CSCI 4830 / 5722
Computer Vision

University of Colorado **Boulder**

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Computer Vision

Dr. Ioana Fleming
Spring 2019
Lecture 4

Reminders

Submissions:

- Homework 1: Sat 1/26 at 6 pm

Readings:

- Szeliski Ch. 3
- P&F Ch. 1 (or 1&2 in 1st edition)

Office hours calendar - on Moodle

Moodle (password: '*vision*')

Linear transformations

A linear transformation from a linear vector space $\mathbf{R}^n$ to $\mathbf{R}^m$ is defined as a map $\mathbf{L}: \mathbf{R}^n \rightarrow \mathbf{R}^m$ such that

- $L(x + y) = L(x) + L(y)$
- $L(\alpha * x) = \alpha * L(x)$

L can be represented as a matrix **A,** such that

$L(x) = A*x$

vector

$\begin{bmatrix} x \\ y \end{bmatrix}$

Linear transformations

Examples:

Identity:

- identity operation:

  x' = x

  y' = y

- With a linear transformation matrix:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

identity matrix

Linear transformations

Examples:

<u>Inversion:</u>

- inversion operation:

  x' = -x

  y' = -y

- With a linear transformation matrix:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

inversion matrix

Linear transformations

Scaling:

- Scaling operation:

  x' = a * x

  y' = b * y

- With a linear transformation matrix:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

scaling matrix

Other Linear transformations matrices

Mirror / reflection:

- About y axis:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

- Over (0,0):

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

Other Linear transformations matrices

Rotation:

- 2D rotation about (0,0):

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

What about ...?

Translation:

- 2D translation operations:

  x' = x + a ←

  y' = y + b ←

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

- Is it possible to have a linear transformation?

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \cancel{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} * \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Linear transformations

- Is it possible to have a 2D linear transformation for translation?

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

- Only linear 2D transformations can be represented with a 2x2 transformation matrix.
- Linear transformations are combinations of: scale, rotation, mirror and shear

Homogeneous coordinates

Conversions for 2D and 3D

Converting to *homogeneous* coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous coordinates

$$\begin{bmatrix} 3 \\ 5 \end{bmatrix} \rightarrow \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} \begin{bmatrix} 6 \\ 10 \\ 2 \end{bmatrix} \begin{bmatrix} 9 \\ 15 \\ 3 \end{bmatrix} \dots$$

Invariant to scaling

$$k\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \end{bmatrix}$$

Homogeneous Coordinates

Cartesian Coordinates

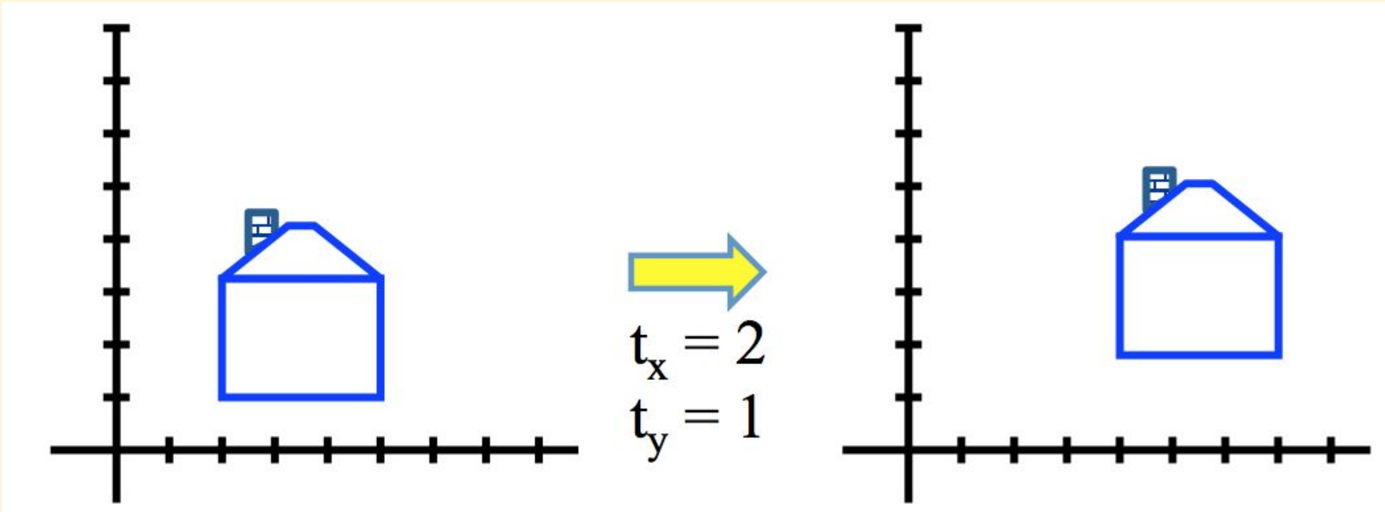
Translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

3×1 3×3 3×1

$x' = x + t_x$

$y' = y + t_y$



Other 2D Transformations

Basic 2D Transformations at 3x3 transformation matrices: translation, rotation, scale, shear

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad - \text{Rot}$$

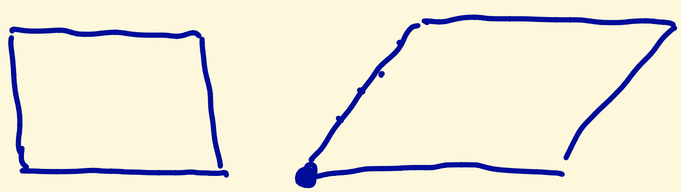
$$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

R+T

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \text{Scale} \longrightarrow$$

$$x' = S_x * x$$
$$y' = S_y * y$$

Shear

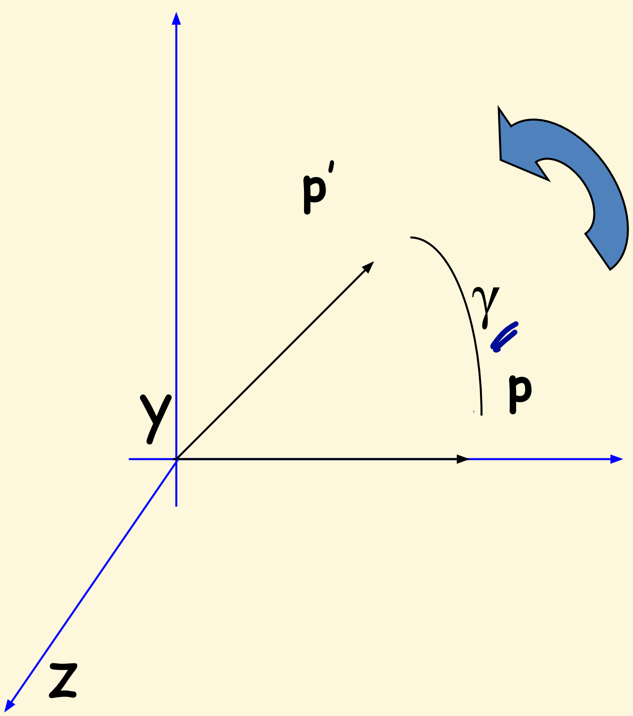


$$\begin{bmatrix} \quad & \quad \end{bmatrix}$$

3D Rotation of Points

$$R^{-1} = \frac{\begin{bmatrix} \cos & \sin \\ -\sin & \cos \end{bmatrix}}{\det(R)} = \begin{bmatrix} \cos & \sin \\ -\sin & \cos \end{bmatrix}$$

Rotation around the coordinate axes, counter-clockwise:



(y,z) $$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

(x,z) $$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$(\overline{xy})$ $$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Euler angles

Rigid transformations

Transformation between two coordinate systems includes a rotation and a translation

$$\begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Before:
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underset{2\times 2}{R} \begin{bmatrix} x \\ y \end{bmatrix} + \underset{2\times 1}{T}$$

With homogeneous coordinates:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} & R & & T \\ 0 & & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

A camera creates an image …

World

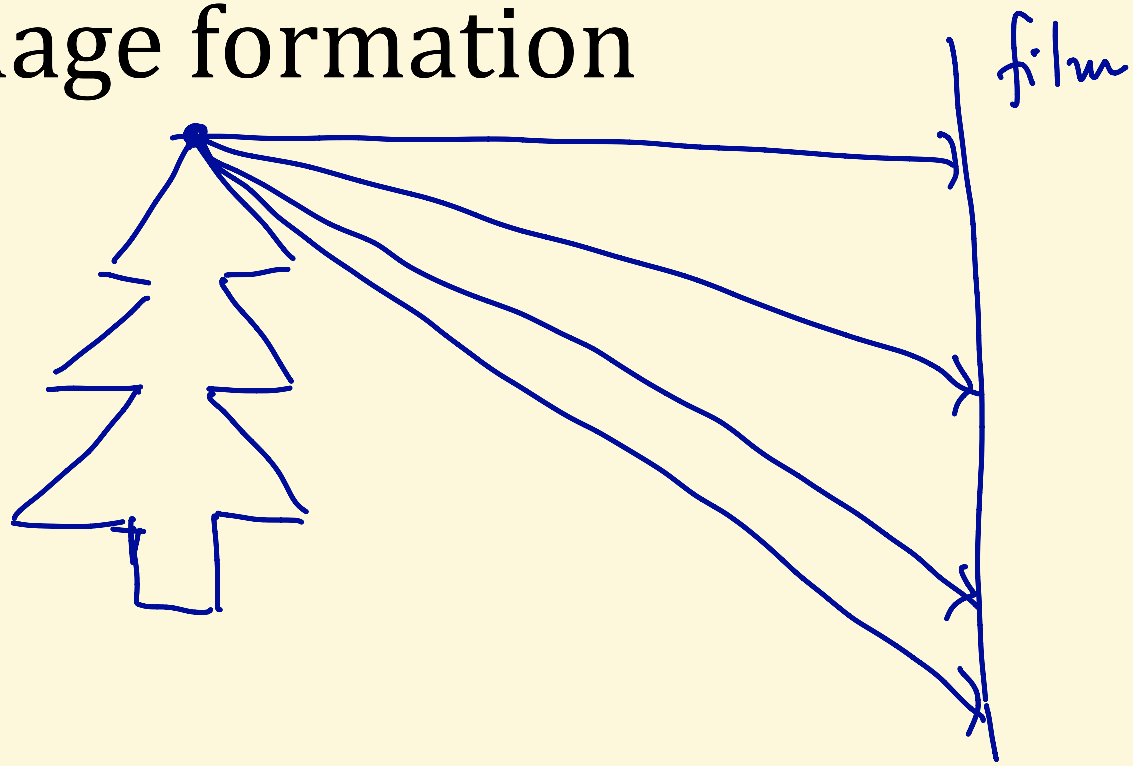
The image intensity I(x,y) measures how much light is captured at pixel (x,y).

- Where does a point (X,Y,Z) in the world get imaged?
- What is the brightness at the resulting point (x,y)?

A camera creates an image …

1. Geometry:
   - Geometry of images captured by cameras
2. Radiometry:
   - What do the intensities in the image tell us about the light in the real world?
3. Simplified models

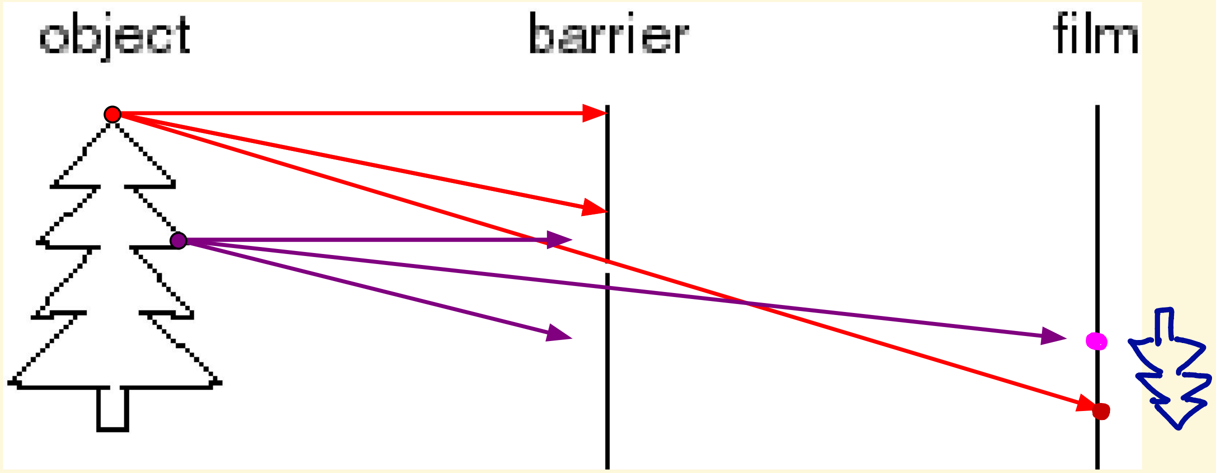
Image formation



Let's design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

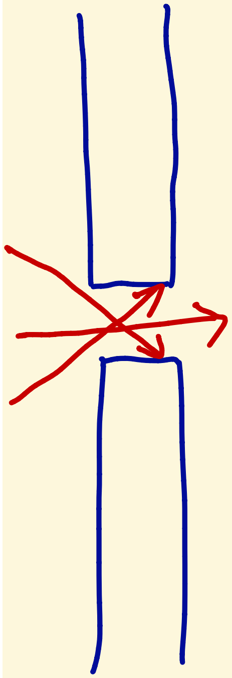
Pinhole camera



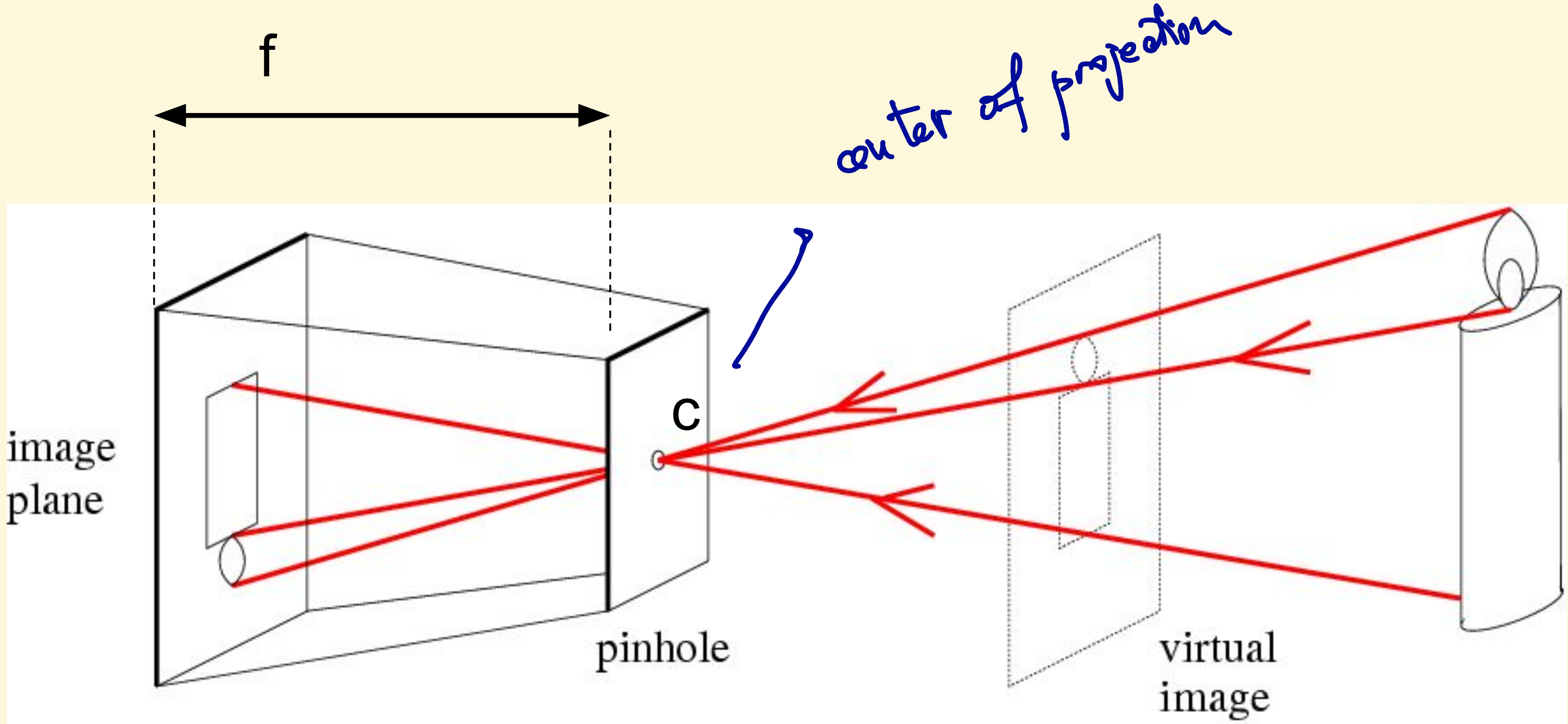
Idea 2: add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**

Vignetting Effect



Pinhole camera



f = focal length
c = center of the camera

Figure from Forsyth

Camera obscura: the pre-camera

- Known during classical period in China (Mozi, China, 470BC to 390BC) and Greece (Aristotle, 384BC to 322BC)

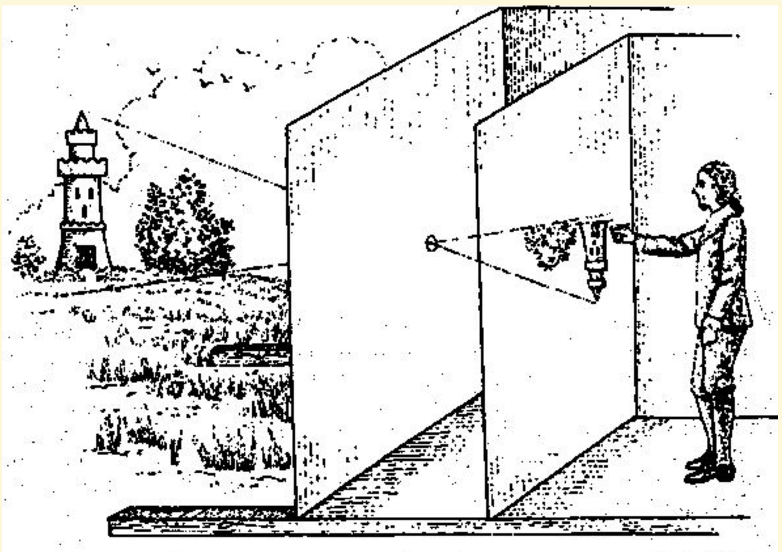
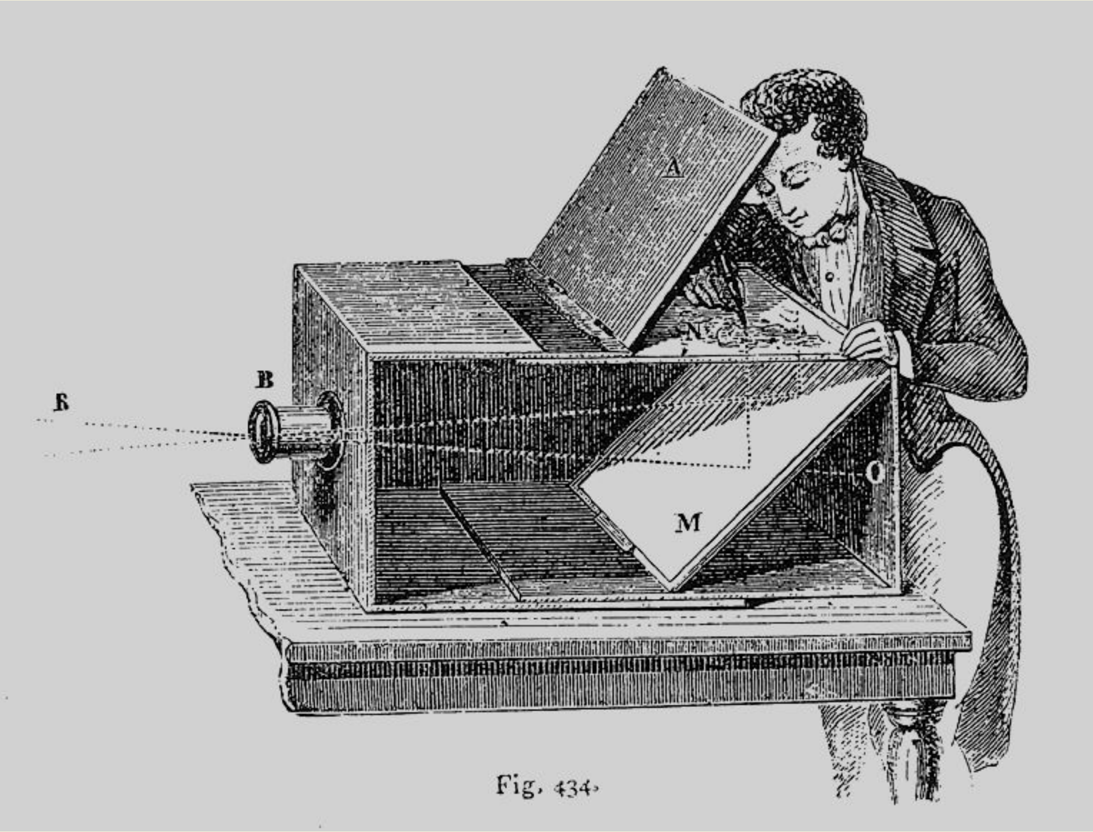


Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys

Camera Obscura

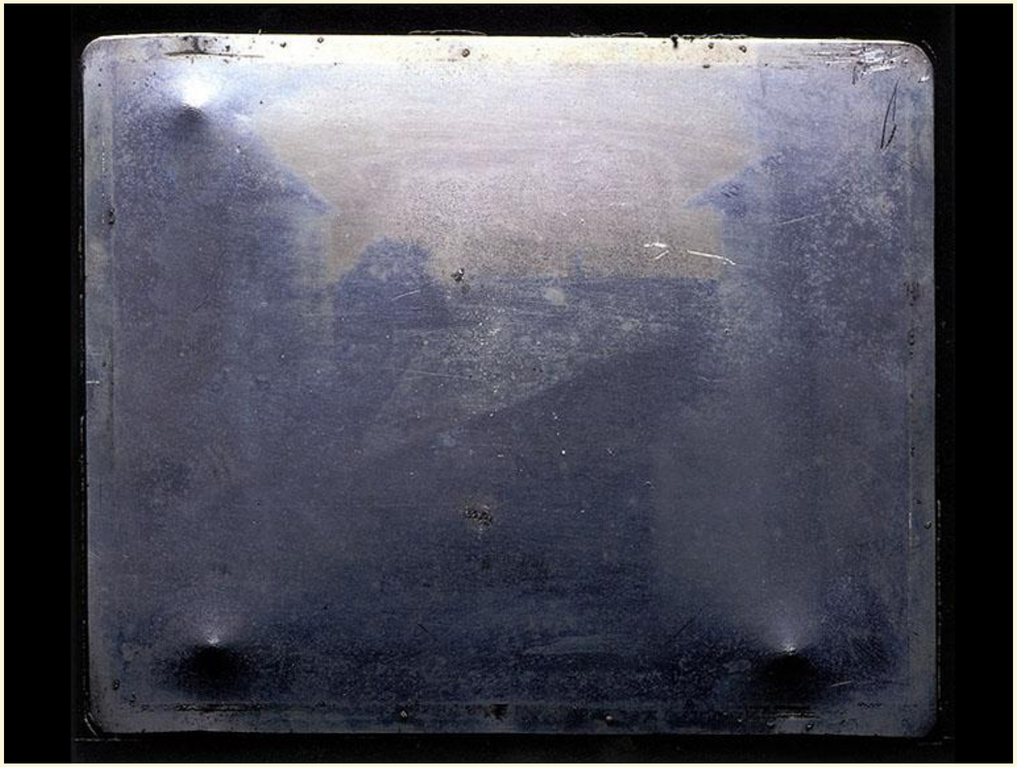


Lens Based Camera Obscura, 1568
A 19th-century artist using a camera obscura to outline his subject

First Photograph

Joseph Niepce, 1826-27

Oldest surviving photograph
- Took 8 hours on pewter plate



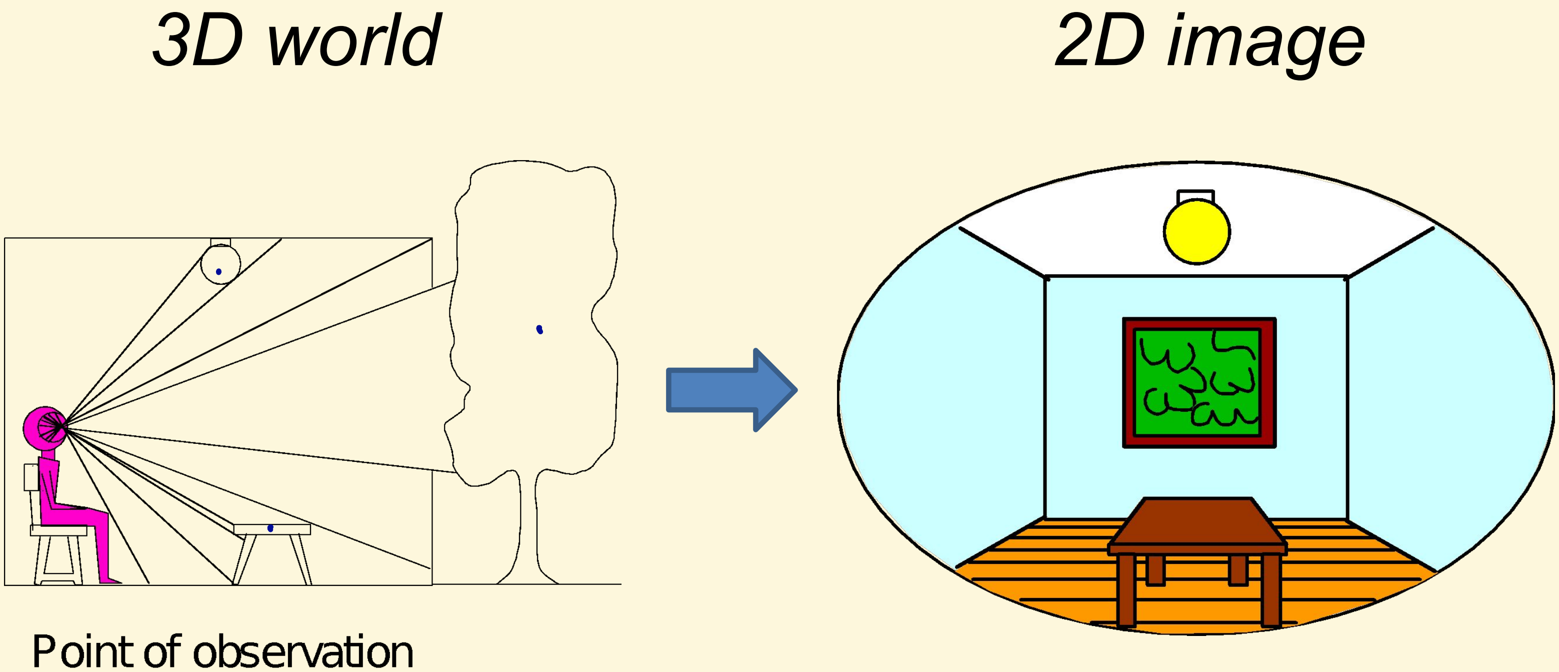
Multiple pinholes

Multiple pinholes

Multiple pinholes

Natural Pinhole Phenomenon

Dimensionality Reduction Machine (3D to 2D)

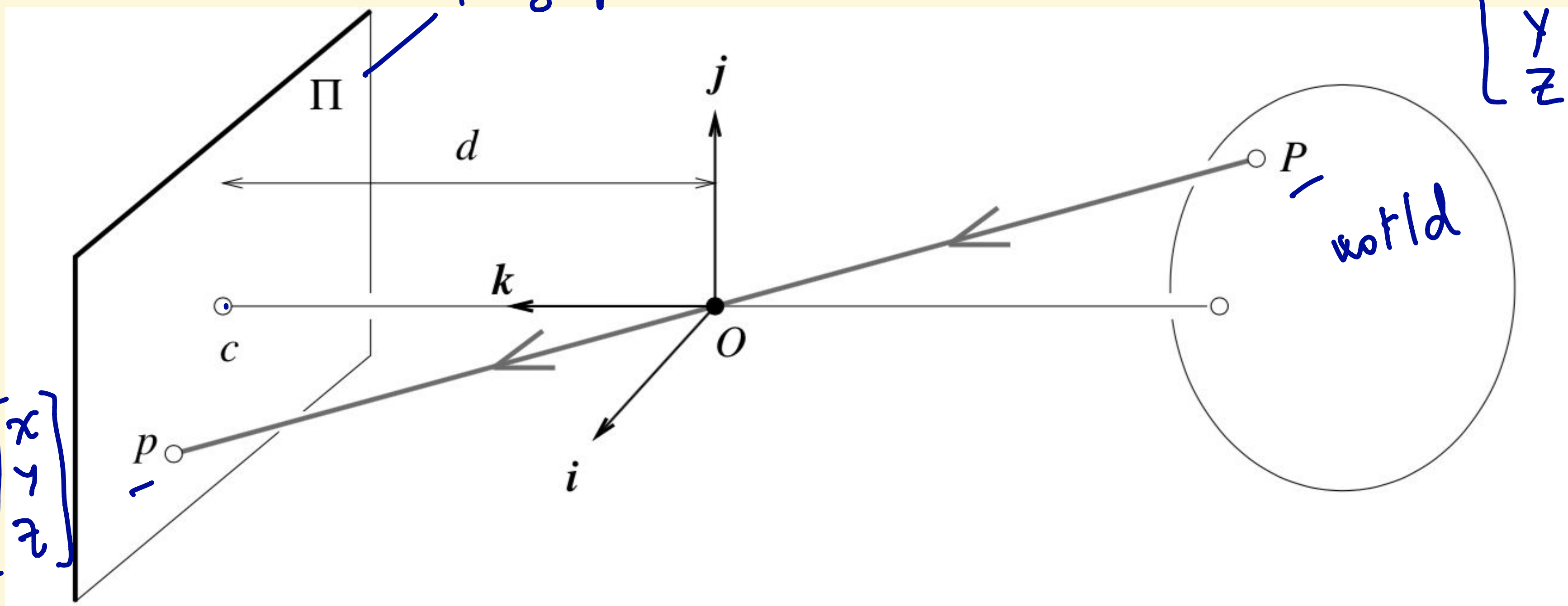


The Pinhole Camera

(O, i, j, k) - camera coord system

image plane - $\parallel (i, j)$

$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$



$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

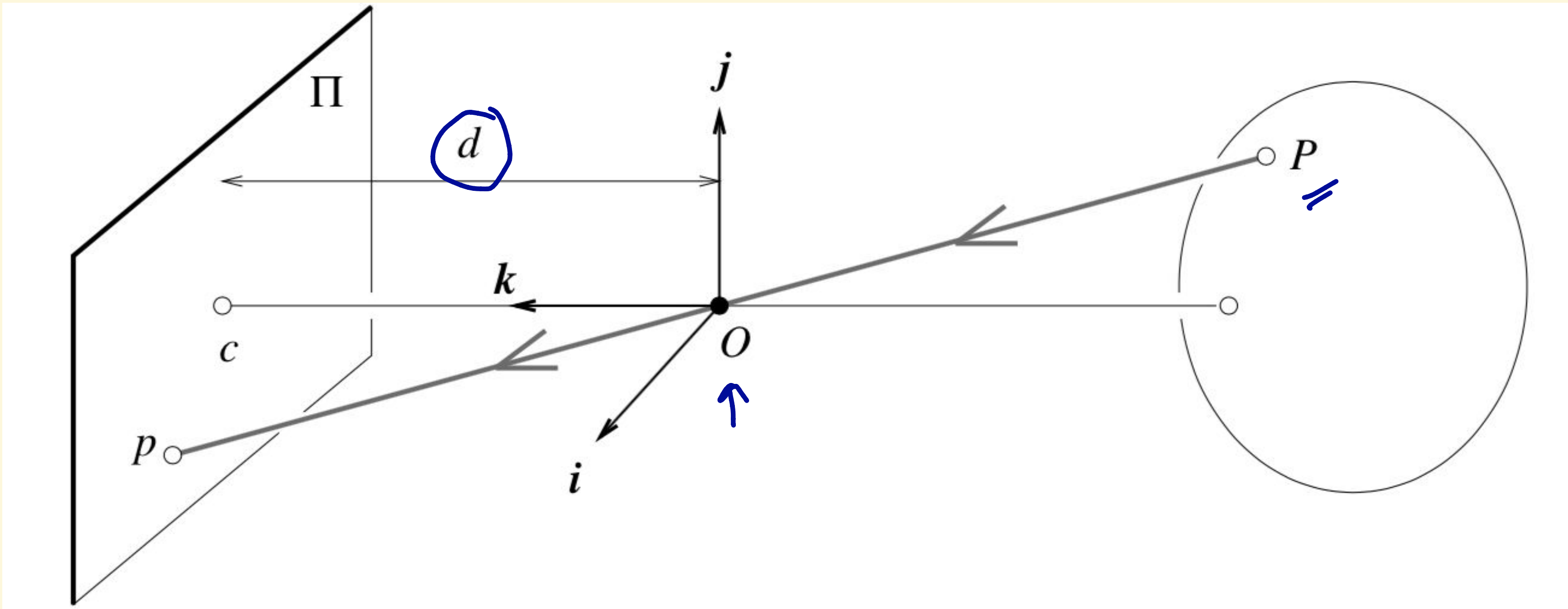
FIGURE 1.4: The perspective projection equations are derived in this section from the collinearity of the point P, its image p, and the pinhole O.

d - distance between Π and O

c - image center

Oc - optical axis $\perp \Pi$

Perspective (pinhole) Projection



P = (X, Y, Z)

$$\begin{cases} x = \lambda X \\ y = \lambda Y \\ d = \lambda Z \end{cases} \iff \lambda = \frac{x}{X} = \frac{y}{Y} = \frac{d}{Z}$$

$$\begin{cases} x = d\dfrac{X}{Z}, \\ y = d\dfrac{Y}{Z}. \end{cases}$$

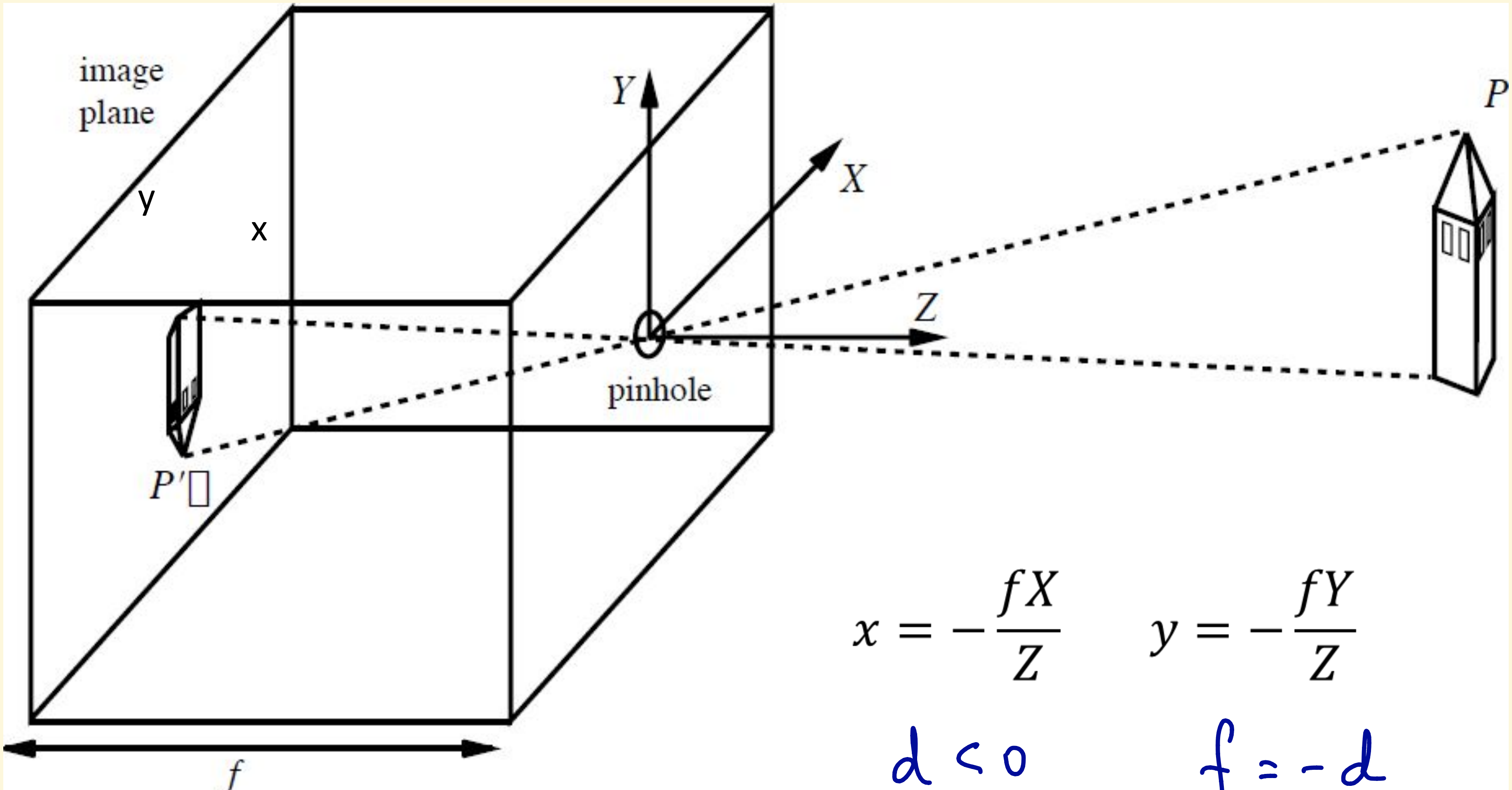
p = (x, y, z) $z = d$

O, P, p colinear

$\vec{Op} = \lambda \vec{OP}$

f

The Pinhole Camera

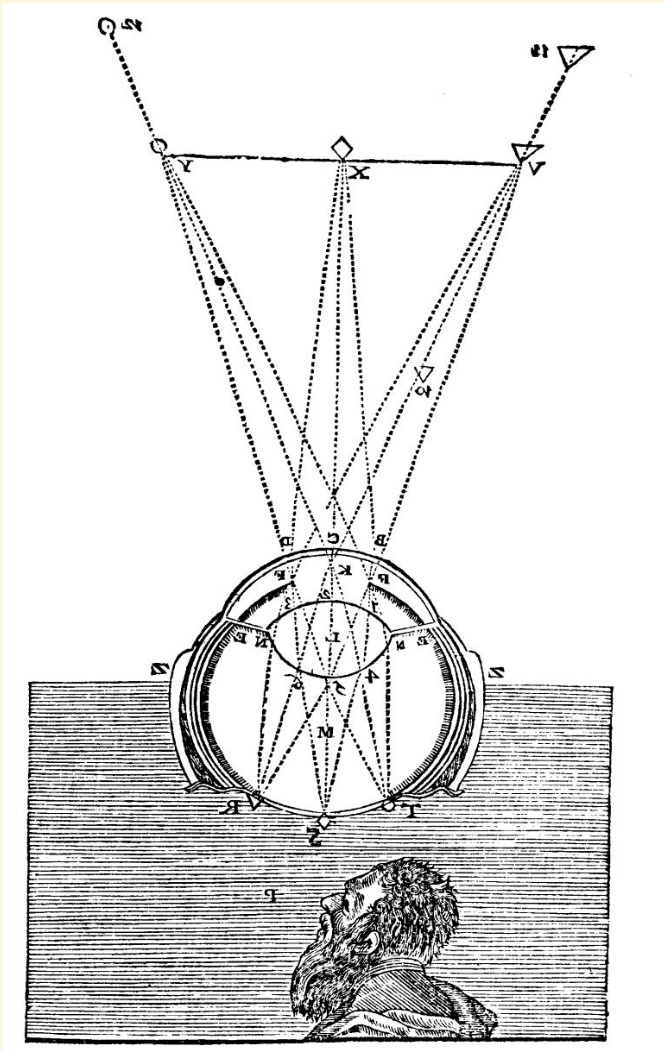


$$x = -\frac{fX}{Z} \qquad y = -\frac{fY}{Z}$$

$d < 0$ $\qquad$ $f = -d$

$f > 0$

The image is inverted



This was pointed out by Kepler in 1604

d is negative!

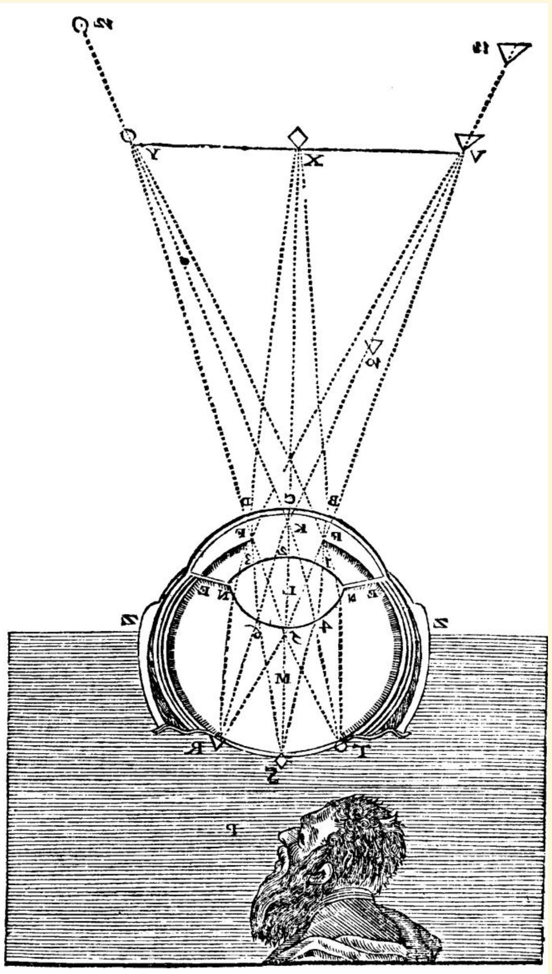
Image plane at distance = focal length from the pinhole

But this is no big deal. The brain can interpret it the right way. And for a camera, software can simply flip the image top-down and right-left. After this trick, we get

$$x = \frac{fX}{Z} \qquad y = \frac{fY}{Z}$$

From Descartes(1637), La Dioptrique

Human retina vs camera retina



From Descartes(1637), La Dioptrique

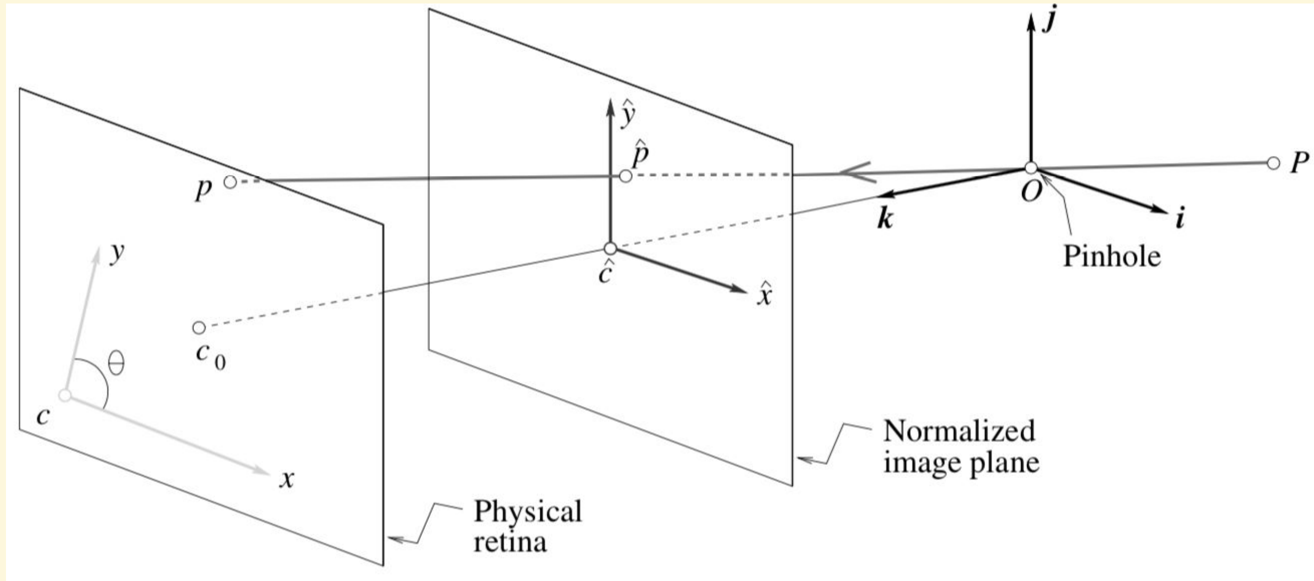


FIGURE 1.14: Physical and normalized image coordinate systems.

- Digital images (and retinas) - spatially discrete, not continuous
- Perspective equation – only valid when all distances are measured in the camera's reference frame, and when image coordinates have their origin at the image center where the axis of symmetry of the camera pierces its retina

Camera retina

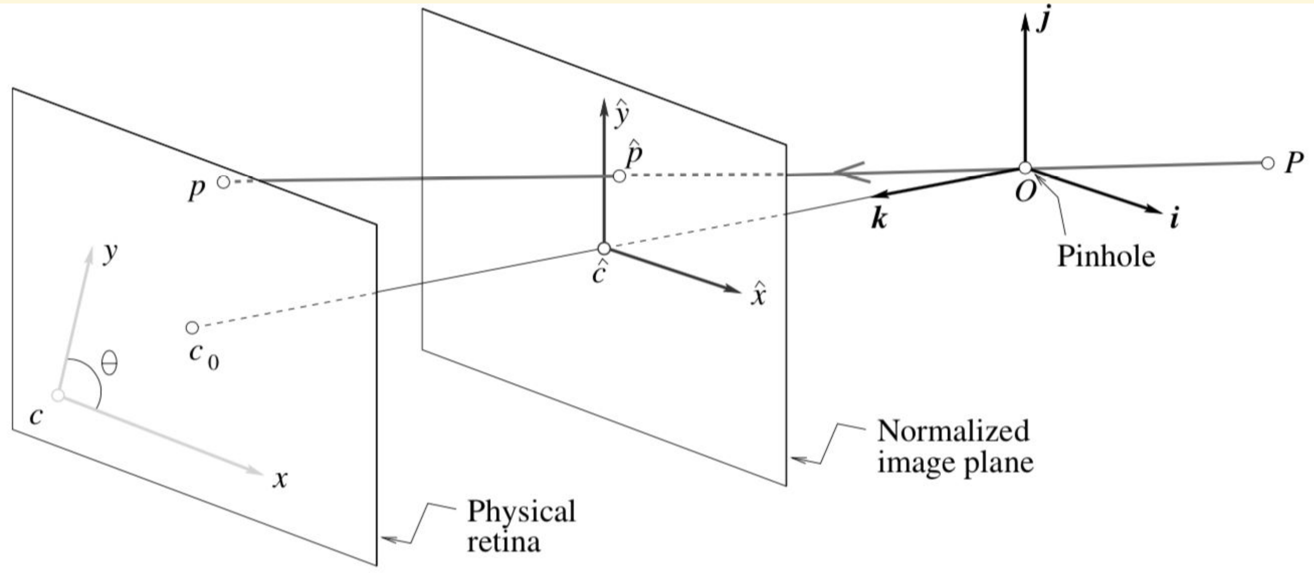


FIGURE 1.14: Physical and normalized image coordinate systems.

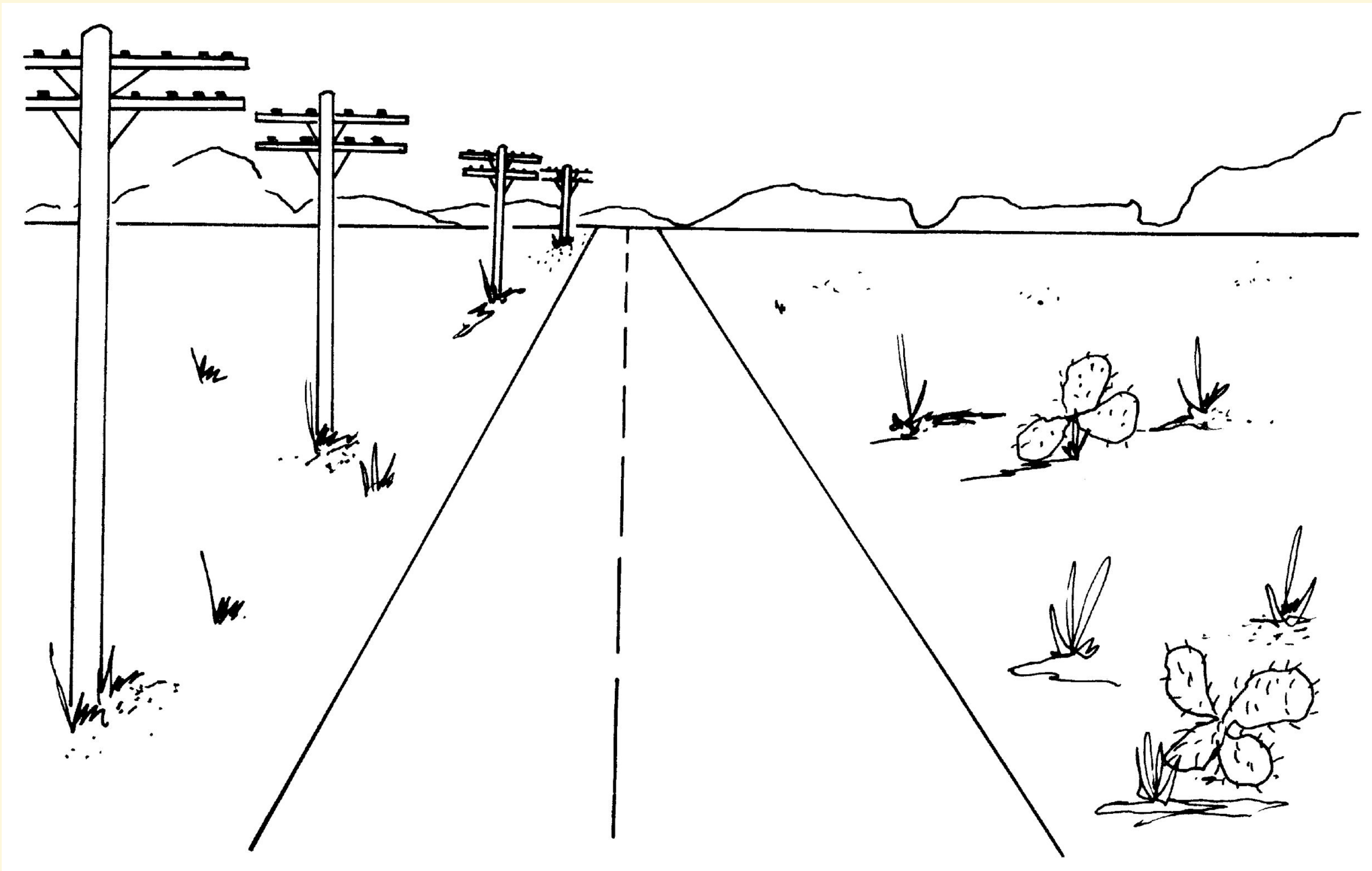
In practice, world and camera coordinates are related by a series of factors:

- focal length
- size of pixels
- position of the image center
- position and orientation of the camera

Intrinsic Parameters (focal length, size of pixels, position of the image center)

Extrinsic Parameters (position and orientation of the camera)

Some perspective phenomena…



Projective Geometry

What is lost?

Length is not preserved

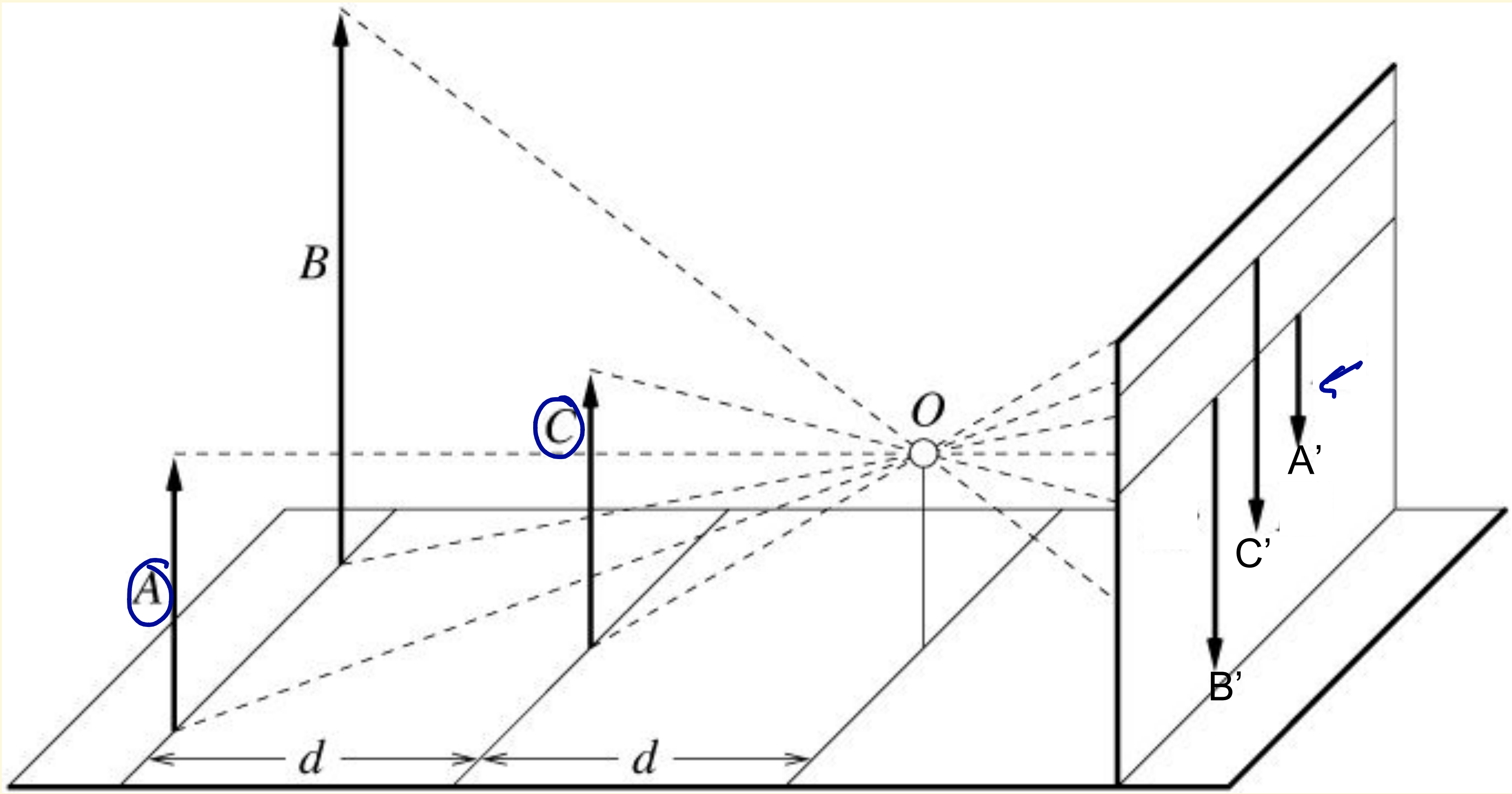


Figure by David Forsyth

Projective Geometry

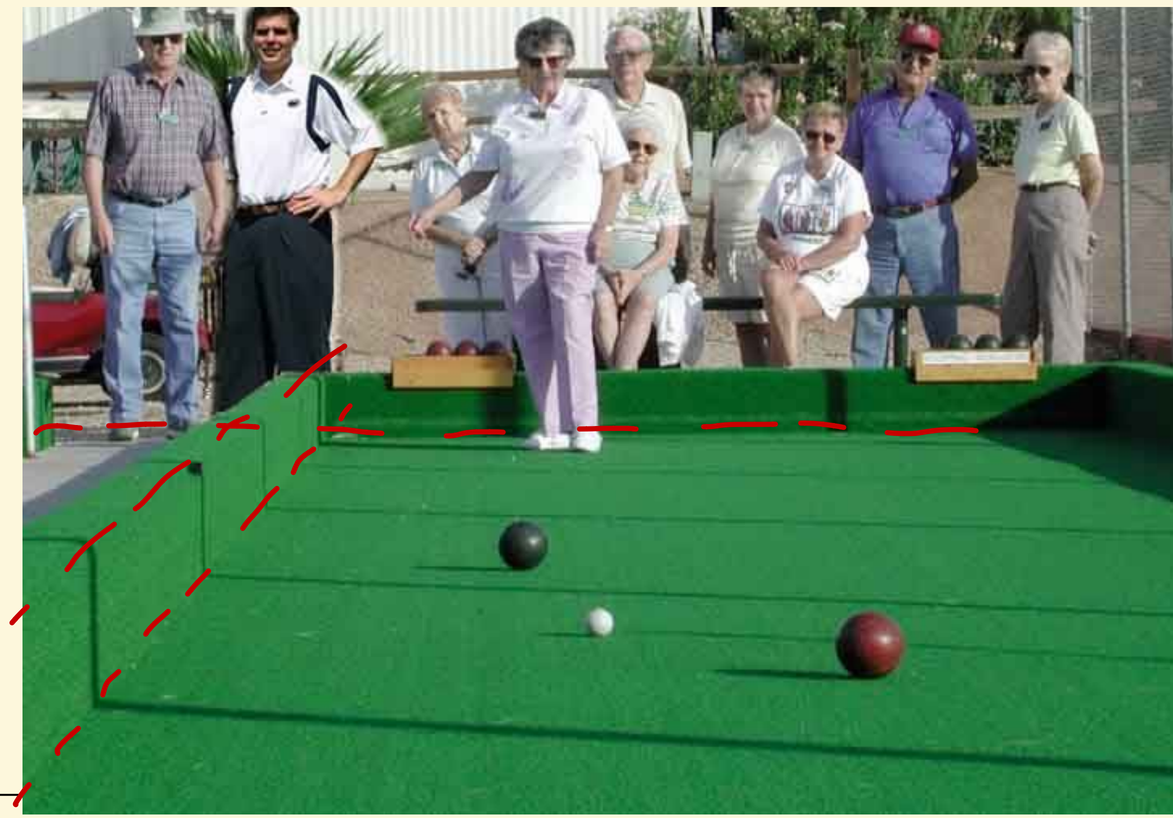
What is lost?

- Length
- Angles

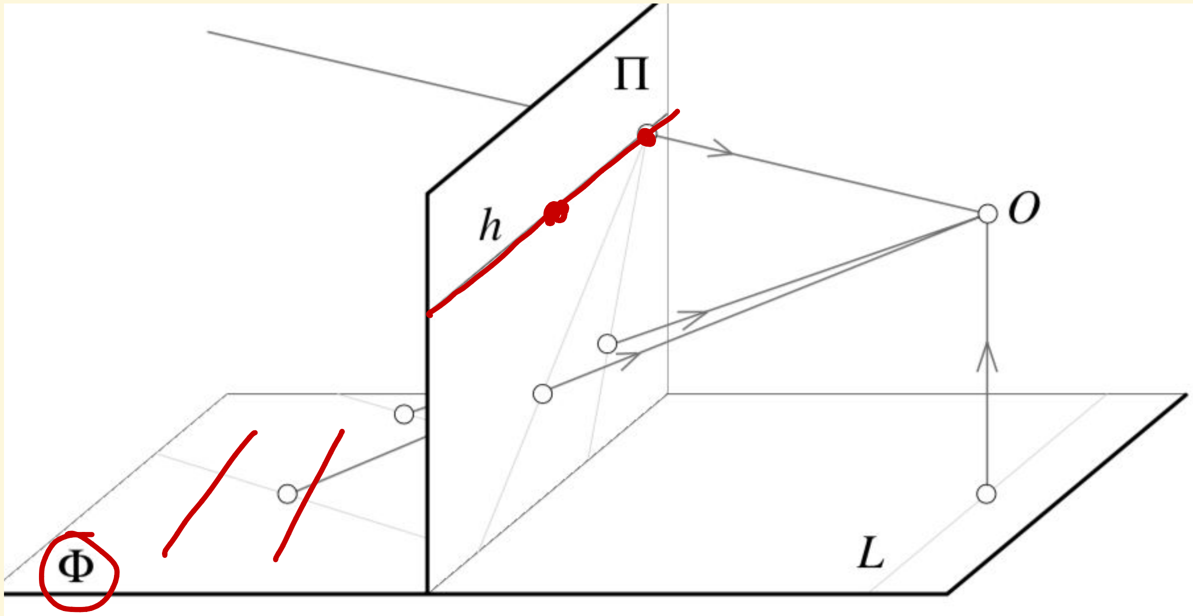
Projective Geometry

What is preserved?

- Straight lines are still straight



Parallel lines



The projections of two parallel lines lying in some plane Φ appear to converge on a horizon line h formed by the intersection of the image plane Π with the plane parallel to Φ and passing through the pinhole. Note that the line L parallel to Π in Φ has no image at all.

Vanishing points and lines

Parallel lines in the world intersect in the image at a "vanishing point"