CSCI 4830 / 5722
Computer Vision

University of Colorado **Boulder**

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Computer Vision

Dr. Ioana Fleming
Spring 2019
Lecture 15

Reminders

Submissions:

- Homework 3: Sat 3/2 at 11 pm

Readings:

- Szeliski:
  - chapter 4.1 (Feature detection – Points and patches)
- P&F:
  - chapter 5 (Local features – corners, SIFT features)

Today

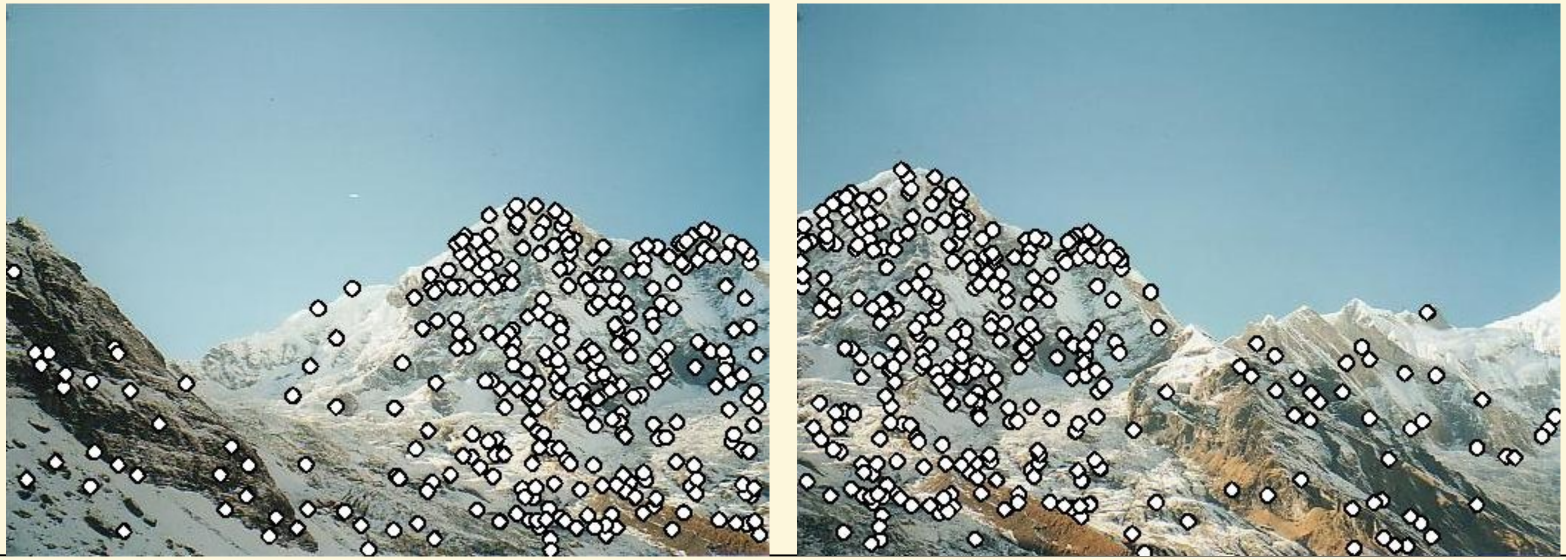
- Interest Points and Corners
- Harris Corner Detection algorithm

How do we build a panorama?

- We need to match (align) images
- Global methods sensitive to occlusion, lighting, parallax effects. So look for local features that match well.
- How would you do it by eye?

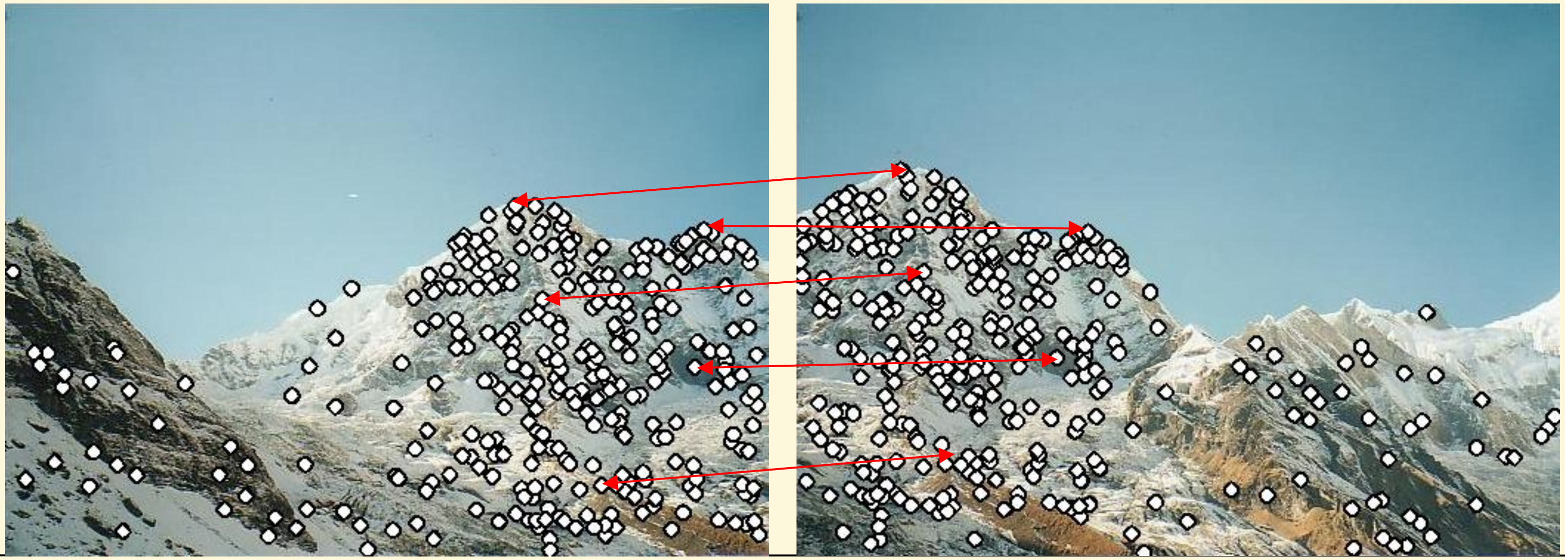
Matching with Features

- Detect feature points in both images



Matching with Features

- Detect feature points in both images
- Find corresponding pairs



Matching with Features

- Detect feature points in both images
- Find corresponding pairs
- Use these pairs to align images

Matching with Features

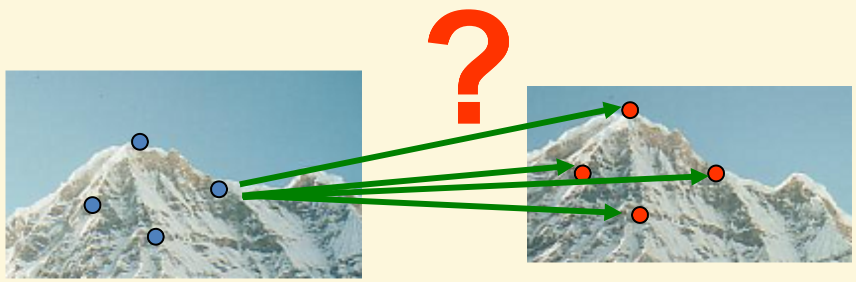
- Problem 1:
  - Detect the *same* point *independently* in both images

no chance to match!

We need a repeatable detector

Matching with Features

- Problem 2:
  - For each point correctly recognize the corresponding one



We need a reliable and distinctive descriptor

This week: interest points

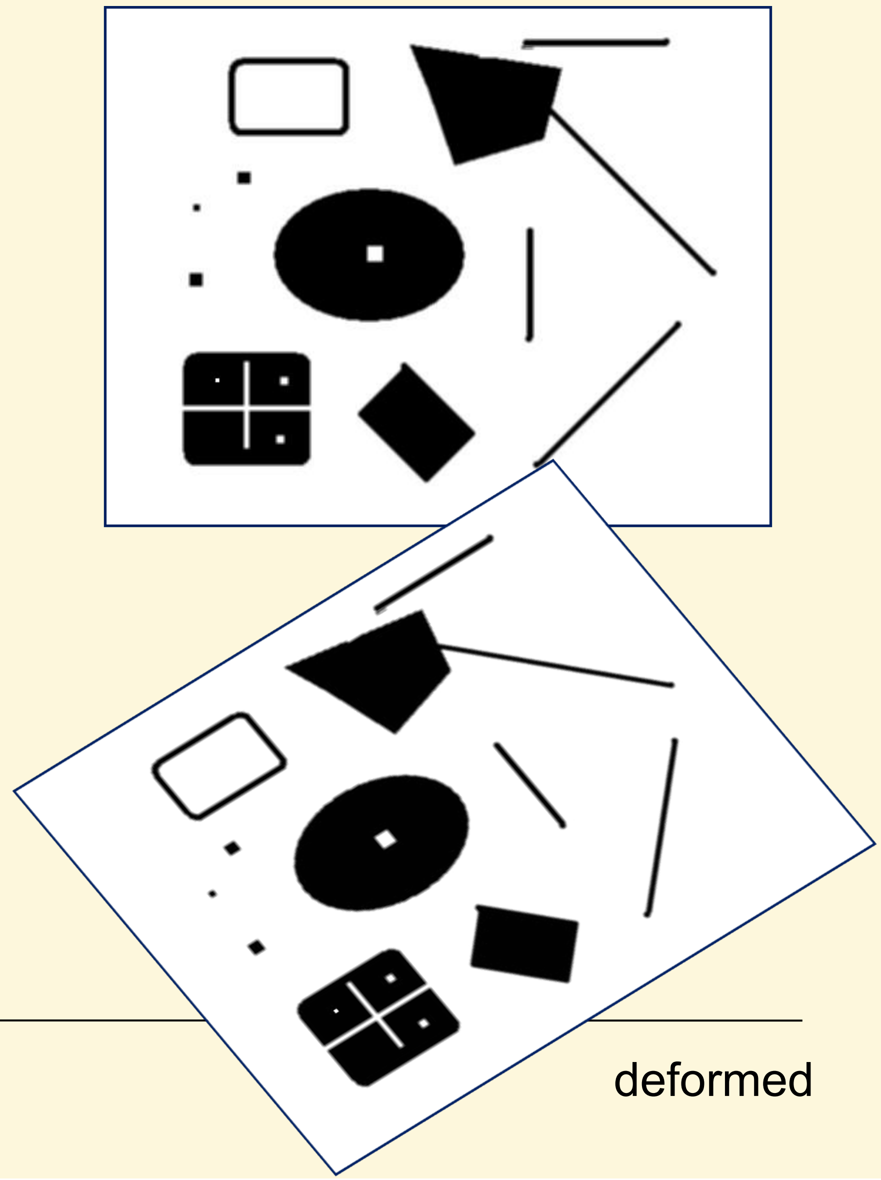
- Note: “interest points” = “keypoints”, also sometimes called “features”

- Many applications
  - tracking: which points are good to track?
  - recognition: find patches likely to tell us something about object category
  - 3D reconstruction: find correspondences across different views

This week: interest points

- Suppose you have to click on some point, go away and come back after I deform the image, and click on the same points again.
  - Which points would you choose?

original

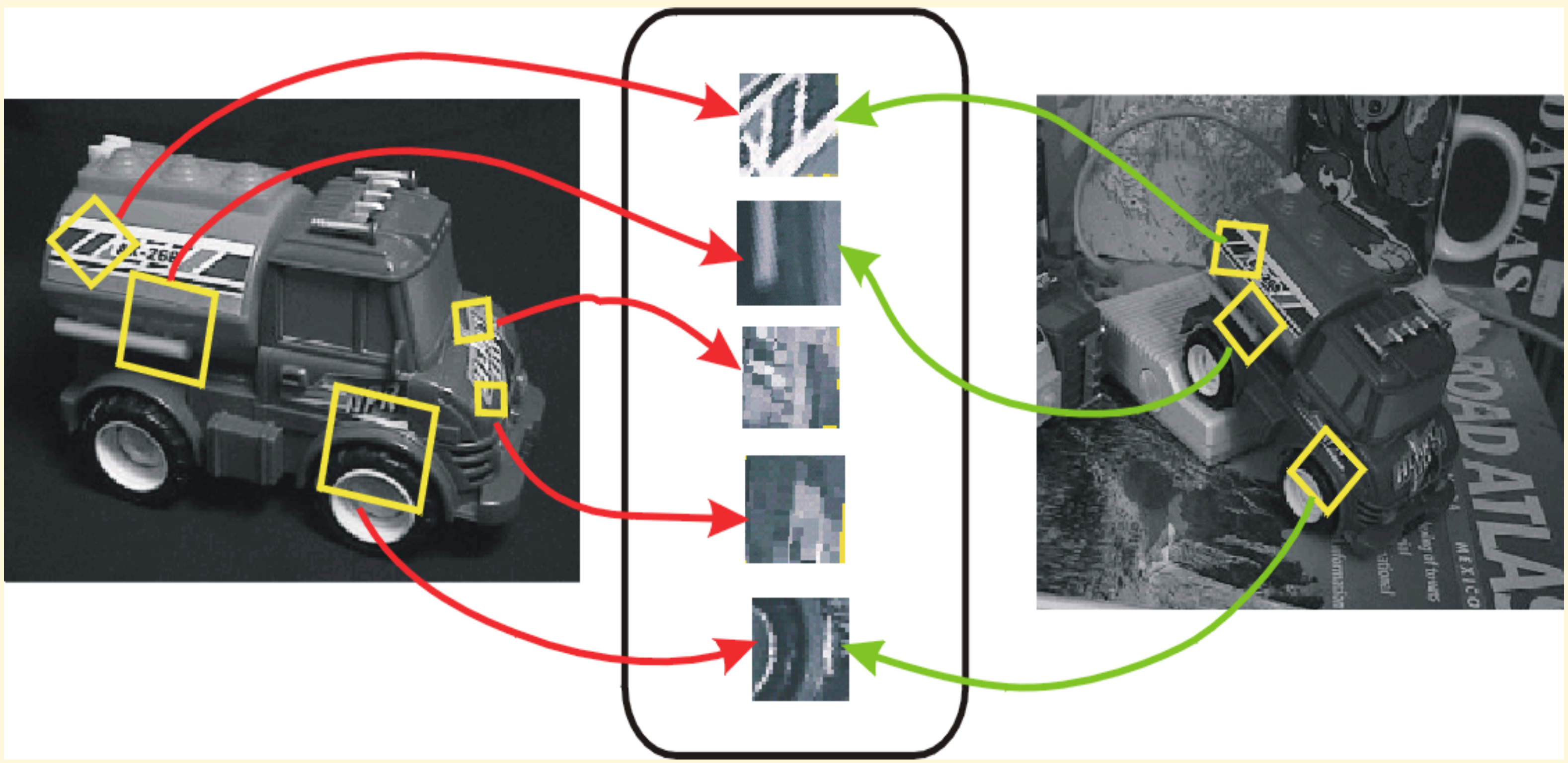


deformed

Invariant Local Features

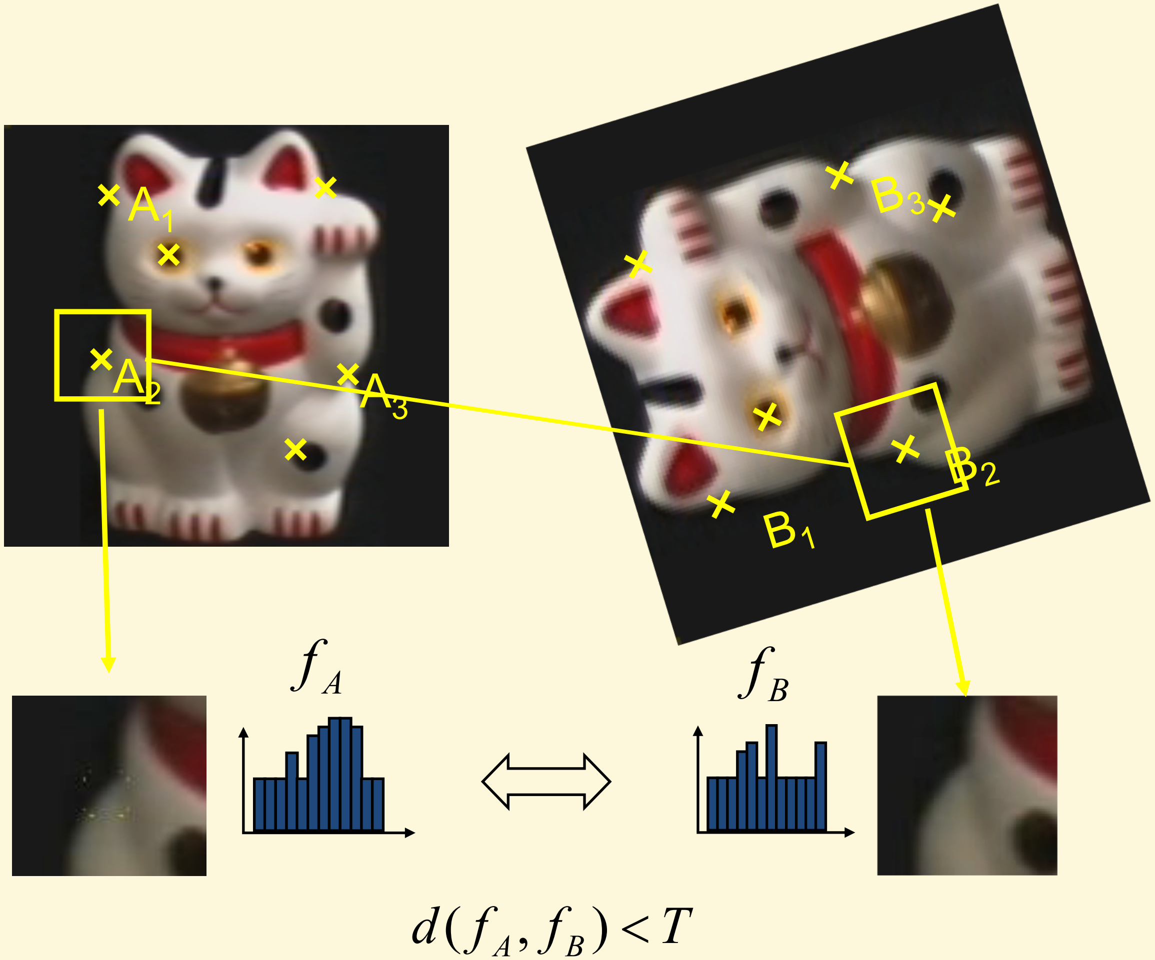
Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters

Features Descriptors



K. Graulman, B. Leibe

Overview of Keypoint Matching

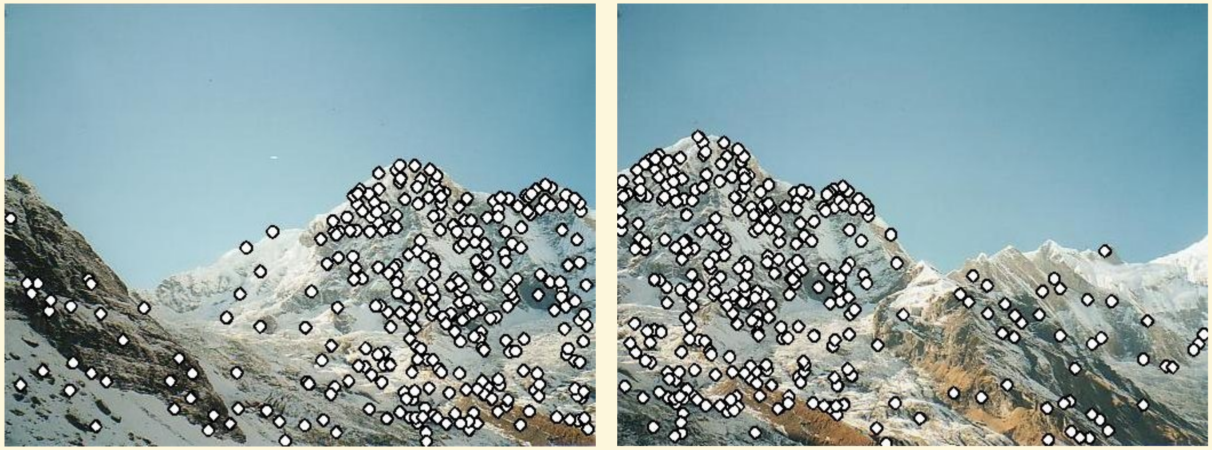


1. Find a set of distinctive key-points
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

Goals for Keypoints

Detect points that are *repeatable* and *distinctive*

Characteristics of good features



- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

K. Grauman, B. Leibe

Many Existing Detectors Available

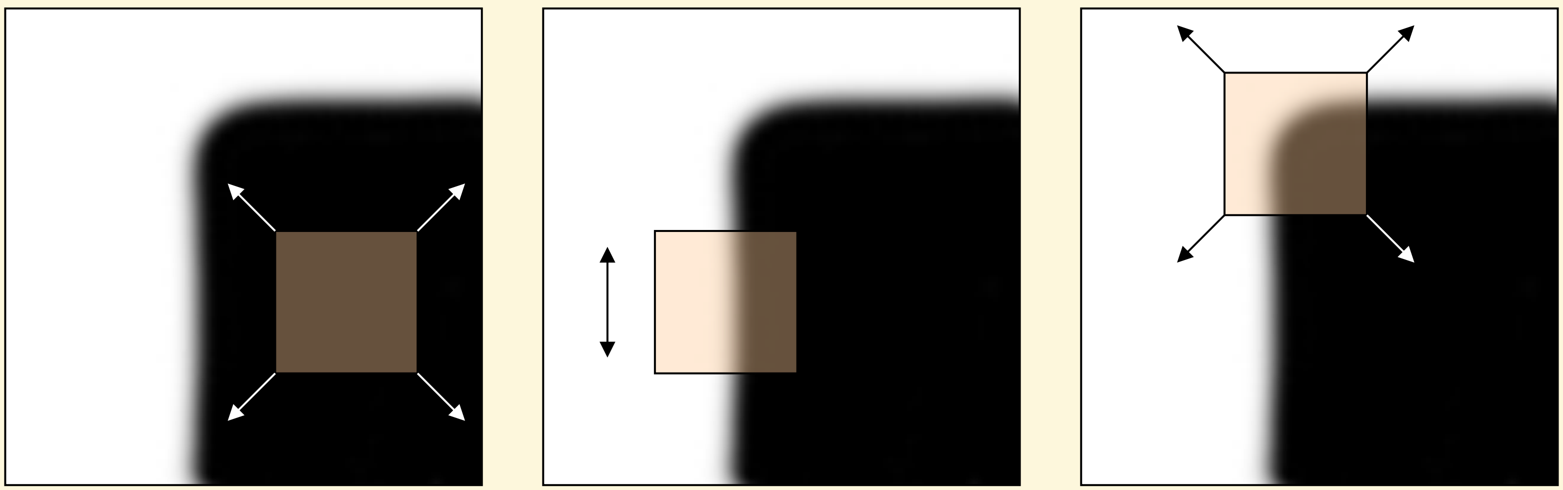
Hessian & Harris	[Beaudet ‘78], [Harris ‘88]
Laplacian, DoG	[Lindeberg ‘98], [Lowe 1999]
Harris-/Hessian-Laplace	[Mikolajczyk & Schmid ‘01]
Harris-/Hessian-Affine	[Mikolajczyk & Schmid ‘04]
EBR and IBR	[Tuytelaars & Van Gool ‘04]
MSER	[Matas ‘02]
Salient Regions	[Kadir & Brady ‘01]
SIFT	[Lowe 2004]
Others…	

Feature extraction: Corners

Slides from Rick Szeliski, Svetlana Lazebnik, and Kristin Grauman

Corners as distinctive interest points

- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



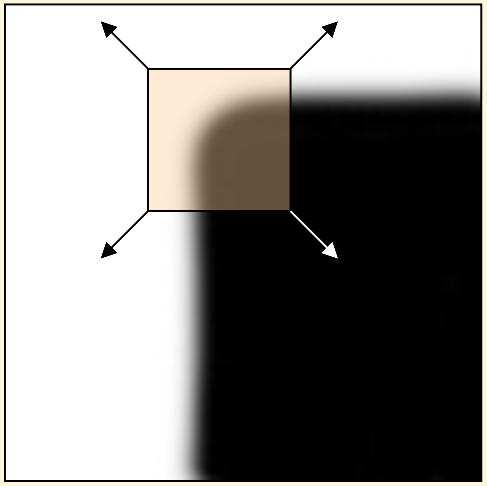
"flat" region: no change in all directions

"edge": no change along the edge direction

"corner": significant change in all directions

Slide credit: Alyosha Efros, Darya Frolova, Denis Simakov

Finding Corners



“corner”

At a corner, we expect two important effects.

- there should be large gradients.
- in a small neighborhood, the gradient orientation should swing sharply.

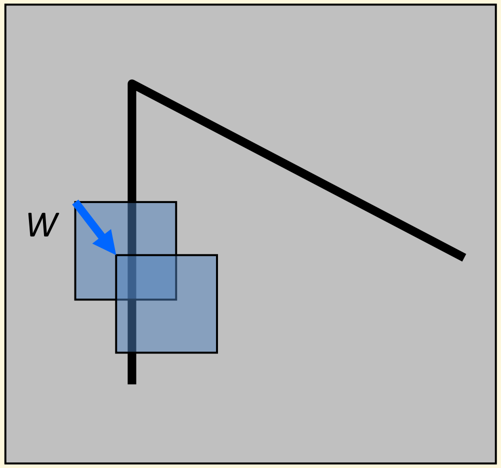
We can identify corners by looking at variations in orientation within a window.

C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“ *Proceedings of the 4th Alvey Vision Conference*: pages 147--151. 1988

Harris corner detection: the math

Consider shifting the window *W* by (*u,v*)

- how do the pixels in W change?
- compare each pixel before and after by summing up the squared differences (SSD)
- this defines an SSD "error" *E*(*u,v*):



$$E(u,v) = \sum_{(x,y)\in W} \left(I(x+u,y+v) - I(x,y)\right)^2$$

Harris Detector: Mathematics

Window-averaged change of intensity induced by shifting the image data by [*u,v*]:

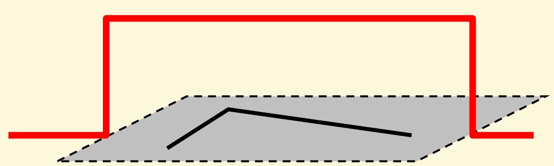
$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

Window function

Shifted intensity

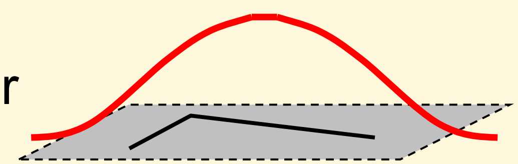
Intensity

Window function $w(x,y)$ =



1 in window, 0 outside

or



Gaussian

Small motion assumption

Taylor Series expansion of *I*:

$$I(x+u, y+v) = I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

If the motion (u,v) is small, then first order approximation is good

$$\begin{aligned} I(x+u, y+v) &\approx I(x,y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \\ &\approx I(x,y) + [I_x \;\; I_y] \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}$$

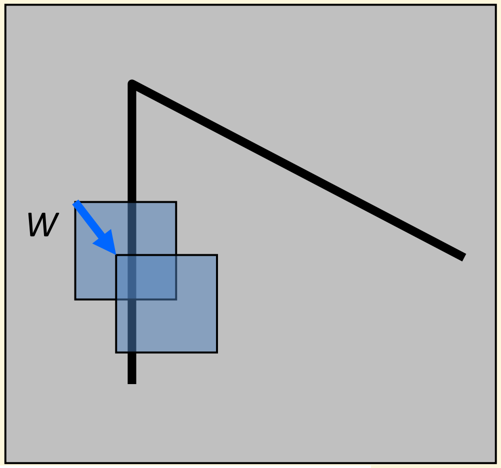
$$\text{shorthand: } I_x = \frac{\partial I}{\partial x}$$

Plugging this into the formula on the previous slide…

Harris corner detection: the math

Using the small motion assumption,
replace I with a linear approximation

(Shorthand: $I_x = \frac{\partial I}{\partial x}$)



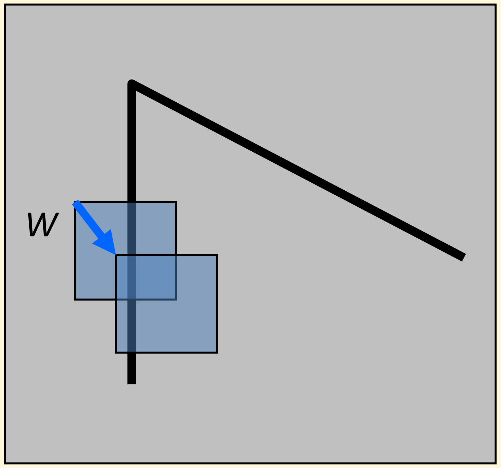
$$
\begin{aligned}
E(u,v) &= \sum_{(x,y)\in W} \left(I(x+u,y+v) - I(x,y)\right)^2 \\
&\approx \sum_{(x,y)\in W} \left(I(x,y) + I_x(x,y)u + I_y(x,y)v - I(x,y)\right)^2 \\
&\approx \sum_{(x,y)\in W} \left(I_x(x,y)u + I_y(x,y)v\right)^2
\end{aligned}
$$

Corner detection: the math

$$E(u,v) \approx \sum_{(x,y)\in W} \left(I_x(x,y)u + I_y(x,y)v\right)^2$$

$$\approx \sum_{(x,y)\in W} \left(I_x^2u^2 + 2I_xI_yuv + I_y^2v^2\right)$$

$$\approx \quad Au^2 + 2Buv + Cv^2$$



$$A = \sum_{(x,y)\in W} I_x^2 \qquad B = \sum_{(x,y)\in W} I_xI_y \qquad C = \sum_{(x,y)\in W} I_y^2$$

- Thus, *E*(*u*,*v*) is locally approximated as a *quadratic form*

The second moment matrix

The surface *E*(*u*,*v*) is locally approximated by a quadratic form.

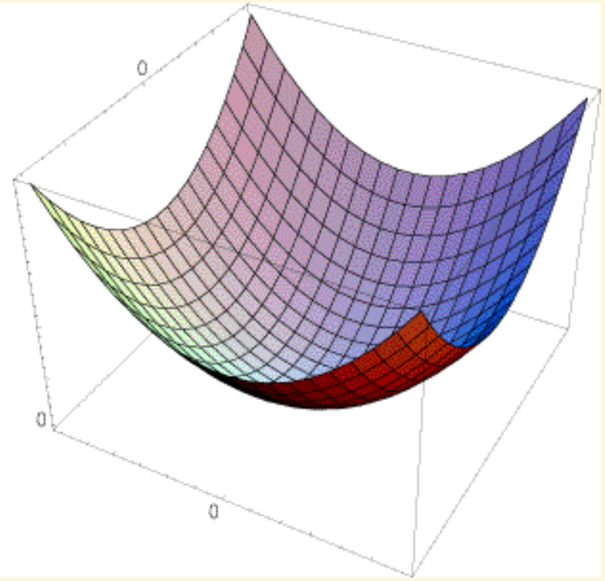
$$E(u,v) \approx Au^2 + 2Buv + Cv^2$$

$$\approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



Taylor series approx to shifted image

(Shorthand: $I_x = \frac{\partial I}{\partial x}$)

$$E(u,v) \approx \sum_{x,y} w(x,y)[I(x,y) + uI_x + vI_y - I(x,y)]^2$$

$$= \sum_{x,y} w(x,y)[uI_x + vI_y]^2$$

$$= \sum_{x,y} w(x,y)\begin{pmatrix} u & v \end{pmatrix} \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

Harris Detector: Mathematics

Expanding I(x,y) in a Taylor series expansion, we have, for small shifts [*u,v*], a *bilinear* approximation:

$$E(u,v) \cong [u,v] \quad M \quad \begin{bmatrix} u \\ v \end{bmatrix}$$

where *M* is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

M is also called "structure tensor"

The second moment matrix

The surface *E*(*u*,*v*) is locally approximated by a quadratic form.

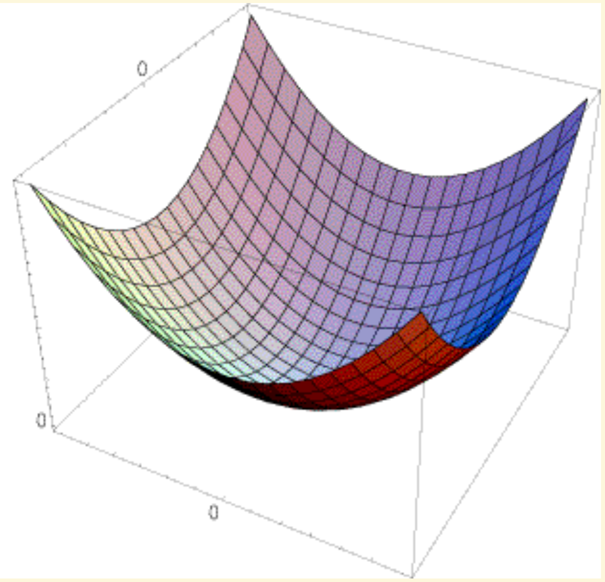
$$E(u,v) \approx Au^2 + 2Buv + Cv^2$$

$$\approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



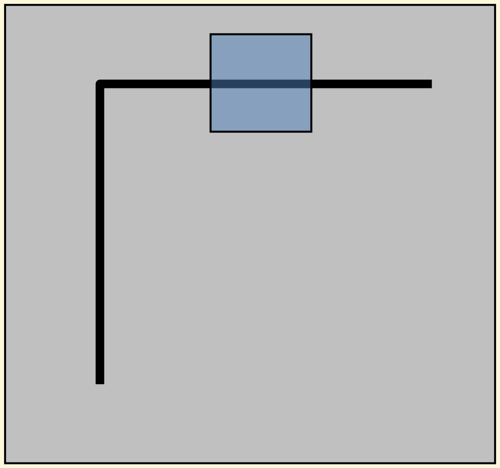
Let's try to understand its shape.

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

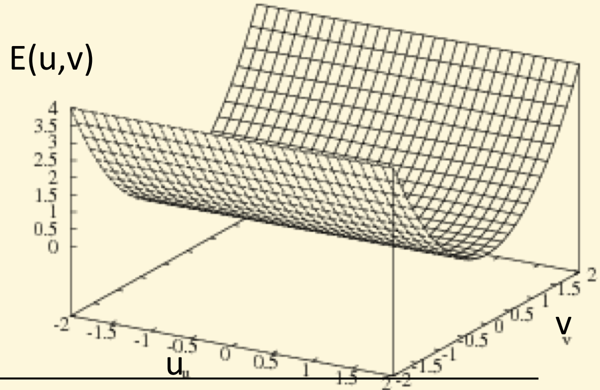
$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



Horizontal edge: $I_x = 0$

$$\mathbf{M} = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}$$

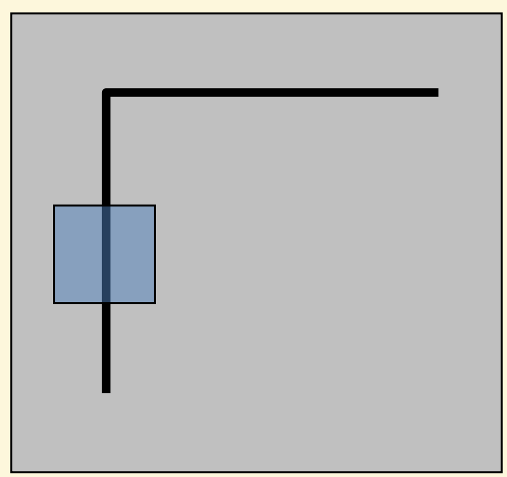


$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$A = \sum_{(x,y)\in W} I_x^2$$

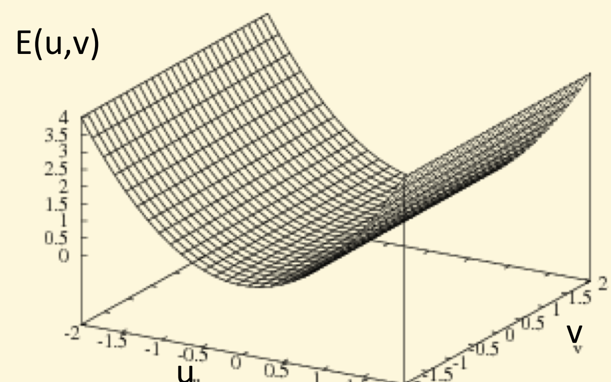
$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



Vertical edge: $I_y = 0$

$$\mathbf{M} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$$



Harris Detector: Mathematics

Intensity change in shifting window: eigenvalue analysis

The shape of M tells us something about the *distribution of gradients* around a pixel

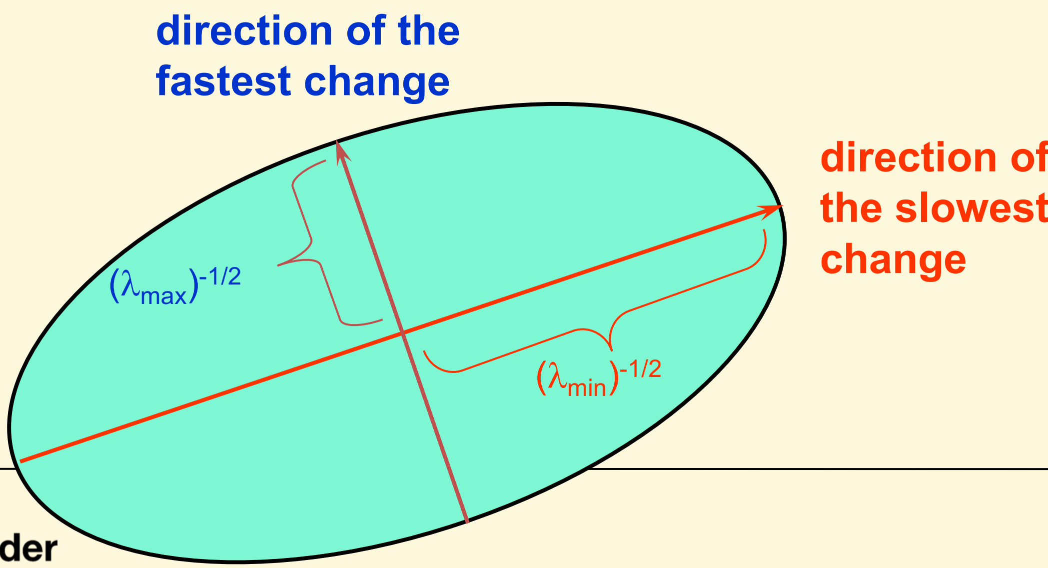
We can visualize M as an ellipse with axis lengths determined by the *eigenvalues* of M and orientation determined by the *eigenvectors* of M

λ_{max}, λ_{min} : eigenvalues of M

Ellipse equation:

$E(u,v)$ = const

$$E(u,v) \cong [u,v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

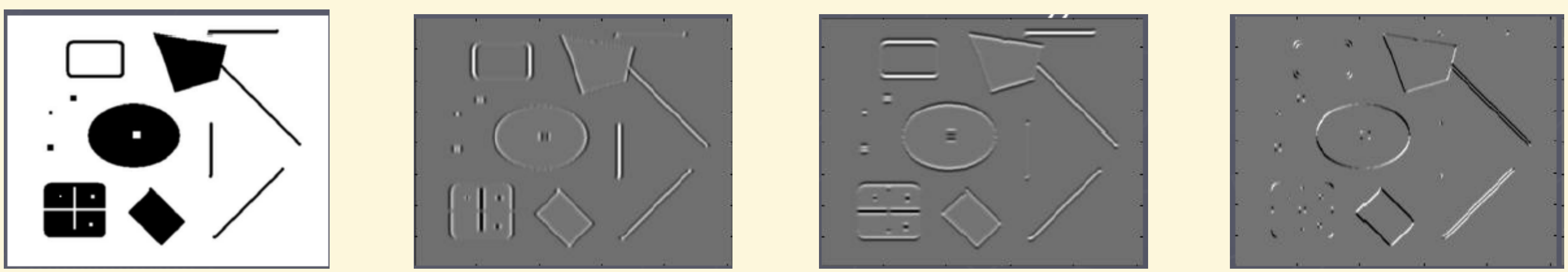


Iso-intensity contour of $E(u,v)$

Corners as distinctive interest points

$$M = \sum w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

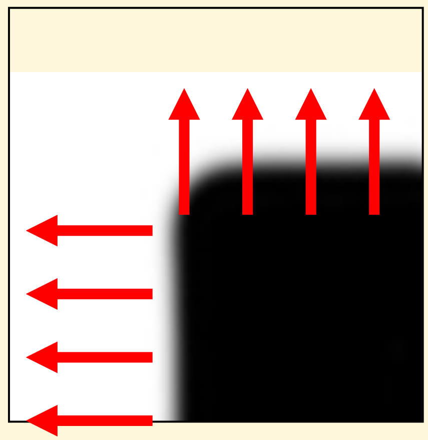
2 x 2 matrix of image derivatives (averaged in neighborhood of a point).



Notation: $I_x \Leftrightarrow \frac{\partial I}{\partial x}$ $\quad I_y \Leftrightarrow \frac{\partial I}{\partial y}$ $\quad I_x I_y \Leftrightarrow \frac{\partial I}{\partial x}\frac{\partial I}{\partial y}$

What does this matrix reveal?

First, consider an axis-aligned corner:



What does this matrix reveal?

First, consider an axis-aligned corner:

$$M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \Rightarrow \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

This means dominant gradient directions align with x or y axis

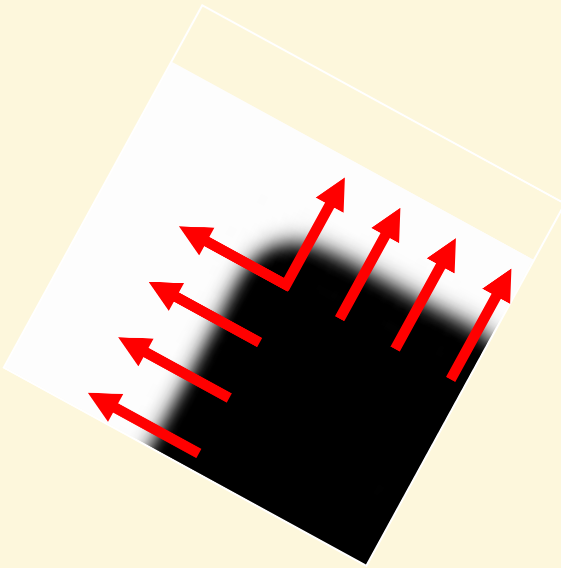
Look for locations where **both** λ's are large.

If either λ is close to 0, then this is **not** corner-like.

What if we have a corner that is not aligned with the image axes?

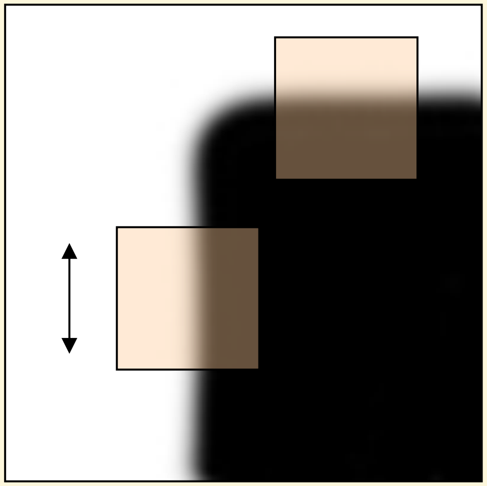
What does this matrix reveal?

Since *M* is symmetric, we have $M = X \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} X^T$



The *eigenvalues* of *M* reveal the amount of intensity change in the two principal orthogonal gradient directions in the window.

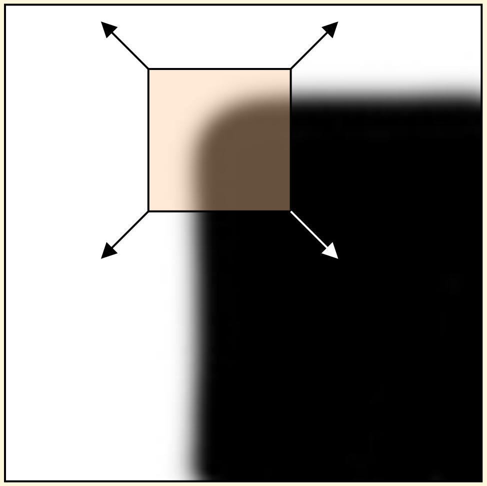
Corner response function



"edge":

$\lambda_1 >> \lambda_2$

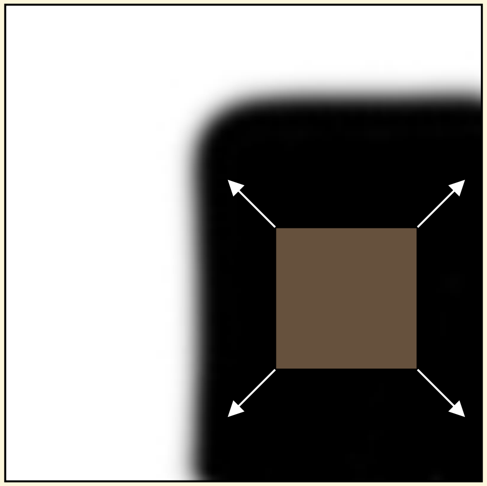
$\lambda_2 >> \lambda_1$



"corner":

λ_1 and λ_2 are large,

$\lambda_1 \sim \lambda_2$;



"flat" region

λ_1 and λ_2 are small;

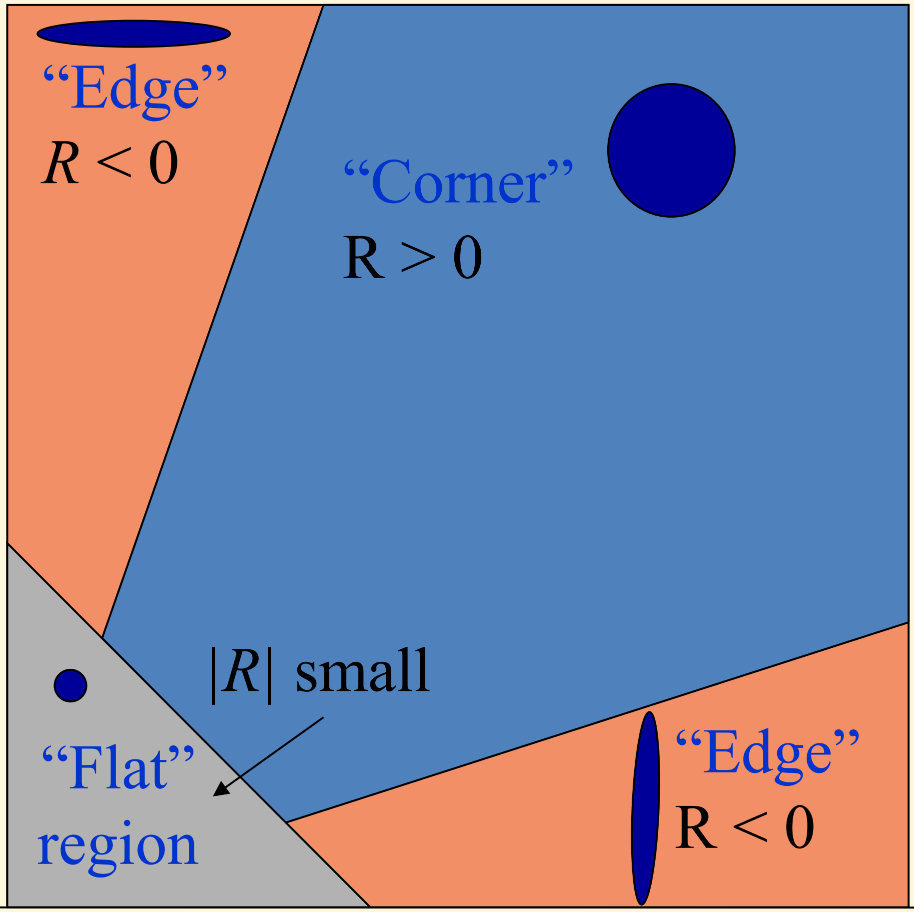
$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

Corner response function

$$R = \det(M) - \alpha\, \mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)

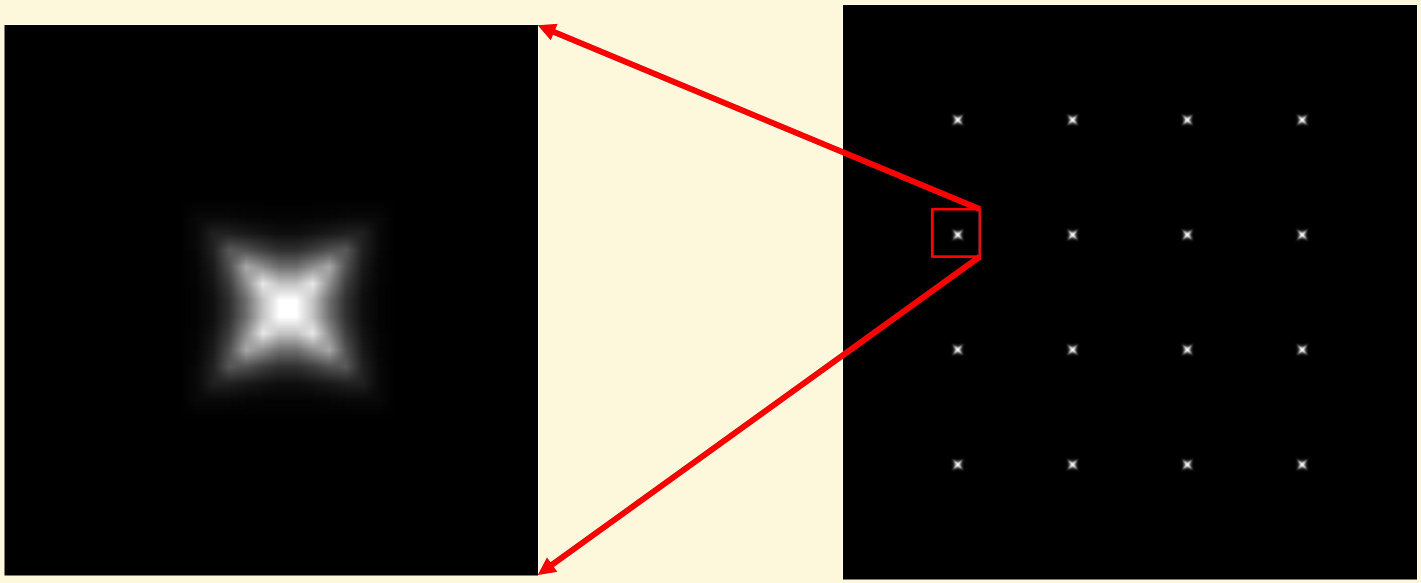
R = “cornerness”



Harris corner detector

1) Filter image with Gaussian window W.
2) Compute magnitude of the gradient everywhere.
3) Compute *M* matrix for each image window, using the same W Gaussian window function
4) Computer the *cornerness (R)* scores for each image window.
5) Find points whose surrounding window gave large corner response (*R* > threshold)
6) Take the points of local maxima (*non-maximum suppression*)

Non-Maximum Supression



Example of Harris application

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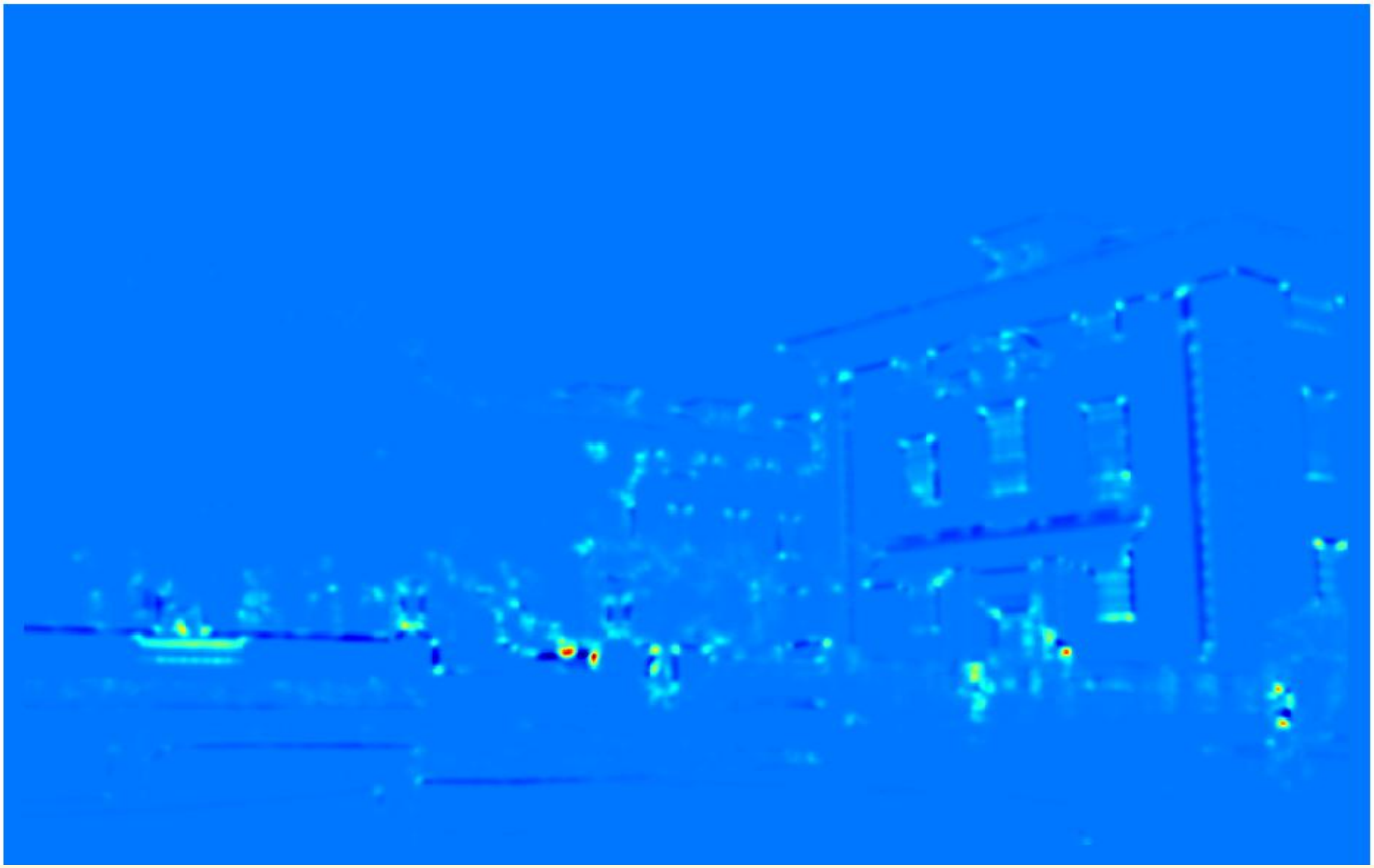
Example of Harris application



Compute corner response at every pixel.

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Example of Harris application

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Properties of the Harris corner detector

- Rotation invariant?

Yes

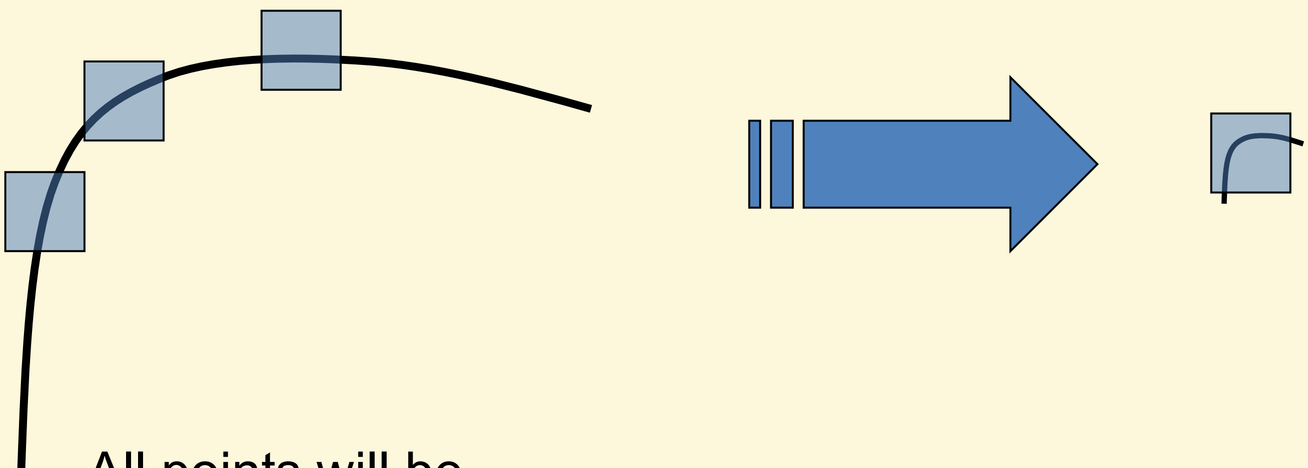
$$M = X \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} X^T$$

- Scale invariant?

Properties of the Harris corner detector

- Rotation invariant? Yes

- Scale invariant? No



All points will be classified as edges

Corner !