

Recurrent neural networks and hierarchical structure

Dieuwke Hupkes

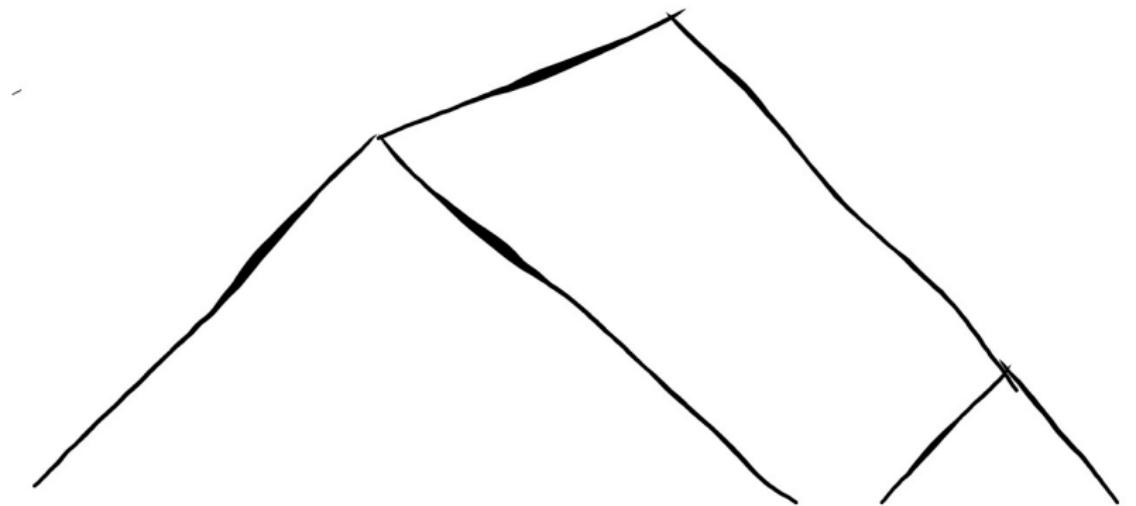
Institute for Logic, Language and Computation
University of Amsterdam

Johns Hopkins University
October 9, 2019

The structure of language

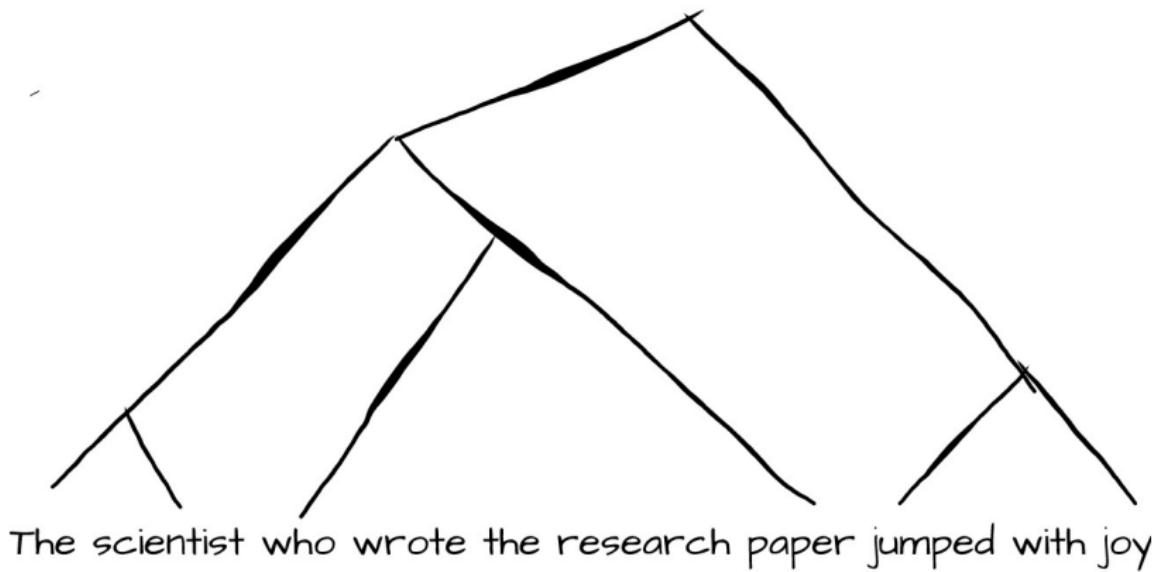
The scientist who wrote the research paper jumped with joy

The structure of language



The scientist who wrote the research paper jumped with joy

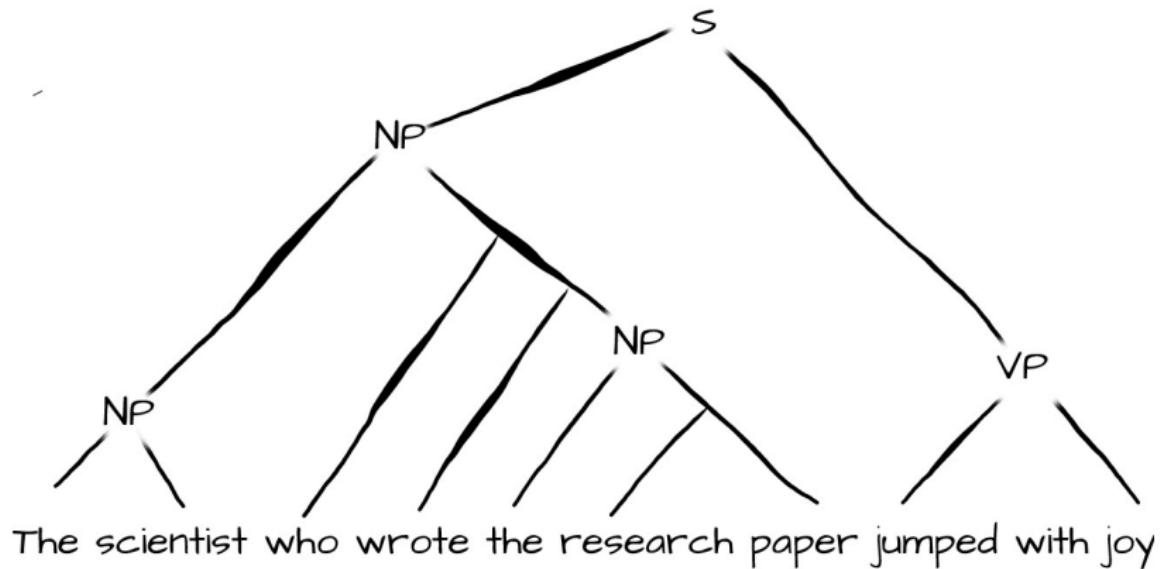
The structure of language



The structure of language



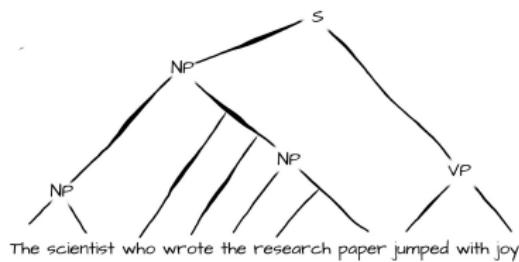
The structure of language



Symbolic structure and the brain



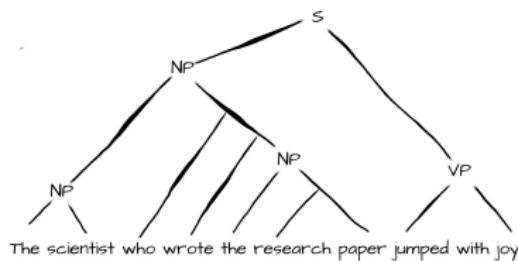
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Symbolic structure and the brain

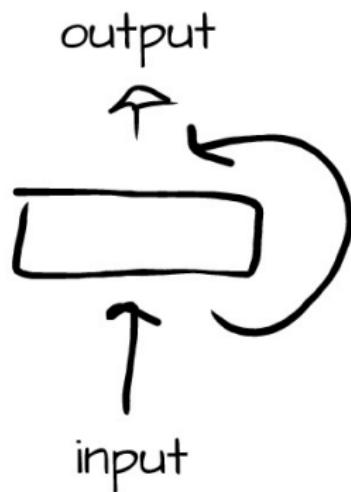


?



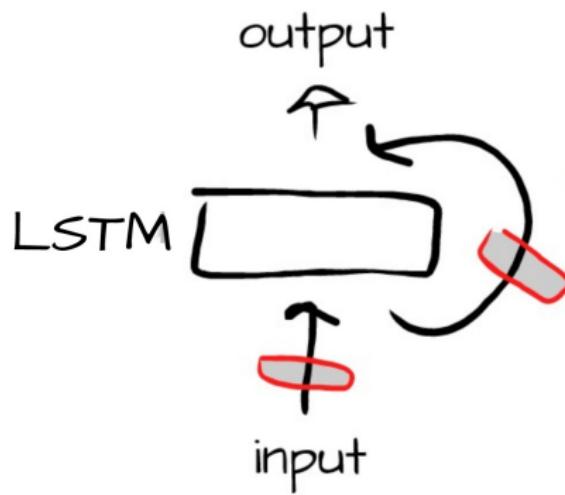
- Language is a product of our brain, but our brains do not have any explicit means to represent rules and symbols, how is this possible?

Simple recurrent network



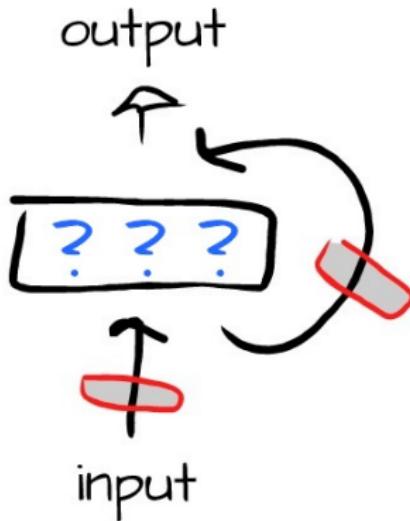
(Elman 1990)

Gated recurrent neural networks



(Hochreiter and Schmidhuber 1997)

Gated recurrent neural networks



- How can hierarchical structure be processed *incrementally*, in *linear time*, by a *recurrent artificial neural network*?

Artificial languages

- The compositionality of neural networks: integrating symbolism and connectionism (Hupkes et al. 2019b)
- Visualisation and ‘diagnostic classifiers’ reveal how recurrent and recursive neural networks process hierarchical structure (Hupkes, Veldhoen, and Zuidema 2018)
- Learning compositionally through attentive guidance (Hupkes et al. 2019a)
- Diagnostic classification and symbolic guidance to understand and improve recurrent neural networks (Hupkes and Zuidema 2017)

Natural language

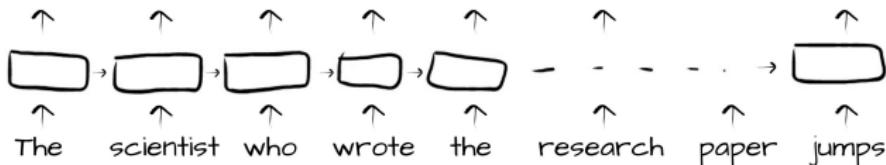
Language modelling

The scientist who wrote the research paper . . . ?

Natural language

Language modelling

The scientist who wrote the research paper ... ?



- Does such a model capture hierarchical structure?

└ The number-agreement task

Subject-verb agreement

The **scientist** who wrote the research paper **jumps**

Subject-verb agreement

The **scientist** who wrote the research paper **jumps**

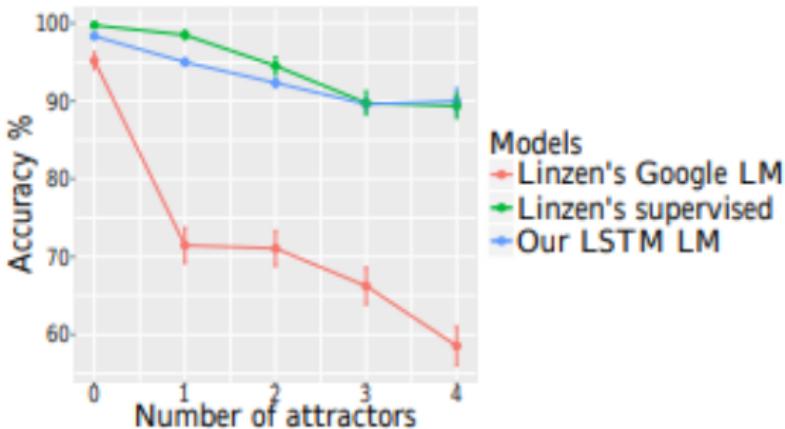
The **scientists** who wrote the research paper **jump**

The number agreement task

The **scientists** who wrote the research paper ... **jump/ jumps?**

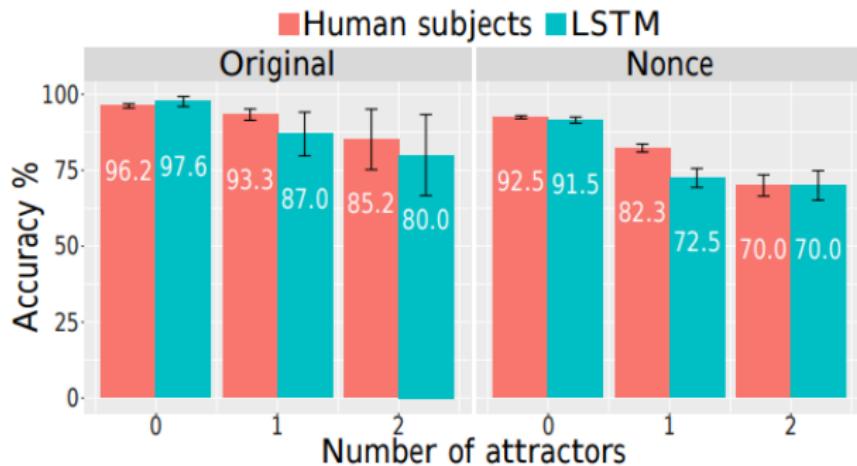
(Linzen, Dupoux, and Goldberg 2016)

Results



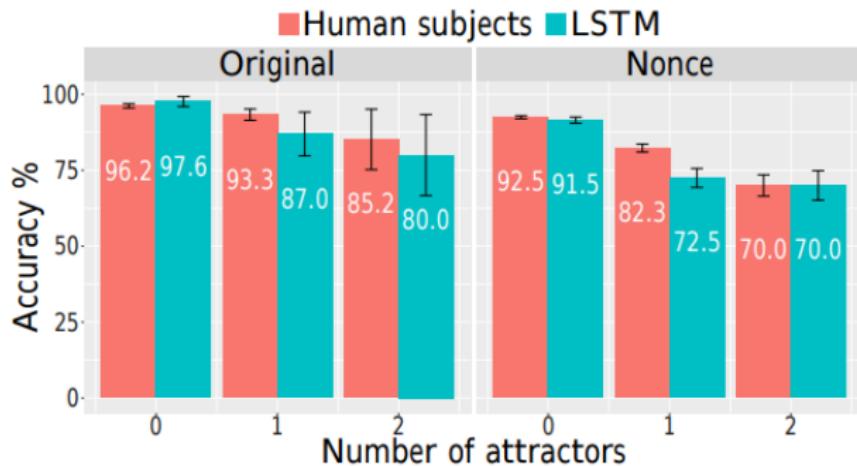
(Gulordava et al. 2018)

Original and nonsensical sentences



(Gulordava et al. 2018)

Original and nonsensical sentences

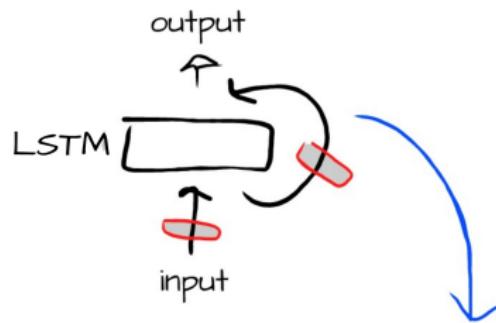


- But *how* do they do this?

└ Diagnostic classification

Diagnostic Classification

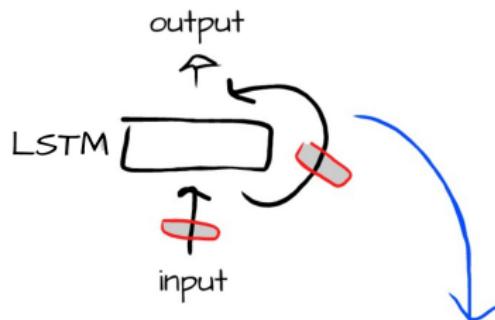
Diagnostic Classification



The scientist who wrote the research paper jumps
[] [] [] [] [] - - - - . []

(Hupkes, Veldhoen, and Zuidema 2018; Veldhoen, Hupkes, and Zuidema 2016)

Diagnostic Classification



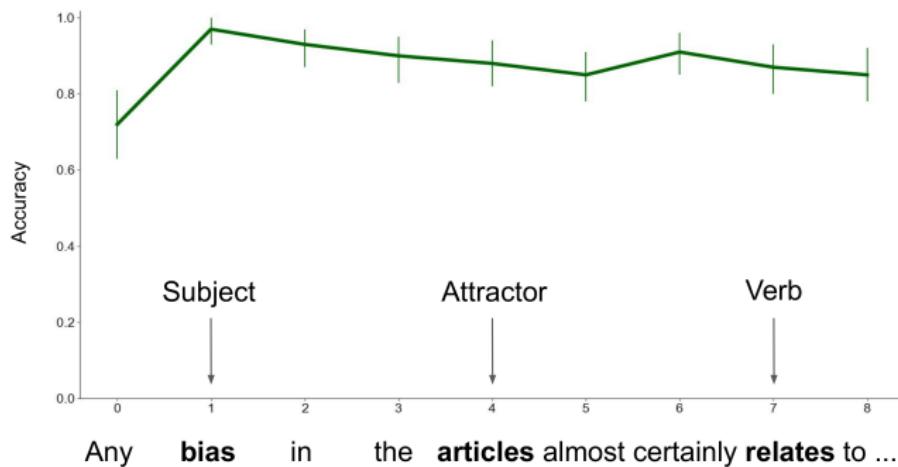
The scientist who wrote the research paper jumps

[] [] [] [] [] - - - - . []
↓ ↓ ↓ ↓ ↓ diagnostic classifier ↓
singular singular singular singular singular singular

(Hupkes, Veldhoen, and Zuidema 2018; Veldhoen, Hupkes, and Zuidema 2016)

Diagnostic Classification

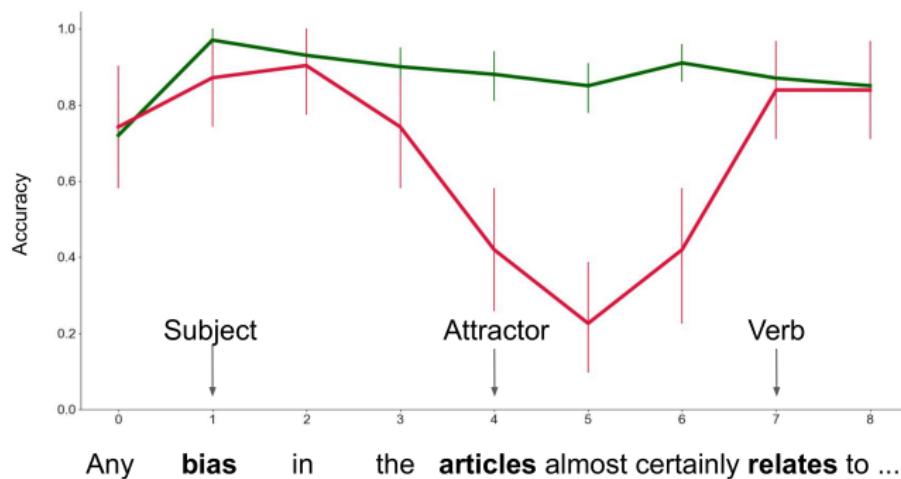
Sentences with correct predictions, h



(Giulianelli, Harding, Mohnert, Hupkes and Zuidema)

Diagnostic Classification

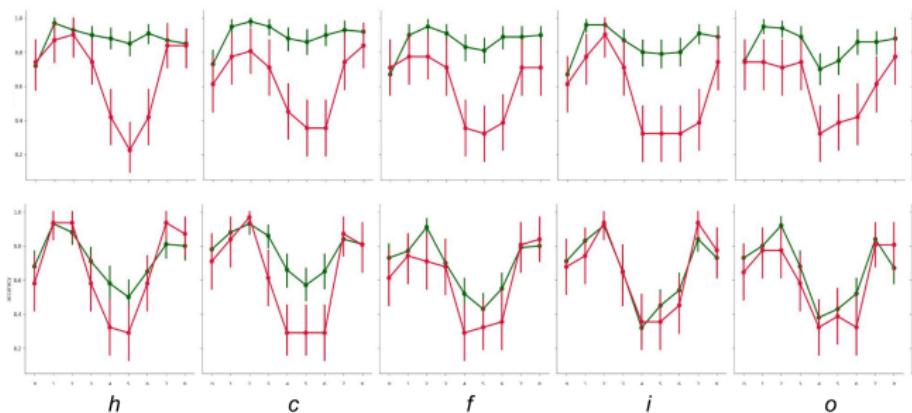
All sentences, h



(Giulianelli et al. 2018)

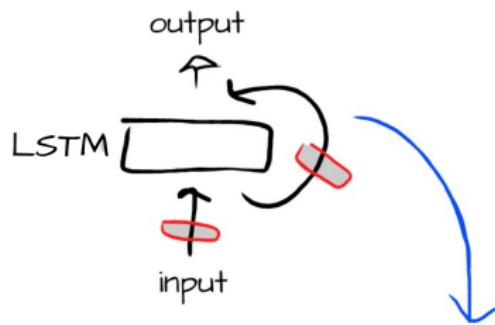
Diagnostic Classification

All sentences, all components



(Giulianelli et al. 2018)

Temporal Generalisation



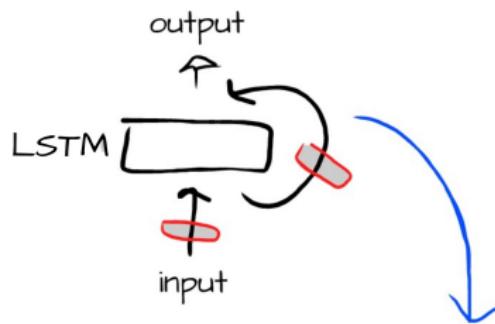
The scientist who wrote the research paper jumps

[] [] [] [] [] - - - - - []

↓
singular

(Giulianelli et al. 2018)

Temporal Generalisation



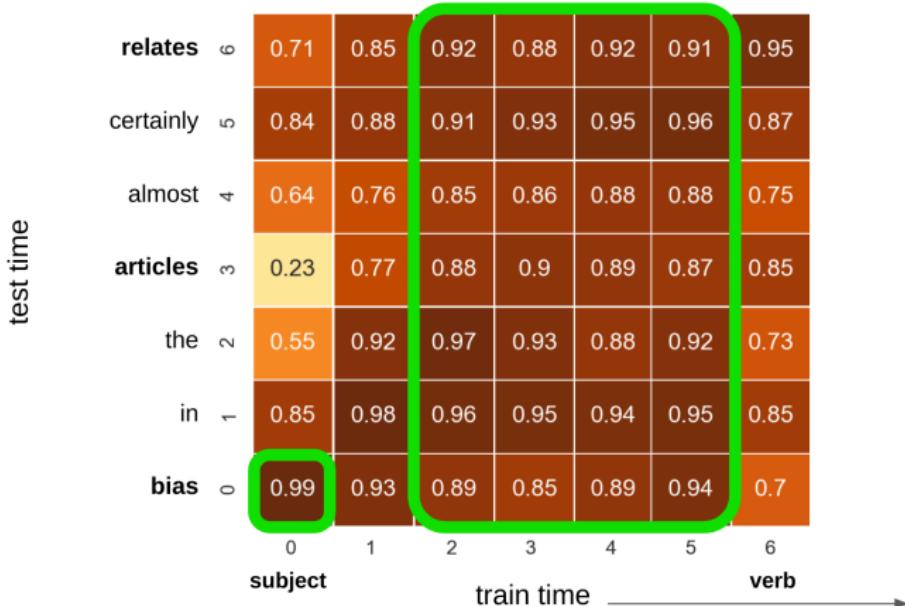
The scientist who wrote the research paper jumps

[] [] [] [] [] - - - - - []

? ? ? ? ?

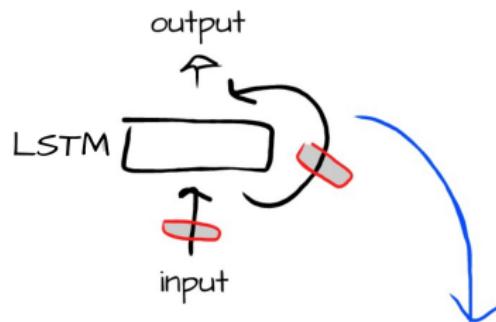
(Giulianelli et al. 2018)

Temporal generalisation matrix



(Giulianelli et al. 2018)

Diagnostic interventions



The scientist who wrote the research paper jumps

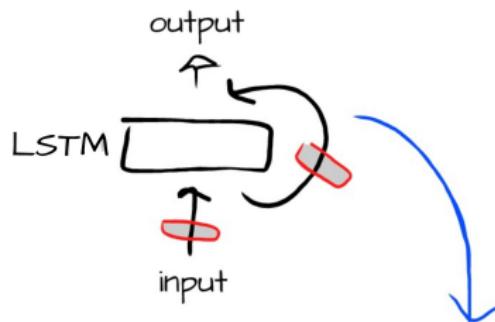
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singular

(Giulianelli et al. 2018)

Diagnostic interventions



The scientist who wrote the research paper jumps

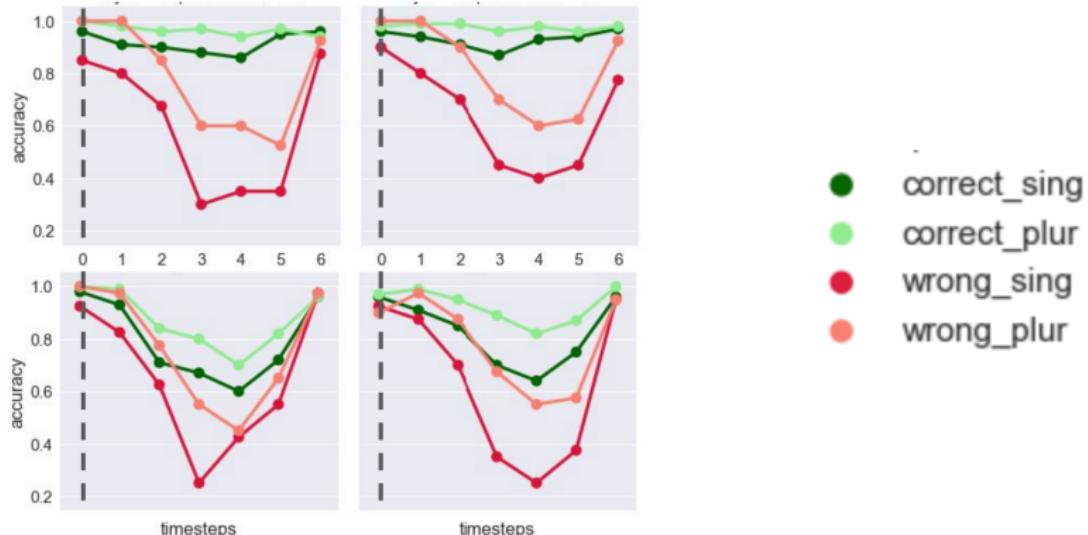
[] [] [] [] [] - - - - - []



singular

(Giulianelli et al. 2018)

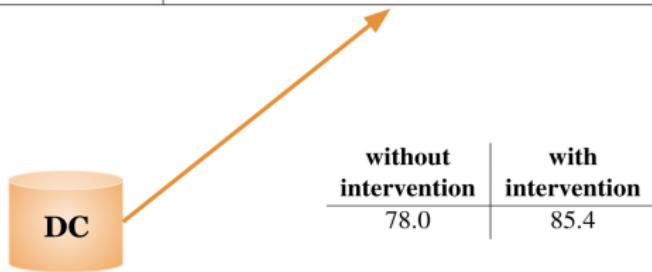
Diagnostic Interventions



(Giulianelli et al. 2018)

Diagnostic interventions, results

Original Intervention	An official estimate issued in 2003 suggests	suggest
-11.05	-8.426	-8.472
-11.05	-8.426	-1.243 -3.951 -5.753 -5.6979



* Overall differences in sentence perplexities are statistically insignificant

(Giulianelli et al. 2018)

Conclusions

With *Diagnostic Classification* we can discover if, when and where information is represented in a recurrent neural network:

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- Number information is stored mostly in the hidden and cell states of the LSTM language model;

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- The model maintains a *deep* and *surface* representation of number;

Conclusions

With *Diagnostic Classification* we can discover if, when and where information is represented in a recurrent neural network:

- Number information is stored mostly in the hidden and cell states of the LSTM language model;
- The model maintains a *deep* and *surface* representation of number;
- The model is indeed distracted by the attractor, but for wrong trials, the encoding already goes wrong *before* the attractor;

Conclusions

With *Diagnostic Classification* we can discover if, when and where information is represented in a recurrent neural network:

- Number information is stored mostly in the hidden and cell states of the LSTM language model;
- The model maintains a *deep* and *surface* representation of number;
- The model is indeed distracted by the attractor, but for wrong trials, the encoding already goes wrong *before* the attractor;
- We can influence the behaviour of the model by *inverting* the diagnostic classifiers.

└ Ablation studies

Ablation Studies

Templates for number-agreement tasks

Simple

Adv

2Adv

CoAdv

NamePP

NounPP

NounPPAdv

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

Templates for number-agreement tasks

Simple the **boy greets** the guy

Adv

2Adv

CoAdv

NamePP

NounPP

NounPPAdv

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

Templates for number-agreement tasks

Simple the **boy** **greets** the guy

Adv the **boy** probably **greets** the guy

2Adv

CoAdv

NamePP

NounPP

NounPPAdv

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

Templates for number-agreement tasks

Simple	the boy greets the guy
Adv	the boy probably greets the guy
2Adv	the boy most probably greets the guy
CoAdv	
NamePP	
NounPP	
NounPPAdv	

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

Templates for number-agreement tasks

Simple

the **boy** **greets** the guy

Adv

the **boy** probably **greets** the guy

2Adv

the **boy** most probably **greets** the guy

CoAdv

the **boy** openly and deliberately **greets** the guy

NamePP

NounPP

NounPPAdv

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

Templates for number-agreement tasks

Simple	the boy greets the guy
Adv	the boy probably greets the guy
2Adv	the boy most probably greets the guy
CoAdv	the boy openly and deliberately greets the guy
NamePP	the boy near Pat greets the guy
NounPP	
NounPPAdv	

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

Templates for number-agreement tasks

Simple	the boy greets the guy
Adv	the boy probably greets the guy
2Adv	the boy most probably greets the guy
CoAdv	the boy openly and deliberately greets the guy
NamePP	the boy near Pat greets the guy
NounPP	the boy near the car greets the guy
NounPPAdv	

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

Templates for number-agreement tasks

Simple	the boy greets the guy
Adv	the boy probably greets the guy
2Adv	the boy most probably greets the guy
CoAdv	the boy openly and deliberately greets the guy
NamePP	the boy near Pat greets the guy
NounPP	the boy near the car greets the guy
NounPPAdv	the boy near the car kindly greets the guy

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

Ablation Results

NA task	Condition	Full Model
Simple	S	100
Adv	S	100
2Adv	S	99.9
CoAdv	S	98.7
namePP	SS	99.3
nounPP	SS	99.2
nounPP	SP	87.2
nounPPAdv	SS	99.5
nounPPAdv	SP	91.2
Simple	P	100
Adv	P	99.6
2Adv	P	99.3
CoAdv	P	99.3
namePP	PS	68.9
nounPP	PS	92.0
nounPP	PP	99.0
nounPPAdv	PS	99.2
nounPPAdv	PP	99.8

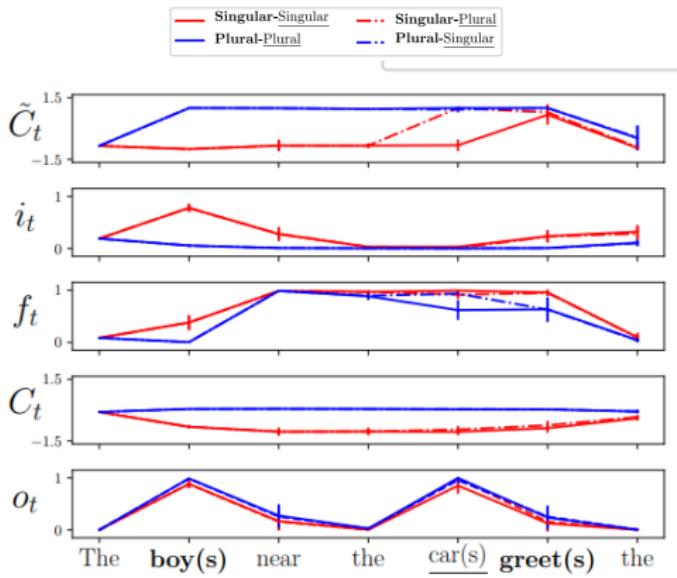
Ablation Results

NA task	Condition	Full Model	Ablated	
			776	988
Simple	S	100	-	-
Adv	S	100	-	-
2Adv	S	99.9	-	-
CoAdv	S	98.7	-	82
namePP	SS	99.3	-	-
nounPP	SS	99.2	-	-
nounPP	SP	87.2	-	54.2
nounPPAdv	SS	99.5	-	-
nounPPAdv	SP	91.2	-	54.0
Simple	P	100	-	-
Adv	P	99.6	-	-
2Adv	P	99.3	-	-
CoAdv	P	99.3	79.2	-
namePP	PS	68.9	39.9	-
nounPP	PS	92.0	48.0	-
nounPP	PP	99.0	78.3	-
nounPPAdv	PS	99.2	63.7	-
nounPPAdv	PP	99.8	-	-

Singular unit behaviour

$$c_t = f_t \circ c_{t-1} + i_t \circ \tilde{c}_t$$

$$h_t = o_t \circ \tanh(c_t)$$



(a) 988 (singular)

(Lakretz et al. 2019)

Diagnostic Classification 2

- Short distance relations?

Diagnostic Classification 2

- Short distance relations?
 - → Diagnostic classifiers to predict *number* information

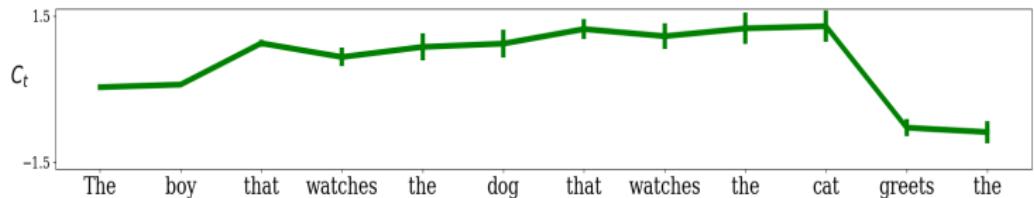
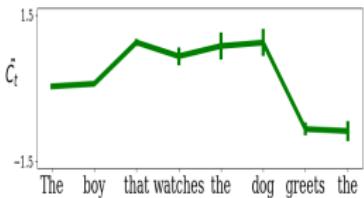
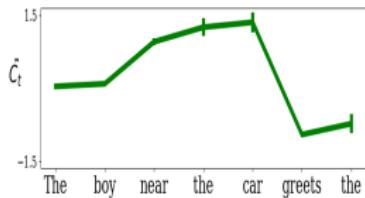
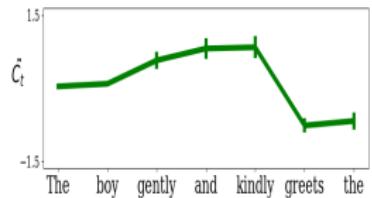
Diagnostic Classification 2

- Short distance relations?
 - → Diagnostic classifiers to predict *number* information
- The syntactic structure?

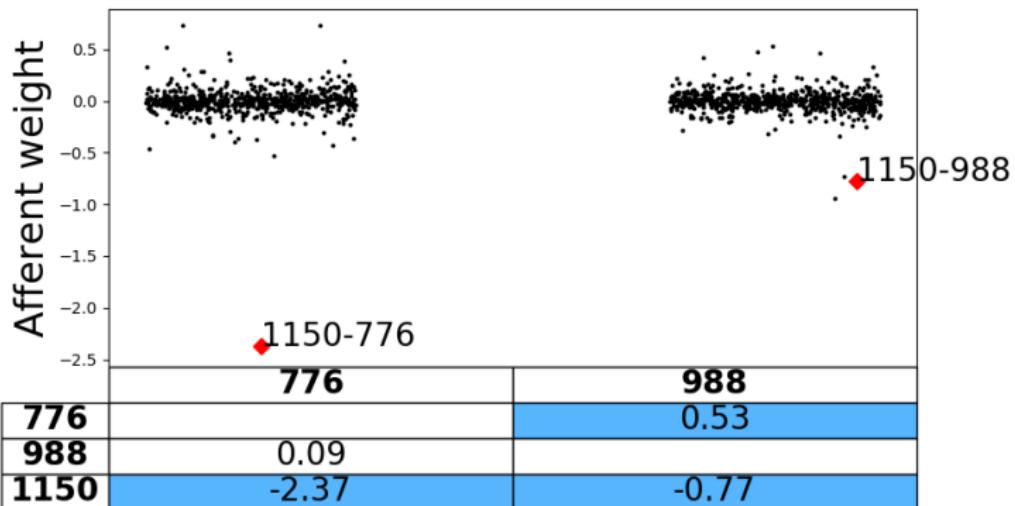
Diagnostic Classification 2

- Short distance relations?
 - → Diagnostic classifiers to predict *number* information
- The syntactic structure?
 - → Diagnostic classifiers to predict *syntactic depth*

Syntax unit 1150, cell activity



Syntax unit 1150, outgoing weights



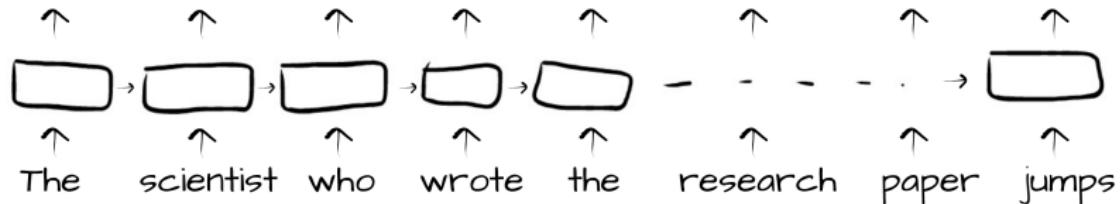
Conclusions

- Using **ablation**, we found that long distance number is encoded locally, in two units;
 - One *singular* unit
 - One *plural* unit
- Using **diagnostic classifiers and ablation**, we found that short distance number is encoded in a distributed fashion;
- Using **diagnostic classification**, we found a number of syntax units, one of which highly interpretable.

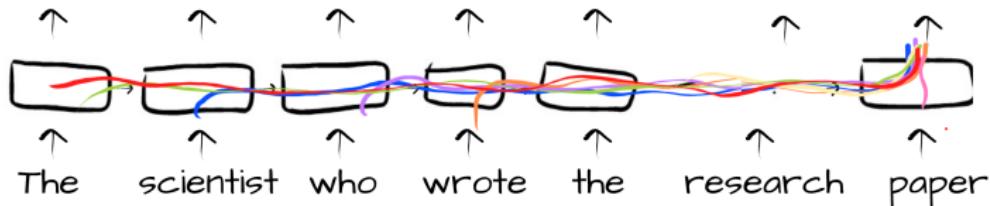
└ Generalised Contextual Decomposition

Generalised Contextual Decomposition

Contextual Decomposition



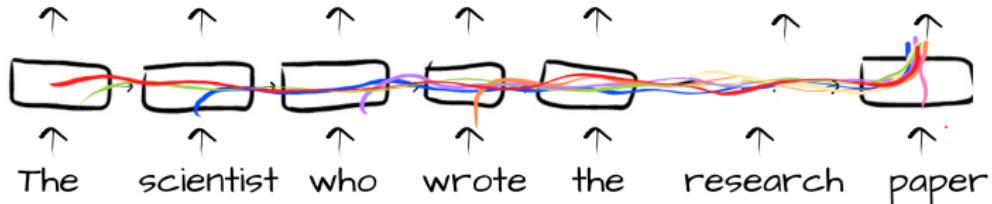
Contextual Decomposition



- Keep track of interactions

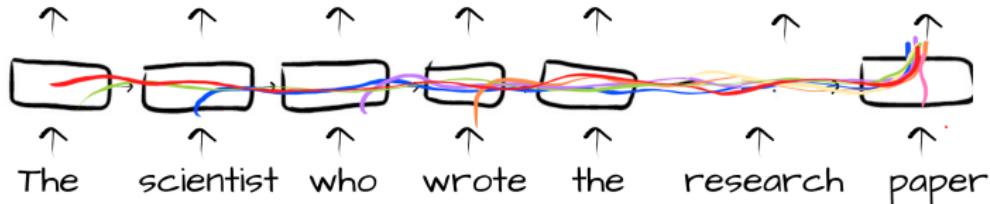
(Murdoch, Liu, and Yu 2018)

Contextual Decomposition



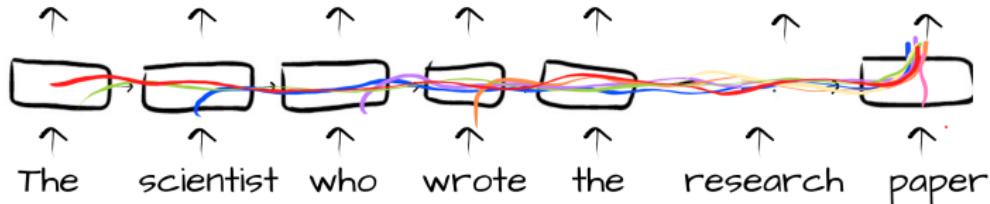
- Keep track of interactions
 - Linear sums: $3 * 2 + 1 * 4$

Contextual Decomposition



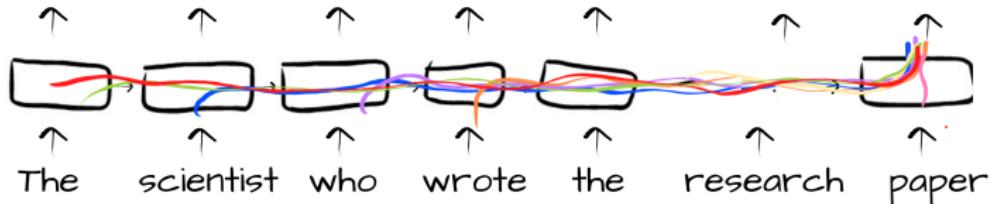
- Keep track of interactions
 - Linear sums: $3 * \textcolor{blue}{2} + 1 * \textcolor{red}{4}$
 - Non-linearities: $\text{TANH}(\textcolor{blue}{10} + \textcolor{red}{20})$

Contextual Decomposition



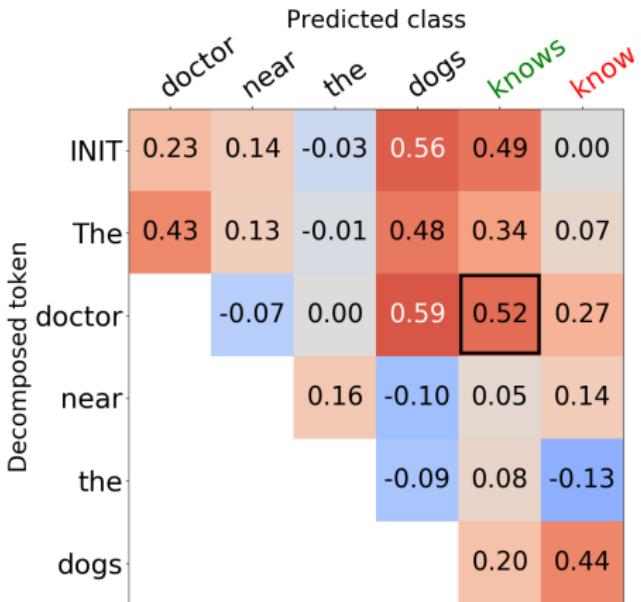
- Keep track of interactions
 - Linear sums: $3 * 2 + 1 * 4$
 - Non-linearities: $\text{TANH}(10 + 20)$
 - Multiplications: $5 * 2$

Contextual Decomposition



- Keep track of interactions
 - Linear sums: $3 * 2 + 1 * 4$
 - Non-linearities: $\text{TANH}(10 + 20)$
 - Multiplications: $5 * 2$
- Which interactions?

Information flow “attention” plots



(Jumelet, Hupkes, and Zuidema 2019)

Singular versus plural

	Predicted class					
	N_{plur}	PREP	the	N_{sing}	\checkmark_{plur}	\checkmark_{sing}
Decomposed token	0.22	0.08	-0.05	0.30	0.12	1.84
INIT	0.24	0.10	-0.02	0.23	0.05	1.33
The	0.34	0.01	0.23	0.58	-0.13	
N_{plur}		0.19	0.27	0.19	0.11	
PREP			0.37	0.08	0.55	
the				0.12	0.57	
N_{sing}						

NounPP – PS

Singular versus plural

	Predicted class						
	N _{plur}	PREP	the	N _{sing}	✓ plur	✗ sing	
Decomposed token	INIT	0.22	0.08	-0.05	0.30	0.12	1.84
The	0.24	0.10	-0.02	0.23	0.05	1.33	
N _{plur}		0.34	0.01	0.23	0.58	-0.13	
PREP			0.19	0.27	0.19	0.11	
the				0.37	0.08	0.55	
N _{sing}					0.12	0.57	

NounPP – PS

	Predicted class						
	N _{sing}	PREP	the	N _{plur}	✓ sing	✗ plur	
Decomposed token	INIT	0.15	0.07	-0.06	0.48	0.75	0.17
The	0.68	0.22	-0.03	0.44	0.51	0.16	
N _{sing}		0.12	-0.01	0.31	0.45	-0.07	
PREP			0.21	0.53	-0.04	0.24	
the				0.10	0.09	0.02	
N _{plur}					0.32	0.74	

NounPP – SP

Pruning information

Task	Condition	GCD	
		FULL	IN
Simple	S	100	73.3
Simple	P	100	100
nounPP	SS	99.2	93.0
nounPP	SP	87.2	90.3
nounPP	PS	92.0	100
nounPP	PP	99.0	100
namePP	SS	99.3	97.7
namePP	PS	68.9	98.3

- FULL: full model accuracy
- IN: information from the subject,

Pruning information

Task	Condition	FULL	GCD	
			IN	INTERCEPT*
Simple	S	100	73.3	97.3
Simple	P	100	100	32.7
nounPP	SS	99.2	93.0	99.8
nounPP	SP	87.2	90.3	98.8
nounPP	PS	92.0	100	0.0
nounPP	PP	99.0	100	7.0
namePP	SS	99.3	97.7	99.4
namePP	PS	68.9	98.3	1.3

- FULL: full model accuracy
- IN: information from the subject,
- INTERCEPT*: only intercept interactions

Pruning information

Task	Condition	FULL	GCD		
			IN	INTERCEPT*	¬INTERCEPT
Simple	S	100	73.3	97.3	69.7
Simple	P	100	100	32.7	100
nounPP	SS	99.2	93.0	99.8	72.7
nounPP	SP	87.2	90.3	98.8	60.5
nounPP	PS	92.0	100	0.0	100
nounPP	PP	99.0	100	7.0	99.8
namePP	SS	99.3	97.7	99.4	76.2
namePP	PS	68.9	98.3	1.3	99.9

- FULL: full model accuracy
- IN: information from the subject,
- INTERCEPT*: only intercept interactions
- ¬INTERCEPT: no intercept interactions

Conclusions

We can use contextual decomposition to track the information flow in recurrent neural networks:

- Plural verbs have a much stronger causal relationship to their plural subject than singular verbs to their singular subject.
- By considering different types of interactions, we find that to predict singular verbs, the model relies heavily on its intercepts
- GCD can also be used in other kinds of scenario's, where behavioural accuracy tests are not possible (anaphora resolution, negative polarity items)!

What's next?

Thanks to my collaborators



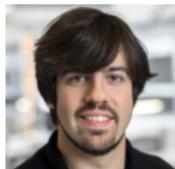
Willem Zuidema



Marco Baroni



Jaap Jumelet



Germán Kruszewski



Yair Lakretz



Sara Veldhoen



Mario Giulianelli



Florian Mohnert



Jack Harding

Special thanks



Willem Zuidema



Jaap Jumelet

Special thanks



Willem Zuidema



Jaap Jumelet

<https://github.com/i-machine-think/diagnnose>

What's next?

What's next?

- Other linguistic questions

What's next?

- Other linguistic questions
 - Negative polarity items (Jumelet and Hupkes 2018; Marvin and Linzen 2018)
 - Filler-gap dependencies (Wilcox et al. 2018, 2019)
 - Reflexive anaphora (Futrell et al. 2019; Jumelet, Hupkes, and Zuidema 2019; Marvin and Linzen 2018)
 - Garden path sentences (Futrell et al. 2019; Van Schijndel and Linzen 2018; Wilcox et al. 2019)
 - Syntactic priming (Prasad, Schijndel, and Linzen 2019; Van Schijndel and Linzen 2018)
 - And many more...
- Other “model” questions

What's next?

- Other linguistic questions
- Other “model” questions
 - Do structural biases help? (Futrell et al. 2018; Wilcox et al. 2019)
 - What is the impact of quantity and quality of training data (Schijndel, Mueller, and Linzen 2019)?

What's next?

- Other linguistic questions
- Other “model” questions
- The ultimate question

What's next?

- Other linguistic questions
- Other “model” questions
- The ultimate question
 - How does this help us to better understand human language processing?

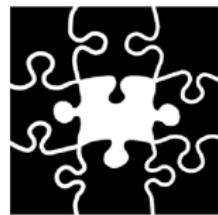
What's next?

- Other linguistic questions
- Other “model” questions
- The ultimate question
 - How does this help us to better understand human language processing?

I'm looking forward to figuring those things out!

Thank you

Thank you for your attention!



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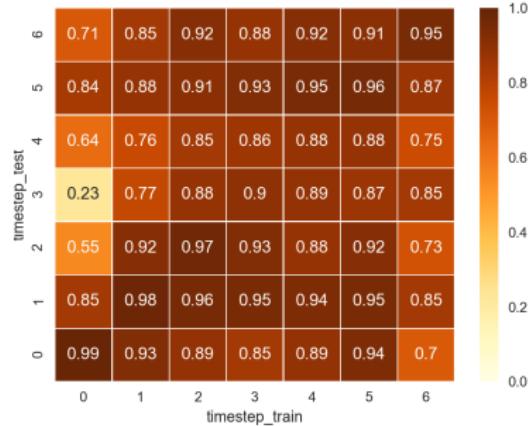
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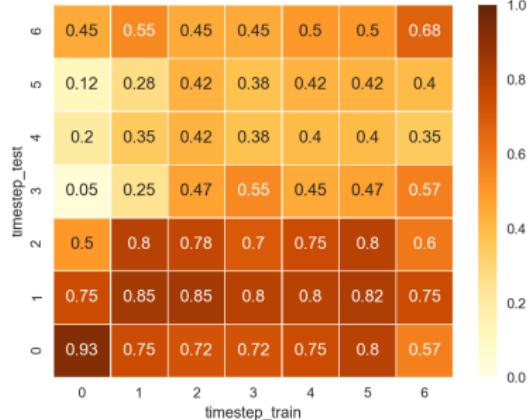
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Appendices

Temporal Generalisation

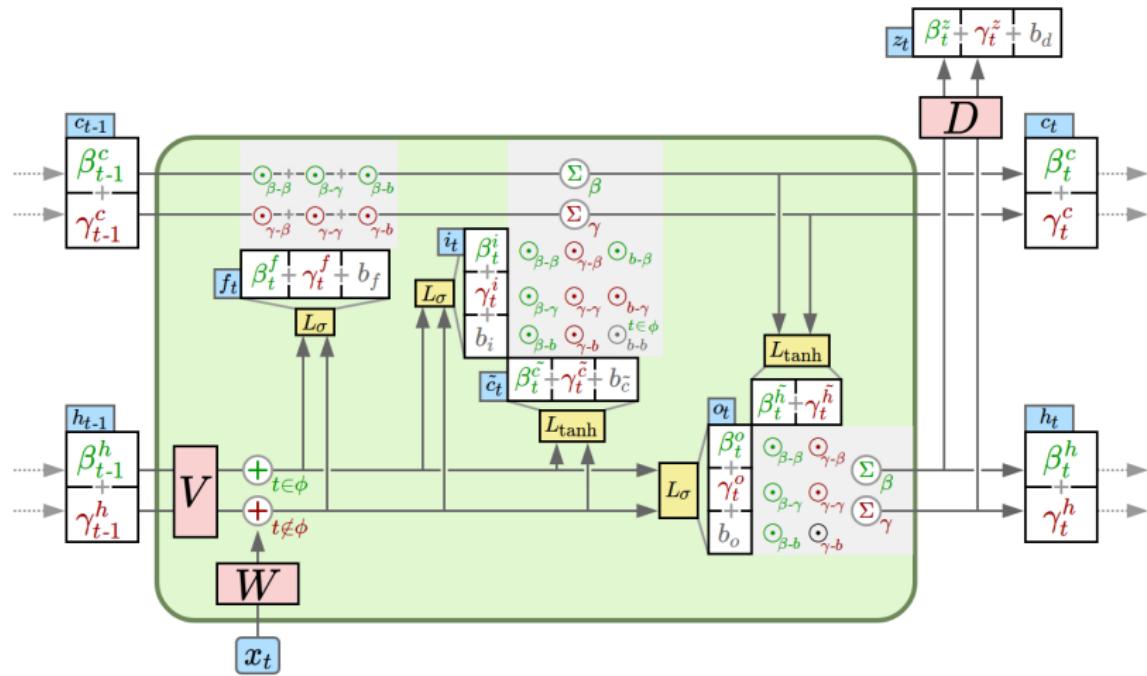


Correct trials



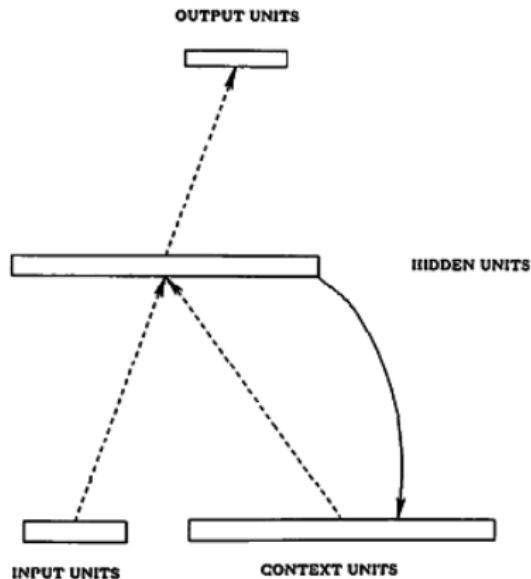
Wrong trials

Generalised Contextual Decomposition



Simple Recurrent Network

$$\mathbf{h}_t = \tanh(\mathbf{Wx}_t + \mathbf{Uh}_{t-1} + \mathbf{b})$$

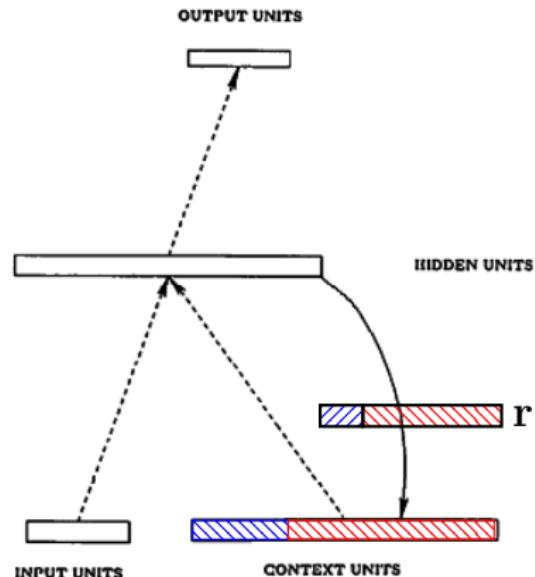


(Elman 1990)

Gated recurrent neural networks

$$\mathbf{h}_t = \tanh(\mathbf{W}\mathbf{x}_t + \mathbf{U}\mathbf{h}_{t-1} + \mathbf{b})$$

$$\mathbf{r}_t = \sigma(\mathbf{W}_r\mathbf{x}_t + \mathbf{U}_r\mathbf{h}_{t-1} + \mathbf{b}_r)$$

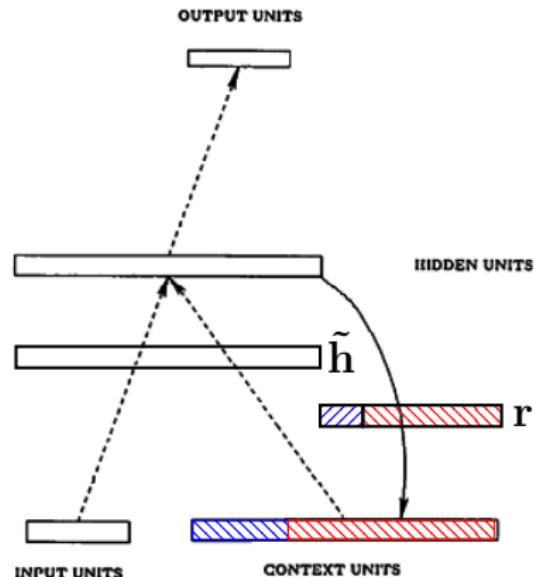


(Cho et al. 2014; Chung et al. 2015)

Gated recurrent neural networks

$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{W}\mathbf{x}_t + \mathbf{U}(\mathbf{r} \odot \mathbf{h}_{t-1}) + \mathbf{b})$$

$$\mathbf{r}_t = \sigma(\mathbf{W}_r \mathbf{x}_t + \mathbf{U}_r \mathbf{h}_{t-1} + \mathbf{b}_r)$$



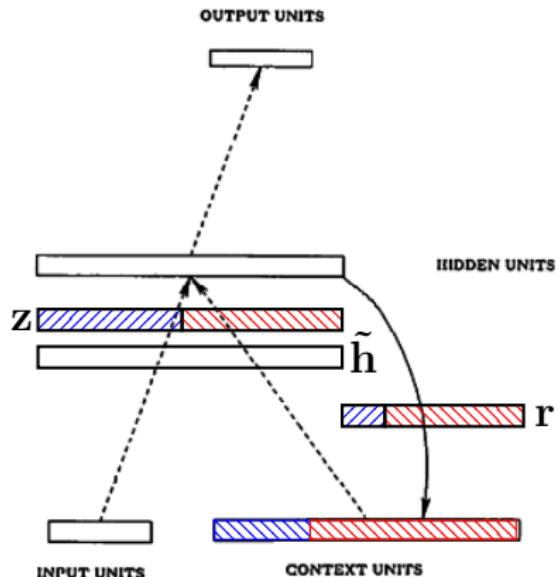
(Cho et al. 2014; Chung et al. 2015)

Gated recurrent neural networks

$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{Wx}_t + \mathbf{U}(\mathbf{r} \odot \mathbf{h}_{t-1}) + \mathbf{b})$$

$$\mathbf{r}_t = \sigma(\mathbf{Wr}_t \mathbf{x}_t + \mathbf{Ur}_{t-1} \mathbf{h}_{t-1} + \mathbf{b}_r)$$

$$\mathbf{z}_t = \sigma(\mathbf{Wz}_t \mathbf{x}_t + \mathbf{Uz}_{t-1} \mathbf{h}_{t-1} + \mathbf{bz})$$



(Cho et al. 2014; Chung et al. 2015)

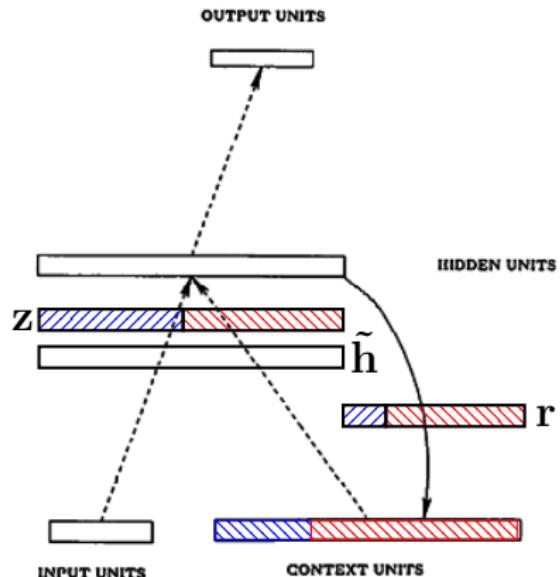
Gated recurrent neural networks

$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{W}\mathbf{x}_t + \mathbf{U}(\mathbf{r} \odot \mathbf{h}_{t-1}) + \mathbf{b})$$

$$\mathbf{r}_t = \sigma(\mathbf{W}_r \mathbf{x}_t + \mathbf{U}_r \mathbf{h}_{t-1} + \mathbf{b}_r)$$

$$\mathbf{z}_t = \sigma(\mathbf{W}_z \mathbf{x}_t + \mathbf{U}_z \mathbf{h}_{t-1} + \mathbf{b}_z)$$

$$\mathbf{h}_t = (1 - \mathbf{z}_t) \odot \mathbf{h}_{t-1} + \mathbf{z}_t \odot \tilde{\mathbf{h}}_t$$



(Cho et al. 2014; Chung et al. 2015)