Sovereign Debt and The Tragedy of the Commons

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Abstract

In this paper I study a tragedy-of-the-commons model: a small open economy composed of different groups that compete for access to government resources and a government can save and borrow from risk-neutral foreigners. I show that the same economic forces that generate overspending in a tragedy-of-the-commons model can also guarantee that a small open economy repays its sovereign obligations. The groups may agree to repay the sovereign debt to avoid having the assets spent inefficiently, even though they can cooperate after defaulting. The fundamental reason for repayment is the same as in Eaton and Gersovitz (1981): countries repay because they would like to borrow again.

1 Introduction

Emerging markets borrow substantial amounts of foreign debt. Most of what they borrow, they eventually repay, even though there are no clear punishments available to creditors besides the threat of eliminating future lending (Eaton and Fernandez, 1995). The absence of clear punishments raises the question of why countries repay their debts.

One of the oldest reasons[1] for repayment of sovereign obligations is the well-known reputational argument: countries choose to repay their debts because they would like to borrow

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[1]Alexander Hamilton, when founding the Treasury Department and consolidating the states debt into a federal one, used the reputational argument to support his view that the external debt accumulated during war of independence should be repaid (see footnote 12 of English, 1996).

again in the future. This argument was first formalized in the economics literature by Eaton and Gersovitz (1981) in the context of a small open economy subject to endowment shocks: by defaulting, the country loses access to international credit markets and is not be able to smooth consumption[2]. The resulting variation in consumption can be painful enough to enforce repayment.[3] The fact that direct sanctions after default are no longer observed in the data makes the reputational argument a very compelling one. Indeed, in his historical analysis of the American State Debts of the 1840s, English (1996) reaches the conclusion that, in spite the lack of sanctions, most states repaid their debts in full in order to maintain access to capital markets, as predicted by the reputational model.

However, in an important theoretical contribution, Bulow and Rogoff (1989) demonstrated that the reputational argument for repayment breaks down when countries can save: by using the asset markets to self-insure, a country can always guarantee higher consumption if it defaults than if it does not. Given the richness of the international financial markets, it is unrealistic to assume that defaulting countries cannot save; and hence a repayment paradox emerges.

The present paper provides an answer to why countries repay their debts in the absence of sanctions. It does so by changing an implicit assumption of the literature on sovereign debt: that a country's decision to default can be modelled as that of a representative agent. The paper breaks with this tradition by recognizing that the repayment of the sovereign debt is not only an economic decision but also a political one.

I set up a simple political economy model in which a small open economy is composed of long-lived political groups with distinct interests. The political groups share common access to a savings technology that is controlled by a government, generating a tragedy of the commons problem regarding the assets of the country. Common-pool models of this type have been used to explain why emerging markets governments over-spend and follow procyclical policies (see Tornell and Velasco, 1992; Tornell and Lane, 1999): domestic groups demand too much spending from the government because each of them enjoys the benefits privately but they all share the costs through the government budget constraint. The main contribution of this paper is to show that the same arguments that lead to over-spending in a political model also provide the incentives for repayment of the sovereign obligations. A political economy model can thus reconcile two seemingly contradictory observations: emerging markets' politicians do not save and spend too much and most of the time they

[2]Several authors have extended the reputation approach to sovereign lending. See for example Atkeson (1991); Grossman and Van Huyck (1988); Worrall (1990).

[3]A recent literature is trying with some success to seriously quantify the implications of Eaton and Gersovitz (1981) model (Aguiar and Gopinath, 2006; Arellano, 2008). In these recent contributions, however, sovereign debt is enforced by an external punishment that is not modeled.

pay back their debts even in the absence of sanctions or clear political costs.

In the model I put forward in this paper, the small open economy has access to assets after default, and a modified version of the theorem by Bulow and Rogoff (1989) holds: after defaulting, there exists a consumption allocation generated only with savings that Pareto improves over the repayment of the debt. Usually, the existence of this default allocation is enough for one to argue that a domestic government would prefer it and default will always ensue. However, in a political game, where multiple groups inside the country are part of the decision making, the allocation after default has to be the result of a political equilibrium. As I will show, this is not generally the case, even when the long-lived domestic groups could in principle cooperate after defaulting.

The political inability of a country to maintain a buffer stock of assets for self-insurance is a critical part of my argument for repayment. In several historical episodes the accumulation of budget surpluses has been shown to be politically impossible. Cole, Dow and English (1995) and English (1996) describe the case of the American States in the 1830s. At that time, the accumulation of a large federal surplus was controversial and at the end, the surplus was distributed to the states. The states did not hold the money for long, and spent or distributed it. A few years later, Benjamin R. Curtis, a supreme court judge, specifically argued that a state's reputation in credit markets was important because American states could not accumulate surpluses, and in an emergency they might need more resources than they could tax in a single year.[4] This is the exact argument formalized in this paper.

However, the above discussion misses an important point: how come the political parties that make the accumulation of surpluses impossible can instead agree on the repayment of debt? In my model, the domestic groups realize that if they were to default they would inefficiently overspend and save too little even though they would all benefit from saving more. Importantly, *the sustainability of a buffer stock can be infeasible not just because the domestic groups do not agree to save today, but also because they might not agree to save in the future.* The main point is to show that these future inefficiencies can be reduced with an appropriately designed external credit line. By granting access to funds only in states where the country really needs them, while curtailing them in times when they are not, a credit line in effect generates a commitment technology that can be preferred to default. However, a non-contingent credit line would not be valued by the groups and would not be repaid: that is, the fact that credit sometimes dries up is a fundamental feature of a sustainable debt contract. It is in the interest of the foreigners to generate these contingencies, as these are

[4] See page 271 of English (1996). English (1996) goes on to say that "The reason for the inability of states to accumulate surpluses is not given by Curtis, but it presumably was the result of U.S. citizens' distrust of government ... If citizens believed that state officials will either steal or waste a large enough share of the surplus, then they will be willing to give up the gain to be had by repudiation."

the features of the debt contract that guarantee its repayment.

In the final section of the paper, I argue that the welfare effects of regulations imposed on the international sovereign markets will depend crucially on why countries repay their debts. In the context of my model, I show that if countries can somehow commit to repay their debts (maybe because of some external enforcement mechanism), the tragedy of commons problems are magnified and welfare may be reduced for two reasons: a country may borrow too much, or domestic cooperation may be negatively affected.

Related Literature

There is an extensive empirical literature documenting that political instability, corruption, and weak property rights go hand-in-hand with lower savings and lower economic growth. For example, in their analysis of international reserve-holding behavior by developing countries, Aizenman and Marion (2004) show that countries with higher indexes of political corruption tend to accumulate lower levels of reserves. They argue that political corruption reduces the size of the buffer stocks held by a government by increasing opportunistic behavior. Regarding the dissipation of positive income shocks, Tornell and Lane (1999) document that when Nigeria, Venezuela, Mexico, Costa Rica, Cote d'Ivoire and Kenya received significant windfalls from their terms of trade, their respective governments increased spending more than proportionally to the increased revenue. Easterly and Levine (1997) provide evidence that linguistic diversity in African countries, a proxy for political fragmentation, is correlated with a range of bad public policies such as low education and little provision of infrastructure, and they argue that ethnic diversity "encourages the adoption of growth-retarding policies that foster rent-seeking behavior and makes it more difficult to form a consensus for growth-promoting public good".

Several explanations of why countries repay their debts have been proposed in the literature. Researchers have studied the possibility that reputation spillovers to other valuable relationships might be costly enough to enforce repayment (Cole and Kehoe, 1995 and Cole and Kehoe, 1997)[5]. Another approach looks at the assets available to the country after default: technological restrictions (Kletzer and Wright, 2000) or collusion among banks (Wright, 2004) might reduce the range of savings mechanisms available to the country after default. Another branch of the literature studies the punishments available to creditors,

[5]For example, Aguiar, Amador and Gopinath (forthcoming) study the dynamics of sovereign debt in a small open economy model with capital accumulation. Debt is sustainable because government default increases the expected expropriation rate and reduces domestic investment.

from military intervention to trade embargoes.[6,7]

The basic model of this paper is based on the dynamic tragedy of the commons models studied by Tornell and Velasco (1992) and Tornell and Lane (1999). The repayment of international debt, the issue of interest here, is not a concern in those papers. Also, they analyze only Markov-perfect equilibria. In this paper, I instead study sub-game perfect equilibria of a dynamic game and provide a full characterization of the efficient equilibria with symmetric payoffs. As will be argued later on, this characterization of the efficient equilibria is necessary to convincingly answer the question of repayment. A related paper is Svensson (2000). He also studies a repeated tragedy of the commons model, focusing on the effects of foreign aid on the collusive behavior of political groups, again a different question from the one I address here..[8]

There is a broad literature on the political economy of fiscal deficits, initiated by Tabellini and Alesina (1990) and Persson and Svensson (1989). In these early papers, the possibility of default on government debt is not considered. Tabellini (1991) and Dixit and Londregan (2000) present models of sustainability of domestic debt. In their models, the lenders are citizens and thus have political rights (they can vote). Here, I instead analyze a model of *sovereign* debt, where lenders reside outside the country and have no political rights.

Dixit, Grossman and Gul (2000) analyze a dynamic political economy model with two political groups that generalizes the model of cooperation and party turnover introduced by Alesina (1988).[9] Similar to this paper, the authors focus on characterizing the properties of efficient sub-game perfect equilibria. However, there are significant differences between their work and mine. First, the issue interest in this paper is debt repayment and its interaction with the political economy structure of a country. Second, the political game I study is dynamic, not only because of an exogenous Markov process affecting a parameter of the model (as in Dixit et al., 2000) but also because of the presence of savings, which are to be determined in equilibrium.[10] Finally, the linear structure of my model allows for a full characterization of the symmetric efficient equilibrium, and can be solved for an arbitrary

[6] Rose (2005) has shown that after a country defaults, its international trade is significantly reduced, identifying a channel through which external creditors might be punishing the defaulting country.

[7] Sandleris (2005) presents a different argument for repayment, based on asymmetric information.

[8] Similarly to this paper, in Svensson (2000), the author characterizes Pareto efficient equilibria. However, the underlying model is not dynamic: the country does not accumulate assets and the endowment process is independent and identically distributed across periods.

[9] The model of Dixit et al. (2000) is also very similar to the risk-sharing models of Kocherlakota (1996) and Thomas and Worrall (1988).

[10] More recent work by Acemoglu, Golosov and Tsyvinski (2008) considers a different dynamic political economy model with self-interested politicians. Although their model includes capital, they focus on characterizing the long-run properties of efficient subgame perfect equilibria and its implications for labor and capital taxation.

number of political groups.

My results are also related to recent work by Gul and Pesendorfer (2004) who develop a theory of preferences for commitment. They study a consumer with these preferences and show how the Bulow and Rogoff (1989) result could be overturned. In contrast, in this paper I study a fully specified political economy model, and the results regarding debt sustainability relate to political variables including the degree of fragmentation of the country and the impatience of different political groups.

The layout of the paper is as follows: I set up the model without debt in Section 2. The model consists of a small economy with different political groups and a government with access to a savings technology. I define the equilibria and show that there exists an efficient equilibrium with symmetric payoffs, which I then characterize. In Section 3, I introduce the possibility of borrowing from foreigners and study the issue of debt repayment. I show that the argument of Bulow and Rogoff (1989) does not hold in general except when there is only one group, or the groups are infinitely patient. I also show that the fragmentation of the country, as defined by the number of political groups, has an inverse-U relationship with the ability to commit to repay the sovereign debt. Section 4 discusses the welfare effects of curtailing borrowing and how these depend crucially on the reason why countries repay their debts. The final section concludes.

2 The Savings-Only Game

In this section I introduce the simple baseline model. I assume throughout this section that the country cannot borrow, but it can save. This assumption will be relaxed, and the question of borrowing and debt repayment will be studied, in the following section.

2.1 The basic model

Time is discrete and runs from 0 to infinity. There are n symmetric groups indexed by $i \in I = \{1, 2, ..., n\}$ that live forever, and a government. The government in the country can save using a linear technology that returns the risk-free rate, R. Let a_t denote the amount saved by the government at the end of period t. The government cannot borrow, $a_t \geq 0$, and is assumed to have no assets initially, $a_{-1} = 0$. In addition to its financial wealth, the government receives an endowment e_t at the beginning of every period t.

The groups receive fiscal provisions from the government. The utility flow of group i

after receiving d_t^i from the government is shown by the following linear function,

$$u_t^i(d_t^i) = \phi_t d_t^i$$

The aggregate parameter ϕ_t captures the benefit to the domestic groups of government provision at any time t. Gains from savings will be introduced in this linear model through variations in ϕ_t. All domestic groups are affected by ϕ_t in the same way: they all agree on the periods in which government spending is relatively more valuable. Hence, the parameter ϕ_t characterizes events that affect all groups in the country in a similar fashion: it can represent a natural disaster, a low aggregate productivity state, or a war.

All groups discount the utility flows generated by government provision exponentially at the same rate β. Given the linear structure, the following restriction on β and R is necessary to bound the payoffs:

Condition 2.1. *The parameters β and R are such that $\beta R < 1$.*

A fiscal demand allocation is defined to be a non-negative and non-stochastic sequence of fiscal demand vectors, $d = \{(d_t^1, ..., d_t^n)\}_{t=0}^{\infty}$. Let $a\,(d) = \{a_{t-1}\,(d)\}_{t=0}^{\infty}$ be a sequence of assets positions generated by d with $a_0 = -1$, that is:

$$a_t(d) = Ra_{t-1}(d) + e_t - \sum_{i=1}^{n} d_t^i$$

for all $t \geq 0$.

An allocation is feasible if the asset positions generated by it are always positive:

Definition 2.2. *A fiscal demand allocation d is **feasible** if the asset positions generated by d are non-negative, that is, $a_t(d) \geq 0$ for all $t \geq 0$.*

2.2 A Desire for Savings

In this linear set up, a desire for savings is introduced by specifying particular processes for the endowment and marginal utility, e_t and ϕ_t. For the rest of the paper, it is assumed that ϕ_t and e_t behave according to the following deterministic rules:

$$\phi_t = \begin{cases} 1 & ; \text{for } t \in T_1 \\ \phi & ; \text{for } t \in T_\phi \\ \phi^2 & ; \text{for } t \in T_{\phi^2} \end{cases}$$

and

$$e_t = \begin{cases} 1 & ; \text{for } t \in T_1 \\ 0 & ; \text{otherwise} \end{cases}$$

for $\phi > 1$, and where $T_1 = \{t|t \mod 3 = 0\}$, $T_\phi = \{t|t \mod 3 = 1\}$, and $T_{\phi^2} = \{t|t \mod 3 = 2\}$. The endowment follows a $1, 0, 0, 1, 0, 0, ...$ pattern, while the aggregate parameter ϕ_t follows a $1, \phi, \phi^2, 1, \phi, \phi^2, ...$ one. The economy is receiving the endowment in states where consumption is less valuable *to all groups*: when $\phi_t = 1$.

The following condition guarantees that saving the endowment for better times is efficient,

Condition 2.3. *The parameters are such that $\beta R \phi > 1$.*

Let a first-best allocation be an allocation that is Pareto optimal in the feasible set: that is, there does not exists another feasible allocation that weakly Pareto dominates it. It follows immediately that,

Proposition 2.4. *Suppose that conditions (2.1) and (2.3) hold. Then, a first-best allocation will have*

$$a_t = \begin{cases} 0 & ; \textit{for } t \in T_{\phi^2} \\ Ra_{t-1} & ; \textit{otherwise.} \end{cases}$$

The result relies on two conditions. Condition (2.1) bounds the payoffs and rules out the need to save when the state is ϕ^2, while condition (2.3) guarantees that it is efficient to save in all other periods.

2.3 The Political Game

The political structure is as follows: the groups compete for the funds in the hands of the government. At the beginning of every period, each group makes a fiscal demand to the government, $\hat{d}_t^i$ for group i and, in return, receives a fiscal provision from the government, d_t^i. One important assumption is that the domestic groups cannot save on their own, and therefore depend on the government to move the country's wealth across time[11].

The government is weak[12], in the sense that it prefers to satisfy the demands of all the groups whenever this is feasible. At any time, if the fiscal demands are such that the government has enough income to satisfy them, $Ra_{t-1} + e_t \geq \sum_{i=1}^n \hat{d}_t^i$, then provisions equal

[11] See the discussion about this assumption in Subsection 3.3.

[12] The political game is a simplified version of that used by Tornell and Velasco (1992) and Tornell and Lane (1999) in their analysis of government expenditures and debt for emerging market.

demands, $d_t^i = \hat{d}_t^t$, and assets evolve according to the budget constraint $a_t = Ra_{t-1} + e_t - \sum_{i=1}^n d_t^i$.

On the other hand, if the fiscal demands are excessive, that is $\sum_{i=1}^n \hat{d}_t^i > Ra_{t-1} + e_t$, then the government cannot provide each group with its desired provision. I assume that each group, in this case, will receive a provision equal to the average of the maximum spending possible: $d_t^i = (Ra_{t-1} + e_t)/n$.

Although specific, this particular political game tries to capture important features of a more general political process. First, the resources in the hands of the government are, in principle, available to all groups. Second, the groups cannot ex-ante commit to any division of the pie. Finally, given the public nature of the government resources, one group is worried that the others might appropriate too big a piece of the fiscal budget without including its members. The particular tie breaking rule used for the case when all groups demand too much can be alternatively justified as a form of government breakdown. For example, it could be assumed that, at a cost, a group can guarantee equal sharing of all the resources of the country. As will be shown in what follows, the focus on equilibria with symmetric payoffs eliminates the importance of the tie-breaking rule, because a deviating group would never want to trigger it.

The history of the game at any time is given by all the previous fiscal demands made by the groups. A (pure) strategy for a group is a mapping from all possible histories to fiscal demands. As usual, subgame perfect equilibrium is such that at any time and after any possible history, a group's fiscal demand is optimal, given the other groups' strategies. Given a subgame perfect equilibrium, a subgame perfect allocation is the fiscal demand allocation that results from the equilibrium play.

Before characterizing the set of equilibria, it turns out to be useful to characterized the worst equilibrium payoff.

2.4 Worst Equilibrium

Let the total-exhaustion strategy profile be given by infinite fiscal demands, $d_t^i = \infty$, after any history. Then the following holds,

Proposition 2.5. *The total-exhaustion strategy profile is a subgame perfect equilibrium of the game. Even more, after any history, this strategy generates the lowest possible subgame perfect payoff to all groups.*

The proof of this last proposition follows directly from noting that when all other groups use the total-exhaustion strategy profile, the payoff of any one group is insensitive to its demands. Also, a group can guarantee itself a minimum payoff when using a total-exhaustion strategy.

There is no cooperation among the groups in the worst equilibrium because of their fears that what is not demanded will be consumed by the other groups. Let W_0 denote the payoff at time 0 to any group in the worst equilibrium, that is:

$$W_0 = \frac{1}{1-\beta^3}\frac{1}{n} \tag{1}$$

Given this simple characterization of the worst equilibrium, it now can be used as the trigger that sustains the best subgame perfect equilibria of the game.

2.5 Efficient Subgame Perfect Equilibria

This subsection characterizes the efficient subgame perfect equilibrium of the savings-only game with symmetric payoffs to all domestic groups. It is shown that such a symmetric equilibrium exists, is 3-period stationary, and can be characterized by the solution to a simple linear program.

Given the characterization of the worst equilibrium, sustainable allocations are defined as follows:

Definition 2.6. *A feasible savings-only allocation d is* ***sustainable*** *if the following* ***sustainability constraints*** *hold:*

$$\sum_{\tau=t}^{\infty}\beta^{\tau-t}\phi_\tau d_\tau^i \geq \phi_t \max\left\{Ra_{t-1}(d)+e_t-\sum_{j\neq i}d_t^j, \frac{Ra_{t-1}(d)+e_t}{n}\right\}+\sum_{\tau=t+1}\beta^{\tau-t}\phi_\tau\frac{e_\tau}{n}. \tag{SC}$$

for all $t \geq 0$ and $i \in I$.

The sustainability constraints ensure that no group prefers to deviate from the prescribed allocation when facing the worst equilibrium payoff as continuation value. A group when deviating can guarantee a flow payoff that is equal to the maximum between forcing equal shares of the resources in the current period, or demanding all of the residual income of the government (that is what is captured by the maximization on the right-hand side). Standard arguments imply that,

Proposition 2.7. *A feasible savings-only allocation is subgame perfect if and only if it is sustainable.*

The next step is to define efficiency,

Definition 2.8. *A savings-only allocation is **efficient** if it is sustainable and there does not exist another sustainable savings allocation that weakly Pareto dominates it as of time 0.*

From the symmetry in the structure of the game, it follows that there exists an efficient allocation that is symmetric:

Proposition 2.9 (Symmetry). *There exists an efficient allocation of fiscal demands d such that $d_t^i = d_t$ for all $i \in I$ and $t \geq 0$.*

In a symmetric allocation, the best deviation by any given group is not to force equal sharing of the government resources but rather to demand all of the residual resources. That is, in a symmetric allocation, $Ra_{t-1} + e_t - (n-1)d_t \geq (Ra_{t-1} + e_t)/n$, given that from the feasibility constraint, $d_t \leq (Ra_{t-1} + e_t)/n$. This simplifies the constraint set, because the sustainability constraints collapse to:

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_\tau d_\tau \geq \phi_t \left(Ra_{t-1}(d) + e_t - (n-1)d_t\right) + \sum_{\tau=t+1} \beta^{\tau-t}\phi_\tau \frac{e_\tau}{n}$$

for all $t \geq 0$, or equivalently, using the budget constraint,

$$\sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\phi_\tau d_\tau \geq \phi_t a_t + \sum_{\tau=t+1} \beta^{\tau-t}\phi_\tau \frac{e_\tau}{n}, \tag{SC'}$$

for all $t \geq 0$.

I now show that there always exists an efficient allocation with symmetric payoffs that does not generate savings at times $t \in T_\phi^2$ (when $\phi_t = \phi^2$), and that such an allocation is stationary:

Proposition 2.10 (No Over Savings). *Suppose that conditions (2.1) and (2.3) hold. Then, in any efficient allocation with symmetric payoffs, d, $a_t(d) = 0$ for all $t \in T_{\phi^2}$. Also, there exists an efficient allocation with symmetric payoffs that is stationary, i.e. if $(t,t') \in T_x \times T_x$ for $x \in \{1, \phi, \phi^2\}$, then $d_t = d_{t'}$. And if $\beta R\phi \geq n$, then the first best symmetric allocation is subgame perfect.*

Recall that what is interesting is the circumstances in which the first best allocation cannot be sustained as an equilibrium of the game. Hence, I will be assuming that,

Condition 2.11. *The parameters are such that $\beta R\phi < n$.*

The efficient sustainable allocation with symmetric payoffs can be completely characterized by the fiscal provisions allocated in the first three periods; the rest of the time, the cycle of provisions simply repeats itself. Thus, the best sustainable savings-only allocation is identified with two numbers: the amounts saved in periods T_1 and T_ϕ. Let V_t represent the present-value utility that a group receives as of time t in an efficient and symmetric savings-only allocation, then

Proposition 2.12. *Suppose assumptions (2.1), (2.3), and (2.11) hold. Then in an efficient allocation with symmetric payoffs, the utilities to the groups at the first periods, V_0, V_1 and V_2, are given by,*

$$(1-\beta^3)V_0 = \frac{1-a_0}{n} + \beta\phi\frac{Ra_0 - a_1}{n} + \beta^2\phi^2\frac{Ra_1}{n} \tag{2}$$

$$(1-\beta^3)V_1 = \phi\frac{Ra_0 - a_1}{n} + \beta\phi^2\frac{Ra_1}{n} + \beta^2\frac{1-a_0}{n} \tag{3}$$

$$(1-\beta^3)V_2 = \phi^2\frac{Ra_1}{n} + \beta\frac{1-a_0}{n} + \beta^2\phi\frac{Ra_0 - a_1}{n} \tag{4}$$

for some a_0 and a_1 such that $0 \leq a_1 \leq Ra_0 \leq R$.

Given Condition (2.3), it follows that the value functions V_0 and V_1 are both increasing in a_0 and a_1: it is first-best efficient to save the maximum amount until the next T_{ϕ^2} period. However, as will be shown below, saving that much might not be sustainable.

In a symmetric allocation, the sustainability constraints in periods T_{ϕ^2} are satisfied, as no savings are done. There are then only two types of sustainability constraints left, those of periods T_1 and T_ϕ. They can be written as:

$$\frac{\theta(\theta-1)}{n}\tilde{a}_1 + \beta^3(V_0 - W_0) \geq \frac{n-\theta}{n}a_0 \tag{5}$$

$$\beta^3(V_0 - W_0) \geq \theta\frac{n-\theta}{n}\tilde{a}_1 \tag{6}$$

where

$$\theta \equiv \beta\phi R$$

$$\tilde{a}_1 \equiv a_1/R$$

Remark 2.13. The difference between V_0 and W_0 is the surplus that cooperation generates in the dynamic game as of any time $t \in T_1$. As can be seen from equations (5) and (6), the size of this surplus limits the ability to sustain savings in earlier periods. That is, as long as $n > \theta$, equations (5) and (6) can be interpreted as representing **endogenous savings limits**. The higher is the benefit of cooperation in future periods, as given by a higher value of $V_0 - W_0$, the more relaxed are the sustainability constraints; and thus a higher level of savings can be sustained.

Plugging the values of W_0 and V_0 from (1) and (2) into equations (5) and (6), the sustainability constraints become:

$$\theta(\theta-1)\tilde{a}_1 \geq ((1-\beta^3)(n-1)+1-\theta)a_0 \tag{SC_0}$$

$$\beta^3(\theta-1)a_0 \geq \theta((1-\beta^3)(n-1)+1-\theta)\tilde{a}_1 \tag{SC_1}$$

The following subsection provides a complete characterization of an efficient and symmetric allocation.

2.6 Solving for an Efficient Allocation

The problem of finding an efficient allocation is finding values for a_0 and a_1 that maximize time 0 utility, are feasible, and satisfy the sustainability constraints (SC_0)-(SC_1). That is,

$$(1-\beta^3)V_0^\star = \max_{0\leq\tilde{a}_1\leq a_0\leq 1} \frac{1}{n}\left\{1-a_0+\theta(a_0-\tilde{a}_1)+\theta^2\tilde{a}_1\right\} \tag{P}$$

subject to (SC_0) and (SC_1).

Let $a_0^\star$ and $\tilde{a}_1^\star$ denote the optimal policies of problem (P).

Let the sets Γ_0 and Γ_1 be defined as:

$$\Gamma_0 = \{(\beta,n,\theta)\in\mathbb{R}^3 | (1-\beta^{3/2})(n-1)+1-\theta>0\}$$
$$\Gamma_1 = \{(\beta,n,\theta)\in\mathbb{R}^3 | (1-\beta^{3})(n-1)+1-\theta\leq 0\}$$

Note that $(\Gamma_0\cap\Gamma_1)=\emptyset$ and $(\Gamma_0\cup\Gamma_1)\cap\mathbb{R}^3\neq\emptyset$. Given these definitions, the solution to Problem (P), and the complete characterization of the efficient allocation is presented in the next theorem,

Theorem 2.14. *Suppose that conditions (2.1), (2.3) and (2.11) hold.*

(i) If $(\beta,n,\theta)\in\Gamma_0$ then the worst equilibrium is the unique equilibrium of the game; that is, $a_0^\star=\tilde{a}_1^\star=0$;

(ii) if $(\beta,n,\theta)\in\Gamma_1$ then the symmetric first best allocation is sustainable, that is, $a_0^\star=\tilde{a}_1^\star=1$;

(iii) otherwise, the efficient and symmetric allocation is such that $a_0^\star=1$ and

$$\tilde{a}_1^\star=\min\left\{\frac{\beta^3(\theta-1)}{\theta((1-\beta^3)(n-1)+1-\theta)},1\right\}. \tag{7}$$

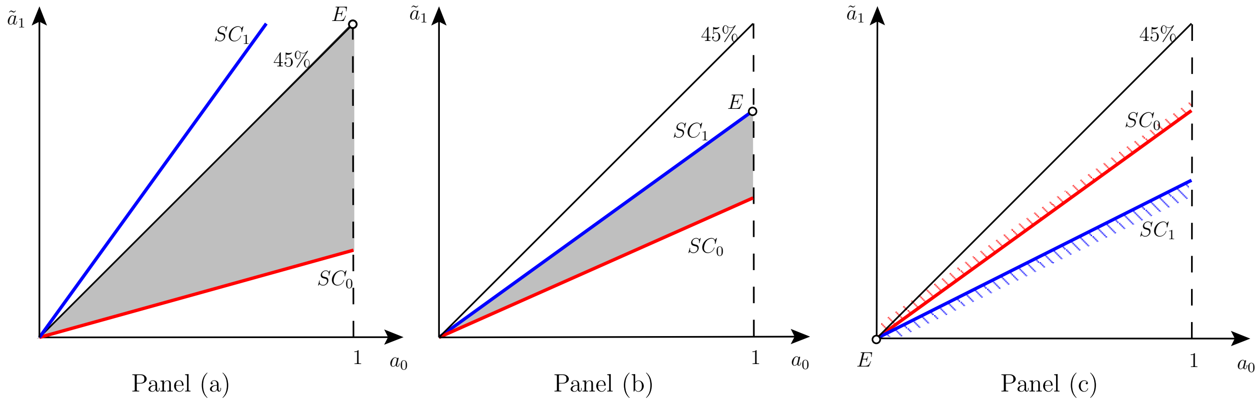

Figure 1: SC_0 denotes the sustainability constraints at T_1 and SC_1, the sustainability constraints at T_ϕ. The feasibility constraint restricts the choices to lie in the triangle formed by the 45 degree line, the horizontal axis and the vertical line $a_0 = 1$. The point E shows the position of the efficient sustainable allocation. Panel (a) illustrates a situation where the first best efficient level of savings is achieved in an equilibrium. Panel (b) illustrates a situation with intermediate values of savings. Panel (c) illustrates a case where the worst equilibrium is also the unique equilibrium.

If $(\beta, n, \theta) \in \Gamma_0$, then no cooperation is possible. The temptations that assets generate are too strong to support any level of savings. When the parameters lie outside Γ_0, some level of savings may by sustainable, but not necessarily the first-best one. The equilibrium is such that in periods T_1 either everything is saved ($a_0 = 1$) or nothing is.

Given conditions (2.1), (2.3), and (2.11), Figure 1 illustrates the result of Theorem 2.14. The sustainability constraints are linear in the $(a_0, a_1/R)$-space and are denoted by SC_0 and SC_1. The first panel illustrates a situation where the first-best level of savings is sustainable. The second panel illustrates a situation where some savings are sustainable but lie strictly below the first-best level. The third panel illustrates a situation where no savings are sustainable.

Understanding the Sustainability Constraints

Suppose that $(\beta, n, \theta) \in \Gamma_0$. This corresponds to a situation where no cooperation is possible (Panel c of Figure 1). Let us rewrite the sustainability constraints once more:

$$\frac{\beta^3(\theta-1)/\theta}{(1-\beta^3)(n-1)+1-\theta} \geq \frac{a_1}{Ra_0} \geq \frac{(1-\beta^3)(n-1)+1-\theta}{\theta(\theta-1)}$$

The first inequality corresponds to the sustainability constraint for periods T_ϕ. This constraint generates an upper bound on a_1: savings cannot be too high, otherwise a group would choose to deviate. The second inequality corresponds to the sustainability constraints in periods T_1. Suppose that $\frac{(1-\beta^3)(n-1)+1-\theta}{\theta(\theta-1)} < 1$, so that there exist strictly positive values a_1 and a_0 that are feasible and such that the sustainability constraint in T_1 would not be binding if those values were chosen. In other words, if the groups *could commit to a given amount a_1 of savings in periods T_ϕ, then some positive level of savings could be sustained in periods T_1*. However, $(\beta, n, \theta) \in \Gamma_0$ guarantees that any level of (a_0, a_1) that is sustainable in periods T_1, will violate sustainability in periods T_ϕ! Thus, the lack of commitment in periods T_ϕ means that nothing but autarky is sustainable. I show next that this commitment can be provided by a contingent credit line. This is the fundamental reason why an appropriately designed credit line can be valuable enough to all groups that they will choose not to default on it.

2.7 The Continuation Game at any Asset Level

It is possible to solve for a symmetric and efficient equilibrium of the continuation game starting at any time t and for any level of initial assets Ra_{t-1}.

Let $\bar{a}_0$ and $\bar{a}_1$ be given by:

$$\bar{a}_0 \equiv \frac{n-1}{n-\theta}\frac{n\beta^3}{n-\theta}(V_0^\star - W_0).$$
$$\bar{a}_1 \equiv R\frac{n\beta^3}{(n-\theta)\theta}(V_0^\star - W_0)$$

These values will serve as upper bounds on the amounts of savings that can be sustained in the symmetric and efficient equilibrium following any continuation. Note that $\bar{a}_0 \geq \bar{a}_1/R$ and that $\bar{a}_0$ and $\bar{a}_1/R$ can be bigger than 1. The characterization of the symmetric efficient equilibrium is in the following proposition:[13]

Proposition 2.15 (Continuation Game and Savings Limits)**.** *Suppose that conditions (2.1), (2.3) and (2.11) hold. For any time t_1 and initial assets Ra_{t_1-1}, there exists an efficient equilibrium of the continuation game starting at t_1 that is symmetric. Let $\hat{t}$ be given by $\hat{t} = \min\{\tau \geq t_1 : \tau \in T_{\phi^2}\}$. The efficient and symmetric allocation can be determined as*

[13] Although the full result of this proposition is not necessary for the upcoming analysis of debt repayment, it is shown here for completeness.

follows:

$$a_t = \begin{cases} a_0^\star & t \in T_1, t \geq \hat{t}, \\ R\tilde{a}_1^\star & t \in T_\phi, t \geq \hat{t}, \\ 0 & t \in T_{\phi^2}, \\ \min\{Ra_{t-1}, \bar{a}_0\} & t \in T_1, t < \hat{t}, \\ \min\{Ra_{t-1}, \bar{a}_1\} & t \in T_\phi, t < \hat{t}. \end{cases}$$

Having characterize the game when the country can save but not borrow, it is time now to introduce sovereign lending.

3 A Contingent Credit Line

The previous section described the equilibrium when the country had no access to external borrowing. In this section, that assumption is relaxed and the possibility of borrowing from foreign creditors is introduced.

It is assumed, as in Bulow and Rogoff (1989), that the foreign investors can and do commit to excluding the country from borrowing again in foreign financial markets if it ever defaults on its sovereign debt. However, foreign investors cannot stop the country from saving after default. This last assumption is a realistic one, as a sovereign nation will have a very complex and opaque set of assets available, even after defaulting, or it might also invest its funds inside its borders.[14]

Bulow and Rogoff (1989) showed under quite general conditions that if countries can save after default, then defaulting on any debt contract is always an arbitrage (as long as the net interest rate is positive).[15] In particular, for any non-trivial debt contract and any allocation that is *feasible* under it, there always exits a state, and an associated *feasible* savings-only allocation starting from that state, that can finance a higher level of consumption at all future dates. That argument is also present here: there exist *feasible* savings-only allocations that would ex-post Pareto dominate any credit line allocation. The question is whether these savings allocations are *sustainable*: that is, whether they can be the outcome of equilibrium play.

[14] Most of the subsequent literature on this topic maintains this assumption. Important exceptions is Kletzer and Wright (2000) and Wright (2004).

[15] The actual restriction of Bulow and Rogoff (1989) is that the wealth of the country is finite, which in this stationary economy imposes the requirement of a positive net interest rate (See Hellwig and Lorenzoni, 2008, for an analysis relaxing this assumption).

For simplicity, I restrict attention to a particular type of credit line[16]. Let $a(d,b)$ be a sequence of asset positions generated by an allocation of fiscal provisions d and *a credit line* b starting with $a_{-1} = 0$. The sequence $a(d,b)$ is defined as

$$a_t(d,b) = Ra_{t-1}(d,b) + e_t + b_t(b) - \sum_{i=1}^{n} d_t^i$$

where

$$b_t(b) = \begin{cases} b & \text{, for } t \in T_{\phi^2}, \\ -Rb & \text{, for } t \in T_1 \text{ and } t > 0, \\ 0 & \text{, otherwise.} \end{cases}$$

The credit line provides access to a total amount of funds b in periods T_ϕ^2. The country is called to repay those funds in the following periods T_1, at a gross interest rate of R. The credit line is closed in periods T_ϕ. The credit line is contingent: it only provides funds in periods where there are most needed, that is T_{ϕ^2}.

Definition 3.1. *An allocation d is **feasible under a credit line** b if $d_t^i \geq 0$ for all $t \geq 0$ and $i \in I$, and the assets positions generated by (d,b) are always positive: $a_t(d,b) \geq 0$ for all $t \geq 0$.*

As mentioned above, the result of Bulow and Rogoff (1989) holds in a modified form:

Lemma 3.2 (Bulow and Rogoff)**.** *For any feasible allocation under a credit line b there exists a savings-only allocation from any positive time $t \in T_1$ that is feasible and that Pareto dominates it as long as $R > 1$.*

The above lemma is a direct application of the theorem of Bulow and Rogoff (1989). However, in this case, a simple proof is as follows: for any credit line b, consider the deviation where in period T_1 the country instead of paying back b to the foreigners, decides to save the amount b up to the following T_{ϕ^2} periods, and keeps doing this for all T_1 periods that follow. Such a strategy increases the funds in periods T_{ϕ^2} by $(R^2 - 1)b > 0$ with respect to the allocation where the debt is always paid back. Allocating those funds equally among all groups generates a strict Pareto improvement.

Note that the above result is independent of the groups' discount factor or the shape of the utility function. This strong feasibility result leads Bulow and Rogoff (1989) to conclude

[16] Although I haven't been able to show this, it is a reasonable conjecture that these type of debt contracts are the best that could be offered.

that direct sanctions are the only mechanism through which significant amounts of borrowing can be supported. Implicit in their argument is the assumption that a country always could implement any feasible allocation if it so chooses. However, is the existence of a feasible Pareto improving allocation enough to guarantee that default will occur? The answer in general is *no*: when the country is composed of groups that lack commitment internally as well as externally, the allocations that need to be considered possible after default have to sustainable as well as feasible.

So, when is a credit line "repayable"? The following definition formally states the notion of repayability of a credit line that will be used in the rest of this paper:

Definition 3.3. *An allocation d is* ***repayable under a credit line*** *b if d is feasible under b, and*

(i) for all $i \in I$ and $t \geq 0$ it is the case that

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_\tau d_\tau^i \geq \phi_t \max\left\{ Ra_{t-1}(d,b) + e_t + (b_t(b))^+ - \sum_{j\neq i} d_t^j, \right. \\ \left. \frac{Ra_{t-1}(d,b) + e_t + (b_t(b))^+}{n} \right\} + \sum_{\tau=t+1} \beta^{\tau-t}\phi_\tau \frac{e_\tau}{n}, \tag{8}$$

(ii) for all $t \geq 0$ and for any sustainable savings allocation $\hat{d}$ starting at t with assets $Ra_{t-1}(d,b)$, there exists an $i \in I$ such that

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_\tau d_\tau^i > \sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_\tau \hat{d}_\tau^i. \tag{9}$$

Part (i) in the definition checks whether the allocation under a credit line b is subgame perfect, given the commitment of the external investors to punish the country by closing the credit line. This is the equivalent of condition (SC) for the case of debt. It is assumed that the payments to foreigners, the positive values of $b_t(b)$, can be appropriated by the local group upon a deviation. This is accordance to our assumption that the government does not care about the foreigners, and will transfer those resources to any local group that demands them.

Part (ii) imposes the restriction that there is no sustainable allocation of the savings-only game that Pareto dominates the credit-line allocation in the continuation game starting at any time. This condition is important: it rules out allocations where all groups in the country will be willing to jointly default; as such, it is in the spirit of Bulow and Rogoff's original argument. Further, it rules out the possibility a credit line is valuable because defaulting on it provides a trigger mechanism that might be otherwise not available in a savings-only equilibrium to the domestic groups. By imposing that the continuation savings-only game be efficient after default, I ensured that the credit-line is valuable for its properties beyond the provision of a trigger.

The objective now is to analyze whether there are any credit-line allocations that are repayable. To proceed, the focus is narrowed to those allocations under a credit line that generate symmetric payoffs to all groups.

3.1 Symmetric Allocations under a Credit Line

The following result relaxes the constraints when focusing on symmetric allocations: when determining whether a symmetric allocation is repayable, it is sufficient to ask whether it is subgame perfect, and whether it is not Pareto dominated by the symmetric and efficient equilibrium of the savings-only game:

Lemma 3.4. *Suppose that d is a symmetric allocation under a credit line b and that d satisfies condition (i) of definition 3.3. If the efficient and symmetric sustainable allocation of the savings-only game d^* starting at t with assets $Ra_{t-1}(d, b)$ is such that*

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_\tau d_\tau \geq \sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_\tau d_\tau^*$$

for all t, then the allocation d is repayable under the credit line b.

If a symmetric allocation returns payoffs above those generated by the efficient symmetric allocation of the continuation savings-only game, then it is repayable. This is particularly useful, because an analytical characterization of that savings-only equilibrium already was provided in the previous section. The following lemma narrows the focus to allocations under a credit line with no savings in periods T_{ϕ^2}:

Lemma 3.5. *Let d_0 be a symmetric and repayable allocation under a credit line b_0. If d_0 prescribes strictly positive savings at some time $t_0 \in T_{\phi^2}$, then there exists a credit line b_1 with $b_1 \leq b_0$ and a repayable allocation under it, d_1, such that savings at all times T_{ϕ^2} are zero and the utility payoffs to all groups under d_1 are at least as high as under d_0.*

Lemma 3.5 formalizes the intuitive result that positive savings in periods T_{ϕ^2} is equivalent to not using all the available credit, and a smaller credit line will suffice.[17] If an allocation

[17] This is intuitive if the allocation is stationary. Lemma 3.5 guarantees that this is the case generally.

were to prescribe positive savings in periods T_{ϕ^2}, then there exists an allocation under a smaller credit line that is also repayable and (weakly) Pareto dominates it: if the groups could choose how much to borrow, they will prefer to borrow a smaller amount. For that reason, I will restrict attention to repayable allocations that do not feature savings in periods T_{ϕ^2} and thus restrict the search to two numbers: the amounts saved in periods $t \in T_1$ and in periods $t \in T_\phi$.

Given an allocation $(a_0, \tilde{a}_1)$ under a credit line b, define the following values[18]:

$$\hat{V}_0 = \frac{(1 - Rb - a_0) + \theta(a_0 - \tilde{a}_1) + \theta^2(\tilde{a}_1 + b/R^2)}{n(1-\beta^3)}$$
$$\beta\hat{V}_1 = \frac{\theta(a_0 - \tilde{a}_1) + \theta^2(\tilde{a}_1 + b/R^2) + \beta^3(1 - Rb - a_0)}{n(1-\beta^3)}$$

A feasible allocation under a credit line b is repayable if and only there exists a_0 and $\tilde{a}_1$ such that:

$$\hat{V}_0 \geq \left(1 - \frac{(n-1)(1-Rb-a_0)}{n}\right) + \beta^3 W_0 \qquad (\tilde{SC}_0)$$
$$\beta\hat{V}_1 \geq \theta\left(a_0 - \frac{(n-1)(a_0 - \tilde{a}_1)}{n}\right) + \beta^3 W_0 \qquad (\tilde{SC}_1)$$
$$\hat{V}_0 \geq V_0^\star \qquad (DC_0)$$
$$\beta\hat{V}_1 \geq \frac{\theta}{n}(a_0 - \min\{a_0, \bar{a}_1/R\}) + \frac{\theta^2}{n}\min\{a_0, \bar{a}_1/R\} + \beta^3 V_0^\star \qquad (DC_1)$$

Constraints ($\tilde{SC}_0$) and ($\tilde{SC}_1$) require that the allocation be subgame perfect (there are the equivalents of constraints (SC_0) and (SC_1) in the savings-only game). Constraints (DC_0) and (DC_1) require that the allocation in the equilibrium path is not dominated by any savings-only allocation.[19]

3.2 Repayability of a credit line

If the first-best savings allocation is an equilibrium of the savings-only game, then no allocation under a credit line would be repayable. This can be seen by noting that the following inequality, which corresponds to (DC_0) in this case, cannot be satisfied for any $b > 0$ and

[18]Here $\tilde{a}_1$ refers to the amount saved in periods T_ϕ divided by R, as in the previous section.

[19]The right-hand side of constraint (DC_1) follows from Proposition 2.15. It is not necessary to check the constraints for periods T_{ϕ^2}, because there are no savings in those periods, and the country is a net receiver of funds from abroad.

for any pair $(a_0,\, \tilde{a}_1)$ such that $1 - Rb \geq a_0 \geq \tilde{a}_1 \geq 0$, as long as $R > 1$:

$$\hat{V}_0 = \frac{(1 - Rb - a_0) + \theta(a_0 - \tilde{a}_1) + \theta^2(\tilde{a}_1 + b/R^2)}{n(1-\beta^3)} < V_0^\star = \frac{\theta^2}{n(1-\beta^3)}$$

So, for a repayable allocation to exist, it must be that $(\beta, n, \theta) \notin \Gamma_1$ and that if $(\beta, n, \theta) \notin \Gamma_0 \cup \Gamma_1$, the first-best efficient level of savings is not an equilibrium of the savings-only game:

$$\frac{\beta^3(\theta - 1)}{\theta((1-\beta^3)(n-1) + 1 - \theta)} < 1 \tag{10}$$

This last condition is equivalent to $1 + (1-\beta^3)(\theta n - 1) > \theta^2$.

When $(\beta, n, \theta) \in \Gamma_0$ the equilibrium set of the savings-only game is a singleton, and the best equilibrum is also the worst. If the slope of constraint (SC_1) is above the 45% line, that is, $\theta^2 \leq 1 + (1-\beta^3)(n-1)$, then the constraint ($\tilde{SC}_1$) is at least as tight as (SC_1), for $R^3 > 1$, and cannot be satisfied for any $b > 0$, $a_0 \geq 0$ and $\tilde{a}_1 \geq 0$. *That is, if the savings level is inefficient because of tight sustainability constraints in periods T_1, then any contingent credit line has no value to the groups in equilibrium and would not be repaid.*

For there to exist a repayable credit line when $(\beta, n, \theta) \in \Gamma_0$, it is necessary that,

$$\theta^2 > 1 + (1-\beta^3)(n-1).$$

In what follows, the two possible cases where debt could be repaid are analyzed: (i) the case when $(\beta, n, \theta) \in \Gamma_0$ and $\theta^2 > 1 + (1-\beta^3)(n-1)$; and (ii) the case when $(\beta, n, \theta) \notin \Gamma_0 \cup \Gamma_1$ and $1 + (1-\beta^3)(\theta n - 1) > \theta^2$.

3.2.1 The case when $(\beta, n, \theta) \in \Gamma_0$.

Here, the unique equilibrium of the savings-only game is the worst and $V_0^\star = W_0$. From Proposition 2.15, it follows that $\bar{a}_1 = 0$, and the left-hand side of constraint (DC_1) becomes:

$$\frac{\theta}{n} a_0 + \beta^3 W_0$$

which is less than $\theta\left(a_0 - \frac{(n-1)(a_0-\tilde{a}_1)}{n}\right) + \beta^3 W_0$, and constraint ($DC_1$) is therefore implied by ($\tilde{SC}_1$). Similarly, constraint (DC_0) is implied by ($\tilde{SC}_0$). Solving for ($\tilde{SC}_0$) and ($\tilde{SC}_1$):

$$\theta(\theta-1)\tilde{a}_1 + \left[\frac{\theta^2}{R^3} - 1 - (1-\beta^3)(n-1)\right] Rb \geq \left[(1-\beta^3)(n-1) + 1 - \theta\right] a_0 \qquad (\tilde{SC}_0')$$

$$\theta\left[(1-\beta^3)(n-1) + 1 - \theta\right]\tilde{a}_1 \leq \left(\frac{\theta^2}{R^3} - \beta^3\right) Rb + \beta^3(\theta-1)a_0 \qquad (\tilde{SC}_1')$$

Note that if $b = 0$, the two inequalities above are equivalent to (SC_0) and (SC_1). Sovereign debt, b, always relaxes ($\tilde{SC}_1'$), the sustainability constraints in periods T_ϕ (this follows from $\theta^2 > 1 > \beta^3 R^3$). However, it might tighten ($\tilde{SC}_0'$), depending on the sign of $\theta^2/R^3 - 1 - (1-\beta^3)(n-1)$. This last point is intuitive: because debt is repaid in periods T_1, one should expect the repayability constraints in those periods to tighten when the interest rate is sufficiently high.

Summarizing, there exists a repayable allocation under a credit line of size b if there exists a pair $(a_0, \tilde{a}_1)$, with $a_0 \geq \tilde{a}_1 \geq 0$ and such that ($\tilde{SC}_0'$) and ($\tilde{SC}_1'$) are satisfied. The following proposition describes when a repayable allocation exists:

Proposition 3.6. *Suppose that $(\beta, n, \theta) \in \Gamma_0$. If $\theta^2 \leq 1 + (1-\beta^3)(n-1)$ then there are no repayable allocations under any non-trivial credit line. If $\theta^2 > 1 + (1-\beta^3)(n-1)$, then a credit line of size $\bar{b}$ is efficient, where:*

$$R\bar{b} = \begin{cases} 1 & ; \textit{for } 1 < R^3 \leq \underline{R}_1 \\ \frac{\theta^2 - (1-\beta^3)(n-1) - 1}{\theta^2} \frac{R^3}{R^3 - 1} & ; \textit{for } \underline{R}_1 \leq R^3 \leq \bar{R}_1 \\ 0 & ; \textit{otherwise.} \end{cases}$$

and

$$1 < \underline{R}_1 = \frac{\theta^2}{1 + (n-1)(1-\beta^3)} < \bar{R}_1 = \frac{\theta^2 - \theta}{1 + (1-\beta^3)(n-1) - \theta}$$

Also, any credit line of size $b < \bar{b}$ admits a repayable allocation.

Figure 2 graphically describes the results obtained above when $\underline{R}_1 \leq R^3 \leq \bar{R}_1$. Panel (a) shows the original situation in the savings-only game, where the worst equilibrium is the unique equilibrium of the game. Panel (b) shows what happens when $b > 0$, that is, when some debt is issued. The introduction of debt affects the old sustainability constraints. In this illustrated case, it relaxes ($\tilde{SC}_1'$) but it tightens ($\tilde{SC}_0'$). However, it generates a non-empty set of repayable allocations (denoted in the graph by the shaded area). The maximum

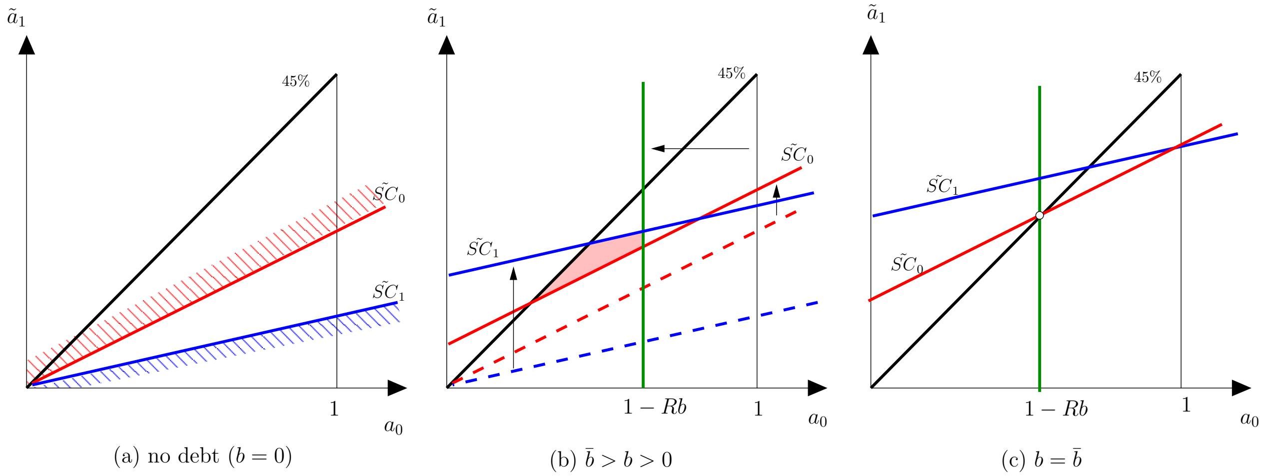

Figure 2: The case when $(\beta, n, \theta) \in \Gamma_0$. This figure illustrates how the introduction of debt generates sustainable allocations. Panel (a) shows the situation with no debt, when the unique equiilibrium is the worst. Panel (b) shows how introducing debt generates a set of repayable allocations (represented by the shaded set). Panel (c) shows how the maximum amount of debt can be computed, as a situation where the repayable set is a singleton (represented by the small circle).

value of b that admits a repayable allocation, $\bar{b}$, occurs when the repayable set is a singleton (shown in Panel (c)).

3.2.2 The case when $(\beta, n, \theta) \notin \Gamma_0 \cup \Gamma_1$.

As discussed above, $\theta^2 < 1 + (1 - \beta^3)(\theta n - 1)$ is a necessary condition for there to exist a repayable allocation when $(\beta, n, \theta) \notin \Gamma_0 \cup \Gamma_1$, because it guarantees that the first-best level of savings is not an equilibrium of the savings-only game. Using the efficient equilibrium of the savings-only game, as given by equation (7), the constraint (DC_0) can be rewritten as:

$$(\theta - 1)(a_0 + \theta \tilde{a}_1) \geq -\left(\frac{\theta^2}{R^3} - 1\right) Rb + \frac{(1 - \beta^3)(n - \theta)(\theta - 1)}{1 + (1 - \beta^3)(n - 1) - \theta} \qquad (DC_0')$$

Constraint ($\tilde{SC}_0'$) can be rewritten as,

$$(\theta - 1)(a_0 + \theta \tilde{a}_1) \geq -\left(\frac{\theta^2}{R^3} - 1\right) Rb + (1 - \beta^3)(n - 1)(a_0 + Rb)$$

The right-hand side of the above equation is less[20] than the right-hand side of (DC_0'); and hence, (DC_0') implies ($\tilde{SC}_0'$). That is, the stronger constraint in periods T_1 is the one enforcing that all the groups do not decide to default jointly.

[20]That follows from $a_0 + Rb \leq 1$ and $(\beta, n, \theta) \notin \Gamma_0$

Before moving to the characterization in this case, one can proceed to show that ($\tilde{SC}_1$) should be holding, unless feasibility impedes it, and that constraint (DC_1) can be safely ignored. This last is intuitive: if the groups were going to default, they would do so before paying back the debt rather than after:

Lemma 3.7. *The following holds:*

(i) If any symmetric allocation is repayable under a credit line of size b, then there exists a repayable allocation where either ($\tilde{SC}_1$) or feasibility, $a_0 \geq \tilde{a}_1$, holds with equality.

(ii) Suppose that (DC_0'), ($\tilde{SC}_1'$) and feasibility hold. If either ($\tilde{SC}_1'$) or feasibility, $a_0 \geq \tilde{a}_1$, holds with equality, then constraint (DC_1) holds as well.

Thus, to determine whether there is a repayable allocation under a credit line b, it suffices to find a pair $(a_0, \tilde{a}_1)$ with $1 - Rb \geq a_0 \geq \tilde{a}_1 \geq 0$, such that inequalities ($\tilde{SC}_1'$) and (DC_0) hold, and either ($\tilde{SC}_1'$) or $a_0 \geq \tilde{a}_1$ holds with equality.

The next proposition characterizes the regions of the parameter space where a credit line can be repaid:

Proposition 3.8. *Let conditions 2.1, 2.3, and 2.11 hold. Suppose that $(\beta, n, \theta) \notin \Gamma_0 \cup \Gamma_1$. If $\theta^2 \geq 1 + (1-\beta^3)(\theta n - 1)$, then no allocation under any non-trivial credit line is repayable. If $\theta^2 < 1 + (1-\beta^3)(\theta n - 1)$, then a credit line of size $\bar{b}$ is efficient, where:*

$$R\bar{b} = \begin{cases} 1 & ;\ \textit{for } 1 < R^3 \leq \underline{R}_2 \\ \left[\frac{(\theta n-1)(1-\beta^3)+1-\theta^2}{(1+(1-\beta^3)(n-1)-\theta)\theta}\right] \frac{\theta-1}{\theta} \frac{R^3}{R^3-1} & ;\ \textit{for } \underline{R}_2 \leq R^3 \leq \bar{R}_2 \\ 0 & ;\ \textit{otherwise.} \end{cases}$$

and

$$1 < \underline{R}_2 = \theta \left(1 + \frac{\beta^3(\theta-1)^2/\theta}{1+(1-\beta^3)(n-1)-\theta} \right)^{-1} < \bar{R}_2 = \frac{\theta(n-1)}{n-\theta}.$$

Also, any credit line of size $b \leq \bar{b}$ admits a repayable allocation.

3.3 Discussion of results

The previous subsections showed that there are situations where a credit line admits a repayable allocation, depending on the parameter space. The following corollary summarizes some of the results contained in Propositions 3.6 and 3.8:

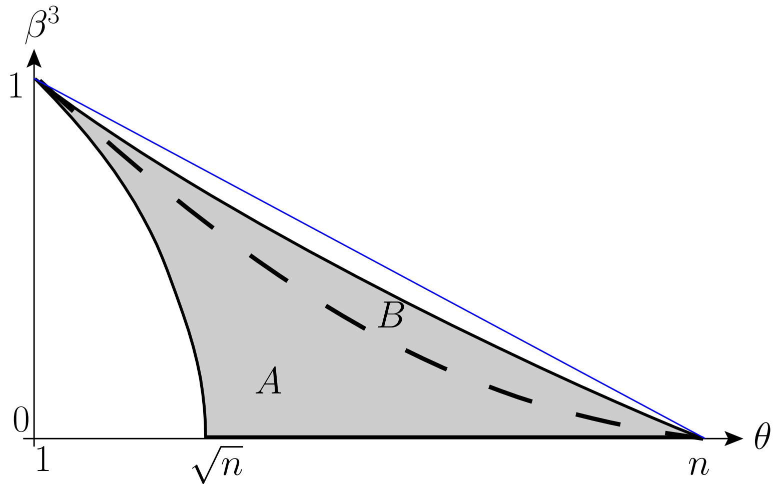

Figure 3: The shaded area represents the set for which any credit line admits a repayable allocation for some R close enough to 1. The upper-right boundary of the shaded set is $\theta^2 = 1 + (1-\beta^3)(\theta n - 1)$, the lower-left boundary is $\theta^2 = 1 + (1-\beta^3)(n-1)$, the dotted line is $\theta = 1 + (1-\beta^{3/2})(n-1)$, and the straight blue line is $\theta = 1 + (1-\beta^3)(n-1)$.

Corollary 3.9. *If* (β, n, θ) *are such that:*

$$1 + (1-\beta^3)(\theta n - 1) > \theta^2 > 1 + (1-\beta^3)(n-1)$$

with $\beta < 1$, $n > \theta > 1$; *then for any credit line* b *there exists a value* $\bar{R} > 1$ *so that for all* $1 < R < \bar{R}$, *the credit line admits a repayable allocation.*

Figure 3 plots the results of Corollary 3.9. The shaded area represents the region of the state space for which repayable allocations exists under any credit line, as long as R is sufficiently small. The area denoted by A is the subset where the worst equilibrium of the savings-only game is also the best, and the area denoted by B is the subset where the best equilibrium is not the worst, but first-best efficiency cannot be achieved.

Note that as β goes to one, the set of θ's under which a repayable allocation exists vanishes: as β goes to one, the domestic groups are sufficiently patient that first-best efficiency can always be achieved, and the credit line is not valued. In similar fashion, for a given $\beta < 1$, when θ is very small, no credit line can ever be repaid, and no savings are ever done: the benefits of the inter-temporal transfer are not sufficient to guarantee either. When θ is high, the benefits of the inter-temporal transfer are large enough to guarantee that the groups on their own can achieve it, and hence the need for a credit line disappears. It is for intermediate values of θ that a credit line can be repaid: where the benefits of cooperation are not sufficiently high to guarantee first-best efficient savings, but not sufficiently low that a credit line is valued.

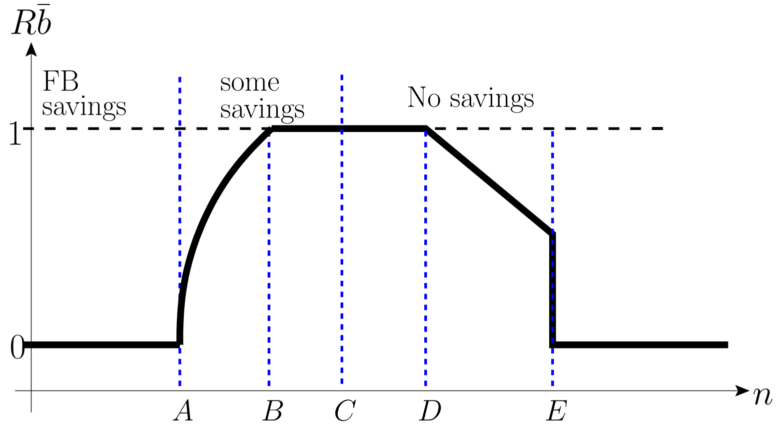

Figure 4: Fragmentation and debt. The figure plots how the maximum amount of debt changes as n increases in the case where $R^3 < \theta$. The values in the x-axis are: $A = \frac{1}{\theta}\left(\frac{\theta^2-1}{1-\beta^3}+1\right)$, $B = 1+\frac{(\theta-1)}{1-\beta^3}\left(\frac{\beta^3(\theta-1)/\theta}{\theta/R^3-1}+1\right)$, $C = \frac{\theta^2/R^3-1}{1-\beta^3}+1$, $D = \frac{(\theta-1)(\theta/R^3+1)}{1-\beta^3}+1$ and $E = \frac{\theta-1}{1-\beta^{3/2}}+1$. For values of n above C, the worst equilibrium is the unique equilibrium of the savings-only game. For $n \in (A, C)$, some savings are done in the savings-only game. For $n \leq A$, first best savings is an equilibrium of the savings-only game.

Fragmentation and repayment

How does the number of groups affect the ability to repay the credit line? When $n = 1$, and there is only one domestic group, the first-best savings-only allocation is an equilibrium; and thus, no credit line would be repaid. When, instead, the country is very fragmented, that is when n is high, no credit line would be ever repaid either. The reason is that the temptation to deviate and consume the current wealth of the government before it is shared in the future by all is just too strong, and the groups cannot commit to saving or repaying the credit line. It is then for intermediate values of fragmentation of the country, that is, intermediate values of n, that the ability to repay a credit line arises. In this case, the credit line offers enough of a commitment device that all groups value it, and can agree in equilibrium to repay it.

The results of Propositions 3.6 and 3.8 allow for a stronger characterization regarding the relation between the amount of debt and the fragmentation of the country. Figure 4 shows how the maximum amount of debt changes with the fragmentation of the country. The model predicts an inverted U-shape relationship between fragmentation and the maximum amount of debt that can be repaid.

Private savings

If the domestic groups could save on their own, and the return on their assets could not be expropriated by the government or other groups, then the first-best savings-only allocation would be an equilibrium of the savings-only game. In that situation, the groups would divide the endowment equally every time it gets realized, and make all the inter-temporal transfers on their own.

One can argue that some of the consumption received by the groups only can be provided through a centralized government. Alternatively, if the government could tax the returns to the private investments made by the groups, then the tragedy-of-the-commons would reappear, as groups would have access to the returns on investments made by others through the government's budget constraint. See, for example, the discussion in Tornell and Velasco (1992).

Illiquid assets

To understand why a credit line is repayable, illiquid bonds are introduced now, and it is shown that if the country had access to a particular type of illiquid assets after default, then no credit line would ever be repaid.

Suppose that after default, the domestic agents could invest in the riskless and liquid bond, but additionally, there is an illiquid bond market that opens in periods T_1 and whose assets pay a return of R^2 in periods T_{ϕ^2}. Importantly, the illiquid bond cannot be traded in periods T_ϕ (that's the illiquid nature of the bond). The presence of this asset would be irrelevant for the equilibrium if there were a representative agent in the economy. However, the following lemma shows that the existence of this illiquid asset eliminates the cyclical demand for external funds, and thus the repayability of any credit line:

Lemma 3.10. *Suppose that (β, n, θ) and R are such that there exists a repayable allocation under credit line b if the illiquid bond market is closed. Then, if the illiquid bond market were open, the first best savings allocation is an equilibrium of the savings-only game.*

What is valuable about a credit line b is its contingent nature: the credit line is only open when the country needs it the most, and it is closed otherwise. That ability to have the credit line closed in intermediate periods provides commitment to the domestic agents not to over-consume in periods T_ϕ. This commitment however can be provided more efficiently by a specific type of illiquid bond.

4 External Enforcement and Welfare

Access to a credit line generates Pareto improvements with respect to the best savings-only allocation both ex-ante (at the time of obtaining the credit) and ex-post (at the time of paying back the debt). The model has no over-borrowing: access to international financial markets is always preferred by all groups at all times.

In what follows I show that the introduction of an external enforcement technology can altered this result by making access to credit welfare reducing in two ways.

First, an exogenous enforcement mechanism can lead to over-borrowing if a tight debt limit is not imposed and the debt issuance is not restricted to the periods when the country needs it the most. In this case, the groups might prefer not to have access to credit.

Second, even if debt issuance is restricted to the periods where it is needed the most and a debt limit is imposed, the introduction of an external enforcement mechanism can destroy domestic cooperation and reduce welfare when compared to the repayable allocation under the same credit-line.

4.1 External Enforcement and Natural Debt Limits

Suppose that the small open economy can now pledge its entire endowment process to the foreigners at any time. Then, the following proposition holds:

Lemma 4.1. *Suppose that conditions 2.1, 2.3 and 2.11 hold. Then the unique equilibrium of the game where the endowment can be pledged in the international financial markets involves maximum indebtedness and zero savings.*

Allowing the country the ability to pledge the endowment leads to a total break down in domestic cooperation. In the initial period, the entire wealth of the country will be spent, and the resulting welfare will be:

$$\frac{R^3}{n(R^3-1)}$$

If the country did not have access to credit at all, then the welfare can be computed according to the results of Theorem 2.14:

$$V_0 = \begin{cases} \frac{1}{n(1-\beta^3)} & ; \text{if}(\beta, n, \theta) \in \Gamma_0 \\ \frac{\theta^2}{n(1-\beta^3)} & ; \text{if } (\beta, n, \theta) \in \Gamma_1 \text{ or } 1 + (1-\beta^3)(\theta n - 1) < \theta^2 \\ \frac{1}{n(1-\beta^3)} \left(1 + \frac{(\theta-1)(n-\theta)(1-\beta^3)}{(1-\beta^3)(n-1)+1-\theta}\right) & ; \text{otherwise} \end{cases}$$

Then the following holds,

Proposition 4.2. *Suppose that conditions 2.1, 2.3 and 2.11 hold. If $(\beta, n, \theta) \notin \Gamma_0$ then there exists an R sufficiently close to $1/\beta$ such that at time 0 all groups strictly prefer having no credit at all rather than having the ability to pledge the entire endowment to the foreigners.*

4.2 External enforcement and the Crowding Out of Domestic Cooperation

In the previous subsection, I have showed that the domestic groups might prefer having no credit at all, rather than having the ability to pledge their entire endowment to the foreigners. The reasoning was that the ability to pledge the entire endowment lead the country to accumulate too much debt, a situation that sometimes was dominated by the best savings-only equilibrium.

In this subsection I make a different point: starting from a repayable allocation under a credit line b, I then ask the question of how the best equilibrium, and welfare, changes if the payments of the credit line b where exogenously enforced. Under certain conditions, domestic agents are better off without the external enforcement technology. Given that it is the original credit line that is enforced, it is not over-borrowing that causes the reduction in welfare, instead the result is due to a collapse in domestic cooperation:

Lemma 4.3. *Suppose that* $(\beta, n, \theta) \in \Gamma_0$*,* $\theta^2 > 1 + (1 - \beta^3)(n - 1)$ *and* $R^3 \in [\underline{R}_1, \bar{R}_1]$*. If a credit line* b *is exogenously enforced, then the unique subgame perfect equilibrium has* $a_0 = a_1 = 0$*.*

From equation ($\tilde{SC}_0'$) it follows that a repayable allocation has to feature $a_0 > 0$ and $a_1 > 0$, given the conditions assumed in Lemma 4.3: $(\beta, n, \theta) \in \Gamma_0$, $\theta^2 > 1 + (1 - \beta^3)(n - 1)$ and $R^3 \in [\underline{R}_1, \bar{R}_1]$. That is, when the credit line is not enforced, the parties can sustain cooperation on their remaining domestic assets: the threat of a deviation which brings about the collapse of the ability to borrow, is enough to make the parties willing to save. However, once borrowing is enforced exogenously, a deviation is not that painful, and the country looses the ability to sustain any domestic savings. Parties will then be worse off:

Proposition 4.4. *Suppose that* $(\beta, n, \theta) \in \Gamma_0$*,* $\theta^2 > 1 + (1 - \beta^3)(n - 1)$ *and* $R^3 \in [\underline{R}_1, \bar{R}_1]$*. Then, at all times, all parties prefer that the credit line is not exogenously enforced.*

That is, the introduction of an exogenous enforcement technology destroys existing domestic cooperation, reducing welfare for a given size of the credit line.

5 Conclusion

This paper extends the reputational reason for sovereign debt repayment originally formalized in Eaton and Gersovitz (1981) by introducing political economy considerations. The main departure with previous work is that the sovereign entity is not assumed to maximize the utility of a representative agent, but instead the political economy interactions that lead to default or repayment are explicitly modeled. The basic structure of the model is based on the tragedy-of-the-commons model introduced by Tornell and Velasco (1992) and Tornell and Lane (1999). The model is simple, but rich enough to demonstrate the importance of the representative agent in generating the results of Bulow and Rogoff (1989). The main implication of the model is that sovereign debt can be repaid in the absence of punishments. The model has novel predictions regarding the interactions of borrowing with political economy variables. I also show that the reason why countries repay their debts can alter the welfare implications of policies aimed at improving the functioning of sovereign debt markets.

A Collection of Proofs

Proof of Proposition 2.9

Standard arguments imply that the set of sustainable payoffs forms a convex, closed, and bounded set (note that the set of sustainable payoffs is bounded above by the first-best payoffs frontier, which is finite given $\beta R < 1$, and below by the zero-consumption allocation).

The symmetric structure of the game then implies that there exists an efficient allocation, d, that delivers the same time-zero payoff to all groups. Let V be that payoff, so that $V_1^i = V$ for all $i \in I$. Now, let a new allocation $\bar{d}$ be such that $\bar{d}_t^i = \sum_j d_t^j / n$. Given that $\bar{d}$ is a convex combination of d, the sustainability constraints will be satisfied, as well as the feasibility constraints. So, $\bar{d}$ is a sustainable allocation that delivers V to all players at time 0 (it is efficient) and has $\bar{d}_t^i$ independent of i for all $t \geq 0$.

Proof of Proposition 2.10

Beginning with last part first: the symmetric first-best allocation is stationary and generates the following sustainability constraints:

$$\frac{1}{n(1-\beta^3)}(\beta\phi R)^2 \geq 1; \;\; \frac{1}{n(1-\beta^3)}\beta\phi^2 R^2 \geq \phi R; \;\; \frac{1}{n(1-\beta^3)}\phi^2 R^2 \geq 0$$

which are all satisfied if $\beta R \phi \geq n$.

For $\beta R\phi < n$, let d be an efficient and symmetric allocation. Suppose that at some time $t_1 \in T_{\phi^2}$, $a_{t_1}(d) > 0$. Consider the lowest $t_2 > t_1$ for which $d_{t_2} > 0$, that is t_2 is the first period after t_1 in which strictly positive consumption is allocated to the groups. Such a period must exist, because otherwise the continuation allocation prescribes zero consumption at all times and sustainability constraints cannot possibly be satisfied in periods $t_1 + 1$ and beyond.

Consider the following feasible perturbation to the allocation d: d_{t_2} by $\varepsilon > 0$ and increase d_{t_1} by $\varepsilon / R^{t_2 - t_1}$, while keeping all other d_τ the same. Note that this change implies that a_τ decreases by $n\varepsilon/R^{t_2-\tau}$ for $t_1 \leq \tau < t_2$, while remaining constant everywhere else. The implied change in time 0 utility of the perturbation is $\beta^{t_1}\varepsilon(\phi^2 - \phi_{t_2}(R\beta)^{t_2-t_1})/R^{t_2-t_1}$. This change is positive as $\phi_{t_2} \leq \phi^2$ and $\beta R < 1$. The perturbation thus generates an allocation that Pareto dominates d as of time 0 (or for any time $\tau \leq t1$).

Now, consider the impact of the change to d on the sustainability constraints as of time τ, as given by (SC'). For times $\tau \geq t_2$, the perturbation to d does not affect the sustainability constraints because neither d_{t+1} nor a_t are affected for $t \geq t_2$. For times $\tau < t_1$, the change strictly increases the equilibrium utilities of the groups (by the same argument used to show that the new allocation Pareto dominates d at time 0), and it does not affects the deviation utility because a_τ has not changed for $\tau < t_1$. At time t_1, again, the left-hand side of the sustainability constraint has increased, as in the previous sentence, while the right-hand side has been reduced; so, the sustainability constraint has been slacked. For times τ such that $t_1 < \tau < t_2$, note that the left-hand side of the sustainability constraints decreases by $\phi_{t_2}\beta^{t_2-\tau}\varepsilon$ but the right-hand side also decreases, by an amount equal to $\phi_\tau n\varepsilon R^{\tau-t_2}$. The sustainability constraint at time τ is relaxed after the perturbation if

$$\phi_{t_2}(\beta R)^{t_2-\tau} < \phi_\tau n \tag{11}$$

holds.

There are three possible cases to consider. First, if $t_2 \in T_1$ then $(\beta R)^{t_2-\tau} < \phi_\tau n$ for all τ, as implied by $\beta R < 1$, $\phi_t \geq 1$ and $n > 1$; hence the sustainability constraints are relaxed. Second, for $t_2 \in T_\phi$, the condition (11) holds for all τ as $(\phi\beta R)(\beta R)^{t_2-\tau-1} < n \leq \phi_\tau n$, which follows from $\phi_\tau \geq 1$, $\beta R < 1$, $\tau < t_2$ and $\phi\beta R < n$. Finally, special care needs to be taken for the case when $t_2 \in T_{\phi^2}$. For this, it is useful to note that when sustainability constraints are holding at times $t \in T_\phi$, the sustainability constraints also are holding at times $t-1$, for times t such that $t_1 < t < t_2$. To see this, note that the sustainability constraint at time $t \in T_\phi$ is:

$$\sum_{k=1}^{\infty} \phi_{\tau+k}\beta^k d_{\tau+k} - \sum_{k=1}^{\infty} \phi_{\tau+k}\beta^k \frac{e_{\tau+k}}{n} \geq \phi a_t, \tag{12}$$

while at time $t-1 \in T_1$, using that $d_t = 0$, the sustainability constraint becomes:

$$\beta\left(\sum_{k=1}^{\infty} \phi_{\tau+k}\beta^k d_{\tau+k} - \sum_{k=1}^{\infty} \phi_{\tau+k}\beta^k \frac{e_{\tau+k}}{n}\right) \geq a_{t-1}. \tag{13}$$

Now, $a_{t-1} = a_t/R$ given that $d_t = 0$ from the budget constraint. The term in brackets in equation (13) is the same as the left-hand side of the equation (12). Given that $\beta R\phi > 1$ by assumption, and $a_t > 0$, it follows that (12) implies (13). Then for $t_2 \in T_\phi^2$, it is sufficient to check that the sustainability constraints hold for $\tau \notin T_1$ after the perturbation. For $\tau \in T_{\phi^2}$, the condition becomes $(\beta R)^{t_2-\tau} < n$, which holds given that $\beta R < 1 < n$. For $\tau \in T_\phi$, the condition becomes $(\phi\beta R)(\beta R)^{t_2-\tau-1} < n$, which again follows from $\beta R < 1 < \phi\beta R < n$. Hence the perturbation generates a sustainable allocation that Pareto dominates the original. It is then the case that in an efficient and symmetric allocation, $a_t(d) = 0$ for all $t \in T_{\phi^2}$.

For the stationarity result: let d be an efficient allocation with symmetric payoffs. From the above result, it is now known that $d_t + d_{t+1}/R + d_{t+2}/R^2 = 1$ for $t \in T_1$. Define

$d_i^* = (1-\beta^3)\sum_{j=0}^{\infty}\beta^{3j}d_{3j+i}$ for $i \in \{0,1,2\}$. Let a new allocation $\hat{d}$ be such that $\hat{d}_t = d^*_{t \bmod 3}$. Note that by construction $\hat{d}$ achieves the same payoffs as d as of time 0. The allocation $\hat{d}$ is feasible, and the linearity of the sustainability constraints guarantees that it is also sustainable.

Proof of Proposition 2.12

The proof follows from the stationarity result and feasibility.

Proof of Theorem 2.14

If $(1-\beta^3)(n-1)+1-\theta \le 0$, that is, when $(\beta,n,\theta) \in \Gamma_1$, then any positive pair $(a_0,\tilde{a}_1)$ satisfy the sustainability constraints, and thus $a_0 = \tilde{a}_1 = 1$ is sustainable.

If $(1-\beta^3)(n-1)+1-\theta > 0$, then the sustainability constraints can be rewritten as,

$$\frac{\beta^3(\theta-1)}{\theta((1-\beta^3)(n-1)+1-\theta)} \ge \frac{\tilde{a}_1}{a_0} \ge \frac{(1-\beta^3)(n-1)+1-\theta}{\theta(\theta-1)} \tag{14}$$

If $\frac{\beta^3(\theta-1)/\theta}{(1-\beta^3)(n-1)+1-\theta} < \frac{(1-\beta^3)(n-1)+1-\theta}{\theta(\theta-1)}$, or equivalently, when $(\beta,n,\theta) \in \Gamma_0$, then there are no positive values of a_1 and a_0 that are sustainable, and thus the worst equilibrium is also the efficient one. If $\frac{\beta^3(\theta-1)/\theta}{(1-\beta^3)(n-1)+1-\theta} \ge \frac{(1-\beta^3)(n-1)+1-\theta}{\theta(\theta-1)}$, then it is optimal to pick $a_0 = 1$ and $\tilde{a}_1$ to be the minimum of between 1 (feasibility) or the left hand side of equation (14).

Proof of Proposition 2.15

The proof of existence follows the argument of the proof of Proposition 2.9. One then can proceed to show that $a_t = 0$ for all $t \in T_{\phi^2}$, using the same arguments as in the proof of Proposition 2.10. With that result in hand, it follows that the continuation game after $\hat{t}$ is the same as the solution found to problem (P), and they share the same savings allocation, $a_0^\star$ and $\tilde{a}_1^\star$. For $t < \hat{t}$, sustainability implies that constraints (SC_0) and (SC_1) have to hold. For $t < \hat{t}$ and $t \in T_\phi$, it is optimal to make a_t as high as possible, implying then that $a_t = \min\{Ra_{t-1}, \bar{a}_1\}$, where $\bar{a}_1$ is the savings limit imposed by (SC_1). Finally, for $t < \hat{t}$ and $t \in T_1$: define the set $H \equiv \{(a_0,a_1) \in \mathbb{R}^+ \times \mathbb{R}^+ | a_1 \le Ra_0; a_1 \le \bar{a}_1; a_0 \le \frac{\theta(\theta-1)}{n-\theta}a_1 + \frac{n\beta^3}{n-\theta}(V_0^\star - W_0)\}$. Set H contains the pairs of a_0 and a_1 that are sustainable. Then the optimal initial values for savings are given by the highest pair of $(a_0,a_1) \in H$ such that $a_0 \le Ra_{t_1-1}$. The upper frontier of H is given by $a_1 \le Ra_0$ together with $a_1 \le \bar{a}_1$, and $a_0 \le \bar{a}_0$. The result then follows.

Proof of Lemma 3.4

Towards a contradiction, suppose that allocation d satisfies all the conditions but is not sustainable under the credit line. Then this means that there exists a time t such that a sustainable, savings-only allocation $\hat{d}$ of the continuation game starting with assets $Ra_{t-1}(d, b)$ at time t satisfies:

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_\tau \hat{d}^i_\tau \geq \sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_\tau d_\tau > \sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_\tau d^*_\tau$$

for all i, and with a strict inequality for at least one i. But then, this implies that d^* is not efficient. A contradiction.

Proof of Proposition 3.5

The proof will proceed through contradiction through a sequence of lemmas. I will look for the best allocation under a credit line of size b under the assumption that such allocation has positive savings at some time in T_{ϕ^2}.

But before doing that, let me state the constraints that need to be satisfied for any repayable allocation. Let $t_1 \in T^2_\phi$. Constraints (8) can be written as:

$$\beta\left(\frac{Ra_{t_1} - a_{t_1+1} + 1 - Rb}{n} + \beta\phi\frac{Ra_{t_1+1} - a_{t_1+2}}{n} + (\beta\phi)^2\frac{Ra_{t_1+2} - a_{t_1+3} + b}{n} + \beta^3 V(a_{t_1+3})\right) \geq \phi^2 a_{t_1} + \beta W_0 \tag{15}$$

$$\beta\left(\phi\frac{Ra_{t_1+1} - a_{t_1+2}}{n} + \beta\phi^2\frac{Ra_{t_1+2} - a_{t_1+3} + b}{n} + \beta^2 V(a_{t_1+3})\right) \geq (a_{t_1+1} + Rb) + \beta^3 W_0 \tag{16}$$

$$\beta\left(\phi^2\frac{Ra_{t_1+2} - a_{t_1+3} + b}{n} + \beta V(a_{t_1+3})\right) \geq \phi a_{t_1+2} + \beta^2 W_0 \tag{17}$$

And constraints (9) can be written as:

$$\phi^2\frac{Ra_{t_1-1} + b - a_{t_2}}{n} + \beta\left(\frac{Ra_{t_1} - a_{t_1+1} + 1 - Rb}{n} + \beta\phi\frac{Ra_{t_1+1} - a_{t_1+2}}{n} + (\beta\phi)^2\frac{Ra_{t_1+2} - a_{t_1+3} + b}{n} + \beta^3 V(a_{t_1+3})\right) \geq \phi^2\left(\frac{Ra_{t_1-1} + b}{n}\right) + \beta V_0^\star \tag{18}$$

$$\frac{Ra_{t_1} - a_{t_1+1} + 1 - Rb}{n} + \beta\phi\frac{Ra_{t_1+1} - a_{t_1+2}}{n} + (\beta\phi)^2\frac{Ra_{t_1+2} - a_{t_1+3} + b}{n}$$
$$+ \beta^3 V(a_{t_1+3}) \geq \frac{Ra_{t_1} + 1 - a_0^d}{n} + \beta\phi\frac{Ra_0^d - a_1^d}{n} + (\beta\phi)^2\frac{Ra_1^d}{n} + \beta^3 V_0^\star \quad (19)$$

$$\phi\frac{Ra_{t_1+1} - a_{t_1+2}}{n} + \beta\phi^2\frac{Ra_{t_1+2} - a_{t_1+3} + b}{n} + \beta^2 V(a_{t_1+3}) \geq \phi\frac{Ra_{t_1+1} - a_2^d}{n} + \beta\phi^2\frac{Ra_2^d}{n} + \beta^2 V_0^\star \quad (20)$$

where $a_0^d = \min\{Ra_{t_1} + 1, \bar{a}_0\}$, $a_1^d = \min\{Ra_0^d, \bar{a}_1\}$, $a_2^d = \min\{Ra_{t_1+1}, \bar{a}_1\}$ and where $V(a_{t_1+3})$ denotes the continuation value at time $t_1 + 3$.

Lemma A.1. *Inequality (18) implies that inequality (19) holds.*

Proof. Inequality (18) can be rewritten as:

$$\beta\left(\frac{Ra_{t_1} - a_{t_1+1} + 1 - Rb}{n} + \beta\phi\frac{Ra_{t_1+1} - a_{t_1+2}}{n} + (\beta\phi)^2\frac{Ra_{t_1+2} - a_{t_1+3} + b}{n} + \beta^3 V_3\right)$$
$$\geq \phi^2\left(\frac{a_{t_1}}{n}\right) + \beta V_0^\star$$

But now, it follows that:

$$\phi^2\left(\frac{a_{t_1}}{n}\right) + \beta V_0^\star \geq \beta\left(\frac{Ra_{t_1} + 1 - a_0^d}{n} + \beta\phi\frac{Ra_0^d - a_1^d}{n} + (\beta\phi)^2\frac{Ra_1^d}{n} + \beta^3 V_0^\star\right)$$

as $a_{t_1} \geq 0$ and it is efficient to spend the resources in periods T_{ϕ^2} in a savings-only game. □

Hence constraint (19) can be safely ignored, as it is implied by (18).

Suppose that d is an efficient repayable allocation under the credit line b. Then, the following must hold:

Lemma A.2. *Suppose that for some time $t_1 \in T_{\phi^2}$, $a_{t_1} > 0$. Then $d_{t_1+1} = d_{t_1+2} = 0$. If $d_{t_1+3} > 0$ then constraint (16) must be holding with strict equality.*

Proof. The proof of this Lemma proceeds by contradiction. Suppose that for some $t_1 \in T_{\phi^2}$, $a_{t_1} > 0$ and $d_{t_1+2} > 0$. Then one could reduce d_{t_1+2} by $\epsilon > 0$ and increase d_{t_1} by ϵ/R^2. This change generates a Pareto improvement for all $t < t_1$ without affecting the deviations values, so all constraints before t_{t_1} are holding after such a perturbation. The change relaxes constraints (A), (16), (18) and leaves unaffected all constraints at times $t > t_1 + 2$. Hence the perturbation is an improvement and generates a repayable allocation, hence $d_{t_1+2} = 0$. A similar argument work for $d_{t_1+1} = 0$.

If $d_{t_1+3} > 0$ and constraint (16) is slack, then one can perturb the allocation by reducing d_{t_1+3} by $\epsilon > 0$, and increasing d_{t_1} by ϵ/R^3. The value of ϵ can be chosen small enough that (16) holds in the modified allocation. Again, such a change now implies that all remaining constraints are relaxed (or unaffected), and hence generates a contradiction of optimality. Thus $d_{t_1+3} > 0$ implies that (16) must be holding with exact equality. □

Let $V(a)$ denote the the best equilibrium value for a group under the credit line b, starting from some time in T_1 with assets a. There will be some savings in periods $t \in T_{\phi^2}$ if

$$\beta V(0) < \beta V(a) - \frac{\phi^2}{n} a$$

for some $a > 0$.

Lemma A.3. *The sequence of a_t for $t \in T_{\phi^2}$ is weakly increasing over time. If $a_t = a_{t+3} > 0$ for some $t \in T_{\phi^2}$, then $a_{t+3\times n} = a_t > 0$ for all $n \in \mathbb{N}$. If $0 < a_{t-3} < a_{t+3}$ then*

$$a_{t+3} - a_{t-3} > R^2(1 - Rb) + b > 0$$

Proof. Suppose that at some time t_1, the equilibrium calls for some positive savings: a_{t_1}. Then, by Lemma A.2, consumption in the next two periods, (periods $t+1$ and $t+2$) must be zero. At time $t_1 + 3$ the budget constraint is:

$$a_{t_1+3} \leq b + R^2(Ra_{t_1} + 1 - Rb)$$

Given that $R > 1$ and $0 \leq Rb \leq 1$, it follows that $a_{t_1+3} = a_{t_1} > 0$ was a feasible choice as of time $t_1 + 3$. Given that $a_{t_1} > 0$ was optimal at time t_1, assets cannot be smaller than a_{t_1} in period $t_1 + 3$. Hence, it follows that the sequence a_t at periods T_{ϕ^2} is weakly increasing over time if a_t was ever positive.

Suppose now that $a_t = a_{t+3}$, for some $t \in T_{\phi^2}$. Then the $t + 6$ peraniod will feature the same problem, (as $d_{t+4} = d_{t+5} = 0$) and it would be optimal to choose $a_{t+6} = a_t$.

If $a_{t-3} < a_t < a_{t+3}$ for $t \in T_{\phi^2}$, then

$$a_{t+3} - a_{t-3} \geq R^2(1 - bR) + b + (R^3 - 1)a_{t-3}$$

that is, a_{t+3} should not have been feasible at time t, and the last result in the Lemma follows. $\square$

So, when the sequence of a_t in periods T_{ϕ^2} increases, it does so by an amount bounded above zero. And if the sequence does not converge, it must be approaching infinity. However, the following lemma guarantees that last cannot be optimal:

Lemma A.4. *There exists a finite $\bar{a}$ such that:*

$$-\frac{\phi^2}{n}a + \beta V(a) < 0 \quad \textit{for all } a \geq \bar{a} \tag{21}$$

Proof. The value of $V(a)$ is bounded above by the first best value:

$$V(a) \leq \beta^2 \left(\frac{\phi^2}{n} R^3 a + \frac{\phi^2 R^2}{n(1-\beta^3)} \right)$$

and thus

$$-\frac{\phi^2}{n}a + \beta V(a) \leq -(1-(\beta R)^3)\frac{\phi^2}{n}a + \beta^3 \frac{\phi^2 R^2}{n(1-\beta^3)}$$

and just let $\bar{a}$ to be some $\bar{a} > \frac{\beta^3 R^2}{(1-\beta^3)(1-(\beta R)^3)}$. $\square$

Lemma A.3 and A.4 implies that if a_{t_0} is ever positive for some $t_0 \in T_{\phi^2}$, then the sequence a_t for $t \in T_{\phi^2}$ must converge to some finite a_2 in finite time. Once the sequence has converged, provisions must be positive at times $t \in T_{\phi^2}$ (that is, consumption cannot be forever zero). But, from Lemma A.2, it follows that constraint (16) must be holding with strict equality:

$$V(a_2) = \beta^2 \phi^2 \frac{R^2(Ra_2 + 1 - Rb) + b - a_2}{n(1-\beta^3)} = 1 + Ra_2 + \beta^3 W_0 \tag{22}$$

And equations (A) and (17) must also hold:

$$\beta\phi^2 \frac{R^2(Ra_2 + 1 - Rb) + b - a_2}{n(1-\beta^3)} \geq \phi R(Ra_2 + 1 - Rb) + \beta^2 W_0 \tag{23}$$

$$\beta^3\phi^2 \frac{R^2(Ra_2 + 1 - Rb) + b - a_2}{n(1-\beta^3)} \geq \phi^2 a_2 + \beta W_0 \tag{24}$$

If $a_2 \geq b$ the consider the following allocation. In the first period T_1, the country defaults and saves $1 + R(a-b)$ consuming Rb. In the following periods T_{ϕ^2}, the country saves $a - b$; in periods T_1, it saves $1+R(a-b)$; and in periods T_ϕ, it saves $R(1+R(a-b))$. This allocation is sustainable and generates a higher payoff (the country consumed Rb in the first period). So a_2 must lie below b.

If $a_2 < b$, then the best allocation under the credit line of size b will feature $a_t = a_2$ for $t \in T_{\phi^2}$. This follows because saving a_2 is feasible at times T_{ϕ^2} given that $a_2 < b$. Now, let $b' = b - a > 0$. Then the stationary allocation $a_0 = 1 - Rb'$, $a_1 = R(1 - Rb')$ and $a_2 = 0$ is a repayable allocation under the credit line of size b'. This follows by noticing that the left hand sides of equations (22), (23), and (24) are the same under the proposed allocation, and the right hand sides have strictly decreased in (22) and (24), while remaining constant in (23). Similarly, constraints (18) and (19) also hold under the new allocation. Note as well that the credit line b' generates the same payoffs as b. Given that I am considering the best allocation under the credit line b, any other allocation with positive savings will deliver a payoff at most as high as under b', which proves the Lemma.

Proof of Proposition 3.6

For a given b, the slope in the $(a_0, \tilde{a}_1)$-space of the boundary of inequality ($\tilde{SC}'_0$) is always higher than the corresponding slope of inequality ($\tilde{SC}'_1$), as $(\beta, n, \theta) \in \Gamma_0$. As discussed above, $\theta^2 > 1 + (1-\beta^3)(n-1)$ is a necessary condition for there to be a repayable credit line when $(\beta, n, \theta) \in \Gamma_0$: this last condition guarantees that the slopes of the boundaries of inequalities ($\tilde{SC}'_0$) and ($\tilde{SC}'_1$) are less than one. For there to exist a repayable allocation, the crossing point of these boundaries, denoted by $(a_0^c, \tilde{a}_1^c)$, has to be such that $a_0^c \geq \tilde{a}_1^c \geq 0$ (see panels (b) and (c) of figure 2). Solving for $(a_0^c, \tilde{a}_1^c)$ delivers:

$$\begin{aligned}
a_0^c =& \frac{n\left[\beta^3(n-1) - n + \theta\right] + (\theta^2/R^3 - \beta^3)(n-1)}{(n-\theta)^2 - \beta^3(n-1)^2} Rb \\
\tilde{a}_1^c =& \frac{\theta(n-\theta)/R^3 - \beta^3(n-1)}{(n-\theta)^2 - \beta^3(n-1)^2} Rb
\end{aligned}$$

where $(\beta, n, \theta) \in \Gamma_0$ implies $(n-\theta)^2 - \beta^3(n-1)^2 > 0$. Thus, for $a_0^c \geq \tilde{a}_1^c$, it is necessary that

$$R^3 \leq \frac{\theta^2 - \theta}{1 + (1-\beta^3)(n-1) - \theta} \tag{25}$$

where $\frac{\theta^2-\theta}{1+(1-\beta^3)(n-1)-\theta} > 1$ given that $1 + (1-\beta^3)(n-1) < \theta^2$. For $\tilde{a}_1^c \geq 0$, it is necessary that $\theta(n-\theta)/R^3 \geq \beta^3(n-1)$, or equivalently:

$$R^3 \leq \frac{\theta(n-\theta)}{\beta^3(n-1)} \tag{26}$$

Note as well that $\frac{\theta(n-\theta)}{\beta^3(n-1)} \geq \frac{\theta^2-\theta}{1+(1-\beta^3)(n-1)-\theta}$ by $(n-\theta)^2 - \beta^3(n-1)^2 > 0$, so inequality (25) implies (26), and thus inequality (26) can be ignored.

From inequalities ($\tilde{SC}_0'$) and ($\tilde{SC}_1'$), it follows that if $\frac{\theta^2}{R^3} - \beta^3 - n(1-\beta^3) \geq 0$, or equivalently:

$$R^3 \leq \frac{\theta^2}{1+(1-\beta^3)(n-1)} \tag{27}$$

then any $Rb \leq 1$, $a_0 = \tilde{a}_1 = 0$ constitute a repayable allocation.

If inequality (27) does not hold, but condition (25) does, the most that can be repaid is obtained when the set that solves inequalities ($\tilde{SC}_0'$), $a_0 \geq \tilde{a}_1$ and $a_0 \leq 1 - Rb$ is a singleton. That is, inequality ($\tilde{SC}_0'$) holds with equality, $a_0 = 1 - Rb$, and $\tilde{a}_1 = a_0$. Using the binding ($\tilde{SC}_0'$), one then can solve for the maximum amount of debt:

$$R\bar{b} = \frac{\theta^2 - (1+(1-\beta^3)(n-1))}{\theta^2} \frac{R^3}{R^3-1} < 1$$

which is less than one and positive given $\theta^2 > 1 + (1-\beta^3)(n-1)$ and that condition (27) does not hold. Condition (25) guarantees that inequality ($\tilde{SC}_1'$) holds.

Finally, note that

$$\frac{\theta^2}{1+(n-1)(1-\beta^3)} < \frac{\theta^2-\theta}{1+(1-\beta^3)(n-1)-\theta}$$

from $\theta^2 > 1 + (n-1)(1-\beta^3)$. This completes the proof.

Proof of Lemma 3.7

Part (i) follows by noting that if there exists a repayable allocation where ($\tilde{SC}_1$) and feasibility are slack, one can always increase $\tilde{a}_1$. Such an increase relaxes (DC_0') and ($\tilde{SC}_1'$). For part (ii), if either ($\tilde{SC}_1'$) or feasibility holds, it follows that

$$\tilde{a}_1 = \min\left\{a_0, \frac{\theta}{(n-\theta)R^3} Rb + \frac{n\beta^3}{\theta(n-\theta)}(\hat{V}_0 - W_0)\right\}$$

and $\tilde{a}_1 \geq \min\{a_0, \bar{a}_1\}$, as $\hat{V}_0 \geq V_0^\star$ and $b \geq 0$. The value in periods T_ϕ under the allocation is:

$$\beta\hat{V}_1 = \frac{\theta}{n}(a_0 - \tilde{a}_1) + \frac{\theta^2}{n}(\tilde{a}_1 + b/R^2) + \beta^3\hat{V}_0$$

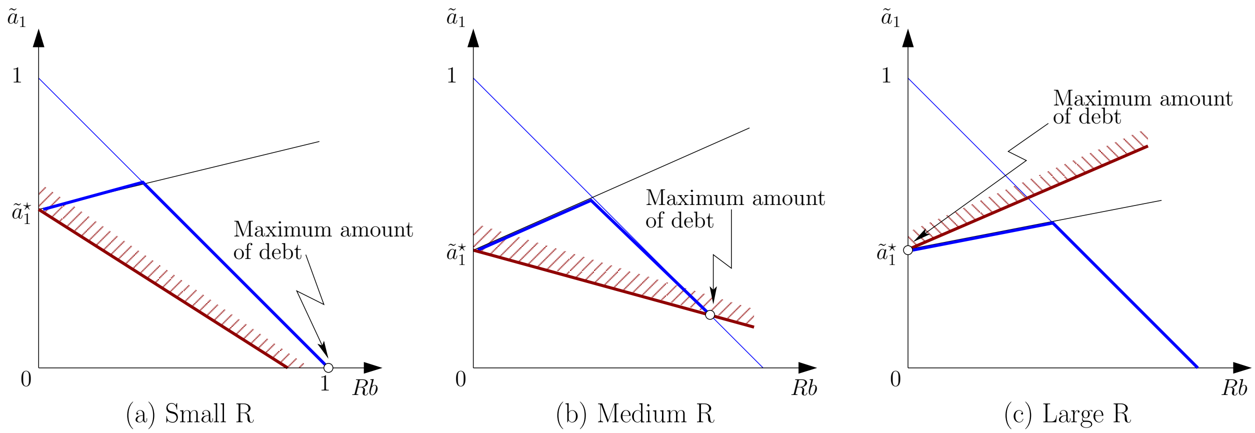

Figure 5: The case when $(\beta, n, \theta) \notin \Gamma_0 \cup \Gamma_1$. The red set represents inequality (28), while the thick blue line is equation (29). The figure illustrates Proposition 3.8, showing the three possible levels of the interest rate. The repayable allocation point with the highest debt level is represented by a small circle.

and $\tilde{a}_1 \geq \min\{a_0, \bar{a}_1/R\}$ implies that constraint (DC_1) is satisfied.

Proof of Proposition 3.8

The set of $(a_0, \tilde{a}_1)$ where both ($\tilde{SC}_1'$) and $a_0 \geq \tilde{a}_1$ hold has a boundary given by the line:

$$\tilde{a}_1(a_0) = \min\left\{a_0, \frac{\left(\frac{\theta^2}{R^3} - \beta^3\right) Rb + \beta^3(\theta-1)a_0}{\theta\left[(1-\beta^3)(n-1)+1-\theta\right]}\right\}$$

Given the above, an repayable allocation exists if the set where $(a_0, \tilde{a}_1(a_0))$ intersects (DC_0') and $0 \leq a_0 \leq 1 - Rb$ is non-empty. Given that the right-hand side of (DC_0') increases in a_0 and $\tilde{a}_1$, it follows that if a repayable allocation exists, an allocation with $a_0 = 1 - Rb$ is also repayable. The constraints then become:

$$\tilde{a}_1 \geq \frac{\beta^3(\theta-1)/\theta}{1+(1-\beta^3)(n-1)-\theta} - \frac{\theta/R^3 - 1}{1-\theta} Rb \tag{28}$$

$$\tilde{a}_1(a_0) = \min\left\{1 - Rb, \frac{\beta^3(\theta-1)/\theta}{1+(1-\beta^3)(n-1)-\theta} + \frac{\left(\frac{\theta}{R^3} - \beta^3\right)}{1+(1-\beta^3)(n-1)-\theta} Rb\right\} \tag{29}$$

Figure 5 plots the three possible situations where the above constraints lie. The set of debt levels under which a repayable allocation exists is convex and always contains 0.

The allocation under a credit line of size $b = 1/R$, which by feasibility implies $\tilde{a}_1 = a_0 = 0$, is repayable iff:

$$\frac{\beta^3(\theta-1)/\theta}{1+(1-\beta^3)(n-1)-\theta} - \frac{\theta/R^3-1}{\theta-1} \leq 0$$

which is equivalent to:

$$R^3 \leq \theta\left[1+\frac{\beta^3(\theta-1)^2/\theta}{1+(1-\beta^3)(n-1)-\theta}\right]^{-1} \tag{30}$$

where $\theta\left[1+\frac{\beta^3(\theta-1)^2/\theta}{1+(1-\beta^3)(n-1)-\theta}\right]^{-1} > 1$, which follows from equation (10).[21]

More generally, a necessary and sufficient condition for the existence of a credit line with a repayable allocation is:

$$\frac{\theta/R^3-\beta^3}{1+(1-\beta^3)(n-1)-\theta} \geq -\frac{\theta/R^3-1}{\theta-1}$$

which is equivalent to,

$$R^3 \leq \frac{\theta(n-1)}{n-\theta} \tag{31}$$

where one can confirm that inequality (30) implies inequality (31).

In the case that $b = 1/R$ is not repayable, the maximum amount of debt that can be sustained obtains at the intersection of the line defined by (28) and $1 - Rb$, which delivers:

$$R\bar{b} = \left[\frac{(\theta n-1)(1-\beta^3)+1-\theta^2}{(1+(1-\beta^3)(n-1)-\theta)\theta}\right]\frac{\theta-1}{\theta}\frac{R^3}{R^3-1}$$

From figure (5), any amount of debt below $\bar{b}$ will also admit a repayable allocation.

Proof of Corollary 3.9

Proposition 3.6 states that a repayable allocation will exist for any credit line of size b, as long as $\theta < 1+(1-\beta^{3/2})(n-1)$ and $\theta > 1+(1-\beta^3)(n-1)$. Proposition 3.8 states that a repayable allocation will exist as long as $1+(1-\beta^3)(n-1) > \theta > 1+(1-\beta^{3/2})(n-1)$ and $\theta^2 < 1+(1-\beta^3)(\theta n-1)$. Now, note that $\theta^2 < 1+(1-\beta^3)(\theta n-1)$ implies that $\theta < 1+(1-\beta^3)(n-1)$. And it follows that a repayable allocation will exist for sufficiently small R, if when $\theta > 1+(1-\beta^{3/2})(n-1)$ it also holds that $\theta^2 < 1+(1-\beta^3)(\theta n-1)$, and if instead $\theta < 1+(1-\beta^{3/2})(n-1)$ then it also holds $\theta^2 > 1+(1-\beta^3)(n-1)$. Given that if

[21] To see this note that $\theta\left(1+\frac{\beta^3(\theta-1)^2/\theta}{1+(1-\beta^3)(n-1)-\theta}\right)^{-1} = \left(\frac{1}{\theta}+\left(1-\frac{1}{\theta}\right)\frac{\beta^3(\theta-1)}{\theta(1+(1-\beta^3)(n-1)-\theta)}\right)^{-1}$ which is less than 1 given equation (10) and $\theta > 1$.

$\theta = 1 + (1-\beta^{3/2})(n-1)$, both $\theta^2 < 1 + (1-\beta^3)(\theta n - 1)$ and $\theta^2 > 1 + (1-\beta^3)(n-1)$, the result follows.

Proof of Lemma 3.10

To show this, assume that the country will invest in periods T_1 the endowment in the illiquid asset. In periods T_ϕ and T_{ϕ^2}, there are no sustainability constraints as no savings are done, and the wealth of the country cannot be appropriated as it is placed in an illiquid bond. In periods T_1 the sustainability constraints are:

$$\frac{\theta^2}{1-\beta^3} \geq 1 + \frac{\beta^3}{1-\beta^3}\frac{1}{n} \tag{32}$$

which is equivalent to: $n\theta^2 \geq 1 + (n-1)(1-\beta^2)$. From corollary 3.9, a credit line of size b admits a repayable allocation if $\theta^2 \geq 1 + (n-1)(1-\beta^3)$. Given that $n \geq 2$, this implies that inequality (32) holds, and that the first-best savings is an equilibrium.

Proof of Lemma 4.1

Given that the entire endowment can be pledged, it is then without loss of generality to consider the equivalent savings-only problem with no endowment but starting with a wealth in period 0 that is equal to the present value of the endowment stream: $Ra_{-1} = 1/(R^3-1)$. As before, in an efficient equilibrium, all the wealth that arrives to periods T_{ϕ^2} will be consumed. In periods T_ϕ then, the sustainability constraint is:

$$\frac{\theta^2 a_1/R}{n} \geq \theta a_1/R$$

which cannot be satisfied if $\theta < n$ for any positive level of assets. In period T_ϕ all assets will be spent. Finally, in periods T_1, the sustainability constraint becomes:

$$\frac{\theta a_0}{n} \geq a_0$$

which again cannot be satisfied for any positive a_0. The unique equilibrium is to borrow the entire endowment and spend it in the first possible period.

Proof of Proposition 4.2

When $(\beta, n, \theta) \notin \Gamma_0$, it follows that $V_0 > R^3/(n(R^3-1))$ when $R = 1/\beta$. By continuity there exists a $R_0 < 1/\beta$ so that for all $R \in (R_0, 1/\beta)$, V_0 is strictly bigger than $R^3/(n(R^3-1)$.

Proof of Lemma 4.3

Given that the credit line is exogenously enforced, without loss of generality, one can solve the problem of how the parties share the remaining endowment after paying back the credit line. That is, suppose that the endowment process is $1 - Rb$ when $t \in T_1$ and 0 otherwise. Given that $(\beta, n, \theta) \in \Gamma_0$, no cooperation is the unique equilibrium of this savings-only game.

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