

# Exchange Rate Policies at the Zero Lower Bound\*

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## Abstract

We study the problem of a monetary authority pursuing an exchange rate policy that is inconsistent with interest rate parity because of a binding zero lower bound constraint. The resulting violation in interest rate parity generates an inflow of capital that the monetary authority needs to absorb by accumulating foreign reserves. We show that these interventions by the monetary authority are costly, and we derive a simple measure of these costs: they are proportional to deviations from the covered interest parity (CIP) condition and the amount of accumulated foreign reserves. Our framework can account for the recent experiences of “safe-haven” currencies and the sign of their observed deviations from CIP.

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# 1 Introduction

Many central banks often manage, implicitly or explicitly, their exchange rate. In a financially integrated world, the path for the exchange rate determines, together with nominal interest rates, the relative desirability of assets denominated in domestic and foreign currency. A long tradition, which dates back at least to [Krugman \(1979\)](#), has focused on how inconsistent fiscal and monetary policies can make domestic assets less attractive than foreign ones and lead to episodes of capital outflows, depletion of foreign reserves, and currency devaluations.

Since the global financial crisis, however, several countries have experienced opposite dynamics, that is, capital inflows, accumulation of foreign reserves, and currency appreciations. The case of Switzerland is emblematic in this respect. Over the period 2010-2017, despite a zero or negative nominal interest rate, Switzerland experienced a large increase in private capital inflows that was accompanied by an equally large increase in holdings of foreign reserves by the Swiss National Bank, which was attempting to prevent an appreciation of the Swiss franc.

In this paper, we argue that episodes of this sort can arise because of a conflict between an exchange rate policy and the zero lower bound constraint on nominal interest rates. To understand our argument, consider a situation in which a monetary authority is pegging the exchange rate, but there are future states of the world in which it would abandon the peg and appreciate. If nominal interest rates are at zero at home and abroad, local currency assets will be attractive to foreigners because the expected future appreciation is not offset by lower domestic interest rates. We show that this force induces capital inflows that *need* to be absorbed by the monetary authority through foreign exchange interventions, and that such unconventional interventions are *costly*. We provide a measure of these costs and show that they can be substantial. For the Swiss franc, the monthly costs of the exchange rate policies carried out by the Swiss National Bank peaked at about 0.6% of monthly gross domestic product. Moreover, our framework can help to rationalize the recent emergence of deviations from covered interest parity for economies with nominal interest rates close to zero.

We formalize this argument in a canonical small open economy model with two main ingredients. First, we assume that foreign financial intermediaries that trade with the domestic economy face potentially binding financial constraints, a feature implying that arbitrage in international financial markets might fail. That is, risk-adjusted returns on domestic currency assets could be higher than those on foreign ones, and only a *finite* amount of capital would flow into the country. Second, we introduce money in the model, which leads to a potentially binding zero lower bound on nominal interest rates, as is standard in monetary models. In such a framework, we study the problem of a benevolent monetary authority that uses its balance sheet to implement a given state-contingent path for its exchange rate.

Let's start from the (risk augmented) interest rate parity condition,

$$(1 + i_t) = \frac{(1 + i_t^*)}{\mathbb{E}_t[e_t/e_{t+1}]} - \text{Cov}_t \left( \Lambda_{t+1}, (1 + i_t) \frac{e_t}{e_{t+1}} \right), \quad (\text{IP})$$

where  $i_t$  and  $i_t^*$  are, respectively, the nominal interest rates on risk-free bonds at home and abroad,  $e_t$  is the exchange rate (the price of foreign currency in terms of domestic currency), and  $\Lambda_{t+1}$  is the financial intermediaries stochastic discount factor. This equation defines the level of  $i_t$  that makes intermediaries indifferent between holding domestic or holding foreign currency bonds given the foreign interest rate and the exchange rate policy.<sup>1</sup>

The Central Bank's exchange rate policy  $(e_t, e_{t+1})$  *does not conflict* with the zero lower bound if equation (IP) holds for some non-negative  $i_t$ , given  $i_t^*$  and  $\Lambda_{t+1}$ . In such a scenario, the monetary authority can always implement the desired exchange rate policy by choosing a level of  $i_t$  that makes intermediaries indifferent between investing in the small open economy or not. We show that, in this case, it is optimal for the monetary authority to choose this particular nominal rate. As a result, interest parity holds, and capital flows between the small open economy and the rest of the world arise only to absorb the desired excess domestic net savings of the private sector.

This implementation, however, is not feasible when the exchange rate policy *conflicts with the zero lower bound*, that is, when there is no non-negative  $i_t$  that is consistent with equation (IP). The zero lower bound then implies that in any equilibrium that implements the exchange rate policy  $(e_t, e_{t+1})$ , interest rate parity will be violated. In this regime, foreign intermediaries have incentives to purchase domestic currency assets, generating a potentially large inflow of capital toward the small open economy. We show that in this situation, the private sector does not have incentives to absorb this inflow of capital, and the monetary authority is forced to issue domestic liabilities and accumulate foreign assets. By issuing high-yielding domestic assets and purchasing low-yielding foreign ones, the trades of the monetary authority induce a resource cost for the small open economy. To implement its desired exchange rate policy, it is optimal for the monetary authority to set interest rates at zero, so as to minimize these costs, while accumulating foreign reserves.

Equation (IP) clarifies the conditions under which a given exchange rate policy might conflict with the zero lower bound on nominal interest rates. The conflict is more likely to arise when (i) the foreign nominal interest rate is low, (ii) there is an expected future appreciation of the domestic currency, or (iii) when the currency of the small open economy is perceived to be a "safe haven", that is, when future appreciations coincide with "bad" times for intermediaries (generating a high covariance between  $\Lambda_{t+1}$  and the exchange rate).

In our view, these three circumstances describe well the environment faced by the Swiss National Bank (SNB) after the global financial crisis. In an effort to dampen the appreciation pressures on the

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<sup>1</sup>The deterministic log-linearized version reduces to  $i_t = i_t^* + \ln(e_{t+1}) - \ln(e_t)$ , which is the usual condition for nominal exchange rate determination in workhorse open-economy models (see Engel, 2014, for a recent survey).

Swiss franc, the SNB established a currency floor vis-à-vis the euro in 2011 and announced that it would not tolerate an exchange rate beyond 1.2 Swiss francs per euro. Such policy was implemented during a period in which interest rates were at zero in all major advanced economies, and the policy itself was not perfectly credible, as financial markets attached a positive probability that the SNB would abandon the floor and appreciate the franc (Jermann, 2017). Moreover, there is evidence that the Swiss franc was expected to appreciate during adverse worldwide economic conditions.<sup>2</sup> Consistent with our reading, the Swiss franc was characterized throughout this period by deviations from covered interest rate parity (CIP) that made Swiss-denominated assets attractive, and the foreign reserves of the SNB jumped from roughly 10% of GDP in 2010 to more than 100% in 2016. In our theory, both observations are symptoms of a conflict between an exchange rate policy and the zero lower bound.

We use the experience of the Swiss franc as a laboratory to measure the costs of an exchange rate policy. Specifically, we show that these resource costs can be approximated by combining balance sheet data from the SNB and observed deviations from CIP. Even though these deviations were on average 50 basis points, the size of the capital flows was large enough to generate substantial losses—on the order of 0.6% of monthly GDP in January 2015.

While offering a prototypical example of a conflict between exchange rate policies and the zero lower bound, the Swiss experience is not an isolated one, and our framework is useful for interpreting the behavior of other advanced economies. As documented in a recent paper by Du, Tepper and Verdelhan (Forthcoming), systematic failures from CIP have occurred for several currencies after 2008. Interestingly, the countries that, according to CIP, had the most attractive currencies were also those with zero (or negative) nominal interest rates and monetary authorities actively pursuing exchange rate policies, as indicated by the massive increase in official holdings of foreign reserves.

Our paper contributes to the literature on exchange rate determination in segmented capital markets. Backus and Kehoe (1989) derive general conditions under which sterilized official purchases of foreign assets do not affect equilibrium allocations and therefore are irrelevant for the determination of the nominal exchange rate—a result in the spirit of the irrelevance of standard open-market operations by Wallace (1981) and Sargent and Smith (1987). A key assumption underlying this irrelevance result is the absence of financial constraints and asset market segmentation that can potentially introduce violations of international arbitrage. We follow the contributions by Alvarez, Atkeson and Kehoe (2009) and Gabaix and Maggiori (2015) in relaxing these assumptions, and we study foreign exchange interventions in the presence of limited international arbitrage.<sup>3</sup>

Fanelli and Straub (2017) consider a real deterministic model in which the government uses foreign exchange intervention to manipulate the domestic real interest rate and manage the terms of trade

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<sup>2</sup> For example, following the intensification of the European debt crisis in May 2012, there was a massive increase in the demand for Swiss francs by international investors. At that stage, speculations that the SNB would abandon the currency floor intensified; see Alice Ross, “Swiss franc strength tests SNB,” *Financial Times*, May 24, 2012, for instance.

<sup>3</sup> Jeanne (2012) studies foreign reserve accumulation as a tool to manage the real exchange rate, but in the context of a real model with a closed capital account for the private sector.

(Costinot, Lorenzoni and Werning, 2014).<sup>4</sup> In such a framework, they derive the optimal intervention and study aspects of credibility and international coordination. Similar to us, they show that deviations from interest parity, due to foreign exchange interventions, generate a cost in the intertemporal resource constraint of the economy. We study instead a monetary model and examine the optimal implementation of a policy for nominal exchange rates with an explicit zero lower bound constraint. In addition, we consider uncertainty and risk premia, which allows us to address the question of whether one should use deviations from covered or uncovered interest rate parity when measuring these costs in the data.

In relation to these costs, Calvo (1991) first raised the warning about the potential costs of sterilized foreign exchange interventions. A mostly empirical literature has subsequently discussed and estimated the “quasi-fiscal” costs of these operations and similarly identified them as a loss in the budget constraint of the government, proportional to the interest parity deviations and the size of the accumulated reserves (see Kletzer and Spiegel 2004, Devereux and Yetman 2014, Liu and Spiegel 2015, and references therein). The common practice in this literature, prominent also in policy discussions about the merits of sterilized interventions, is to use deviations from the uncovered interest rate parity (UIP) condition when computing these costs.<sup>5</sup> Our paper clarifies that this practice might lead to biases: as will become clear from our analysis, using deviations from UIP in these calculations is equivalent to computing the ex-ante net costs from foreign exchange interventions without appropriately discounting them.

The failure of CIP since 2008 has been documented in detail by Du, Tepper and Verdelhan (Forthcoming). They provide evidence that such deviations and the resulting failure of arbitrage were due to balance sheet constraints on financial intermediaries, likely induced by tighter banking regulations following the financial crises. They also uncover a negative cross-country relation between nominal interest rates and deviations from CIP, meaning that currencies that were most attractive were also characterized by lower interest rates. To the best of our knowledge, our paper is the first to provide a formal framework for interpreting these findings and investigating their welfare implications. Specifically, we provide a theory where failures from CIP arise from the binding balance sheet constraints of financial intermediaries, which explains why positive CIP deviations appear for some currencies and not others, and we explain their connections to official holdings of foreign reserves and low interest rates.

Finally, our work is related to the literature that studies unconventional policies when monetary policy is constrained, either by a zero lower bound or by a fixed exchange rate regime. Correia, Farhi, Nicolini and Teles (2013), Adao, Correia and Teles (2009), and Farhi, Gopinath and Itskhoki (2014) emphasize how various schemes of taxes and subsidies can achieve the same outcomes that would

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<sup>4</sup>Another related paper in this regard is Cavallino (2016), who studies the role of foreign exchange interventions in response to exogenous capital flow shocks.

<sup>5</sup>See, for example, Adler and Mano (2016) and Sarno and Taylor (2001) for reviews of the literature.

prevail in the absence of constraints to monetary policy. [Schmitt-Grohé and Uribe \(2016\)](#) and [Farhi and Werning \(2012\)](#) study capital controls as second-best policy instruments to deal with capital flows under a fixed exchange rate regime. In contrast to these studies, we investigate foreign exchange interventions as a tool to implement a given exchange rate policy at the zero lower bound.<sup>6</sup> There are limitations and benefits associated with these different policies, and more research is needed to tease out the appropriate policy mix.<sup>7</sup>

The structure of the paper is as follows. Section 2 introduces the model, while Section 3 characterizes the monetary equilibria for a given exchange rate policy. In Section 4 we introduce the problem of the monetary authority, characterize the optimal balance sheet policy, and conduct a comparative statics analysis. Section 5 shows how to measure the costs of foreign exchange rate interventions, and Section 6 presents empirical evidence. Section 7 concludes. Throughout the paper, we assume that the monetary authority wishes to implement an exogenous exchange rate target. In the online Addendum A we endogenize this target in a model with sticky wages and show that our implementation results continue to hold in this environment.

## 2 The model

We consider a small open economy that lasts for two periods, indexed by  $t \in \{1, 2\}$ . There is an uncertain state  $s \in \{s_1, \dots, s_N\} \equiv S$  that is realized at  $t = 2$ , and we denote by  $\pi(s) \in (0, 1]$  the probability that state  $s$  occurs. There is only one good, and no production.

The small open economy is inhabited by a representative household and a monetary authority. The rest of the world is populated by a mass of financial intermediaries that can purchase domestic and foreign assets.

The household receives an endowment of the consumption good,  $(y_1, \{y_2(s)\})$ , and decides on a consumption allocation,  $(c_1, \{c_2(s)\})$ .<sup>8</sup> In addition, the household also receives a lump-sum transfer (or a tax, if negative) of  $\{T_2(s)\}$  from the monetary authority in the second period.<sup>9</sup>

There is an international financial market with a full set of Arrow-Debreu securities, indexed in foreign currency. The price level in the international financial markets is normalized to one, so that foreign prices are effectively quoted in units of the consumption good. Let  $q(s)$  be the price, in foreign

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<sup>6</sup>For other work exploring the open economy dimension of the zero lower bound, see [Krugman \(1998\)](#), [Cook and Devreux \(2013\)](#), [Svensson \(2003\)](#), [Benigno and Romei \(2014\)](#), [Acharya and Bengui \(2015\)](#), [Fornaro \(Forthcoming\)](#), [Caballero, Farhi and Gourinchas \(2015\)](#), [Eggertsson, Mehrotra, Singh and Summers \(2016\)](#), and [Corsetti, Kuester and Müller \(2016\)](#). For the interaction of the zero lower bound with safe haven currencies, see [Gourinchas and Rey \(2016\)](#).

<sup>7</sup>In a previous working paper version of the paper, [Amador, Bianchi, Bocola and Perri \(2017\)](#), we showed that, when appropriately designed, capital controls and negative nominal interest rates can reduce the costs of foreign exchange interventions.

<sup>8</sup>We use the following notation: a vector of the form  $(x_1, \{x_2(s)\})$  denotes an  $x_1$  value at  $t = 1$  and a value of  $x_2(s)$  at  $t = 2$  conditional on the state  $s$ .

<sup>9</sup>Here we are not modeling the fiscal authority for simplicity. More realistically, one can think that the monetary authority transfers resources (or gets recapitalized) by the Treasury, and those resources are then passed on to the household.

currency as of period 1, of the Arrow-Debreu security that pays one unit of foreign currency in state  $s$  in period 2, and zero in all others. The price  $q(s)$  is exogenous and taken as given by all agents.

The small open economy has its own currency in circulation, as well as a full set of Arrow-Debreu securities denominated in domestic currency. We denote by  $p(s)$  the domestic currency price in period 1 of the domestic Arrow-Debreu security that pays one unit of domestic currency in state  $s$  in period 2, and zero otherwise. There is a nominal exchange rate in periods 1 and 2,  $(e_1, \{e_2(s)\})$ , which denotes the amount of domestic currency necessary to purchase a unit of foreign currency at any period and state. Goods trade is costless, and as a result, the law of one price holds: the domestic price level at any state is equal to the exchange rate.

**The domestic households.** The budget constraint of the domestic household in the initial period is

$$y_1 = c_1 + \sum_{s \in S} \left[ q(s)f(s) + p(s)\frac{a(s)}{e_1} \right] + \frac{m}{e_1}, \quad (1)$$

where  $f(s)$  and  $a(s)$  denote the purchases of domestic and foreign Arrow-Debreu securities,  $m$  are money holdings, and where we have assumed that all initial asset positions of the households are zero.

In period 2 at state  $s$ , the budget constraint of the household becomes

$$y_2(s) + T_2(s) + f(s) + \frac{a(s) + m}{e_2(s)} = c_2(s) \text{ for all } s \in S. \quad (2)$$

Domestic households can purchase and sell any amount of domestic securities. They can also purchase unrestricted non-negative amount of foreign assets. However, we assume that the household cannot short-sell foreign securities:

$$f(s) \geq 0, \text{ for all } s \in S. \quad (3)$$

This assumption guarantees that the financial constraints of the financial intermediaries will matter for the equilibrium allocation. The zero in the above equation, however, is not important, as all our results would survive if domestic households had a strictly positive borrowing limit in foreign currency.

The household's problem is to choose  $(c_1, \{c_2(s)\}, m, \{f(s), a(s)\})$ , subject to the budget constraints, to maximize the following utility function:

$$u(c_1) + h\left(\frac{m}{e_1}\right) + \beta \sum_{s \in S} \pi(s)u(c_2(s)), \quad (4)$$

where  $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$  for  $\sigma > 0$ , and  $h$  is an increasing, differentiable, and concave function, with a



satiation level  $\bar{x}$  (i.e.,  $h(x) = h(\bar{x})$  for all  $x \geq \bar{x}$ ).

**The foreign intermediaries.** There is a mass one of foreign financial intermediaries, which are owned by foreign households. They start the period with some amount of capital,  $\bar{w} > 0$ , which they use to purchase domestic assets, including money, issued by the small open economy and foreign financial assets. They choose their portfolio  $(m^*, \{a^*(s), f^*(s)\})$  and dividend stream  $(d_1^*, \{d_2^*(s)\})$  to maximize the expected discounted present value of dividends:

$$d_1^* + \sum_{s \in S} \pi(s) \Lambda(s) d_2^*(s), \quad (5)$$

where  $\Lambda(s) = q(s)/\pi(s)$  is the stochastic discount factor in the foreign markets.

In the initial period, their budget constraint is

$$\bar{w} = \frac{m^*}{e_1} + \sum_{s \in S} \left[ \frac{p(s)a^*(s)}{e_1} + q(s)f^*(s) \right] + d_1^*. \quad (6)$$

In period 2 at state  $s$ , their budget constraint is

$$d_2^*(s) = \frac{m^* + a^*(s)}{e_2(s)} + f^*(s). \quad (7)$$

These intermediaries cannot issue negative dividends in the first period and have limited ability to borrow in both domestic and foreign financial markets:

$$d_1^* \geq 0, f^*(s) \geq 0, \text{ and } a^*(s) \geq 0 \text{ for all } s \in S. \quad (8)$$

As was the case for the household, the zero in these constraints is not critical for our results, and its only role is to make certain expressions in the paper less cumbersome. The important assumption here is that there are some limits in the ability of the intermediaries to issue equity or to borrow.

**The monetary authority.** We impose that the monetary authority has a given nominal exchange rate objective, which we denote by  $(e_1, \{e_2(s)\})$ . In general, an exchange rate objective would arise from the desire to achieve a particular inflation or output target. In Addendum A, we study optimal exchange rate policies in a model with wage rigidities. For the moment, however, we simply assume that the monetary authority follows this objective and we define an equilibrium *given*  $(e_1, \{e_2(s)\})$ . This allows us to transparently illustrate the role of the balance sheet of the monetary authority in determining the nominal exchange rate.

To achieve its exchange rate objective, the monetary authority issues money and a state uncorrelated bond denominated in domestic currency,  $(M, A)$ . We denote by  $\bar{p}$  the price of the risk-free



domestic bond. It also purchases foreign reserves in the form of an uncontingent bond denominated in foreign currency,  $F$ , at price  $\bar{q}$ . As with the households, we restrict  $F \geq 0$ .<sup>10</sup>

In the second period, the monetary authority withdraws the money from circulation and redistributes the returns of its portfolio holdings to the domestic household. The associated budget constraints are

$$\frac{\bar{p}A + M}{e_1} = \bar{q}F, \quad (9)$$

$$T_2(s) = F - \frac{A + M}{e_2(s)} \quad \text{for all } s \in S \quad (10)$$

for periods 1 and 2 respectively.

The prices of the domestic and foreign uncontingent bond, which can be replicated from the set of domestic and foreign Arrow-Debreu securities, respectively, are

$$\bar{p} = \sum_{s \in S} p(s) \equiv \frac{1}{1+i} \quad \bar{q} = \sum_{s \in S} q(s) \equiv \frac{1}{1+i^*}, \quad (11)$$

where we have defined the domestic and international risk-free interest rate as  $i$  and  $i^*$ .

**Monetary equilibrium.** An *equilibrium given an exchange rate policy*  $(e_1, \{e_2(s)\})$  is a household's consumption profile,  $(c_1, \{c_2(s)\})$ , and its asset positions,  $(m, \{a(s), f(s)\})$ ; intermediaries' dividends policy,  $(d_1^*, \{d_2^*(s)\})$ , and its asset positions,  $(m^*, \{a^*(s), f^*(s)\})$ ; the monetary authority's transfer to the households,  $\{T_2(s)\}$ , and its asset positions,  $(M, F, A)$ ; and domestic asset prices  $\{p(s)\}$ , such that

1. The domestic household chooses consumption and portfolio positions to maximize utility, (4), subject to the budget constraints, (1) and (2), as well as the no-borrowing constraints, (3), while taking prices  $\{q(s), p(s)\}$  and transfers  $\{T_2(s)\}$  as given.
2. Intermediaries choose the dividend policy and portfolio positions to maximize their objective, (5), subject to their budget constraints, (6) and (7), as well as the non-negativity restriction on their asset holdings, and first-period dividends, (8) while taking prices  $\{q(s), p(s)\}$  as given.
3. The purchases of assets by the monetary authority, and its transfers to the households satisfy its budget constraints, (9) and (10) for all  $s \in S$ , together with (11) and the non-negativity restriction on foreign reserves,  $F \geq 0$ .

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<sup>10</sup>In this paper, we restrict the monetary authority to issue or buy only state uncontingent securities (risk-free bonds). In Amador, Bianchi, Bocola and Perri (2018), we study the portfolio choices of the monetary authority in an environment that does not feature such a restriction.

4. Domestic asset markets clear:

$$a(s) + a^*(s) = A \quad \text{for all } s \in S, \quad (12)$$

$$m + m^* = M. \quad (13)$$

The above definition does not specify an objective function for the monetary authority. For a given exchange rate policy  $(e_1, \{e_2(s)\})$ , there are potentially many possible monetary equilibria, indexed by particular balance sheet positions for the monetary authority. Our objective is to study how a benevolent monetary authority that maximizes the household's welfare sets its balance sheet optimally in order to implement  $(e_1, \{e_2(s)\})$ . Before studying this problem, though, it is useful to first characterize some useful properties of monetary equilibria.

### 3 Characterizing monetary equilibria

This section characterizes monetary equilibria. We start by defining a “first-best” consumption allocation, which will be a useful benchmark for the optimal policy of the monetary authority. We then move to describe some key equilibrium conditions and present a characterization of the monetary equilibria.

#### 3.1 First best in a real economy

We define the *first-best* consumption allocation as the allocation  $(c_1^{fb}, \{c_2^{fb}(s)\})$  that solves

$$\max_{c_1, \{c_2(s)\}} \left\{ u(c_1) + \beta \sum_{s \in S} \pi(s) u(c_2(s)) \right\} \quad (14)$$

subject to

$$y_1 - c_1 + \sum_{s \in S} q(s)(y_2(s) - c_2(s)) \geq 0. \quad (15)$$

In what follows, we impose an assumption that guarantees this consumption allocation could be implemented as a monetary equilibrium, absent the zero lower bound constraint:

**Assumption 1.** *Intermediary capital is such that*

$$\sum_{s \in S} q(s) \max \{ y_2(s) - c_2^{fb}(s), 0 \} \leq \bar{w}.$$

This condition guarantees that the intermediaries have enough capital to cover the external gross

liability/inflow position of the economy generated by the first-best allocation.<sup>11</sup>

The first-best allocation equalizes the ratio of marginal utility in the first period to marginal utility in the second period across states, adjusted by prices and probabilities. That is,

$$\frac{\beta\pi(s)}{q(s)} \frac{u'(c_2^{fb}(s))}{u'(c_1^{fb})} = 1$$

for all  $s \in S$ .

The property of equalizing this ratio, but not necessarily to one, is shared by a different type of consumption allocations which, under certain conditions, will be part of any monetary equilibrium. We define them as “equal gaps” consumption allocations.

**Definition 1.** *We say that a consumption allocation features equal gaps if it satisfies*

$$\frac{\beta\pi(s)}{q(s)} \frac{u'(c_2(s))}{u'(c_1)} = \frac{\beta\pi(s')}{q(s')} \frac{u'(c_2(s'))}{u'(c_1)}, \quad (16)$$

for all  $s, s' \in S$ .

These consumption allocations feature no intratemporal distortions in the second period, just as the first best, but may feature an intertemporal distortion. An alternative way of interpreting these allocations is that the second-period consumption allocation is the solution to the following static planning problem, indexed by  $C_2$ :

$$U(C_2) \equiv \max_{\{c_2(s)\}} \left\{ \sum_{s \in S} \pi(s) u(c_2(s)) \text{ subject to } \bar{q}C_2 = \sum_{s \in S} q(s)c_2(s) \right\}, \quad (SP)$$

where  $C_2$  are the second period expenditures necessary to purchase the consumption bundle  $\{c_2(s)\}$ . If an equilibrium features equal gaps, we only need to determine initial consumption, and the second-period aggregate  $C_2$ . Along with the prices of foreign securities, this is sufficient to characterize the second-period consumption in every state. It is also useful to define an “average” of the second-period endowment,  $Y_2$ :

$$Y_2 \equiv \frac{\sum_{s \in S} q(s)y_2(s)}{\bar{q}}. \quad (17)$$

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<sup>11</sup>From the budget constraints of the households and the monetary authority, we have that  $y_2(s) - c_2^{fb}(s) + f(s) + F = x^*(s)$ , where  $x^*(s) \geq 0$  represents the payoff to intermediaries on their domestic investments in state  $s$ . Given that  $f(s) \geq 0$  and  $F \geq 0$ , and  $x^*(s) \geq 0$ , it follows that  $x(s) \geq \max\{y_2(s) - c_2^{fb}(s), 0\}$ . In the first-best allocation, domestic state prices would be equalized with foreign ones, and thus summing over across states, using the state price  $q(s)$ , we get that the total domestic investments made by the intermediaries must be  $\sum_{s \in S} q(s)x^*(s) \geq \sum_{s \in S} q(s) \max\{y_2(s) - c_2^{fb}(s), 0\}$ . But the total domestic investments of the intermediaries cannot be bigger than  $\bar{w}$  as of time 1, and so  $\sum_{s \in S} q(s)x^*(s) \leq \bar{w}$ , generating the condition in Assumption 1.

### 3.2 Equilibrium conditions

We now discuss the key equilibrium conditions of the model, starting with the optimality conditions for the household.

**Household optimality and domestic prices.** The household solves a standard consumption-saving problem, with multiple assets (domestic and foreign securities) and potentially binding borrowing constraints. Recall that these constraints apply only when the household borrows in foreign currency. Because of that, the first-order condition of the household with respect to domestic securities holds with equality and implies that their price is given by

$$p(s) \frac{e_2(s)}{e_1} = \frac{\beta \pi(s) u'(c_2(s))}{u'(c_1)} \quad (18)$$

for all  $s \in S$ .

Their optimality condition with respect to foreign asset  $s$  might instead hold with inequality because of the borrowing constraint,

$$q(s) \geq \frac{\beta \pi(s) u'(c_2(s))}{u'(c_1)}, \quad (19)$$

for all  $s \in S$ . When the above condition holds with strict inequality for some  $s$ , the household chooses not to invest in the associated foreign security, that is,  $f(s) = 0$ , because this security is strictly dominated by the domestic one.

**The zero lower bound on the nominal interest rate.** The household also chooses its money holdings. The household's optimality condition with respect to money holdings can then be written as

$$h' \left( \frac{m}{e_1} \right) = u'(c_1) \left( 1 - \sum_{s \in S} p(s) \right) = u'(c_1) \frac{i}{1+i}, \quad (20)$$

where we have used the definition of the risk-free rate on a nominal bond in (11).

Note that equation (20) implies that domestic nominal interest rates cannot be negative. Because  $h' \geq 0$  and  $u' \geq 0$ , we must have that  $i \geq 0$  in any monetary equilibrium.

**Intermediary's optimality and profits.** The intermediary chooses investment in foreign and domestic securities, including money. Let us denote by  $\Pi$  their period 1 profits, that is, the difference between the expected discounted present value of their dividends and their initial capital. Because they share the same stochastic discount factor that prices the foreign securities, investing in foreign

assets yields no profits. However, investing in domestic ones may, depending on the equilibrium prices. In particular, their profits  $\Pi$  are

$$\Pi = \frac{m^*}{e_1} \left[ \sum_{s \in S} \frac{q(s)e_1}{e_2(s)} - 1 \right] + \sum_{s \in S} \frac{p(s)a^*(s)}{e_1} \left[ \frac{q(s)e_1}{p(s)e_2(s)} - 1 \right] \quad (21)$$

where  $m^*$  and  $a^*$  are non-negative and such that  $m^*/e_1 + \sum_s p(s)a^*(s)/e_2(s) \leq \bar{w}$ .

The term in square brackets are return differentials. The first, is the return differential of holding money and the foreign nominal risk-free bond.<sup>12</sup> The second, is the return differential between domestic and foreign Arrow-Debreu securities.

Given the linearity of their objective function, the optimal portfolio decision of intermediaries is to channel all of their wealth into the domestic security that yields the largest differential return.

**The intertemporal resource constraint.** We can obtain an intertemporal resource constraint in this economy by consolidating the household and the monetary authority budget constraints. Specifically, solving out for  $f(s)$  using the household's budget constraint in the second period, and plugging it back into the household's first-period budget constraint, we obtain:

$$y_1 = c_1 + \sum_{s \in S} \left[ q(s) \left( c_2(s) - y_2(s) - T_2(s) - \frac{a(s) + m}{e_2(s)} \right) + p(s) \frac{a(s)}{e_1} \right] + \frac{m}{e_1}$$

Using the budget constraints of the monetary authority, we have that the transfer in the second period can be expressed as

$$T_2(s) = \frac{1}{\bar{q}} \left[ \frac{\bar{p}A + M}{e_1} \right] - \frac{A + M}{e_2(s)}.$$

Using this in the previous equation, and collecting terms, we obtain

$$y_1 = c_1 + \sum_{s \in S} \left[ q(s) \left( c_2(s) - y_2(s) + \frac{A - a(s) + M - m}{e_2(s)} \right) + p(s) \frac{A - a(s)}{e_1} \right] + \frac{M - m}{e_1}.$$

Market clearing implies that  $A(s) - a(s) = a^*(s)$  and  $M - m = m^*$ , and thus we obtain the following condition that must hold in any equilibrium:

$$y_1 - c_1 + \sum_{s \in S} q(s)(y_2(s) - c_2(s)) - \Pi = 0 \quad (22)$$

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<sup>12</sup> To see this, we can use  $q(s) = \pi(s)\Lambda(s)$  to obtain

$$\sum_{s \in S} \frac{q(s)e_1}{e_2(s)} - 1 = \mathbb{E} \left[ \Lambda(s) \left( \frac{e_1}{e_2(s)} - (1 + i^*) \right) \right].$$

This equation is similar to the first-best intertemporal resource constraint, equation (15), but adjusted to incorporate a potential loss for the small open economy,  $\Pi$ . When foreign intermediaries make profits by purchasing domestic assets, someone in the small open economy is taking the opposite side and incurring a loss. This loss is always non-negative because the intermediaries can always choose a portfolio yielding zero profits. That is, in equilibrium,  $\Pi \geq 0$ .

**Gross capital flows and trade balance.** Using the household budget constraint in the first period, as well as the monetary authority budget constraints, we obtain the following equality, linking the trade deficit to the evolution of the net foreign asset position:

$$\underbrace{c_1 - y_1}_{\text{trade deficit}} = \underbrace{\frac{m^* + \sum_s p(s)a^*(s)}{e_1}}_{\text{foreign liabilities}} - \underbrace{\left[ \sum_s q(s)f(s) + F \right]}_{\text{foreign assets}}. \quad (23)$$

### 3.3 Monetary equilibria featuring equal gaps

Under certain conditions, equal gaps allocations are the only possible equilibrium outcome. We proceed to show this next. Toward this end, we make the following assumption:

**Assumption 2.** *The parameters are such that*

$$\left[ \max_{s_1, s_2} \left( \frac{\pi(s_1)q(s_2)}{q(s_1)\pi(s_2)} \right)^{1/\sigma} - 1 \right] + \frac{\bar{q} \max_{s_1, s_2} \{y_2(s_1) - y_2(s_2)\}}{y_1 + \bar{q}Y_2} \leq \frac{\bar{w}}{y_1 + \bar{q}Y_2}$$

where  $Y_2$  is defined in (17).

This assumption is satisfied when the variation in the second period endowment and the variation in  $\pi(s)/q(s)$  (which determines the variation in consumption in the second period) are not large, or when the capital of foreign intermediaries is sufficiently large relative the value of the country's endowment. For example, if both the second period endowment and  $\pi(s)/q(s)$  are constant, the assumption is satisfied for any intermediary capital level.

We then have the following result:

**Lemma 1.** *Suppose that Assumption 2 holds. Then the consumption allocation of any monetary equilibrium features equal gaps.*

When a consumption allocation features equal gaps, the intermediary's problem simplifies. Using condition (16), we must have that excess returns on all domestic securities are equalized:

$$0 \leq \frac{q(s')e_1}{p(s')e_2(s')} - 1 = \sum_{s \in S} \left[ \frac{p(s)}{\bar{p}} \times \left( \frac{q(s)e_1}{p(s)e_2(s)} \right) \right] - 1 = \sum_{s \in S} \frac{q(s)e_1}{e_2(s)}(1 + i) - 1 \quad (24)$$

for any  $s' \in S$ . The first inequality follows from the household's optimality conditions, (18) and (19), which require that  $p(s) \leq q(s)e_1/e_2(s)$ . The first equality follows from the definition of equal gaps and that  $p(s)/\bar{p}$  sums to one (by definition of  $\bar{p}$ ). The second equality follows from the definition of  $i$ .

Let us define  $\Delta(i)$  to be the right-hand term of (24):

$$\Delta(i) \equiv \sum_{s \in S} \frac{q(s)e_1}{e_2(s)}(1 + i) - 1. \quad (25)$$

In an equal gaps allocation,  $\Delta(i)$  captures the profits per unit of capital. When  $\Delta(i) > 0$ , intermediaries optimally invest all of their wealth in domestic securities. When  $\Delta(i) = 0$ , intermediaries make zero profits. Thus, we can write their profits as

$$\Pi = \Delta(i) \times \bar{w}.$$

This expression also captures the losses for the small open economy.

The value of  $\Delta(i)$  has another interpretation. Consider a simpler problem where the intermediaries decide between two assets. It can invest in the domestic risk-free nominal bond with return  $i$ , or in the foreign currency risk-free bond with return  $i^*$ . The difference in payoffs between these two assets, from the perspective of an intermediary, is

$$\mathbb{E} \left[ \Lambda(s) \left( \frac{e_1}{e_2(s)}(1 + i) - (1 + i^*) \right) \right] = \left[ \sum_{s \in S} q(s) \frac{e_1}{e_2(s)} \right] (1 + i) - 1 = \Delta(i) \quad (26)$$

And thus  $\Delta(i)$  is the “risk-adjusted” difference between the domestic and foreign risk-free bond returns. When  $\Delta(i) = 0$ , we say that *interest parity* holds. However, in our model it could be that  $\Delta(i) > 0$ . Such violation of interest parity can arise because intermediaries and households face potentially binding borrowing constraints.

Focusing attention to equal gap allocations is additionally helpful because equilibria within this class can be described by just three values: initial consumption,  $c_1$ , the second period consumption expenditures,  $C_2$ , and money balances,  $m$ .

**Lemma 2** (Characterization of Equilibrium). *Under Assumptions 1 and 2, a consumption allocation  $(c_1, \{c_2(s)\})$  and money holdings  $m$  are part of an equilibrium given the exchange rate policy  $(e_1, \{e_2(s)\})$*



if and only if there exists an  $i$  such that

$$y_1 - c_1 + \bar{q}(Y_2 - C_2) = \Delta(i)\bar{w} \quad (27)$$

$$\frac{\bar{q}u'(c_1)}{\beta U'(C_2)} = 1 + \Delta(i) \geq 1, \quad (28)$$

$$h'\left(\frac{m}{e_1}\right) = u'(c_1)\frac{i}{1+i}, \quad (29)$$

and  $\{c_2(s)\}$  solves the static planning problem (SP) given  $C_2$ ; and where  $Y_2$  and  $U$  are defined in (SP) and (17). Household welfare in this equilibrium is

$$u(c_1) + h(m/e_1) + \beta U(C_2). \quad (30)$$

Equation (28), the novel addition in this lemma, represents the household's Euler equation for foreign assets. Here we have used the envelope condition for the static planning problem, (SP), with the equal gaps condition, (24). Recall that (29) implicitly imposes the zero lower bound.

Note that equations (27) and (28) have a solution only if  $\Delta(i)\bar{w} < y_1 + \bar{q}Y_2$ . Intuitively, the losses need to be lower than the present value of the country's endowment in order to have positive consumption. Moreover, first-period consumption  $c_1$  is below the first best, and it is decreasing in  $\Delta(i)$  and  $\bar{w}$ . An increase in  $\bar{w}$  when  $\Delta(i) > 0$  induces a negative income effect that pushes households to consume less today. An increase in  $\Delta(i)$  generates a similar negative income effect, but also a negative substitution effect which further reduces first-period consumption. As this result is useful for the analysis to follow, we summarize it below.

**Corollary 1.** *Suppose  $\Delta(i)\bar{w} < y_1 + \bar{q}Y_2$ . There is a unique pair  $(c_1, C_2)$  that solves (27) and (28). When  $\Delta(i) = 0$ ,  $c_1$  coincides with the first-best consumption. In addition,  $c_1$  strictly decreases with  $\Delta(i)$  and strictly decreases in  $\bar{w}$  for  $\Delta(i) > 0$ .*

## 4 The problem of the monetary authority

We now study the problem of the monetary authority. Section 4.1 characterizes the monetary equilibrium that maximizes the welfare of domestic households, which we refer to as the “best equilibrium.” Section 4.2 describes the balance sheet policy that allows the monetary authority to implement the best equilibrium. We conclude the section with a graphical illustration of the main results, and with a discussion of comparative statics.

## 4.1 Best equilibrium

The objective of the monetary authority is to choose an equilibrium, given an exchange rate policy  $(e_1, \{e_2(s)\})$ , that maximizes the domestic household's welfare. Given Lemma 2, the problem of the monetary authority can be formulated as follows

$$\begin{aligned} \max_{c_1, C_2, m, i} \quad & \{u(c_1) + h(m/e_1) + \beta U(C_2)\} \\ \text{subject to} \quad & (27), (28), \text{ and } (29). \end{aligned} \tag{MP}$$

We refer to the solution for (MP) as a “best equilibrium.” Note that even though the monetary authority's problem seems deterministic, uncertainty and risk play a role, as they determine the shape of  $\Delta(i)$ , thus affecting the intertemporal resource constraint (27).

The solution to (MP) can be characterized by two cases depending on the exchange rate policy and its effect on  $\Delta(0)$ .

First, consider the case in which the exchange rate policy is such that  $\Delta(0) < 0$ . Then, there exists a non-negative domestic nominal interest rate,  $\tilde{i}$ , such that  $\Delta(\tilde{i}) = 0$ . We can show that in such a scenario, the monetary authority sets  $i = \tilde{i}$  and implements the first-best allocation.

**Proposition 1.** *Suppose Assumptions 1 and 2 hold. If  $\Delta(0) < 0$ , then the best equilibrium features  $(c_1^{fb}, C_2^{fb}, m, i)$  where*

$$\begin{aligned} C_2^{fb} &= \sum_{s \in S} q(s) c_2^{fb} / \bar{q}, \\ i &> 0 \text{ and such that } \Delta(i) = 0, \\ m &\text{ such that } h'(m/e_1) = u'(c_1^{fb}) \frac{i}{1+i}. \end{aligned}$$

Importantly, the above solution cannot be an equilibrium if  $\Delta(0) > 0$ : in this case, there is no non-negative nominal interest rate consistent with interest rate parity. The following proposition describes the optimal solution in this case, which is our main result.

**Proposition 2.** *Suppose Assumptions 1 and 2 hold. If  $\Delta(0) > 0$  and  $\Delta(0)\bar{w} < y_1 + \bar{q}Y_2$ , then the best equilibrium features  $(c_1, C_2, m, i)$  such that*

$$\begin{aligned} i &= 0, \quad m \geq \bar{m}, \text{ and} \\ (c_1, C_2) &\text{ are the unique solution to (27) and (28).} \end{aligned}$$

That is, the best equilibrium features zero nominal interest rates, a failure of interest rate parity, and a consumption allocation distorted away from the first best. In this case, the monetary authority is trying to implement an exchange rate policy that makes domestic assets attractive even if nominal

interest rates were set to zero,  $\Delta(0) > 0$ . As  $\Delta(i)$  increases with  $i$ , any equilibrium necessarily features a deviation from interest rate parity. Intermediary capital will flow into the country, generating the losses captured by  $\Delta(i)\bar{w}$ . By setting the lowest possible domestic interest rate,  $i = 0$ , and thus selecting the lowest possible  $\Delta(i)$ , the monetary authority alleviates the costs associated with this capital inflow.

Before turning to study the implementation analysis, it is useful to discuss the conditions under which  $\Delta(0) > 0$  is more likely to emerge. For this purpose, we can write  $\Delta(0)$  as follows:

$$\begin{aligned}\Delta(0) &= \mathbb{E} \left[ \Lambda(s) \left( \frac{e_1}{e_2(s)} - (1 + i^*) \right) \right] \\ &= \frac{\mathbb{E}[e_1/e_2(s)]}{1 + i^*} - 1 + \mathbb{Cov} \left( \Lambda(s), \frac{e_1}{e_2(s)} \right)\end{aligned}\tag{31}$$

Three main forces determine whether  $\Delta(0) > 0$ : the rate of appreciation of the domestic currency,  $\mathbb{E}[e_1/e_2(s)]$ , the foreign interest rate,  $i^*$ , and the covariance of the appreciation rate with the stochastic discount factor of the intermediaries. Holding everything else constant, the zero lower bound is more likely to be a problem for the monetary authority when the expected appreciation is high, the foreign interest is low, and the covariance term is positive.

These results are intuitive. A high expected appreciation of the currency or a low foreign interest rate makes the domestic asset more attractive for a given nominal rate. The same occurs if the domestic currency tends to appreciate in bad states of the world for the foreigners, a property referred as “safe haven” in the literature.

The above can also help us to understand how external factors beyond  $i^*$  affect  $\Delta(0)$ .<sup>13</sup> Consider a situation in which the variance of the exchange rate is fixed; and the correlation between the exchange rate and  $\Lambda(s)$  is also fixed but strictly positive. If the variance of  $\Lambda(s)$  increases, then  $\Delta(0)$  increases as well. Monetary authorities of safe-haven currencies are thus more likely to face a conflict between their exchange rate policy and the zero lower bound constraint when the international price of risk increases (that is, the variance of  $\Lambda(s)$  increases).<sup>14</sup>

## 4.2 Implementation

We now study the role of the monetary authority’s balance sheet for the implementation of the best equilibrium, that is we characterize the positions  $F$ ,  $M$ , and  $A$  underlying the best equilibrium of the previous section. It turns out that we only need to characterize  $F$ : the value of  $M$  is, in fact, pinned

<sup>13</sup>Rey (2013) has argued that there is a global financial cycle. This external force can drive variation in long interest rates and equity prices, given a fixed domestic short interest rate. Relatedly, in our model, the small open economy is affected by the compensation for risk required from foreign intermediaries. That is, even though domestic good prices (i.e., exchange rates in our model) and all nominal interest rates were expected to remain the same, variation in  $\{\Lambda(s)\}_{s \in S}$  that changes the covariance in (31) has real effects at home.

<sup>14</sup>A related interesting point made by Hassan, Mertens and Zhang (2016) is that a central bank that induces a real appreciation in bad times lowers its risk premium in international markets and increases capital accumulation.

down by the households' demand for money, while the total amount bought in domestic securities  $A$  follows from the budget constraint of the monetary authority.

We first consider the case discussed in Proposition 1, where the monetary authority optimally chooses an allocation that maintains interest parity, and operates away from the zero lower bound.

**Corollary 2** (Implementation away from the zero lower bound). *Suppose Assumptions 1 and 2 hold. If  $\Delta(0) \leq 0$ , the monetary authority implements the best equilibrium with any  $F \in [0, (y_1 - c_1^{fb} + \bar{w})/\bar{q}]$ .*

In this first scenario, accumulating reserves is not necessary to implement the exchange rate policy. Moreover, interest parity holds and the accumulation of foreign reserves does not affect the equilibrium outcomes (locally), thus mirroring the classic irrelevance result of Backus and Kehoe (1989). The reason for this irrelevance is that, as long as the intervention is not too large, there is sufficient intermediary capital for private agents to undo the interventions of the monetary authority.

We next consider the case discussed in Proposition 2, where the zero lower bound binds, and the monetary authority chooses an allocation that violates interest parity. In this case, it is necessary for the monetary authority to engage in foreign reserve accumulation. It optimally does so by selecting the minimum amount of reserves necessary to sustain its exchange rate policy. We summarize it in the following corollary.

**Corollary 3** (Implementation at the zero lower bound). *Suppose Assumptions 1 and 2 hold. If  $\Delta(0) > 0$ , the monetary authority implements the best equilibrium with  $F = (y_1 - c_1 + \bar{w})/\bar{q} > 0$ , where  $c_1$  is the best equilibrium first-period consumption.*

Why does the monetary authority need to accumulate foreign reserves? In the best equilibrium when  $\Delta(0) > 0$ , domestic assets strictly dominate foreign ones. As a result, the capital of foreign intermediaries flows to the small open economy. This capital must be absorbed by either a trade deficit or by capital outflows. That is, from equation (23),

$$\underbrace{\sum q(s)f(s)}_{\text{capital outflow}} + \underbrace{\bar{q}F + (c_1 - y_1)}_{\text{trade deficit}} = \underbrace{\bar{w}}_{\text{capital inflow}} \quad (32)$$

From Lemma 1 we know that the trade deficit is lower in the best equilibrium than it is in the first best, as  $c_1 < c_1^{fb}$ . Because capital inflows are higher in the best equilibrium relative to the first best, they must be absorbed by an outflow of resources. Domestic households have no incentives to purchase foreign assets because, under the best equilibrium, those are dominated by domestic ones. So, they set  $f(s) = 0$  for all  $s$ . It follows that the best equilibrium *must* feature an accumulation of foreign reserves by the monetary authority,  $F > 0$ .

An important observation is that the necessity of foreign reserve accumulation by the monetary authority is independent of the sign of the trade balance in the resulting equilibrium. *Both a trade deficit and a trade surplus are possible outcomes.*

### 4.3 A simple illustration

We now provide a graphical illustration of the key results of this section. To this end, we leverage the results of Lemma 2 and describe the consumption allocation that arises in the best equilibria using a simple diagram in the  $(c_1, C_2)$  space, where  $C_2$  represents the value of the second-period consumption allocation  $\{c_2(s)\}$ .

In both panels of Figure 1, the thick solid lines represent indifference curves, that is, combinations of  $(c_1, C_2)$  delivering the same level of welfare,

$$u(c_1) + \beta U(C_2),$$

with  $U(C_2)$  defined in (SP). The thin solid lines delimit the set of feasible allocations to the small open economy in the first-best problem, that is, those that satisfy (15). The tangency between the indifference curves and this feasibility line represents the first-best consumption allocation, denoted by  $(c_1^{fb}, c_2^{fb})$ . In both panels, we denote the endowment point  $(y_1, Y_2)$  by  $Y$ , and the consumption allocation in the best equilibrium by  $E$ .

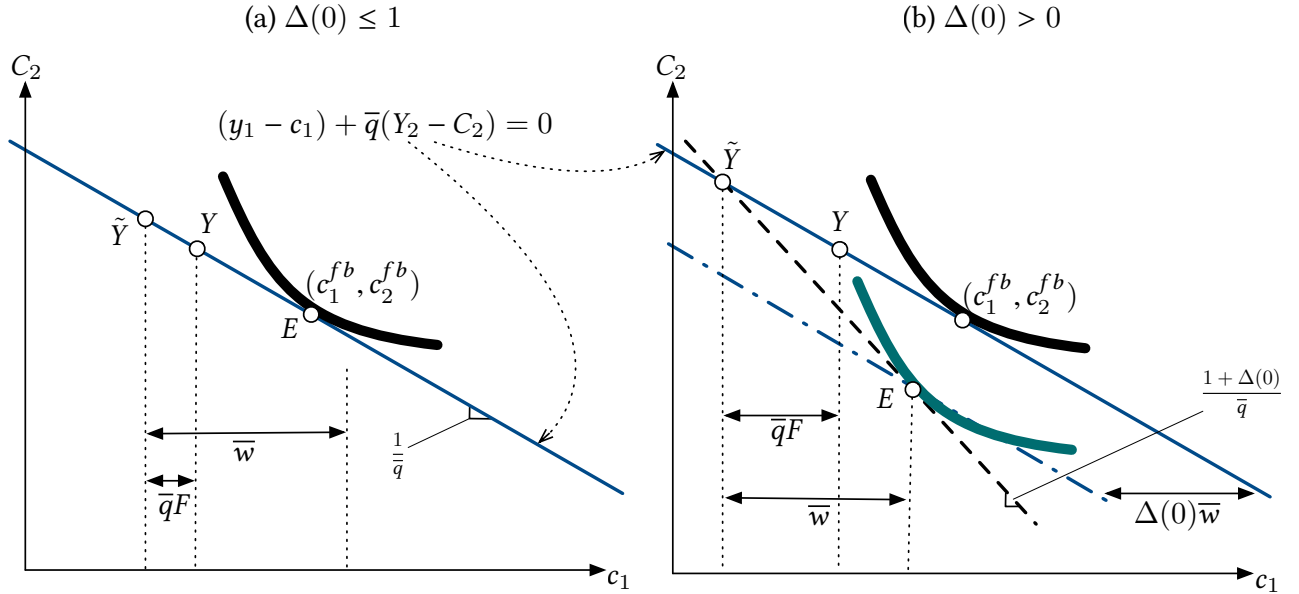


Figure 1: Reserves ( $F$ ) and the best equilibrium

Panel (a) describes the case in which  $\Delta(0) \leq 0$ . As discussed in Proposition 1, the best equilibrium features the first-best consumption allocation and the nominal interest rate that guarantees  $\Delta(i) = 0$ . The graph is also useful in understanding why changes in foreign reserves are locally irrelevant, as we discussed in Corollary 2. Specifically, foreign reserves holdings  $F$  by the monetary authority shift the endowment point from point  $Y$  to point  $\tilde{Y} = (y_1 - F, Y_2 + F/\bar{q})$ . When  $F$  is small (that is,  $F < y_1 + \bar{w} - c_1^{fb}$ ), these interventions have no effects on the equilibrium consumption allocation

because the private sector undoes the external position taken by the monetary authority by borrowing more from foreigners.

Panel (b), instead, describes the case in which  $\Delta(0) > 0$ . As discussed in Proposition 2, the best equilibrium features a nominal interest rate equal to 0 and deviations from interest parity given by  $\Delta(0)$ . The dash-dotted line represents the constraint (27) evaluated at  $i = 0$ . This line is parallel to the first-best feasibility constraint, but reduced by a magnitude  $\Delta(0)\bar{w}$ , which captures the profits of foreign intermediaries and the losses for the small open economy. The best equilibrium is the point on this line where the slope of the indifference curve satisfies (28) with  $i = 0$ . This slope is  $(1 + \Delta(0))/\bar{q}$  and is represented in the figure by the dashed line. This dashed line is also useful for understanding the role of reserves. In particular, its intersection with the first-best feasibility constraint, denoted by  $\tilde{Y}$ , determines the magnitude of the foreign reserve accumulation that is necessary to implement the best equilibrium. The figure shows that it is useful to decompose the welfare reduction that arises as a consequence of the exchange rate policy into two channels: a resource loss, captured by the parallel shift in the thin solid line, and the intertemporal distortion, captured by the steeper dashed line.

In this section, we assumed that the monetary authority takes as given the exchange rate policy to focus on the optimal implementation. Clearly, there are reasons why the monetary authority might choose these exchange rate policies in the first place, and one may worry that, in a more general model where the exchange rate is endogenous, the monetary authority might choose an implementation that is not the best. In Addendum A, however, we show that this concern is not valid in our setup. That is, even though the monetary authority optimally chooses an exchange rate policy, it will carry it out using the best implementation described in this section.

## 4.4 Comparative statics

Let us briefly discuss two comparative statics of the model by zooming into the two terms that determine the losses:  $\bar{w}$  and  $\Delta(i)$ .<sup>15</sup>

Consider first an increase in intermediary capital,  $\bar{w}$ , in a situation in which  $\Delta(0) > 0$ , and the monetary authority sets  $i = 0$  to implement the best equilibrium. As can be seen from equation (27), an increase in  $\bar{w}$  increases the losses because intermediaries are able to obtain higher profits. Because of the higher losses and the fact that there are no changes in the intertemporal distortion, equation (28), households are unambiguously worse off. We can also see from Figure 1, panel (b), that an increase in intermediary capital induces a higher reserve accumulation by the monetary authority. If intermediaries are better capitalized, the interventions done by the monetary authority to reverse the capital inflows need to be larger.<sup>16</sup>

<sup>15</sup>For more detail on the arguments, we refer the reader to an earlier version of this paper (Amador et al., 2017).

<sup>16</sup>It is important to highlight that a higher intermediary's capital is not beneficial in part because there is already enough capital to finance the first-best consumption (Assumption 2).

The second comparative statics refer to the role of the exchange rate policy, the foreign interest rate, and the role of risk, all captured in  $\Delta(0)$ . From equation (31),  $\Delta(0)$  increases when i) there is a larger expected appreciation of the domestic currency, ii) the covariance of the appreciation rate with the stochastic discount factor of the intermediaries is larger, and iii)  $i^*$  is lower. For a given  $\bar{w}$ , the increase in  $\Delta(0)$  has two effects on the best equilibrium. It increases the magnitude of the losses in (27) and increases the intertemporal distortion as compared to the first-best allocation, as seen in (28).<sup>17</sup> As a result, the domestic households are unambiguously worse off. Similar to the discussion above, a larger  $\Delta(0)$  also requires a larger reserve accumulation by the monetary authority.

This discussion highlights that if a country is better integrated with the international financial markets (that is, financial intermediaries can invest more resources in it) or if its currency has better hedging properties (that is, it is a safe haven), then the larger the interventions required to implement the exchange rate policy under a binding zero lower bound, and the larger the associated costs.

## 5 Measuring the costs of foreign exchange interventions

In the previous sections, we have shown that certain exchange rate policies require the monetary authority to actively intervene in foreign exchange markets and that these interventions are costly for the small open economy. We have identified two distinct welfare costs associated with these interventions: an intertemporal distortion in the consumption allocation and a resource cost. This latter in our stylized model is the product of two objects: the deviations from interest rate parity,  $\Delta(i)$ , and the amount of capital that foreign intermediaries devote to the small open economy,  $\bar{w}$ . In this section, we show how to use available data to measure this second cost.

### 5.1 Measuring $\Delta(i)$

In the literature, measuring return differentials on bonds denominated in different currencies can be done in two ways: the uncovered interest parity condition (UIP) and the covered interest parity condition (CIP). An important question is which of these two should be used as a proxy for  $\Delta(i)$  when measuring the costs of foreign exchange interventions. A standard practice in the literature is to use deviations from the UIP condition; see, for example, [Adler and Tovar Mora \(2011\)](#) and [Adler and Mano \(2016\)](#). In what follows, we show that UIP deviations are, in general, not the right empirical counterpart to  $\Delta(i)$ . We next show that, under reasonable assumptions, CIP deviations should be used instead to proxy for  $\Delta(i)$ .

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<sup>17</sup>A reduction in  $i^*$ , which is equivalent to an increase in  $\bar{q}$ , also affects the resource constraint, equation (27). One may have conjectured that whether such a reduction is beneficial would depend on whether the economy is a net external lender or a borrower. However, we can show that with a binding zero lower bound, a reduction in  $i^*$  unambiguously reduces welfare, even for a net external borrower. The key is that with a binding zero lower bound, a net external borrower effectively borrows at a rate higher than  $i^*$ , and the monetary authority ends up saving at a lower interest rate.



We start by rewriting the resource loss per unit of capital inflow,  $\Delta(i)$  in equation (26), as follows:

$$\Delta(i) = \underbrace{\left\{ \frac{1+i}{1+i^*} \mathbb{E} \left[ \frac{e_1}{e_2(s)} \right] - 1 \right\}}_{\text{UIP deviation}} + \underbrace{\text{Cov} \left[ \frac{q(s)}{\pi(s)}, \frac{e_1}{e_2(s)} \right]}_{\text{risk premium}}.$$

From the above equation, we can immediately see that deviations from UIP would be an imperfect measure of  $\Delta(i)$  as long as the risk premium component is different from zero.

A simple example might be useful in explaining why UIP should not be used to measure the costs of foreign exchange interventions. Consider a situation in which  $\Delta(i) = 0$  but the deviations from UIP are negative. In our model, this occurs when the currency of the small open economy has good hedging properties (when it appreciates in bad times for foreign financial intermediaries). Assume also that the monetary authority in period 1 accumulates foreign reserves and finances this accumulation by issuing a domestic currency risk-free bond. The returns from this strategy per unit of foreign bond purchased are

$$r_2(s) = 1 - \frac{1+i}{1+i^*} \frac{e_1}{e_2(s)}.$$

It is clear from this example that the monetary authority makes profits on average from this strategy because it is shorting assets with low yields and purchasing high-yielding ones. That is,  $\mathbb{E}[r_2(s)] > 0$ . However, it should also be clear that these profits, when appropriately discounted, equal zero from an ex-ante perspective. Indeed, we have

$$\mathbb{E} \left[ \frac{\beta u'(c_2(s))}{u'(c_1)} r_2(s) \right] = \mathbb{E} [\Lambda(s) r_2(s)] = \Delta(i).$$

Thus, if  $\Delta(i) = 0$ , the monetary authority does not gain or lose anything from this strategy: it is purchasing a riskier asset than the one it is shorting, and the profits it receives in expectation exclusively reflect a fair compensation for undertaking such risk. Note that in the above derivation, we have used our assumption of complete international financial markets in that the domestic and the foreign stochastic discount factor are equivalent in every state of the world. In Addendum B we show that this assumption is not needed for the result: even with arbitrary incomplete markets, a monetary authority would not gain or lose anything in our example as long as domestic agents could freely purchase risk-free domestic and foreign bonds. This point is related to the result in Backus and Kehoe (1989), who show that with perfect capital mobility foreign exchange interventions can be ineffective even under incomplete markets.

If UIP deviations are not the right measure, could we use deviations from the CIP to proxy for  $\Delta(i)$ ? To examine CIP deviations within our model, we need to open a forward exchange rate market. Given that we have complete markets in each of the two regions (domestic and foreign asset markets), we could open such a forward market in either of the two.

Let us consider then, the price of a forward exchange rate contract in the international financial markets.<sup>18</sup> The idea is to consider the following trade. A foreign household has a claim to a unit of domestic currency in period 2. She would like to exchange it for a claim to a constant amount of foreign currency in period 2. Let  $\hat{e}$  denote the price of this contract (i.e., the forward exchange rate). The value  $\hat{e}$  must satisfy the following condition:

$$\sum_{s \in S} q(s) \left[ \frac{1}{e_2(s)} - \frac{1}{\hat{e}} \right] = 0, \quad (33)$$

which implies that the forward exchange rate equals

$$\hat{e} = \frac{\sum_{s \in S} q(s)}{\sum_{s \in S} \frac{q(s)}{e_2(s)}}$$

From the definition of  $\Delta(i)$ , we have that

$$\Delta(i) = \left[ \sum_{s \in S} \frac{q(s)e_0}{e_1(s)} \right] (1+i) - 1 = \underbrace{\frac{1+i}{1+i^*} \frac{e_0}{\hat{e}} - 1}_{\text{CIP deviation}}$$

Hence, direct observation of a CIP deviation provides a correct estimate of a loss per unit of capital inflow,  $\Delta(i)$ .

This distinction between UIP and CIP is an important one. Going back to our previous example, a safe-haven currency might experience negative deviations from UIP and, at the same time, observe positive deviations from CIP. If we were to use deviations from the UIP condition, we would incorrectly conclude that the small open economy is gaining from foreign exchange interventions while, in reality, these interventions are costly. As we discuss in our empirical application, this situation is indeed relevant when studying the experience of the Swiss National Bank.

The literature also discusses an alternative interpretation to the safe haven. Consider the case in which a *safety premium* arises not from the risk properties of domestic assets vis-à-vis foreign ones, but rather from foreigners having a strict preference for holding the asset perceived to be safe. As a result, they are willing to hold this asset even when its risk-adjusted rate of return lies strictly below that of foreign ones. In this case, for example, the SNB can, by creating monetary liabilities and accumulating US assets, generate ex-ante discounted profits as long as this safety premium on its monetary liabilities vis-à-vis US dollar assets is strictly positive.<sup>19</sup> However, under this interpretation

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<sup>18</sup>Under Assumption 2, that is, under equal gaps, it does not matter in which market the forward contracts are traded, as they both deliver the same prices. The reason is that under equal gaps, foreign markets and households share the same ratio of marginal utilities across states in period 2 (that is, the only distortion is intertemporal). A forward contract is a trade across states in period 2, and thus, the forward price should be the same in both markets.

<sup>19</sup>This argument is similar to how seigniorage generates revenue for the government in monetary models.

with perfect arbitrage, the CIP deviation between the Swiss franc and the US dollar would be negative: foreigners should be indifferent between holding a Swiss asset at a lower rate of return, and holding an equivalent US security and selling forward its dollar return back into Swiss francs. As we show next, there is no evidence of a negative CIP deviation between Swiss francs and US dollars. Note that this does not contradict the argument that safe asset demand and supply considerations played an important role during and after the financial crisis of 2008—a point argued strongly in a recent literature, summarized in [Caballero, Farhi and Gourinchas \(2017\)](#). Rather, our point is that the measured difference in rates of return does not justify the view that the safety premium on Swiss francs was particularly higher *than for other safe assets* (i.e., US securities) during this period.

## 5.2 Measuring $\bar{w}$

In order to measure the costs of foreign exchange interventions, we need to measure the amount of capital that foreign intermediaries can invest in the small open economy,  $\bar{w}$ . Unfortunately, this object cannot be directly measured. However, we show that we can use additional equilibrium relations of the model in order to approximate the resource costs using the foreign reserves accumulated by the monetary authority.

Specifically, we can rewrite the intertemporal resource constraint of the small open economy as follows:<sup>20</sup>

$$y_1 - c_1 + \frac{\bar{q}}{1 + \Delta(i)}(Y_2 - C_2) = \underbrace{\frac{\Delta(i)}{1 + \Delta(i)}\bar{q}F}_{\text{alternative measure}}, \quad (34)$$

which corresponds to the dashed line in panel (b) of Figure 1 when  $i = 0$ . Thus, we can approximate the resource loss using the reserves accumulated by the monetary authority, and multiplying the amount of foreign reserves by the CIP deviation.

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<sup>20</sup>Using the budget constraint of the households and the government, market clearing, and solving out this time for  $a(s)$ , we can obtain

$$y_1 = c_1 + \sum_{s \in S} \frac{p(s)e_2(s)}{e_1} (c_2(s) - y_2(s)) + \sum_{s \in S} \left( q(s) - \frac{p(s)e_2(s)}{e_1} \right) (f(s) + F) + \left[ \sum_{s \in S} p(s) - 1 \right] \frac{m^*}{e_1}.$$

The last term is zero, as intermediaries do not hold money unless at zero domestic interest rates. Using equal gaps, (24), and the definitions of  $\Delta(i)$  in (25), and  $Y_2$  and  $C_2$ , we get:

$$y_1 - c_1 + \frac{\bar{q}}{1 + \Delta(i)}(Y_2 - C_2) = \frac{\Delta(i)}{1 + \Delta(i)} \left( \sum_{s \in S} q(s)f(s) + \bar{q}F \right).$$

Households' optimality with respect to foreign assets implies that if  $\Delta(i) > 0$ ,  $f(s) = 0$ , and the result in (34) follows.

### 5.3 Infinite horizon and balance sheet composition

Two final aspects remain to be addressed regarding our measurement of the costs. First, so far, we have studied a two-period model. The lack of a multiperiod framework makes it difficult to uncover, for example, whether it is the flows or the stocks of reserves that matter in measuring the costs. Second, while in our analysis we have restricted the monetary authority to issue or purchase risk-free domestic and foreign bonds, in practice the balance sheet of central banks contains several types of assets and liabilities that differ, for example, by currency of denomination and maturity. A relevant question is whether and how we should account for these different financial assets when computing the costs of interventions.

We tackle these two issues by extending our setting to an infinite horizon economy. Let  $s^t$  now index the history of state realizations up to time  $t$ . Let  $F(s_{t+1}, s^t)$  denote the realized value of the portfolio of foreign reserves in the subsequent state  $(s_{t+1}, s^t)$ . This value  $F(s_{t+1}, s^t)$  is allowed to be state dependent to account for all the potentially different maturity, currency of denomination, or risk properties of the underlying assets held by the monetary authority. However, independently of the underlying securities that make up the portfolio, the value of the foreign reserve portfolio at the end of period  $t$  is

$$\sum_{s_{t+1} \in S} q(s_{t+1}, s^t) F(s_{t+1}, s^t).$$

In Appendix B, we show that under allocations satisfying equal gaps, and taking as given future policies, we can write the resource losses for the small open economy between periods  $t$  and  $t + 1$  in a way that is analogous to equation (34)

$$\tilde{y}(s^t) - c(s^t) + \frac{\bar{q}(s^t)(\tilde{Y}_2(s^t) - C_2(s^t))}{1 + \Delta(s^t)} = \frac{\Delta(s^t)}{1 + \Delta(s^t)} \sum_{s_{t+1} \in S} F(s_{t+1}, s^t) q(s_{t+1}, s^t), \quad (35)$$

where  $\tilde{y}(s^t)$  and  $\tilde{Y}_2(s^t)$  are the “effective endowments” in period  $t$  and  $t + 1$ , respectively (see equations (B.8) and (B.9) in Appendix B). The former is constructed by summing the initial net foreign asset position to the period  $t$  endowment, while the latter also consolidates the value of the next-period endowment with the next-period savings policy.

We wish to emphasize two main points. First, our measure of resource costs can be interpreted more generally as the *one-period ahead* costs incurred by the monetary authority taking as given future policies. Second, to approximate the losses at any period, we just need to compute the one-period deviations from CIP and the end-of-period value of the *stock* of *total* reserves. The composition of the monetary authority’s balance sheet is irrelevant in equation (35) because arbitrage returns are equalized across all securities under an equal gap allocation. Thus, the market value of total reserves is enough to compute the losses.

## 6 Empirical analysis

In this section, we first use our theoretical results to quantify the resource cost of foreign exchange interventions in the case of Switzerland over the period 2010-2017. We argue below that Switzerland during this period is a good example of the economic circumstances analyzed in this paper: an interest rate close to or at its lower bound, an explicit exchange rate policy, large accumulation of reserves by the SNB, and persistent and significant CIP deviations.

We then discuss how our framework is useful for understanding the patterns of CIP deviations, interest rates, and foreign reserve accumulation by central banks observed after the financial crisis for major international currencies. We finally show that similar patterns were also observed in another (rare) early episode of interest rates at their lower bound, that is, Switzerland in the late 1970s.

### 6.1 The case of the Swiss franc: 2010-2017

Following the global financial crisis, as shown in Panel (a) of Figure 2, the Swiss franc appreciated by roughly 25% against the euro. The Swiss National Bank perceived this appreciation to be damaging for the Swiss economy and, to counteract it, it established a currency floor of 1.2 Swiss francs per euro in 2011.<sup>21</sup> The SNB kept this floor until January 2015, when the floor was abandoned and the Swiss franc appreciated by 15% vis-à-vis the euro.

As Panel (b) of Figure 2 shows, throughout the 2010-2017 period, nominal interest rates in Switzerland were at or below zero. Moreover, all throughout this period there were times in which financial markets assigned a non-trivial probability of franc appreciations (Jermann, 2017), and these expectations of appreciation were correlated with bad economic conditions in Europe and worldwide.<sup>22</sup> The SNB experience during the 2010-2017 period is well described by our simple model: A Central Bank with an interest rate at the zero bound, pursuing an exchange policy that makes its own domestic currency assets attractive relative to a reference foreign currency. Our theoretical analysis predicts that, under these circumstances, we should observe foreign reserve accumulation by the SNB, concurrent with strictly positive CIP deviations for the Swiss franc. Panel (c) of Figure 2 shows that this is indeed the case.

Panel (c) of the figure reports the (annualized) three-month CIP deviations between the Swiss franc and the euro, along with a monthly series for the stock of foreign reserves held by the SNB as a fraction of annual (trend) Swiss GDP.<sup>23</sup> The panel shows that CIP deviations were virtually absent

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<sup>21</sup>The 2011 Q3 SNB Quarterly Bulletin stated that with the 1.20 floor “the SNB is taking a stand against the acute threat to the Swiss economy and the risk of deflationary development that spring from massive overvaluation of the Swiss franc.”

<sup>22</sup>As the European crisis deepened following the Greek elections of May 2012, there was increased speculation that the SNB could impose capital controls or abandon the currency floor. See, for example, Alice Ross and Haig Simonian, “*Swiss eye capital controls if Greece goes*,” *Financial Times*, May 27, 2012 and the article mentioned in footnote 2.

<sup>23</sup>For this analysis, we use the CIP deviations with respect to the euro, as this was the currency used for the floor on the Swiss franc. The deviations with respect to the US dollar are even larger.

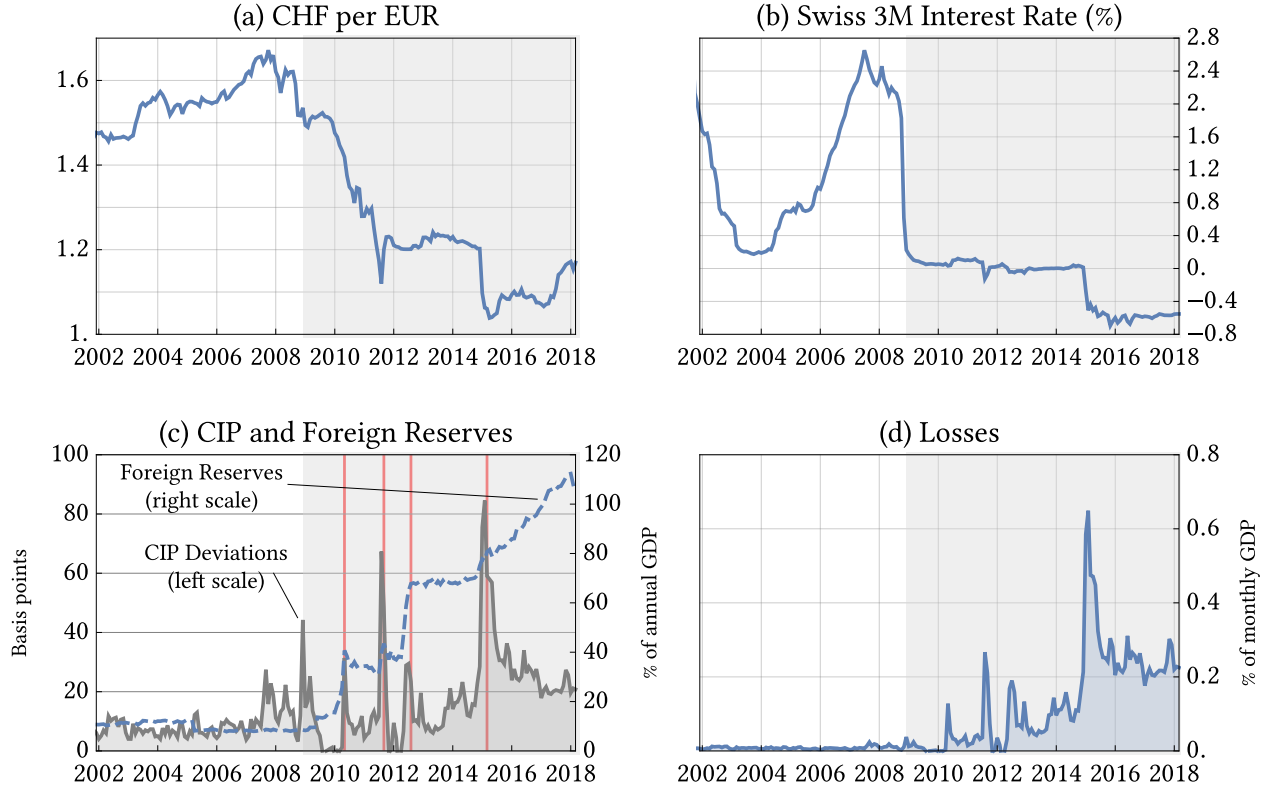


Figure 2: Exchange Rate, interest rate, foreign reserves, CIP deviations, and losses

Note: The shaded areas represent the months in which the Swiss interest rate was below 0.5%.

before the 2008 financial crisis, and that these deviations spiked during the crisis.<sup>24</sup> More interestingly for our purpose, the panel shows that, starting in 2010, there is a tight connection between large *positive* CIP deviations and increases in SNB holdings of foreign reserves. First, each of the four post-crisis spikes in the CIP deviations (denoted by the vertical lines in the panel) corresponds to large increases in reserve accumulation (which brought Swiss foreign reserves from 10% to 80% of GDP). Also, over the 2016-2017 period, historically sizable CIP deviations (between 20 and 40 basis points) have corresponded with additional reserve accumulation, bringing SNB reserves to 110% of GDP.

In view of our discussion in Section 5, we can use these series to measure the resource costs associated with these foreign exchange interventions. Specifically, we let a period be a month and calculate the costs in period  $t$ :

$$\text{losses}_t = \frac{\Delta_t}{1 + \Delta_t} \times \frac{F_t}{GDP_t}, \quad (36)$$

<sup>24</sup>See, for example [Baba and Packer \(2009\)](#), for a discussion of how tightening financial constraints might explain the deviations during the financial crisis.

where

$$\Delta_t = \left[ \frac{(1 + i_t^{\text{CHF},3\text{M}}) e_t}{(1 + i_t^{\text{EUR},3\text{M}}) \hat{e}_t^{3\text{M}}} \right]^{1/3} - 1,$$

where  $i_t^{\text{CHF},3\text{M}}$  is the three-month Swiss franc denominated overnight index swap (OIS);  $i_t^{\text{EUR},3\text{M}}$  is the OIS rate on euro denominated swaps;  $e_t$  denote the mid-point of the spot exchange rate between the Swiss franc and the euro; and  $\hat{e}_t^{3\text{M}}$  is the mid-point of the three-month forward exchange rate between the Swiss franc and the euro;  $F_t$  is the value of the stock of foreign reserves held by the SNB (in current Swiss francs), and  $GDP_t$  is the monthly (trend) Swiss nominal GDP (in current Swiss francs).<sup>25</sup>

The loss as a fraction of monthly GDP is reported in panel (d) of Figure 2. As can be seen, the costs of foreign exchange interventions after 2010 were significant, reaching around 0.6% of monthly GDP around January 2015, the month when the SNB decided to abandon the currency floor vis-à-vis the euro.

## 6.2 CIP deviations, foreign reserves, and interest rates across countries

While the recent Swiss experience provides a clear example of the economic forces studied in this paper, we now argue that our results are also useful for interpreting other experiences. In a recent paper, Du et al. (Forthcoming) have documented that well after the financial crisis of 2008, substantial deviations from CIP have been persistently observed for several advanced economies. In this section, we use the same set of countries studied by these authors and document that (i) positive deviations from CIP are concentrated in countries/periods where the nominal interest rate is close to zero, and (ii) deviations from CIP are positively related to foreign reserves accumulated by the monetary authority. These results support the idea that some of the CIP deviations observed after the financial crises are due to a conflict between exchange rate policies and the zero lower bound on nominal interest rates.

We collect data on exchange rates (both spot and forward rates) against the US dollar, and on the nominal interest rate (OIS) for the Japanese yen, Danish krone, Swedish krona, Canadian dollar, British pound, Australian dollar, New Zealand dollar, and the US dollar, over the 2010-2018 period. We also collect data on total foreign exchange reserves held by monetary authorities in these countries.<sup>26</sup>

<sup>25</sup> The OIS spot and forward rates (both bids and asks) are at a daily frequency, obtained from Bloomberg, and averaged over their respective months. The data on foreign reserves and GDP are from the IMF International Financial Statistics and OECD Quarterly National accounts, respectively. Foreign reserves are at a monthly frequency, while the GDP series is quarterly. To obtain monthly trend GDP, we HP-filtered the GDP series and imputed a monthly value from its trend. We choose to use the three-month CIP deviation rather than the one-month CIP deviation because there is less high frequency variation in the former.

<sup>26</sup> CIP gaps are computed as in Du et al. (Forthcoming) with the only difference being that we use OIS interest rates as opposed to LIBOR, as LIBOR rates for some countries are no longer recorded after 2013. The set of countries is also the same with the exception of Norway, which we exclude from our sample as it has no OIS rate. Prior to this period, and with the exception of the 2008-2009 financial crises, CIP deviations were essentially zero for all of these currencies. We exclude from our analysis the very volatile period of the financial crisis. The dollar exchange rate and OIS data were collected at a daily frequency from Bloomberg and were averaged over their respective months. Data sources and methodology for computing the reserve to GDP ratio are the same used for the Swiss series and are detailed in footnote 25.



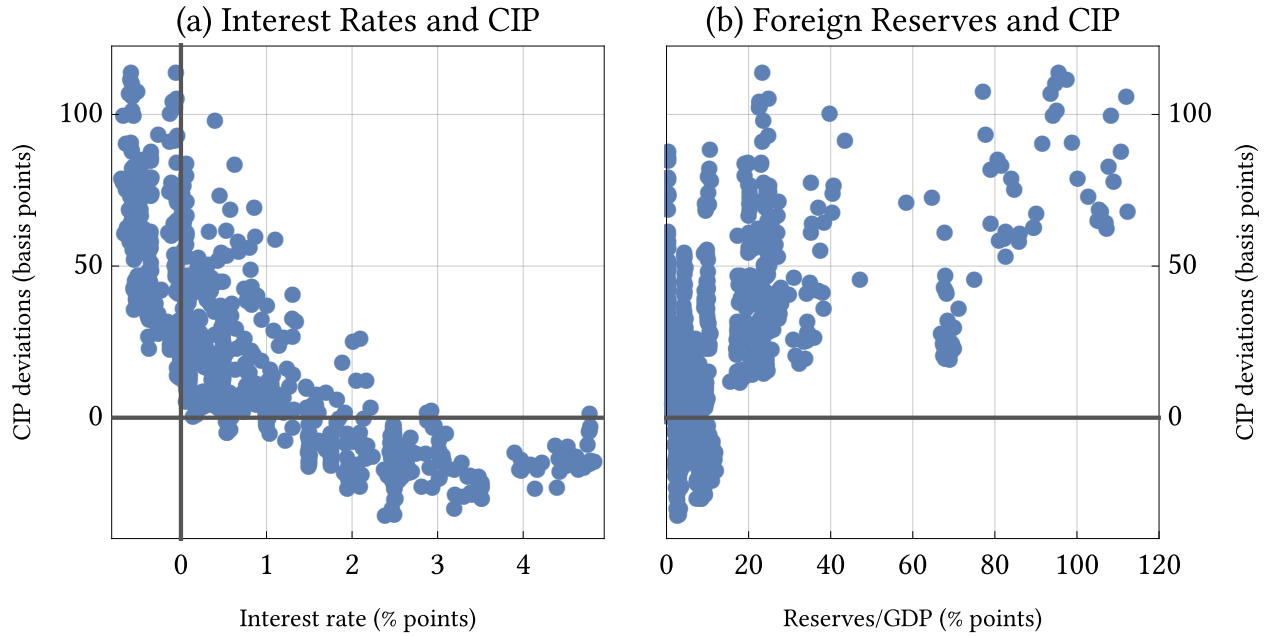


Figure 3: Relation between CIP gaps, reserves, and interest rates

Panel (a) of Figure 3 plots the observations of monthly CIP deviations for each of these currencies (with respect to the dollar) against their corresponding nominal interest rates. The panel shows that CIP deviations are positive for countries and time periods characterized by very low nominal interest rates, whereas they tend to be small when nominal interest rates are positive. A negative relation between CIP gaps and nominal interest rates has also been documented by [Du et al. \(Forthcoming\)](#). This graph highlights the non-linearity of the relations: CIP deviations are large only when interest rates are close to zero. This finding lends support to our result that CIP deviations are only part of an optimal equilibrium when the zero lower bound constraint on the nominal interest rate binds.

Panel (b) of Figure 3 plots these monthly CIP deviations against the corresponding level of foreign reserves (normalized by trend GDP). The figure shows a positive relationship between the level of foreign reserves held by the monetary authority and the deviations from CIP. This empirical finding, which to the best of our knowledge has not been previously noted in the literature, is consistent with the mechanism at the heart of our model, whereby the monetary authority is able to sustain a positive CIP deviation by accumulating a sufficiently large position in foreign assets.

We complement these last two panels with Table 1, which shows the results of regressing the monthly CIP deviations on measures of foreign reserves. The table shows that the positive association between CIP deviations and foreign reserves is robust. Specifically, this association holds whether we include country and time fixed effects, whether we drop the Swiss franc from the sample, and whether

we do the analysis with the level or the first difference in foreign reserves.<sup>27</sup>

Table 1: CIP Deviations and Foreign Reserves

	(1)	(2)	(3)	(4)	(5)	(6)
	no fixed effects	with fixed effects	excluding Switzerland	no fixed effects	with fixed effects	excluding Switzerland
$(F/Y)_t$	73.15*** (16.70)	41.46*** (9.88)	208.95*** (37.81)			
$\Delta(F/Y)_t$				438.75*** (99.91)	190.01*** (10.68)	181.15** (72.99)
country/time FE	No	Yes	Yes	No	Yes	Yes
$N$	890	890	791	890	890	791
$R^2$	0.29	0.84	0.84	0.03	0.83	0.82

Clustered (at country level) standard errors in parentheses. \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Notes: Dependent variable is the (annualized) CIP deviation in basis points.  $F/Y$  is the value of foreign reserves divided by (annualized) trend GDP.  $\Delta(F/Y)$  is the monthly difference in  $F/Y$  with respect to previous month. The sample includes monthly observations from January 2010 to March 2018 for the Swiss franc, Japanese yen, Danish krone, Swedish krona, Canadian dollar, British pound, Australian dollar, New Zealand dollar, and euro. The regressions include a constant term, and we report the adjusted  $R^2$  in columns (2)-(3) and (5)-(6).

Finally, the experience of Switzerland in the late 1970s provides another interesting episode of exchange rate policy in an environment with very low interest rates.<sup>28</sup> Panel (a) of Figure 4 shows the monthly time series for the Swiss franc against the US dollar for the period 1977-1979, and it shows that the Swiss franc had been steadily appreciating against the US dollar, just as it did in the aftermath of the 2007-2009 crisis.<sup>29</sup> In an effort to prevent further appreciations, the SNB initially reduced the domestic rate, which by the end of 1978 reached levels close to zero (see the shaded area in panel (b) of the figure). At this point, just as it did in 2011, the SNB announced a temporary floor between the Swiss franc and the deutsche mark, and, to maintain the floor, it engaged in large foreign exchange interventions. Panel (c) of Figure 4 shows the monthly time series of foreign reserves (excluding gold, as a fraction of trend GDP), together with deviations from CIP, calculated in the same way as in the previous section.<sup>30</sup>

Panel (c) shows that the ratio of foreign reserves to GDP increased by over 10% of GDP, and around the same time, the deviations from CIP increased by over 50 basis points. By mid-1979, the interna-

<sup>27</sup>We use both the level and the changes in foreign reserves in the specifications because, differently from the calculations of the losses, our simple two period model does not determine which one is the appropriate measure.

<sup>28</sup>See Claire Jones, "Swiss tried to put ceiling on franc before", *Financial Times*, September 6, 2011, for a description of the macroeconomic environment in Switzerland at the time.

<sup>29</sup>Over the period 1977-1978, the Swiss franc also appreciated 30% against the deutsche mark.

<sup>30</sup>The only difference is our data source, since Bloomberg data are not available for this early period. Three-month nominal interest rates are interbank rates from the OECD *Main Economic Indicators*, and daily spot and three-month forward rates are provided by the SNB.

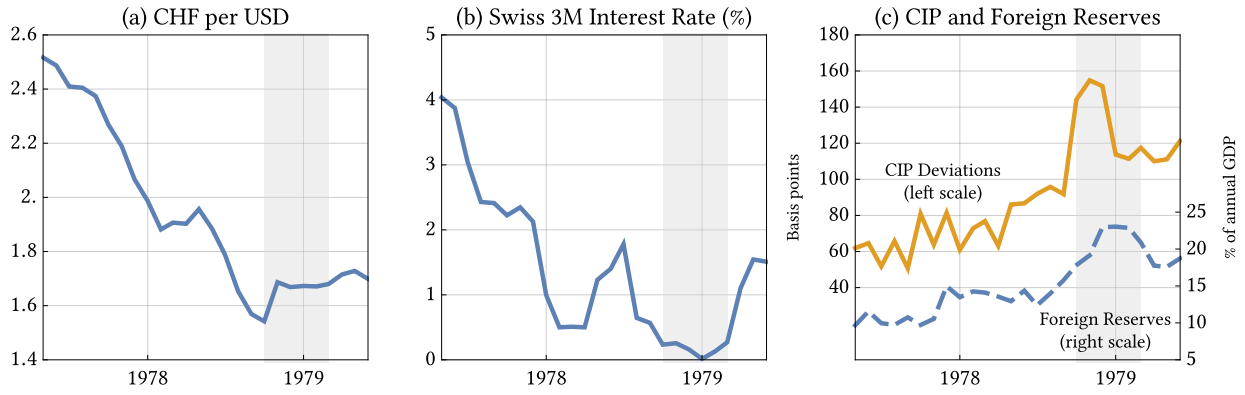


Figure 4: Foreign reserves, interest rates, and CIP deviations: Switzerland 1977-1979

Note: The shaded areas represent the months in which the Swiss interest rate was below 0.5%.

tional macroeconomic conditions changed substantially, and the SNB was able to avoid appreciation of the currency while maintaining a positive interest rate. As a consequence, both the level of foreign reserves and the deviations from covered interest parity abated.

## 7 Conclusions

This paper studied the problem of a monetary authority pursuing an exchange rate policy that is inconsistent with interest rate parity because of a binding zero lower bound constraint. We have shown that even if monetary policy is constrained, it can still achieve an independent exchange rate objective by using foreign exchange interventions that result in observable deviations from arbitrage in capital markets. These interventions, however, are costly from the point of view of the domestic economy. We show how these costs can be measured and document that they were substantial in the recent experience of the Swiss National Bank. Moreover, the main predictions of our theory are consistent with the behavior of foreign reserves, nominal interest rates, and deviations from the covered interest rate parity conditions for a panel of advanced economies.

The analysis could be extended in several directions. One interesting question relates to reserve management: given that reserves accumulation is a necessary tool for conducting an exchange rate policy at the zero lower bound, what are the optimality principles that should govern its asset allocations? In [Amador et al. \(2018\)](#) we introduce a foreign reserve portfolio for the monetary authority and characterize the trade-offs that the monetary authority faces.

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## A Omitted Proofs

### Proof of Lemma 1

Suppose that Assumptions 1 and 2 hold. Then the consumption allocation of any monetary equilibrium feature equal gaps.

*Proof.* Suppose we have an equilibrium allocation that features unequal gaps. Let

$$\kappa(s) \equiv \frac{q(s)e_1}{p(s)e_2(s)} - 1 = \frac{q(s)}{\beta\pi(s)} \frac{u'(c_1)}{u'(c_2(s))} - 1.$$

Note that this implies that

$$c_2(s) = \left( \frac{(1 + \kappa(s))\beta\pi(s)}{q(s)} \right)^{1/\sigma} c_1.$$

Let  $\bar{\kappa} \equiv \max_{s \in S} \{\kappa(s)\}$ . Given that  $\kappa(s) \geq 0$  for all  $s$ , It follows that  $\bar{\kappa} > 0$  (or else the gaps are all equalized). Let  $\bar{S} \equiv \{s | \kappa(s) = \bar{\kappa}\}$ . Let  $S_0 \equiv \{s | \kappa(s) = 0\}$ . And let  $\underline{S}$  be their complement,  $\underline{S} \equiv S/(\bar{S} \cup S_0)$ .

The intermediaries' problem implies that  $a^*(s) = 0$  for all  $s$  such that  $\kappa(s) < \bar{\kappa}$ . In addition,  $m^* = 0$ ; and  $\sum p(s)a^*(s)/e_1 = \bar{w}$ . Intermediaries' profits are

$$\Pi = \bar{\kappa}\bar{w}$$

From the households' problem,  $f^*(s) = 0$  for all  $s$  such that  $\kappa(s) > 0$ . From the trade balance equation, (23),

$$c_1 - y_1 = \bar{w} - \left( \sum_{s \in S_0} q(s)f(s) + \bar{q}F \right).$$

Using the budget constraints, market clearing, and that  $m^* = 0$ , we can obtain a similar equation for  $c_2(s)$ : For all  $s \notin \bar{S}$ :

$$c_2(s) = \begin{cases} y(s) + F & s \in \underline{S} \\ y(s) + F - a^*(s)/e_2(s) & s \in \bar{S} \\ y(s) + F + f(s) & s \in S_0. \end{cases}$$

From  $\sum p(s)a^*(s)/e_1 = \bar{w}$ , we have that

$$\bar{w} = \sum_{s \in \bar{S}} p(s)a^*(s)/e_1 = \frac{1}{1 + \bar{\kappa}} \sum_{s \in \bar{S}} q(s) \frac{a^*(s)}{e_2(s)}.$$

So it follows that

$$\sum_{s \in \bar{S}} q(s)c_2(s) = \sum_{s \in \bar{S}} q(s)y(s) + \sum_{s \in \bar{S}} q(s)F - (1 + \bar{\kappa})\bar{w}.$$

Substituting out  $c_2(s)$  as a function of  $\kappa(s)$  and  $c_1$  delivers

$$\begin{aligned} \sum_{s \in \bar{S}} q(s) \left( \left( \frac{(1 + \bar{\kappa})\beta\pi(s)}{q(s)} \right)^{1/\sigma} c_1 \right) &= \sum_{s \in \bar{S}} q(s)y_2(s) + \sum_{s \in \bar{S}} q(s)F - (1 + \bar{\kappa})\bar{w} \\ c_1(1 + \bar{\kappa})^{1/\sigma} \sum_{s \in \bar{S}} q(s) \left( \frac{\beta\pi(s)}{q(s)} \right)^{1/\sigma} &= \sum_{s \in \bar{S}} q(s)y_2(s) + \sum_{s \in \bar{S}} q(s)F - (1 + \bar{\kappa})\bar{w} \end{aligned} \quad (\text{A.1})$$

The intertemporal resource constraint implies

$$c_1 + \sum q(s)c_2(s) = y_1 + \sum q(s)y_2(s) - \bar{\kappa}\bar{w}$$

And using the value of  $c(s)$  above and the definition of  $Y_2$  and  $\bar{q}$ , we obtain:

$$c_1 = \frac{y_1 + \bar{q}Y_2 - \bar{\kappa}\bar{w}}{1 + \sum q(s) \left( \frac{(1 + \kappa(s))\beta\pi(s)}{q(s)} \right)^{1/\sigma}}$$



For any  $s_0 \notin \bar{S}$ ,

$$\left( \frac{(1 + \kappa(s_0))\beta\pi(s_0)}{q(s_0)} \right)^{1/\sigma} c_1 \geq y_2(s_0) + F$$

where the above follows from  $f(s_0) \geq 0$ . Using (A.1) to substitute out  $F$ , we get

$$\left[ \left( \frac{(1 + \kappa(s_0))\beta\pi(s_0)}{q(s_0)} \right)^{1/\sigma} - \frac{1}{\sum_{s \in \bar{S}} q(s)} (1 + \bar{\kappa})^{1/\sigma} \sum_{s \in \bar{S}} q(s) \left( \frac{\beta\pi(s)}{q(s)} \right)^{1/\sigma} \right] c_1 \geq y_2(s_0) + \frac{1}{\sum_{s \in \bar{S}} q(s)} \left[ (1 + \bar{\kappa})\bar{w} - \sum_{s \in \bar{S}} q(s)y_2(s) \right]$$

Then,

$$\begin{aligned} & \overbrace{\left[ \left( \frac{1 + \kappa(s_0)}{1 + \bar{\kappa}} \right)^{1/\sigma} \frac{\left( \frac{\pi(s_0)}{q(s_0)} \right)^{1/\sigma}}{\frac{1}{\sum_{s \in \bar{S}} q(s)} \sum_{s \in \bar{S}} q(s) \left( \frac{\pi(s)}{q(s)} \right)^{1/\sigma}} - 1 \right]}^{\leq \max_{s_1, s_2} \left\{ \frac{\pi(s_1)q(s_2)}{\pi(s_2)q(s_1)} \right\} - 1} \overbrace{\left[ (1 + \bar{\kappa})^{1/\sigma} \sum_{s \in \bar{S}} q(s) \left( \frac{\beta\pi(s)}{q(s)} \right)^{1/\sigma} c_1 \right]}^{= \sum_{s \in \bar{S}} q(s)c_2(s) \leq y_1 + \bar{q}Y_2} \\ & + \underbrace{\sum_{s \in \bar{S}} q(s)y_2(s) - \left( \sum_{s \in \bar{S}} q(s) \right) y_2(s_0)}_{\leq \bar{q} \max_{s_1, s_2} (y_2(s_1) - y_2(s_2))} \geq \underbrace{(1 + \bar{\kappa})\bar{w}}_{> \bar{w}} \end{aligned}$$

where we have used that  $\bar{w} > 0$  and  $\bar{\kappa} > 0$  to obtain the strict inequality in the last term. Thus,

$$\left[ \max_{s_1, s_2} \left( \frac{\pi(s_1)q(s_2)}{q(s_1)\pi(s_2)} \right)^{1/\sigma} - 1 \right] + \frac{\bar{q} \max_{s_1, s_2} \{y_2(s_1) - y_2(s_2)\}}{y_1 + \bar{q}Y_2} > \frac{\bar{w}}{y_1 + \bar{q}Y_2}$$

which contradicts Assumption 2, proving the claim.  $\square$

## Proof of Lemma 2

Under Assumptions 1 and 2, a consumption allocation  $(c_1, \{c_2(s)\})$  and money holdings  $m$  are part of an equilibrium given the exchange rate policy  $(e_1, \{e_2(s)\})$  if and only if there exists an  $i$  such that:

$$y_1 - c_1 + \bar{q}(Y_2 - C_2) = \Delta(i)\bar{w} \quad (27)$$

$$\frac{\bar{q}u'(c_1)}{\beta U'(C_2)} = 1 + \Delta(i) \geq 1, \quad (28)$$

$$h' \left( \frac{m}{e_1} \right) = u'(c_1) \frac{i}{1 + i}, \quad (29)$$

and  $\{c_2(s)\}$  solves the static planning problem (SP) given  $C_2$ ; and where  $Y_2$  and  $U$  are defined in (SP) and (17). Household welfare in this equilibrium is

$$u(c_1) + h(m/e_1) + \beta U(C_2). \quad (30)$$

*Proof.* We'll prove the necessary and sufficient parts independently.

**The “only if” part.** Equation (29) follows immediately from the household first-order condition with respect to money balances.

From Lemma 1, we know that equal gaps allocations are the only possible equilibrium under Assumption 2. As a result,  $\{c_2(s)\}$  solves problem (SP) with  $\bar{q}C_2 = \sum_s q(s)c_2(s)$ . Note also that, (SP) implies  $\bar{q} \frac{\pi(s)}{q(s)} u'(c_2(s)) = U'(C_2)$ .

Let  $1 + \kappa \equiv \frac{q(s)e_1}{p(s)e_2(s)} = \frac{q(s)}{\beta\pi(s)} \frac{u'(c_1)}{u'(c_2(s))}$  which holds for any  $s$ . Note that  $\kappa \geq 0$ , from (19). Under equal gaps, it follows that  $\kappa = \Delta(i)$ , and thus  $1 + \Delta(i) \geq 1$ . The definition of  $\kappa$  implies that (28) holds.

From the resource constraint, (22), we have

$$y_1 - c_1 + \sum_{s \in S} q(s)(y_2(s) - c_2(s)) = \Delta(i)\bar{w}.$$

Using  $\bar{q}C_2 = \sum_s q(s)c_2(s)$ , and the definition of  $Y_2$ , delivers (27).

**The “if” part.** Consider  $C_2$ ,  $c_1$ ,  $i$ , and  $m$  that solves (27)-(29). Let  $\{c_2(s)\}$  be the associated solution to the (SP) problem.

Let us conjecture an equilibrium with the following properties:

$$\begin{aligned} p(s) &= \frac{\beta\pi(s)u'(c_2(s))e_1}{u'(c_1)e_2(s)}; & f^*(s) &= f(s) = d_1^* = m^* = 0; & M &= m \\ F &= \frac{1}{\bar{q}}(y_1 - c_1 + \bar{w}); & A &= \frac{e_1\bar{q}F - M}{\bar{p}}; & T_2(s) &= F - \frac{A + M}{e_2(s)} \\ a^*(s) &= e_2(s)[y(s) + F - c_2(s)]; & a(s) &= A - a^*(s); & d_2^*(s) &= \frac{a^*(s)}{e_2(s)} \end{aligned}$$

The conjectures above guarantee that the budget constraints of intermediaries, households, and the monetary authority are holding, as well as market clearing in both money and domestic securities.

We need to show that  $F \geq 0$ , and  $a^*(s) \geq 0$ , and that the households and intermediaries are optimizing.

Toward showing that  $F \geq 0$ , we note that  $c_1 \leq c_1^{fb}$  (an implication of the results in Corollary 1). In addition, from Assumption 1,

$$\sum_{s \in S} q(s) \max\{y_2(s) - c_2^{fb}(s), 0\} \leq \bar{w}$$

From the resource constraint of the first-best problem, we can rewrite the above as

$$c_1^{fb} - y_1 + \sum_{s \in S} q(s) \max\{c_2^{fb}(s) - y_2(s), 0\} \leq \bar{w}.$$

So  $c_1^{fb} \leq y_1 + \bar{w}$ . It follows that  $c_1 \leq c_1^{fb} \leq y_1 + \bar{w}$ , and thus  $F \geq 0$ .

Towards  $a^*(s) \geq 0$ , note that, using (28),

$$\begin{aligned}
\frac{a^*(s)}{e_2(s)} &= y_2(s) + F - c_2(s) = y_2(s) + \frac{1}{\bar{q}}(y_1 - c_1 + \bar{w}) - \left( \frac{(1 + \Delta(i))\beta\pi(s)}{q(s)} \right)^{1/\sigma} c_1 \\
&= \frac{1}{\bar{q}}(y_1 + \bar{q}Y_2) \left\{ \frac{\bar{q}(y_2(s) - Y_2)}{y_1 + \bar{q}Y_2} + 1 + \frac{\bar{w}}{y_1 + \bar{q}Y_2} - \left[ 1 + \bar{q} \left( \frac{(1 + \Delta(i))\beta\pi(s)}{q(s)} \right)^{1/\sigma} \right] \frac{c_1}{y_1 + \bar{q}Y_2} \right\} \\
&= \frac{y_1 + \bar{q}Y_2}{\bar{q}} \left\{ \frac{\bar{q}(y_2(s) - Y_2)}{y_1 + \bar{q}Y_2} + 1 + \right. \\
&\quad \left. + \frac{\bar{w}}{y_1 + \bar{q}Y_2} - \frac{1 + \bar{q} \left( \frac{(1 + \Delta(i))\beta\pi(s)}{q(s)} \right)^{1/\sigma}}{1 + ((1 + \Delta(i))\beta)^{1/\sigma} \sum q(s) \left( \frac{\pi(s)}{q(s)} \right)^{1/\sigma}} \frac{y_1 + \bar{q}Y_2 - \bar{w}}{y_1 + \bar{q}Y_2} \right\}
\end{aligned}$$

where we have used in the last step that

$$c_1 = \frac{y_1 + \bar{q}Y_2 - \bar{w}}{1 + ((1 + \Delta(i))\beta)^{1/\sigma} \sum q(s) \left( \frac{\pi(s)}{q(s)} \right)^{1/\sigma}}.$$

Now note that

$$\frac{1 + \bar{q} \left( \frac{(1 + \Delta(i))\beta\pi(s)}{q(s)} \right)^{1/\sigma}}{1 + ((1 + \Delta(i))\beta)^{1/\sigma} \sum q(s) \left( \frac{\pi(s)}{q(s)} \right)^{1/\sigma}} \leq \max_{s_1, s_2} \left( \frac{\pi(s_1)q(s_2)}{\pi(s_2)q(s_1)} \right)^{1/\sigma}. \quad (\text{A.2})$$

To see this, let  $R \equiv \max_{s_1, s_2} \left( \frac{\pi(s_1)q(s_2)}{\pi(s_2)q(s_1)} \right)^{1/\sigma}$ . Note that  $R \geq 1$ . Now, notice that

$$R \geq \frac{\bar{q} \left( \frac{(1 + \Delta(i))\beta\pi(s)}{q(s)} \right)^{1/\sigma}}{((1 + \Delta(i))\beta)^{1/\sigma} \sum q(s) \left( \frac{\pi(s)}{q(s)} \right)^{1/\sigma}} = \frac{\left( \frac{\pi(s)}{q(s)} \right)^{1/\sigma}}{\frac{\sum q(s) \left( \frac{\pi(s)}{q(s)} \right)^{1/\sigma}}{\sum q(s)}}.$$

Then, it follows that (using  $R \geq 1$  in the second step):

$$\begin{aligned}
0 &\leq R \left( ((1 + \Delta(i))\beta)^{1/\sigma} \sum q(s) \left( \frac{\pi(s)}{q(s)} \right)^{1/\sigma} \right) - \bar{q} \left( \frac{(1 + \Delta(i))\beta\pi(s)}{q(s)} \right)^{1/\sigma} \\
&\Rightarrow 0 \leq (R - 1) + R \left( ((1 + \Delta(i))\beta)^{1/\sigma} \sum q(s) \left( \frac{\pi(s)}{q(s)} \right)^{1/\sigma} \right) - \bar{q} \left( \frac{(1 + \Delta(i))\beta\pi(s)}{q(s)} \right)^{1/\sigma},
\end{aligned}$$

which rearranging delivers (A.2).

Now, going back to  $a^*(s)/e_2(s)$  above, we have:

$$\frac{a^*(s)}{e_2(s)} \geq \frac{y_1 + \bar{q}Y_2}{\bar{q}} \left[ -\frac{\bar{q} \max_{s_1, s_2} (y_2(s_1) - y_2(s_2))}{y_1 + \bar{q}Y_2} + \frac{\bar{w}}{y_1 + \bar{q}Y_2} - \left[ \max_{s_1, s_2} \left( \frac{\pi(s_1)q(s_2)}{\pi(s_2)q(s_1)} \right)^{1/\sigma} - 1 \right] \right],$$

where we used that  $c_1 \leq y_1 + \bar{q}Y_2$ . Assumption 2 implies that  $a^*(s) \geq 0$ .

The final step is to check the optimality of the households and intermediaries. Given the domestic security

prices we conjectured, the households are on their Euler equation for domestic securities, and their money balances are consistent with optimality given (29). Given that  $q(s) \geq \frac{p(s)e_2(s)}{e_1}$  (which follows from equal gaps and  $\Delta(i) \geq 0$ ), we have that  $f(s) = 0$  is optimal for the household.

For the intermediaries, note that given that  $q(s) = (1 + \Delta(i))\frac{p(s)e_2(s)}{e_1}$ , then from (21), the intermediaries are indifferent between any of the domestic securities. Note that  $i \geq 0$ , implying that  $m^* = 0$  is also consistent with intermediaries' optimality.

**Household's utility.** In any equal gaps allocation, the utility of the households equals (30) by the definition of  $U$  in problem (SP).  $\square$

## Proof of Corollary 1

Suppose  $\Delta(i)\bar{w} < y_1 + \bar{q}Y_2$ . There is a unique pair  $(c_1, C_2)$  that solves (27) and (28). When  $\Delta(i) = 0$ ,  $c_1$  coincides with the first-best consumption. In addition,  $c_1$  strictly decreases with  $\Delta(i)$  and strictly decreases in  $\bar{w}$  for  $\Delta(i) > 0$ .

*Proof.* Uniqueness of  $(c_1, C_2)$  follows from the strict concavity of  $u(\cdot)$  and  $U(\cdot)$  (the latter follows from standard arguments). It is also straightforward to verify that equation (27) and (28) are the solution to the problem that defines the first best consumption allocation when  $\Delta(i) = 0$ . To demonstrate the comparative static results, we can combine equation (27) with (28) to obtain

$$\frac{\bar{q}u'(c_1)}{\beta U' \left( \frac{y_1 - c_1 + \bar{q}Y_2 - \Delta(i)\bar{w}}{\bar{q}} \right)} = (1 + \Delta(i)).$$

Total differentiation of the above expression leads to the following expression:

$$\begin{aligned} \frac{\partial c_1}{\partial \Delta(i)} &= \frac{1}{\bar{q}} \left\{ \frac{1 - \bar{w}U''(C_2)/U'(C_2)}{u''(c_1)/u'(c_1) + (1/\bar{q})U''(C_2)/U'(C_2)} \right\} \\ \frac{\partial c_1}{\partial \bar{w}} &= -\Delta(i) \frac{1}{\bar{q}} \left\{ \frac{U''(C_2)/U'(C_2)}{u''(c_1)/u'(c_1) + (1/\bar{q})U''(C_2)/U'(C_2)} \right\}. \end{aligned}$$

The first expression is always negative because of the strict concavity of  $u(\cdot)$  and  $U(\cdot)$ , implying that  $c_1$  strictly decreases with  $\Delta(i)$ . The second expression tells us that  $c_1$  strictly decreases with  $\bar{w}$  when  $\Delta(i) > 0$ .  $\square$

## Proof of Proposition 1

Suppose Assumptions 1 and 2 hold. If  $\Delta(0) < 0$  then the best equilibrium features  $(c_1^{fb}, C_2^{fb}, m, i)$  where

$$\begin{aligned} C_2^{fb} &= \sum_{s \in S} q(s)c_2^{fb}/\bar{q}, \\ i &> 0 \text{ and such that } \Delta(i) = 0, \\ m &\text{ such that } h'(m/e_1) = u'(c_1^{fb}) \frac{i}{1+i} \end{aligned}$$

*Proof.* The monetary authority cannot implement an interest rate  $i$  such that  $\Delta(i) < 0$ . Thus, we must have

$\Delta(i) \geq 0$ . Because  $\Delta(0) < 0$ , and  $\Delta(\cdot)$  is increasing in  $i$ , there exists a level of  $i$  such that  $\Delta(i) = 0$ . Denote this level by  $\bar{i}$ . Any equilibrium must feature  $i \geq \bar{i}$ .

By Lemma 2, we know that if  $i = \bar{i}$ , the resulting consumption allocation in the monetary equilibrium equals that of the first best, and money demand satisfies

$$h'(m/e_1) = u'(c_1^{fb}) \frac{\bar{i}}{1 + \bar{i}}.$$

Consider now any  $\tilde{i} > \bar{i}$ , and denote by  $(\tilde{c}_1, \tilde{C}_2, \tilde{m})$  the resulting allocation in the monetary equilibrium. By the definition of first best, we know that  $u(\tilde{c}_1) + \beta U(\tilde{C}_2)$  cannot be higher than the utility level achieved at  $(c_1^{fb}, C_2^{fb})$ . Moreover, by corollary 1, we know that  $\tilde{c}_1 \leq c_1^{fb}$  which implies, by equation (29), that  $\tilde{m}/e_1$  is smaller than the value achieved at  $\bar{i}$ . Thus, because  $h(\cdot)$  is strictly increasing, the utility from real money balances under  $\tilde{i}$  is also lower than that achieved at  $\bar{i}$ . It follows that households' welfare is maximized at  $\bar{i}$ , thus proving the result.  $\square$

## Proof of Proposition 2

Suppose Assumptions 1 and 2 hold. If  $\Delta(0) > 0$  and  $\Delta(0)\bar{w} < y_1 + \bar{q}Y_2$ , then the best equilibrium features  $(c_1, C_2, m, i)$  such that

$$i = 0, \quad m \geq \bar{m}, \quad \text{and} \\ (c_1, C_2) \text{ are the unique solution to (27) and (28).}$$

*Proof.* Because of the zero lower bound constraint, we have that  $i \geq 0$ . Denote by  $(c_1, C_2, m)$  the consumption allocation and money demand that is achieved in a monetary equilibrium with  $i = 0$ . Following the same steps as in the proof of Proposition 1, we can verify that the welfare of the representative household is maximized when the monetary authority sets  $i = 0$ . Consider any  $\tilde{i} > 0$ , and denote by  $(\tilde{c}_1, \tilde{C}_2, \tilde{m})$  the associated allocation in the monetary equilibrium. First, because  $\Delta(\tilde{i}) > \Delta(0)$ , we have that  $u(\tilde{c}_1) + \beta U(\tilde{C}_2)$  is below  $u(c_1) + \beta U(C_2)$ . Second, money demand is satiated at zero interest rates, which implies that  $h(\tilde{m}/e_1)$  is lower than the one achieved at  $i = 0$ . It follows that the best equilibrium features  $i = 0$ .  $\square$

## Proof of Corollary 2

*Implementation away from the ZLB:* Suppose Assumptions 1 and 2 hold. If  $\Delta(0) < 0$ , the monetary authority implements the best equilibrium with any  $F \in [0, (y_1 - c_1^{fb} + \bar{w})/\bar{q}]$ .

*Proof.* The first observation we make is that by Lemma 1, the monetary equilibrium implemented features equal gaps. Hence, for given  $i$ , we have that  $(c_1, C_2, m)$  solve (27)-(29). We now argue that if  $F \in [0, (y_1 - c_1^{fb} + \bar{w})/\bar{q}]$ , we must have  $\Delta(i) = 0$ . Suppose by contradiction that  $\Delta(i) > 0$ . From Corollary 1, we have that  $c_1 < c_1^{fb}$ . In addition, from the trade balance equation,

$$c_1^{fb} > c_1 = y_1 + \bar{w} - \left( \sum_{s \in S} q(s)f(s) + \bar{q}F \right) \geq c_1^{fb} - \sum_{s \in S} q(s)f(s) = c_1^{fb}$$

The first equality follows from the fact that  $\Delta(i) > 0$  implies that  $\sum_{s \in S} \frac{p(s)a^*(s)}{e_1} = \bar{w}$  and  $m^* = 0$ . The first inequality follows from  $F \leq (y_1 - c_1^{fb} + \bar{w})/\bar{q}$ , and the last equality follows from household optimality implies  $f(s) = 0$  in an equal gaps equilibrium.

Now that we proved that  $\Delta(i) = 0$ , the next step is to show that the consumption allocations correspond to the first best. We know from Corollary 1 that there is a unique pair of  $c_1, C_2$  that solves (27) and (28). Since (27) is the resource constraint, the first-best allocation (15) and (28) together with (SP) imply that (16) is satisfied, and it follows that  $c_1 = c_1^{fb}$ ,  $C_2 = C_2^{fb}$ , and  $c_2(s) = c^{fb}(s)$ . Finally, we know that  $i$  is such that  $\Delta(i) = 0$  and  $m$  follows from (29).  $\square$

## Proof of Corollary 3

*Implementation at the ZLB: Suppose Assumptions 1 and 2 hold. If  $\Delta(0) > 0$ , the monetary authority implements the best equilibrium with  $F = (y_1 - c_1^* + \bar{w})/\bar{q} > 0$ , where  $c_1^*$  is the best equilibrium first-period consumption.*

*Proof.* The first observation we make is that by Lemma 1, the monetary equilibrium implemented features equal gaps. Let us first show that  $F$  induces  $c_1 = c_1^*$ . From the trade balance equation, (23), we have

$$c_1 = y_1 + \bar{w} - \left( \sum_{s \in S} q(s)f(s) + \bar{q}F \right) = c_1^* \quad (\text{A.3})$$

where we substituted the value of  $F$  and used that  $f(s) = 0$  in an equal gaps equilibrium.

Next, we show that  $i = 0$ . Suppose  $i > 0$ . From Corollary 1, we would have  $c_1 < c_1^*$ , contradicting (A.3). Hence,  $i = 0$ . Using  $\Delta(0)$  and  $c_1 < c_1^*$ , we can obtain  $C_2$  as the unique solution to (27). Finally, since  $i = 0$ , we must have that  $m \geq \bar{m}$ .  $\square$

## B Resource costs in an infinite horizon model

In this appendix, we derive the one-period ahead resource loss (35) in an infinite horizon version of the model. Time is indexed by  $t = 0, 1, \dots$ . We denote by  $s^t$  the history of states up to time  $t$ ; that is,  $s^t = s_0, (s_1, \dots, s_t)$ .

The budget constraint for households in state  $s^t$

$$y(s^t) + T(s^t) + f(s^t) + \frac{a(s^t) + m(s^{t-1})}{e(s^t)} = c(s^t) + \sum_{s_{t+1} \in S} \frac{p(s^t)}{e(s^t)} a(s^t, s_{t+1}) + \frac{m(s^t)}{e(s^t)} + \sum_{s_{t+1} \in S} q(s_{t+1}, s^t) f_{t+1}(s_{t+1}, s^t). \quad (\text{B.1})$$

The government budget constraint in state  $s^t$  is

$$\frac{A(s^t) + M(s^{t-1})}{e(s^t)} + F(s^t) = T(s^t) + \frac{M(s^t)}{e(s^t)} + \sum_{s_{t+1} \in S} \frac{p(s_{t+1}, s^t)}{e(s^t)} A(s_{t+1}, s^t) + q(s_{t+1}, s^t) F_{t+1}(s_{t+1}, s^t). \quad (\text{B.2})$$

Combining the households' and the government's budget constraints (B.1)-(B.2), we obtain

$$\begin{aligned} \frac{A(s^t)}{e(s^t)} + \frac{a(s^t)}{e(s^t)} + m(s^{t-1}) - M(s^{t-1}) + F(s^t) + f(s^t) + y(s^t) = c(s^t) + \sum_{s_{t+1} \in S} \frac{p(s_{t+1}, s^t)}{e(s^t)} [A(s_{t+1}, s^t) + \\ + a(s_{t+1}, s^t)] + \frac{m(s^t) - M(s^t)}{e(s^t)} + \sum_{s_{t+1} \in S} q(s_{t+1}, s^t) (f(s_{t+1}, s^t) + F(s_{t+1}, s^t)) ]. \quad (\text{B.3}) \end{aligned}$$

Updating one period forward and rearranging,

$$\begin{aligned} \frac{A(s_{t+1}, s^t)}{e(s_{t+1}, s^t)} + \frac{a(s_{t+1}, s^t)}{e(s_{t+1}, s^t)} = c(s_{t+1}, s^t) - [y(s_{t+1}, s^t) + F(s_{t+1}, s^t) + f(s_{t+1}, s^t) + m(s^t) + M(s^t)] \\ \sum_{s_{t+1} \in S} \frac{p(s_{t+2}, s^{t+1})}{e(s_{t+1}, s^t)} [A(s_{t+2}, s^{t+1}) + a(s_{t+2}, s^{t+1})] + \frac{m(s^{t+1})}{e(s_{t+1}, s^t)} - \frac{M(s^{t+1})}{e(s_{t+1}, s^t)} + \\ + \sum_{s_{t+1} \in S} [q(s_{t+2}, s^{t+1}) (f(s_{t+2}, s^{t+1}) + F(s_{t+2}, s^{t+1})) ]. \quad (\text{B.4}) \end{aligned}$$

Substituting (B.4) into (B.3) and rearranging

$$\begin{aligned} \frac{A(s^t) + a(s^t)}{e(s^t)} + F(s^t) + f(s^t) + y(s^t) + m(s^{t-1}) + M(s^{t-1}) - c(s^t) = \\ \sum_{s_{t+1} \in S} \left[ q(s_{t+1}, s^t) - \frac{p(s_{t+1}, s^t)e(s_{t+1}, s^t)}{e(s^t)} \right] (f(s^t, s_{t+1}) + F(s^t, s_{t+1})) + \\ \left( 1 - \sum_{s_{t+1} \in S} p(s_{t+1}, s^t) \right) \left( \frac{M(s^t) + m(s^t)}{e(s^t)} \right) \\ + \sum_{s_{t+1} \in S} \frac{p(s_{t+1}, s^t)}{e(s^t)} \left[ c(s_{t+1}, s^t) - y(s_{t+1}, s^t) + \sum_{s_{t+1} \in S} \frac{p(s_{t+2}, s^{t+1})}{e(s_{t+1}, s^t)} [A(s_{t+2}, s^{t+1}) + a(s_{t+2}, s^{t+1})] \right. \\ \left. + \frac{M(s^{t+1})}{e(s_{t+1}, s^t)} + \frac{m(s^{t+1})}{e(s_{t+1}, s^t)} + \sum_{s_{t+1} \in S} [q(s_{t+2}, s^{t+1}) (f(s_{t+2}, s^{t+1}) + F(s_{t+2}, s^{t+1}))] \right] \end{aligned}$$

Using market clearing  $A(s^t) + a(s^t) = a^*(s^t)$ ,  $m(s^t) + m^*(s^t) = M(s^t)$ , we obtain

$$\begin{aligned} \frac{a^*(s^t)}{e(s^t)} + F(s^t) + f(s^t) + y(s^t) - \frac{m^*(s^{t-1})}{e(s^t)} - c(s^t) = \\ \sum_{s_{t+1} \in S} \left[ q(s_{t+1}, s^t) - \frac{p(s_{t+1}, s^t)e(s_{t+1}, s^t)}{e(s^t)} \right] (f(s^t, s_{t+1}) + F(s^t, s_{t+1})) + i \frac{m^*(s^t)}{e(s^t)} \\ \sum_{s_{t+1} \in S} \frac{p(s_{t+1}, s^t)e(s_{t+1}, s^t)}{e(s^t)} \left[ c(s_{t+1}, s^t) - y(s_{t+1}, s^t) + \sum_{s_{t+1} \in S} \frac{p(s_{t+2}, s^{t+1})}{e(s_{t+1}, s^t)} a^*(s_{t+2}, s^{t+1}) \right. \\ \left. - \frac{m^*(s^{t+1})}{e(s_{t+1}, s^t)} + \sum_{s_{t+1} \in S} [q(s_{t+2}, s^{t+1}) (f(s_{t+2}, s^{t+1}) + F(s_{t+2}, s^{t+1}))] \right]. \quad (\text{B.5}) \end{aligned}$$

Let us define

$$\Delta(s^t) \equiv \sum_{s \in S} \frac{q(s_{t+1}, s^t)e(s^t)}{e(s_{t+1}, s^t)} (1 + i(s^t)) - 1,$$

Notice that under equal gaps

$$\sum_{s_{t+1} \in S} \left[ q(s_{t+1}, s^t) - \frac{p(s_{t+1}, s^t)e(s^{t+1})}{e(s^t)} \right] [F(s_{t+1}, s^t) + f(s_{t+1}, s^t)] = \frac{\sum_{s_{t+1} \in S} q(s) [F(s_{t+1}, s^t) + f(s_{t+1}, s^t)] \Delta(s^t)}{1 + \Delta(s^t)} \quad (\text{B.6})$$

and

$$\sum_{s_{t+1} \in S} \frac{p(s_{t+1}, s^t)e(s_{t+1}, s^t)}{e(s^t)} [y(s_{t+1}, s^t) - c(s_{t+1}, s^t)] = \frac{\bar{q}(\tilde{Y}_2(s^t) - C_2(s^t))}{1 + \Delta(s^t)} \quad (\text{B.7})$$

Let us define

$$\tilde{y}(s^t) \equiv a^*(s^t) + F(s^t) + f(s^t) + y(s^t) - \frac{m^*(s^{t-1})}{e(s^t)} \quad (\text{B.8})$$

and

$$\begin{aligned} \tilde{Y}_2(s^t) \equiv Y_2(s^t) + \sum_{s_{t+1} \in S} \frac{p(s_{t+2}, s^{t+1})}{e(s_{t+1}, s^t)} [a^*(s_{t+2}, s^{t+1})] + \\ \sum_{s_{t+1} \in S} \frac{p(s_{t+1}, s^t)}{e(s^t)} \sum_{s_{t+1} \in S} [q(s_{t+2}, s^{t+1}) (f(s_{t+2}, s^{t+1}) + F(s_{t+2}, s^{t+1}))], \end{aligned} \quad (\text{B.9})$$

where  $Y_2$  and  $C_2$  are defined analogously as in (SP) and (17). Using expressions (B.6)-(B.9) and substituting the optimality conditions for  $m^*(s^t)$  and  $f(s^t)$  into (B.5), we obtain the following resource constraint

$$\tilde{y}(s^t) - c(s^t) + \frac{\bar{q}(\tilde{Y}_2(s^t) - C_2(s^t))}{1 + \Delta(s^t)} = \sum_{s_{t+1} \in S} F(s_{t+1}, s^t) q(s_{t+1}, s^t) \frac{\Delta(s^t)}{1 + \Delta(s^t)}. \quad (\text{B.10})$$



# Online Addendum to “Exchange Rate Policies at the Zero Lower Bound”

By Manuel Amador, Javier Bianchi, Luigi Bocola, and Fabrizio Perri

## A Optimal Exchange Rate Policy

In this appendix, we allow the monetary authority to choose its exchange rate policy  $(e_1, e_2(s))$ , in addition to its balance sheet. This approach allows us to verify the robustness of the insights obtained earlier, but now in an environment in which exchange rate policies and foreign exchange interventions are jointly determined. In line with the analysis above, we will show that when the ZLB does not bind, the monetary authority will implement the chosen path for the exchange rate by varying nominal interest rates rather than by accumulating foreign reserves. When the ZLB binds, the monetary authority may instead find it optimal to incur losses from foreign exchange interventions in order to depreciate its exchange rate.

### A.1 Environment

We extend our basic SOE model to include non tradable (NT) goods, endogenous production, and a nominal rigidity that takes the form of sticky wages. In particular, wages, denoted by  $p^w$ , are fixed (and constant) in domestic currency,  $p_1^w = p_2^w = \bar{p}^w$ .<sup>31</sup> We follow the usual tradition in New Keynesian models of working with a cashless limit, where the value of real money balances in the utility vanishes. For simplicity, we consider a deterministic setup.<sup>32</sup>

**Firms.** Tradable and non-tradable goods are produced with a production function that uses labor,  $l$ . Taking as given prices and wages, firms in the tradable and non-tradable sector maximize profits

$$\begin{aligned}\Pi_t^T &\equiv \max_{l_T} \left(l_t^T\right)^\alpha - \frac{\bar{p}^w}{e_t} l_t^T, \\ \Pi_t^N &\equiv \max_{l_N} p_t^N (l_t^N)^\alpha - \frac{\bar{p}^w}{e_t} l_t^N,\end{aligned}$$

where  $l_t^T, l_t^N$  represent labor demands in each sector,  $p_t^N$  is the price of non-tradables expressed in foreign currency and  $\bar{p}^w/e_t$  represents the wage in foreign currency. The first-order conditions lead to standard labor demand equations:

$$l_t^N = \left(\frac{\alpha p_t^N e_t}{\bar{p}^w}\right)^{1/(1-\alpha)}, \quad (\text{A.1})$$

$$l_t^T = \left(\frac{\alpha e_t}{\bar{p}^w}\right)^{1/(1-\alpha)}. \quad (\text{A.2})$$

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<sup>31</sup>We could allow wages to be upward flexible, without material changes to our results, as long as there is some cost from having high inflation in period 2. Sticky prices in non-tradable goods would deliver essentially the same results as sticky wages.

<sup>32</sup>It is relatively straightforward to extend our results to uncertainty. In particular, a shock to second period productivity would translate into a stochastic exchange rate policy in period 2.

**Households.** Households' preferences over tradable and non-tradable consumption,  $c^T$  and  $c^N$ , and labor,  $n$ , are given by

$$\sum_{t=1,2} \beta^{t-1} [\phi \log(c_t^T) + (1 - \phi) \log(c_t^N) + \chi \log(1 - n_t)] . \quad (\text{A.3})$$

Households solve essentially the same problem as in the previous version of the model. They face a portfolio in domestic and foreign bonds, and in addition they choose the amount of tradable and non-tradable consumption. In line with the sticky wage assumption, we assume that households are off their labor supply, and work as many hours as firms demand at the given wage. Hence, the household problem consists of choosing  $\{c_1^T, c_1^N, c_2^T, c_2^N, f, a\}$  to maximize (A.3) subject to the following budget constraints

$$\begin{aligned} \bar{p}^w n_1 + \Pi_1^T + \Pi_1^N + T_1 &= c_1^T + c_1^N p_1^N + \frac{a}{e_1} + f \\ \bar{p}^w n_2 + \Pi_2^T + \Pi_2^N + T_2 &= c_2^T + c_2^N p_2^N + f(1 + i^*) + \frac{a(1 + i)}{e_2} \end{aligned}$$

and  $f \geq 0$ . In addition to the intertemporal Euler conditions, the household problem features an intratemporal Euler equation that equates the relative price of non-tradables to the marginal rate of substitution:

$$p_t^N = \frac{1 - \phi}{\phi} \frac{c_t^T}{c_t^N}. \quad (\text{A.4})$$

In equilibrium, the market for non-tradable goods clears:

$$y_t^N = c_t^N, \quad (\text{A.5})$$

and households supply labor to meet the labor demand,  $n_t = l_t^T + l_t^N$ . Notice also that combining (A.1), (A.4), and (A.5) yields a NT employment allocation as a function of the exchange rate and the level of tradable consumption given by

$$\hat{l}^N(c_t^T, e_t) = \left( \frac{1 - \phi}{\phi} \right) \frac{\alpha c_t^T e_t}{\bar{p}^w}. \quad (\text{A.6})$$

**Foreign Intermediaries.** The problem of foreign intermediaries is exactly as described in Section 2 (equations 5-8).

## A.2 Monetary Authority problem.

The objective of the monetary authority is to choose the monetary equilibrium that delivers highest welfare. The monetary authority chooses an exchange rate policy  $(e_1, e_2)$ , in addition to a nominal interest rate  $i$  and a foreign asset position  $F$ . The key difference with the analysis in the previous section is that now the monetary

authority optimally chooses  $(e_1, e_2)$ . The optimality conditions for the path of the exchange rate are respectively:

$$\begin{aligned}
& \underbrace{\frac{\partial \hat{l}_1^T}{\partial e_1}}_{\text{Keynesian Channel}} \underbrace{\left( \lambda(1+i^*)\alpha \hat{l}^T(e_1)^{\alpha-1} - \frac{\chi}{1-n_1} \right)}_{\text{Labor Wedge}} \\
& + \underbrace{\frac{\partial \hat{l}_1^N}{\partial e_1}}_{\text{Keynesian Channel}} \underbrace{\left( \frac{1-\phi}{c_1^N} \alpha \hat{l}^N(e_1, c_1^T)^{\alpha-1} - \frac{\chi}{1-n_1} \right)}_{\text{Labor Wedge}} \leq \underbrace{\frac{\lambda}{e_2} \bar{w}}_{\text{Intervention Loss}} + \underbrace{\xi c_1^T \beta}_{\text{Interest Rate Distortion}}, \quad (\text{A.7})
\end{aligned}$$

and

$$\begin{aligned}
& \underbrace{\frac{\partial \hat{l}_2^T}{\partial e_2}}_{\text{Keynesian Channel}} \underbrace{\left( \lambda \alpha \hat{l}^T(e_2)^{\alpha-1} - \frac{\beta \chi}{1-n_2} \right)}_{\text{Labor Wedge}} \\
& + \underbrace{\beta \frac{\partial \hat{l}_2^N}{\partial e_2}}_{\text{Keynesian Channel}} \underbrace{\left( \frac{1-\phi}{c_2^N} \alpha \hat{l}^N(e_2, c_2^T)^{\alpha-1} - \frac{\chi}{1-n_2} \right)}_{\text{Labor Wedge}} \leq \underbrace{-\frac{\lambda e_1}{e_2} \bar{w}}_{\text{Intervention Loss}} - \underbrace{\xi c_2^T}_{\text{Interest Rate Distortion}} \quad (\text{A.8})
\end{aligned}$$

where  $\lambda, \xi$  denote, respectively, the Lagrange multipliers associated with the resource constraint and the domestic Euler equation, and  $\hat{l}^T(e_1)$  denote the employment equilibrium function equation given by (A.2). In a solution in which the central bank intervenes in the asset markets, (A.8) and (A.7) hold with equality. The left-hand side of (A.7) indicates the benefits of depreciating the exchange rate in period 1: by depreciating the exchange rate, the monetary authority can increase labor demand, and this has positive effects on welfare to the extent that production is inefficiently low (when there are positive labor wedges in the tradable and non-tradable sectors). The right-hand side indicates the potential costs from depreciating the exchange rate, which is composed of the two terms we analyzed in Section 2. The first term represents the intervention losses. Given  $i$  and  $e_2$ , an increase in  $e_1$  raises the expected appreciation rate of the domestic currency, which opens a wider gap in the interest parity condition. As we have shown in equation (22), this produces losses for the SOE, which are proportional to the foreign wealth of investors. The second term is the loss due to the distortion in the consumption-saving decisions of domestic households. A rise in  $e_1$  increases the real interest rate, and distorts consumption toward the second period.

Equation (A.8) is analogous to (A.7), with the key difference that the two terms on the right hand side have the opposite sign. That is, a higher  $e_2$  reduces the real return of domestic bonds and reduces both the intervention losses and the interest rate distortions. Because the right hand side is negative, this indicates that the monetary authority at the optimum allows for a non-positive labor wedge in the second period, as long as there is also a positive labor wedge in the first period. Putting together (A.7) and (A.8) indicates that the monetary authority trades-off a positive labor wedge in the first period against a negative labor wedge in the second period. While away from the ZLB, the monetary authority can offset these wedges by cutting down the nominal interest rate, this is not the case at the ZLB. Below, we solve the model numerically and show the role of foreign exchange interventions once the economy hits the ZLB.

### A.3 Results of optimal exchange rate policies

We now present a numerical illustration and discuss the optimal policy of the monetary authority. Figure 5 reports key variables in the monetary authority solution as a function of the discount factor of the households  $\beta$ . When  $\beta$  increases, households become more patient and reduce their current consumption. In absence of a policy response by the monetary authority, this shift would depress output in period 1: by reducing their demand for non-tradable consumption, the price of non-tradable goods would drop, leading to a decline in the demand of labor in the non-tradable sector (see equation (A.6)). The response of the monetary authority to this increase in households' patience is to depreciate the exchange rate; by doing so, the monetary authority can stimulate labor demand and restore efficient production. Importantly, this policy is achieved initially with a reduction in nominal interest rates and without accumulating foreign assets (see panel (d) and (e) of the figure). This mirrors our results in Proposition 1.

This response of the monetary authority, however, is not always feasible. For sufficiently high values of  $\beta$ , the nominal interest rates that would allow the monetary authority to achieve the desired exchange rate policy is negative. Initially, the monetary authority sets nominal interest rates at zero and tolerates the output inefficiencies induced by high discounting of the households: we can see from panel (b) that output starts dropping as a function of  $\beta$ . Eventually, however, the welfare costs of the recession are so large that the monetary authority becomes willing to bear the losses from foreign exchange interventions in order to depreciate the exchange rate and moderate the output gaps. The threshold at which the monetary authority intervenes is higher when the level of foreign wealth is higher, in line with our results that a deviation from interest parity generates a first-order loss proportional to  $\bar{w}$ . Once the monetary authority intervenes, however, it requires a larger accumulation of foreign assets. This in turn generates a nonmonotonic relationship between  $F$  and  $\bar{w}$ , as illustrated in panel (e) of Figure 5.<sup>33</sup>

The lessons learned in the model with an exogenous exchange rate policy carry over to this more general environment. For example, we showed that, when operating at the ZLB, a higher level of foreign wealth unambiguously decreased households' welfare when the exchange rate policy was given. In this new environment, in which the monetary authority optimally chooses its exchange rate policy, a similar result holds. However, there is a caveat: the monetary authority may eventually stop intervening and give up on its exchange rate policy if the foreign wealth is large enough. Figure 6 shows a simulation for a case in which the monetary authority is operating at the ZLB. As can be seen, higher wealth strictly reduces domestic welfare, up to the point where the monetary authority stops intervening.

## B Losses with incomplete financial markets

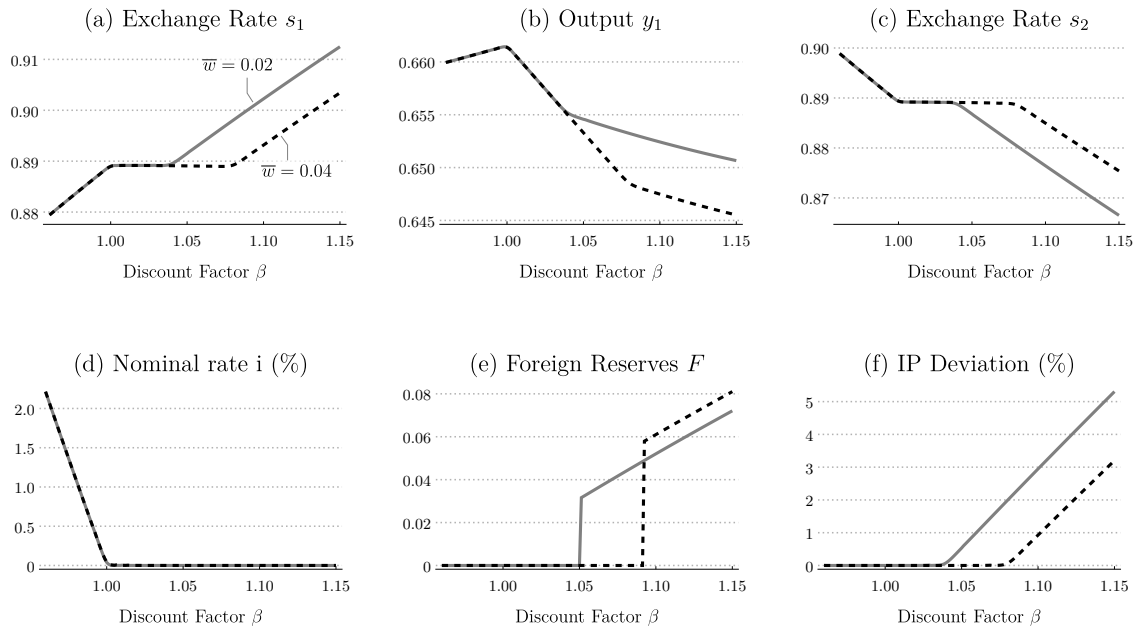
In this section we derive the losses of foreign exchange rate interventions in environments that, differently from the analysis in the main text, do not feature a full set of Arrow-Debreu securities. Specifically, we modify our setting and allow for arbitrary incomplete financial markets by introducing "wedges" in the Euler equations of domestic household and foreign financial intermediaries.

Let  $s^t$  be a history of states up to time  $t$  and  $s_{t+1}$  the realization of the state at time  $t + 1$ . We denote by  $q^f(s^t, s_{t+1})$  the stochastic discount factor for foreigners given a history  $(s^t, s_{t+1})$  and by  $q^d(s^t, s_{t+1})$  the stochastic

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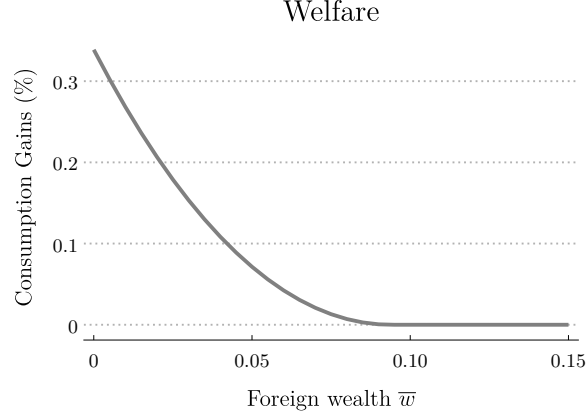
<sup>33</sup>The policies conducted by several developed economies following the global financial crisis have a natural interpretation through the lens of our model. Facing a slump and deflationary pressures, central banks first lowered interest rates before engaging in accumulation of foreign assets to stimulate employment via a weakening of the domestic currency. For the case of Switzerland, in particular, our model suggests that in response to the European monetary authority's quantitative easing, the Swiss National Bank faced larger losses from sustaining a depreciated exchange rate (because of a combination of lower  $i^*$  and higher  $\bar{w}$ ) and hence let the currency appreciate in January 2015.

Figure 5: Optimal interventions with endogenous exchange rate policy



Note: Numerical illustration for two different values of foreign wealth for a range of discount factors. Parameter values are as follows  $\phi = 0.5, p^w = 1, \chi = 1, \alpha = 0.7, i^* = 0$  and low and high values for  $\bar{w} = \{0.02, 0.04\}$ . The discount factor is represented by the x-axis. Output in panel (b) denotes the sum of tradable and non-tradable output expressed in units of tradables.

Figure 6: **Welfare and foreign wealth under the ZLB**



Note: Parameter values are just as in Figure 5 with  $\beta = 1.2$ . The x-axis represents different values of foreign wealth  $\bar{w}$ . The y-axis represents the welfare generated by the optimal intervention policy relative to a policy where the monetary authority does not accumulate reserves. Welfare gains are represented as percentage increases in permanent consumption of both non-traded and tradable goods.

discount factor for domestic agents. We let  $e(s^t)$  be the value of the exchange rate (franc per dollar) given an arbitrary history  $s^t$ . The foreign price level is normalized to 1 in every period.

We assume that that domestic and foreign agents can trade one-period risk-free bonds issued in domestic and foreign currency. Specifically, their demand for these assets satisfies the Euler equations

$$\bar{q}(s^t) = \mathbb{E}_t[q^f(s^t, s_{t+1})]\mu_t^f \quad \bar{q}(s^t) = \mathbb{E}[q^d(s^t, s_{t+1})]\mu_t^d \quad (\text{B.1})$$

$$\bar{p}(s^t) = \mathbb{E}_t \left[ q^f(s^t, s_{t+1}) \frac{e(s^t)}{e(s^t, s_{t+1})} \right] v_t^f \quad \bar{p}(s^t) = \mathbb{E}_t \left[ q^d(s^t, s_{t+1}) \frac{e(s^t)}{e(s^t, s_{t+1})} \right] v_t^d. \quad (\text{B.2})$$

When  $\mu_t^f = \mu_t^d = v_t^f = v_t^d = 1$ , these conditions collapse to canonical Euler equations. Our formulation, however, is more general, as the “wedges” in the above equations are not restricted to be equal to 1. In this fashion, we allow for the possibility that the trading of these assets is influenced by other factors- for instance, leverage constraints or convenience yields induced by preferences for certain types of bonds.

Besides this assumption, we don't place further restriction on  $q^d(s^t, s_{t+1})$  and  $q^f(s^t, s_{t+1})$ . Thus, the following results do not require  $q^d(s^t, s_{t+1})$  and  $q^f(s^t, s_{t+1})$  to be equal in every state of the world, that is, we allow for the possibility of incomplete spanning in international financial markets.

Consider now the balance sheet of a monetary authority that can trade in domestic and foreign bonds. We denote by  $F_{t+1}$  the holdings of foreign bonds and by  $A_{t+1}$  the holdings of domestic bonds chosen at date  $t$ . To simplify the algebra, we assume that the monetary authority finances the purchase of foreign bonds by issuing domestic liabilities (and vice versa)

$$\bar{q}(s^t)F_{t+1} = -\bar{p}(s^t)\frac{A_{t+1}}{e(s^t)}. \quad (\text{B.3})$$

Given this assumption, the gains or losses for the monetary authority in state  $(s^t, s_{t+1})$  are

$$L(s^t, s_{t+1}) = F_{t+1} \left[ 1 - \frac{\bar{q}(s^t)}{\bar{p}(s^t)} \frac{e(s^t)}{e(s^t, s_{t+1})} \right].$$

We can then derive an expression for the expected discounted value of the monetary authority's gains or losses from asset purchases.

**Proposition 3.** *Assume that equations (B.1)-(B.2) are satisfied. Consider a sequence  $\{F_t, A_t\}$  satisfying equation (B.3). Then, for each  $t$ , we must have*

$$\mathbb{E}_t \left[ q^d(s^t, s_{t+1}) L(s^t, s_{t+1}) \right] = F_{t+1} \bar{q}(s^t) \left[ \frac{v_t^d - \mu_t^d}{v_t^d \mu_t^d} \right]. \quad (\text{B.4})$$

*Proof.* Pick an arbitrary period  $t$ . Taking expectations over  $s_{t+1}$  we have

$$\mathbb{E}_t [q^d(s^t, s_{t+1}) L(s^t, s_{t+1})] = F_{t+1} \sum_{s_{t+1}} \pi(s_{t+1}|s^t) q^d(s^t, s_{t+1}) \left[ 1 - \frac{\bar{q}(s^t)}{\bar{p}(s^t)} \frac{e(s^t)}{e(s^t, s_{t+1})} \right]. \quad (\text{B.5})$$

Because of equation (B.1), we have that

$$\sum_{s_{t+1}} \pi(s_{t+1}|s^t) q^d(s^t, s_{t+1}) = \frac{\bar{q}(s^t)}{\mu_t^d}.$$

Because of equation (B.2), we have that

$$\sum_{s_{t+1}} \pi(s_{t+1}|s^t) q^d(s^t, s_{t+1}) \frac{\bar{q}(s^t)}{\bar{p}(s^t)} \frac{e(s^t)}{e(s^t, s_{t+1})} = \frac{\bar{q}(s^t)}{\bar{p}(s^t)} \frac{\bar{p}(s^t)}{v_t^d}.$$

Substituting these results in equation B.5 we obtain

$$\mathbb{E}_t \left[ q^d(s^t, s_{t+1}) L(s^t, s_{t+1}) \right] = F_{t+1} \bar{q}(s^t) \left[ \frac{v_t^d - \mu_t^d}{v_t^d \mu_t^d} \right],$$

demonstrating the result.  $\square$

This result has two main implications. First, from equation (B.4) we can see that if domestic and foreign agents are on their frictionless Euler equations (all the wedges are equal to 1), the expected discounted gains or losses on the positions of the monetary authority always equal zero. This would be true even in the presence of sizable deviations from uncovered interest rate parity. Thus, an implication of Proposition 3 is that one cannot compute the gains or losses from central banks' interventions by using deviations from UIP. Second, under the assumptions we made on forward exchange markets in Section 5.1, we can verify that deviations from covered interest rate parity capture the term  $(v_t^d - \mu_t^d)/(v_t^d \mu_t^d)$ , and they are the correct measure of deviation from interest rate parity to use when computing the costs of foreign exchange interventions.

A simple example is useful in understanding why our argument of Section 5.1 generalizes in this setting. Consider a situation in which all agents are on their frictionless Euler equation, and suppose that the monetary authority purchases foreign currency bonds and issues domestic currency securities. Assume also that the real interest rate on foreign assets is higher than the one on domestic assets, that is

$$\frac{1}{\bar{q}(s^t)} \frac{\mathbb{E}_t[e(s^t, s_{t+1})]}{e(s^t)} > \frac{1}{\bar{p}(s^t)}.$$

This, for example, arises if the domestic currency is a good hedge for foreigners, that is, if its value appreciates in states in which  $q_t^f(s^t, s_{t+1})$  is high.

Clearly, the monetary authority makes money on average with this strategy, because it shorts a low-yielding asset and purchases a high yielding one. However, when appropriately discounted, the trade does not generate a profit. Indeed, domestic agents can replicate this trade. and we must have

$$\mathbb{E}_t \left[ q^d(s^t, s_{t+1}) \left( \frac{1}{\bar{q}(s^t)} \frac{e(s^t, s_{t+1})}{e(s^t)} - \frac{1}{\bar{p}(s^t)} \right) \right] = 0.$$

Thus, if domestic agents cannot profit from this trade, neither can the monetary authority.