CSCI 4830 / 5722
Computer Vision

University of Colorado **Boulder**

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Computer Vision

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Spring 2019
Lecture 2

Reminders

Submissions:

- Homework 1: Sat 1/26 at 6 pm

Moodle, Matlab, Piazza

Disabilities forms

Readings:

- Szeliski 2.1 before Friday

What is Computer Vision?

<u>Extracting information from images</u>

Methods for: acquiring, processing, analyzing and interpreting images.

Digital images

- from a camera (usually)
- divided into smaller elements (pixels)
- stored as a matrix of numbers, usually positive (0-255)
- 8-bit, 12-bit, 16-bit, or float
- color image has 3 channels: R, G, B

What is Computer Vision?

Extracting information from images

Besides pixel information, what else can we use?

- information about the camera parameters
  - if known
  - if not, derive them from information from multiple images
  - use differences between images

Image Transformations: 2D

Motivation:

- mosaics

Image Transformations: 2D

Motivation:

- medical imaging: multi-modality registration /fusion

Image registration:
"the determination of a one-to-one mapping between the coordinates in one space and those in another, such that points in the two spaces that correspond to the same anatomical point are mapped to each other."

Calvin Maurer '93

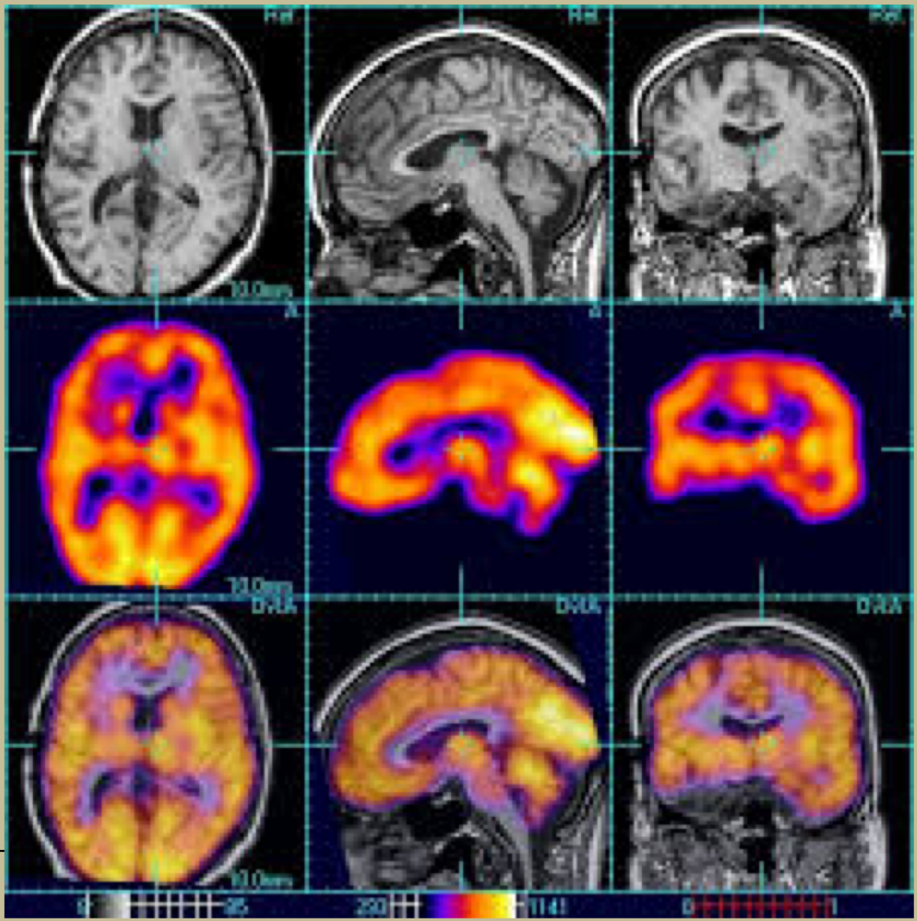


Image Transformations: 2D

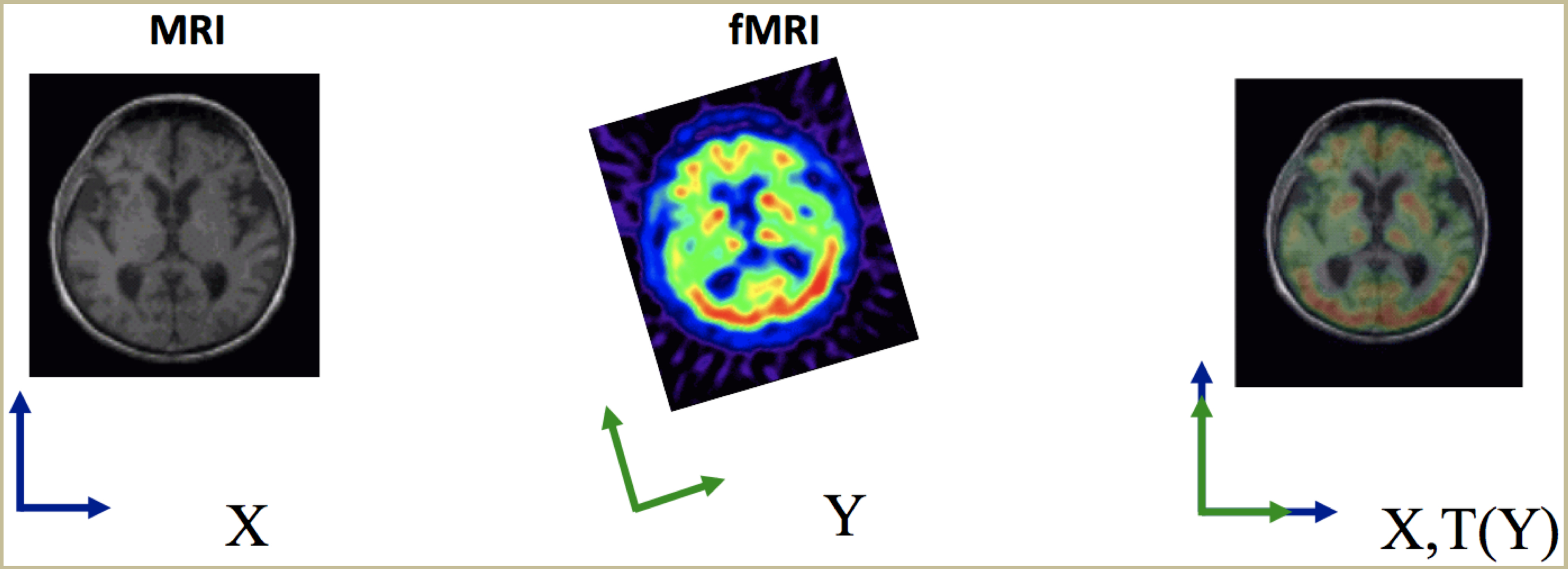


Image Transformations: 2D

Solution: fit the parameters of some transformation according to a set of corresponding/matching “feature” pairs

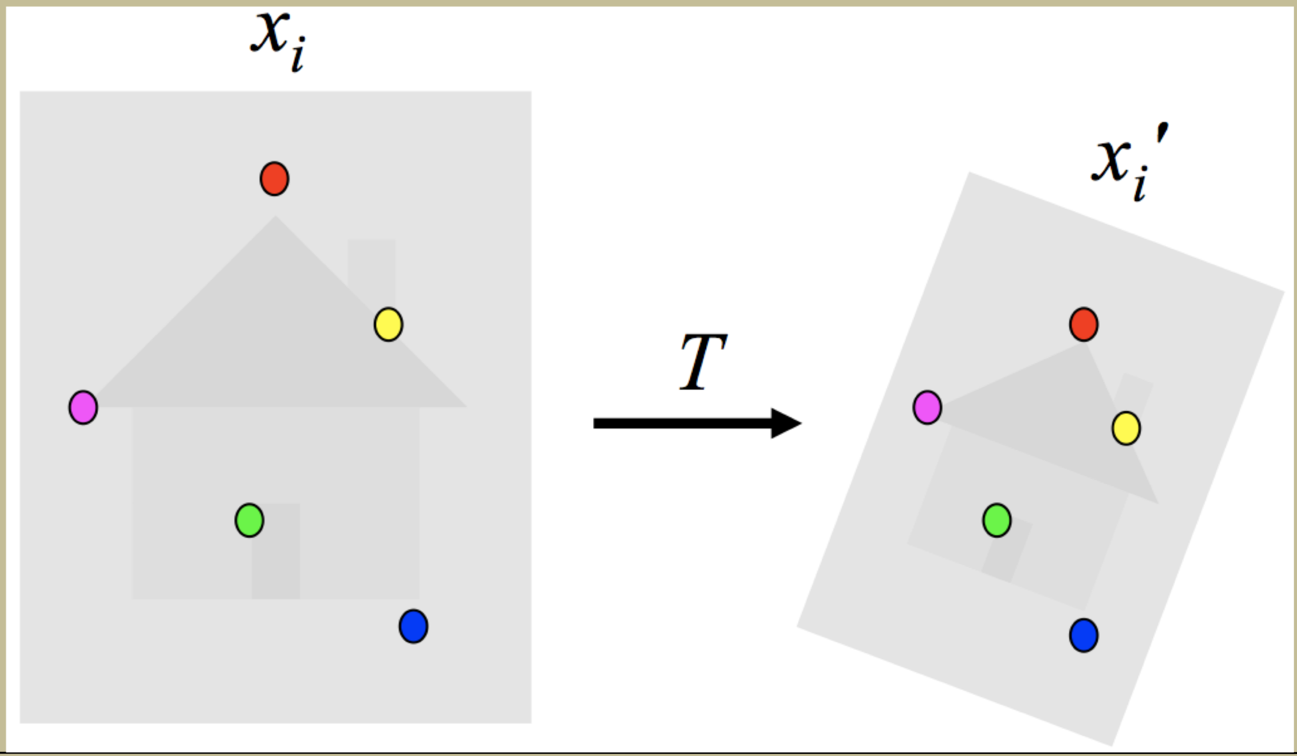


Image Transformations: 2D

Transformations: use "location" of each corresponding feature pair.

Location: image coordinates vs world coordinates

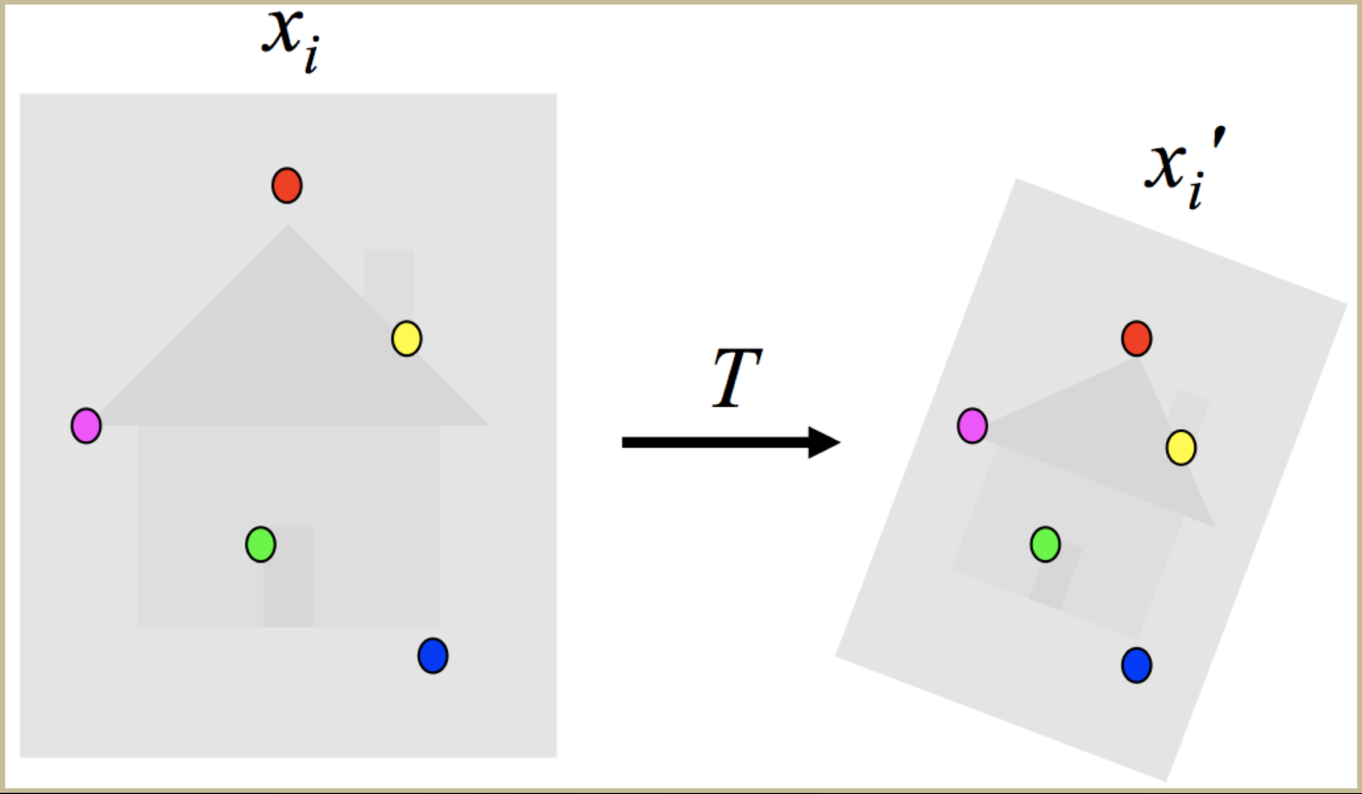
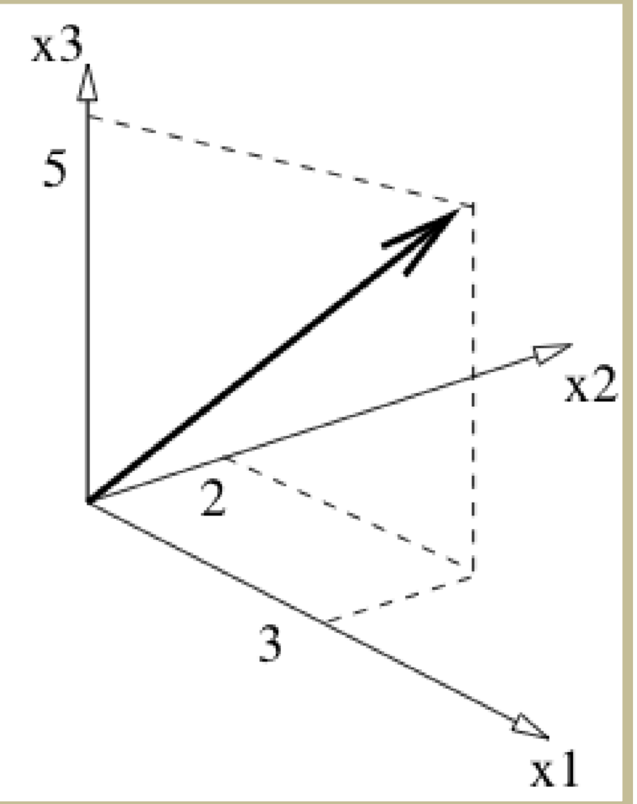


Image coordinates, Points, Vectors

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{pmatrix}$$

e.g.

$$\boldsymbol{x} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$$



The **length** of *x*, a.k.a. the **norm** or **2-norm** of *x*, is

$$\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

e.g.,

$$\|x\| = \sqrt{3^2 + 2^2 + 5^2} = \sqrt{38}$$

Vector Arithmetic

Addition To add two vectors together, add them componentwise

```
>> v1 = [3 2 5]'

v1 =

     3
     2
     5

>> v2 = [1 3 2]'

v2 =

     1
     3
     2
```

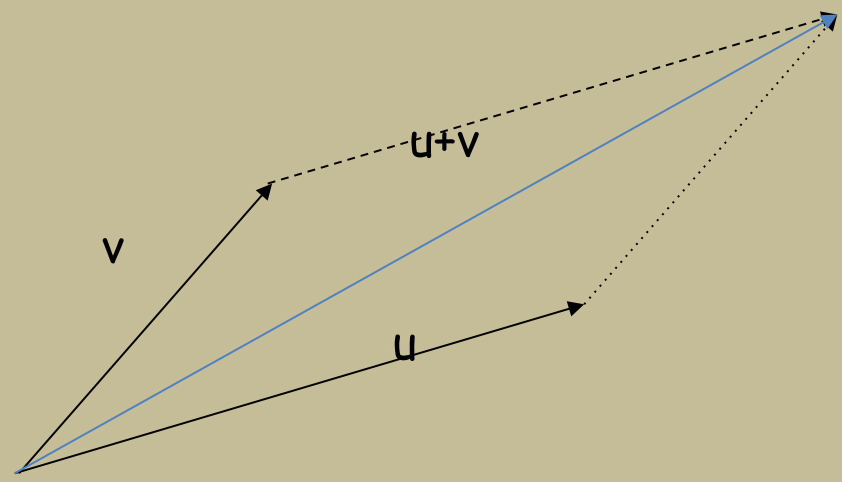
```
>> v1+v2

ans =

     4
     5
     7
```

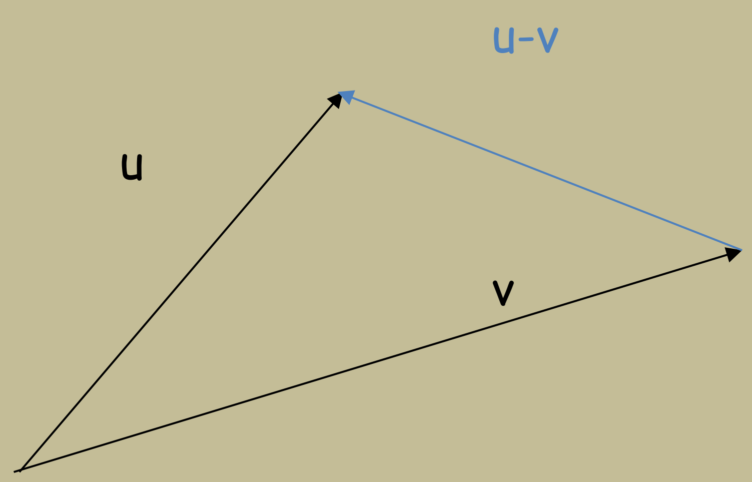
Vector Addition

$$u + v = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}$$



Vector Subtraction

$$u - v = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} - \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} u_1 - v_1 \\ u_2 - v_2 \end{bmatrix}$$



Scaling a vector

$$\boldsymbol{z} = \alpha \boldsymbol{x}$$

for a scalar α then

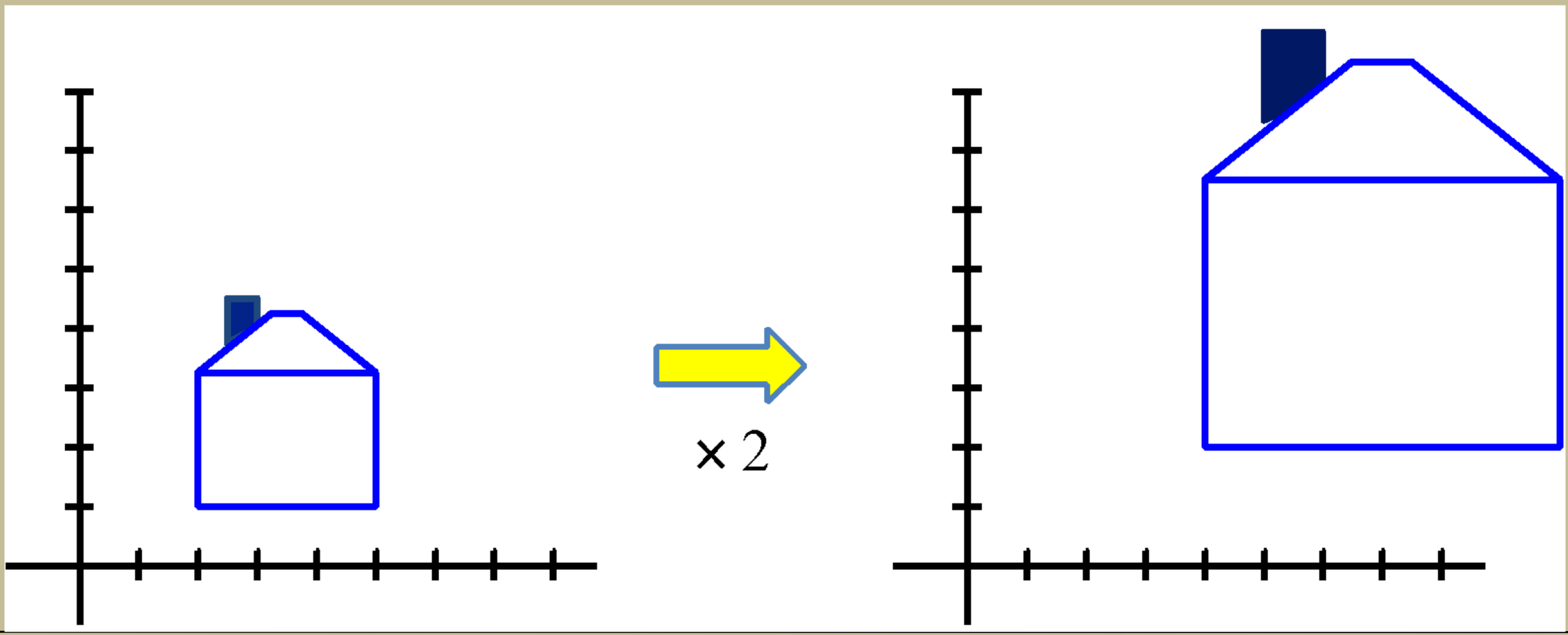
$$\boldsymbol{z} = \alpha \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3\alpha \\ 5\alpha \\ 2\alpha \end{pmatrix}$$

(This is just like stretching/shrinking the vector by a factor α

Scaling an object

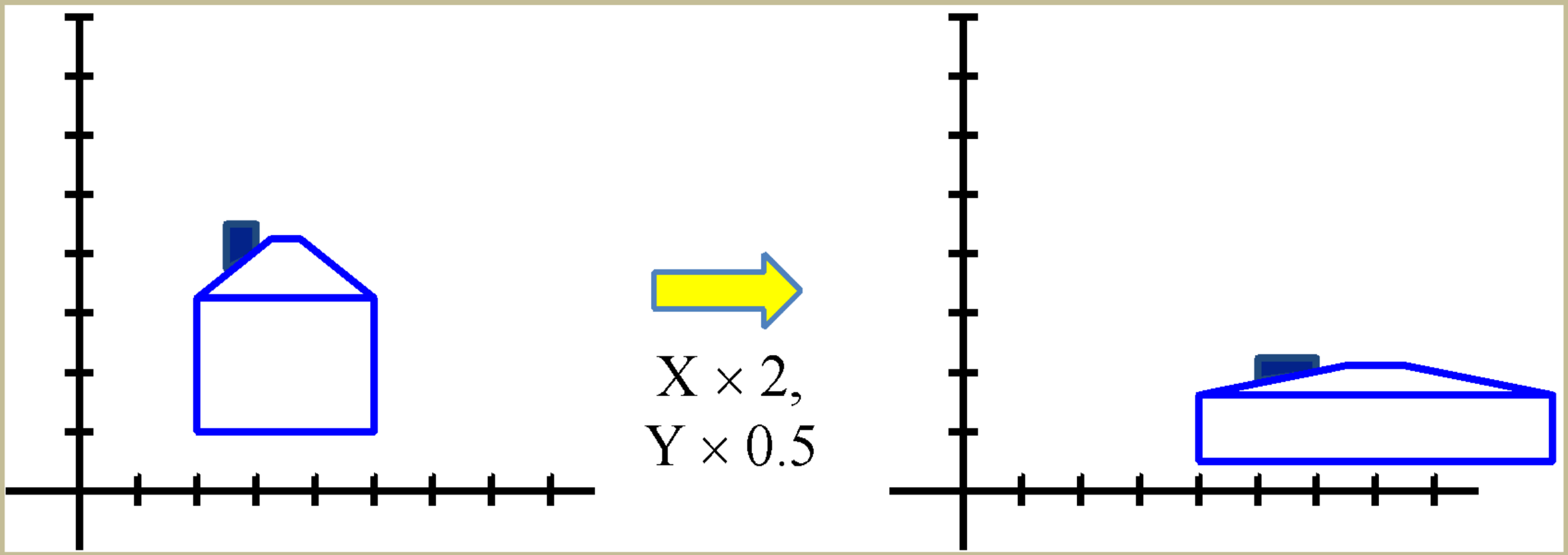
Scaling a coordinate means multiplying each of its components by a scalar.

Uniform scaling means this scalar is the same for all components:



Scaling an object

Non-uniform scaling means different scalar per component



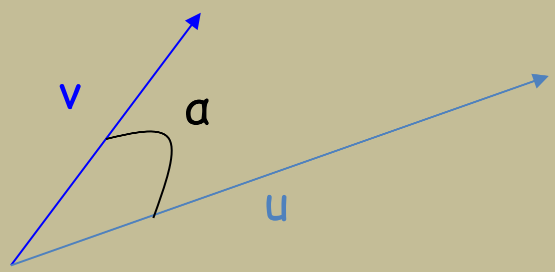
Inner product of two vectors

$$a = \begin{bmatrix} 6 \\ 2 \\ -3 \end{bmatrix} \qquad b = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$$

$$a \cdot b = a^T b \quad = \begin{bmatrix} 6 & 2 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix} \quad = 6 \cdot 4 + 2 \cdot 1 + (-3) \cdot 5 \quad = 11$$

Inner (dot) Product

$$u^T v = \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_1 \end{bmatrix} = u_1.v_1 + u_2.v_2$$



The inner product is a SCALAR.

$$u^T v = \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \|u\| \|v\| \cos \alpha$$

$$u^T v = 0 \Leftrightarrow u \perp v$$

In matlab use sum(a.*b)

```
>> a = [1 2 3]

a =

     1     2     3

>> b = [2 3 4]
```

```
b =

     2     3     4

>> sum(a.*b)

ans =

    20
```

Matrices

A matrix is an $n \times M$ element array

$$A = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1m} \\ a_{21} & & & \\ . & & & \\ . & & & \\ . & & & \\ a_{N1} & ... & ... & a_{NM} \end{bmatrix}$$

transpose of matrix A (written A^T) is a_{ji} (A with rows and columns flipped)

Transpose of a Matrix

Transpose:

$$C_{m\times n} = A^T{}_{n\times m} \qquad (A+B)^T = A^T + B^T$$

$$c_{ij} = a_{ji} \qquad (AB)^T = B^T A^T$$

Examples:

$$\begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 2 & 5 \end{bmatrix} \qquad \begin{bmatrix} 6 & 2 \\ 1 & 5 \\ 3 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 & 3 \\ 2 & 5 & 8 \end{bmatrix}$$

If $A^T = A$, we say A is **symmetric.**

Transpose of a Matrix: properties

$$(A^T)^T = A$$

$$(A+B)^T = A^T + B^T$$

$$(AB)^T = B^T A^T$$

$$(rA^T) = rA^T$$

Transpose in Matlab

In Matlab use A′

```
>> A = [1 2 ; 3 4]

A =

     1     2
     3     4

>> A'

ans =

     1     3
     2     4
```

Matrix Multiplication Example

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 10 & 2 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Matrix Multiplication Example

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 10 & 2 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$a_{11} = 1 * 4 +$$

Matrix Multiplication Example

$$\begin{bmatrix} \color{red}{1} & \color{red}{2} & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} \color{red}{4} & 5 \\ \color{red}{10} & 2 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$a_{11} = 1 * 4 + 2 * 10 +$$

Matrix Multiplication Example

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 10 & 2 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} 30 & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$a_{11} = 1 * 4 + 2 * 10 + 3 * 2 = 30$$

Matrix Multiplication Example

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 10 & 2 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} 30 & 39 \\ a_{21} & a_{22} \end{bmatrix}$$

$$a_{12} = 1 * 5 + 2 * 2 + 3 * 10 = 39$$

Matrix Multiplication Example

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 10 & 2 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} 30 & 39 \\ 34 & a_{22} \end{bmatrix}$$

$$a_{12} = 3 * 4 + 2 * 10 + 1 * 2 = 34$$

Matrix Multiplication Example

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 10 & 2 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} 30 & 39 \\ 34 & 29 \end{bmatrix}$$

$$a_{12} = 3 * 5 + 2 * 2 + 1 * 10 = 29$$

Matrix Product

Product:

$$C_{n\times p} = A_{n\times m} B_{m\times p}$$

A and B must have compatible dimensions

$$c_{ij} = \sum_{k=1}^{m} a_{ik} b_{kj}$$

In Matlab: `>> A*B`

Example:

$$\begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 17 & 29 \\ 19 & 11 \end{bmatrix} \qquad \boxed{\begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} \boxed{} \end{bmatrix}}$$

Matrix Multiplication is **not** commutative: $A_{n\times n} B_{n\times n} \neq B_{n\times n} A_{n\times n}$

Matrix Addition

$$A_{n \times m} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ & \dots & & \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix}$$

Sum:

$$C_{n \times m} = A_{n \times m} + B_{n \times m}$$

$$c_{ij} = a_{ij} + b_{ij}$$

A and B must have the same dimensions

Example:

$$\begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 4 & 6 \end{bmatrix}$$

Determinant of a Matrix

Determinant: A must be square

$$\det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

Example:

$$\det\begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} = 2 - 15 = -13$$

$$\det\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Determinant in Matlab

In matlab use det(A)

```
>> A = [ 1 2 ; 3 4]

A =

     1     2
     3     4

>> det(A)

ans =

    -2
```

Inverse of a Matrix

- If A is a square matrix, the **inverse** of *A*, called A^{-1}, satisfies

$$AA^{-1} = I \quad \text{and} \quad A^{-1}A = I,$$

- Where *I*, the **identity matrix**, is a diagonal matrix with all 1's on the diagonal.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse of a 2D Matrix

For a 2-D matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{|A|}$$

Example:

$$\begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix}^{-1} = \frac{1}{28}\begin{bmatrix} 5 & -2 \\ -1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix} = \frac{1}{28}\begin{bmatrix} 5 & -2 \\ -1 & 6 \end{bmatrix} \cdot \begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix} = \frac{1}{28}\begin{bmatrix} 28 & 0 \\ 0 & 28 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Inverses in Matlab

in Matlab use inv(A)

```
>> A = [1 2 ; 3 4]

A =

     1     2
     3     4

>> inv(A)

ans =

   -2.0000    1.0000
    1.5000   -0.5000
```

Trace of a matrix

trace of a Matrix is Tr(A) =

$$\sum_{i=1}^{N} a_{ii}$$

(the sum of the diagonal entries)

In matlab use trace(A)

Next lecture

- Continue image transformations in 2D and Linear Algebra Review
- Readings: Szeliski 2.1