementations.
- Based on a group of projective transformations called *homographies.*

Notation

Image points

$$\widetilde{\mathbf{m}} = [u, v, 1]^T \text{ and } \boxed{\widetilde{\mathtt{M}} = [X, Y, Z, 1]^T.}$$

world

$$s\tilde{m} = A[R \quad T]\tilde{M}$$

$$\mathbf{A} = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

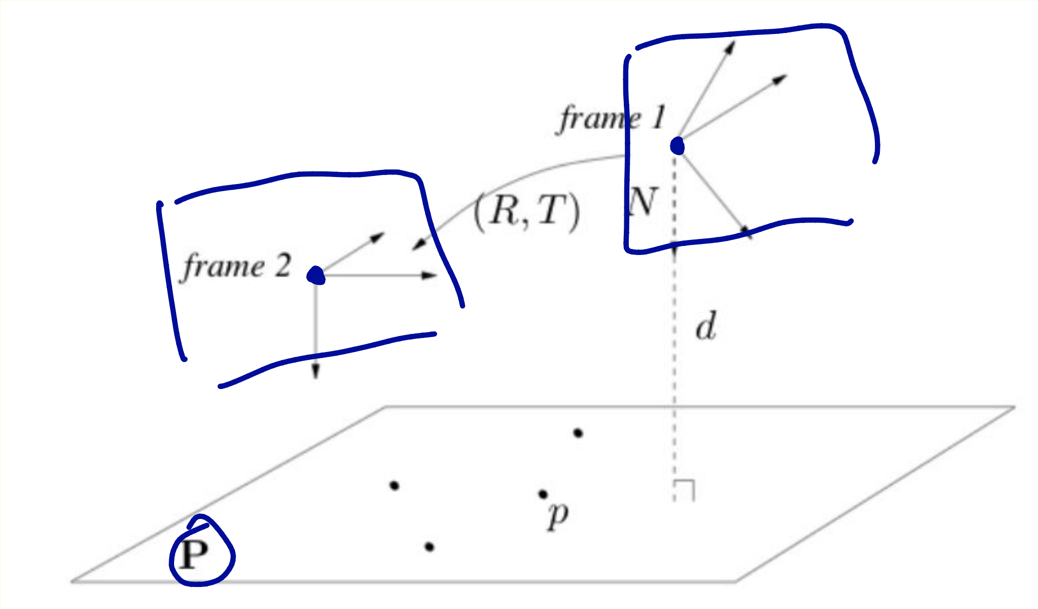
- s = arbitrary scale factor
- (u0, v0) = coordinates of the principal point
- α & β = scale factors in the u and v axes
- γ = skew factor between the two axes

Planar Homographies

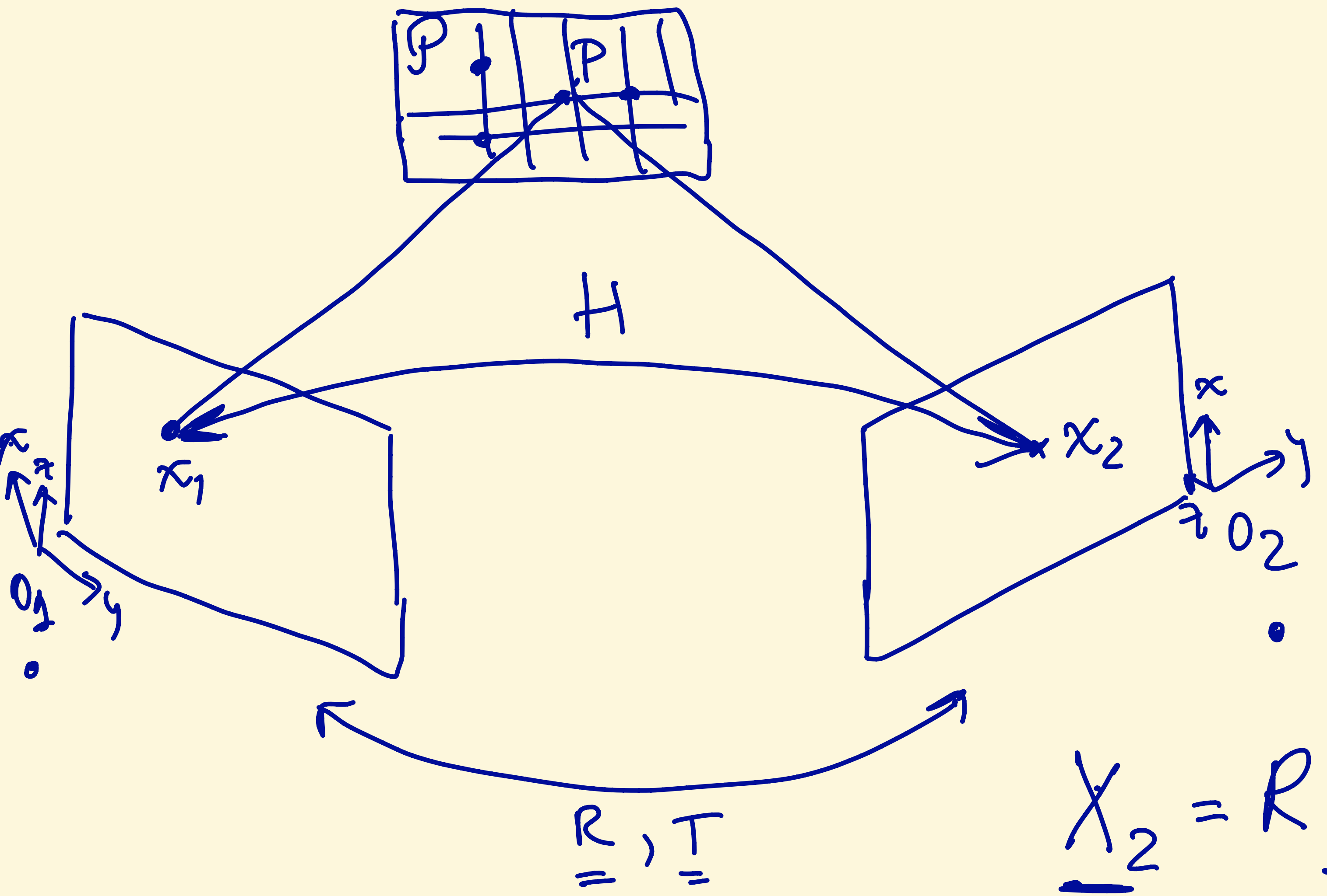
First Fundamental Theorem of Projective Geometry: There exists a unique homography H that performs a change of basis between two projective spaces of the same dimension.

$$s\tilde{m} = H\tilde{M}$$

If two cameras image planar points from the same scene, then those cameras are related through a *homography*.

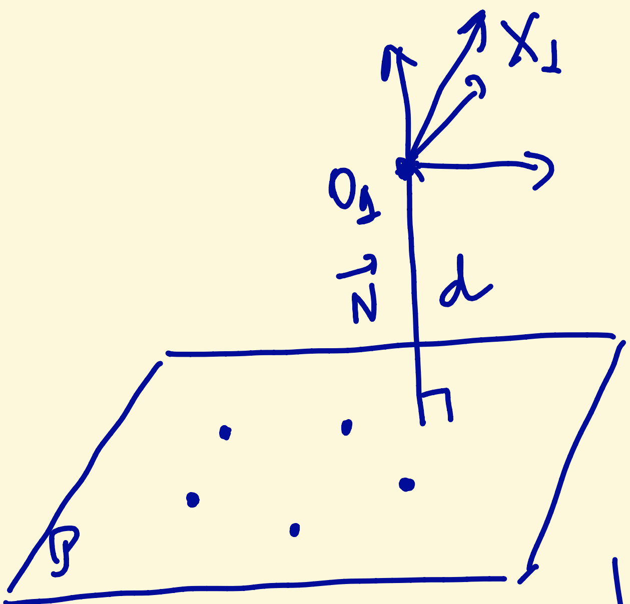


Planar Homographies



$$\underline{X}_2 = R\,\underline{X}_1 + T$$

Planar Homographies



x_1

d – dist between 1st camera O and the Plane

$N \to$ unit normal to plane P

$\vec{X_1} \to$ a vector x_1

$$X_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \qquad N = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$N^T X_1 = d \longrightarrow \boxed{\frac{N^T X_1}{d} = 1}$$

$$a x_1 + b y_1 + c z_1 = d$$

$$X_2 = R X_1 + T = R X_1 + T * 1 = R X_1 + T \frac{N^T X_1}{d} =$$

$$= \underbrace{\left(R + T \frac{N^T}{d}\right)}_{\longrightarrow H} X_1$$

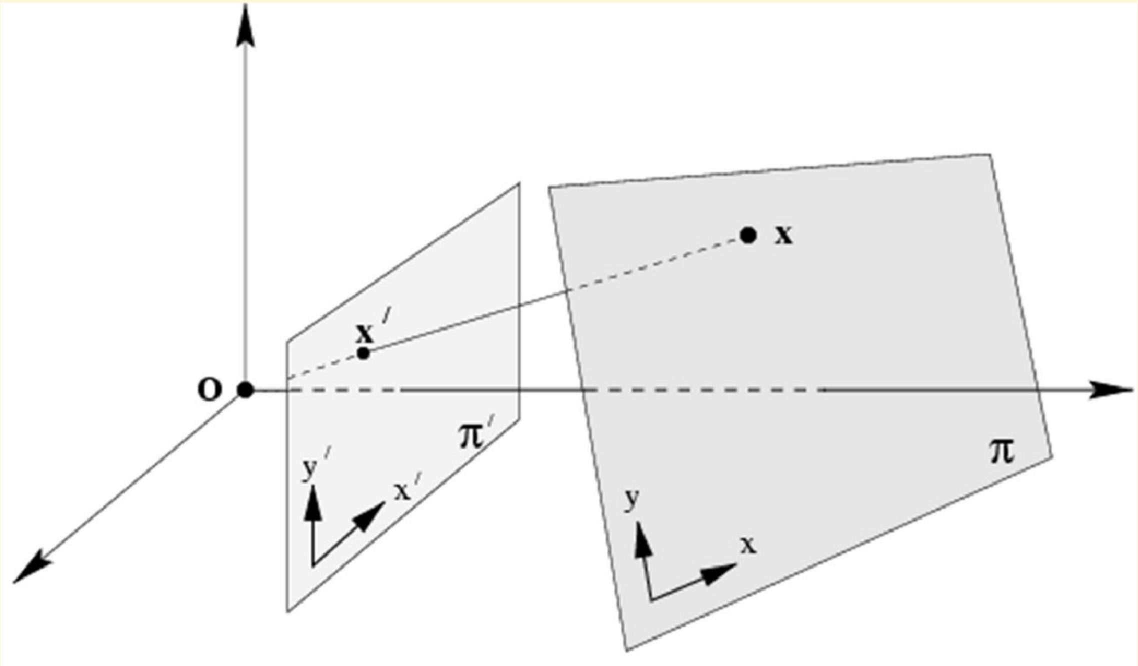
3x3

H: 3 r, 3 t, N, d

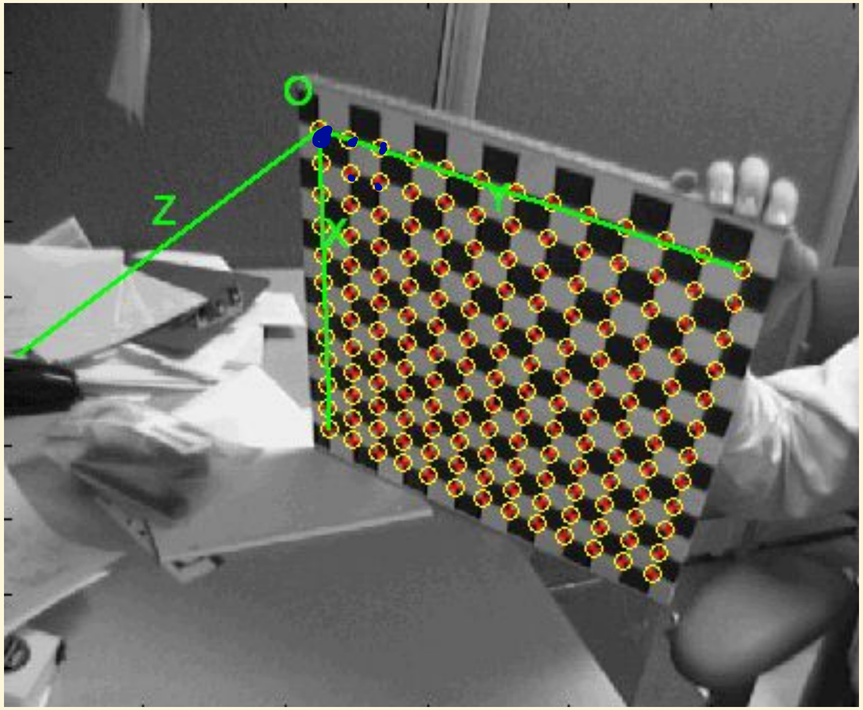
$$\frac{T}{d}$$

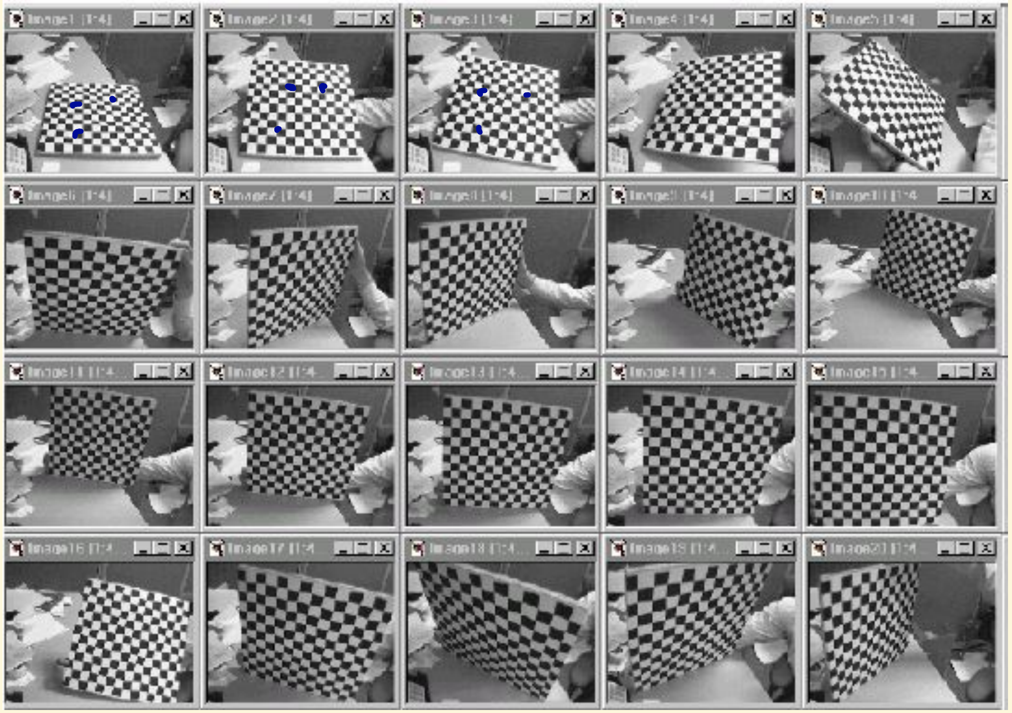
Planar Homographies

- Homography = transformation between 2 spaces of the same dimension
- In this case, 2 planes



Multi-Plane Calibration





Trick: set the world coordinate system to the corner of the checkerboard

Planar Homographies

$m = A[R\ T]M$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underset{\substack{\uparrow \\ \text{intrinsics}}}{A} \begin{bmatrix} R & & & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = A \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \qquad Z \rightarrow 0$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = A \underbrace{\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{bmatrix}}_{H} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \rightarrow \begin{matrix} (0,5) & (0,10) \\ (5,0) & (5,5 \end{matrix}$$

scale

$sm = HM$

Planar Homographies

$$s[u \quad v \quad 1]^T = A[r_1 \quad r_2 \quad r_3 \quad t][X \quad Y \quad Z \quad 1]^T$$

$$\begin{aligned}
s[u \quad v \quad 1]^T &= A[r_1 \quad r_2 \quad r_3 \quad t][X \quad Y \quad 0 \quad 1]^T \\
s[u \quad v \quad 1]^T &= A[r_1 \quad r_2 \quad t][X \quad Y \quad 1]^T \\
s[u \quad v \quad 1]^T &= H[X \quad Y \quad 1]^T
\end{aligned}$$

$$s\tilde{m} = H\tilde{M}$$

Planar Homographies - Estimating a homography

$$m = \lambda H M \qquad m = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

$$\underbrace{m \otimes m}_{} = \lambda\, m \otimes H M$$

$$0 = m \otimes H M$$

$$\hat{m} H M = 0$$

$$\begin{bmatrix} 0 & -1 & v \\ 1 & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} h_1^T M \\ h_2^T M \\ h_3^T M \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 - h_2^T M + v\, h_3^T M = 0 \\ h_1^T M + 0 - u\, h_3^T M = 0 \end{bmatrix}$$

$$\begin{bmatrix} 0^T & M^T & -vM^T \\ M^T & 0^T & -uM^T \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = 0$$

$$a \otimes b = \hat{a} * b \rightarrow$$

$\hat{a} \rightarrow$ skew-symmetric matrix

rank 2

$$\hat{a} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

- $h_1 = [h_{11} \; h_{12} \; h_{13}]$
- $h_2 = \cdots$
- $h_3 = \cdots$

$$h_1^T M = M^T h_1$$

Estimating a Homography

$$\begin{aligned} x_2 &= \lambda H x_1 \\ x_2 \times x_2 &= \lambda x_2 \times H x_1 \\ \hat{x}_2 H x_1 = 0 \quad &\text{or} \quad \lambda = 0 \end{aligned}$$

Now $H = \begin{bmatrix} h_1^\top \\ h_2^\top \\ h_3^\top \end{bmatrix} \in \mathbb{R}^{3\times 3}, h_i \in \mathbb{R}^3$. Hence $Hx_1 = \begin{bmatrix} h_1^\top x_1 \\ h_2^\top x_1 \\ h_3^\top x_1 \end{bmatrix} \in \mathbb{R}^{3\times 1}$. Let $x_2 = \begin{bmatrix} x \\ y \\ w \end{bmatrix}$. Then,

$$\hat{x}_2 = \begin{bmatrix} 0 & -w & y \\ w & 0 & -x \\ -y & x & 0 \end{bmatrix}.$$

$$\hat{x}_2 H x_1 = \begin{bmatrix} 0^\top & -wx_1^\top & yx_1^\top \\ wx_1^\top & 0^\top & -xx_1^\top \\ -yx_1^\top & xx_1^\top & O^\top \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = 0 \quad \Rightarrow \quad \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} h = 0$$

Homework 2 - mosaics