03 Exercise Notebook 3

December 31, 2024

1 Exercise 3 - Brian Chen (bc604)

In this exercise, you will analyse a dataset obtained from the London transport system (TfL). The data is in a filled called `tfl_readership.csv` (comma-separated-values format). As in Exercise 2, we will load and view the data using `pandas`.

[1]:
```
# If you are running this on Google Colab, uncomment and run the following␣
↪lines; otherwise ignore this cell
# from google.colab import drive
# drive.mount('/content/drive')
```

[2]:
```
import math
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
```

[3]:
```
# Load data
df_tfl = pd.read_csv('tfl_ridership.csv')
# If running on Google Colab change path to '/content/drive/MyDrive/
↪IB-Data-Science/Exercises/tfl_ridership.csv'

df_tfl.head(13)
```

[3]:
```
       Year Period        Start          End Days  Bus cash (000s)  \
0   2000/01   P 01  01 Apr '00  29 Apr '00   29d              884
1   2000/01   P 02  30 Apr '00  27 May '00   28d              949
2   2000/01   P 03  28 May '00  24 Jun '00   28d              945
3   2000/01   P 04  25 Jun '00  22 Jul '00   28d              981
4   2000/01   P 05  23 Jul '00  19 Aug '00   28d              958
5   2000/01   P 06  20 Aug '00  16 Sep '00   28d              984
6   2000/01   P 07  17 Sep '00  14 Oct '00   28d             1001
7   2000/01   P 08  15 Oct '00  11 Nov '00   28d              979
8   2000/01   P 09  12 Nov '00  09 Dec '00   28d              971
9   2000/01   P 10  10 Dec '00  06 Jan '01   28d              912
10  2000/01   P 11  07 Jan '01  03 Feb '01   28d              943
11  2000/01   P 12  04 Feb '01  03 Mar '01   28d              975
12  2000/01   P 13  04 Mar '01  31 Mar '01   28d              974
```

	Bus Oyster PAYG (000s)	Bus Contactless (000s)	\
0	0	0	
1	0	0	
2	0	0	
3	0	0	
4	0	0	
5	0	0	
6	0	0	
7	0	0	
8	0	0	
9	0	0	
10	0	0	
11	0	0	
12	0	0	

	Bus One Day Bus Pass (000s)	Bus Day Travelcard (000s)	…	\
0	210	231	…	
1	214	205	…	
2	209	221	…	
3	216	241	…	
4	225	248	…	
5	243	236	…	
6	205	216	…	
7	199	221	…	
8	184	212	…	
9	192	211	…	
10	193	186	…	
11	194	210	…	
12	186	204	…	

	Tube Contactless (000s)	Tube Day Travelcard (000s)	\
0	0	655	
1	0	605	
2	0	650	
3	0	708	
4	0	730	
5	0	702	
6	0	639	
7	0	668	
8	0	640	
9	0	631	
10	0	556	
11	0	617	
12	0	584	

	Tube Season Travelcard (000s)	Tube Other incl free (000s)	\
0	1066	200	

```
1                            1168                            217
2                            1154                            212
3                            1196                            214
4                            1165                            165
5                            1164                            151
6                            1286                            196
7                            1298                            220
8                            1302                            242
9                             993                            195
10                           1259                            234
11                           1237                            246
12                           1262                            266

    Tube Total (000s)  TfL Rail (000s)  Overground (000s)  DLR (000s)  \
0                2509                0                  0          96
1                2598                0                  0          93
2                2623                0                  0          98
3                2761                0                  0         105
4                2643                0                  0         103
5                2608                0                  0         100
6                2763                0                  0         107
7                2819                0                  0         113
8                2839                0                  0         114
9                2359                0                  0          90
10               2634                0                  0         110
11               2688                0                  0         120
12               2699                0                  0         119

    Tram (000s)  Air Line (000s)
0          45.8              0.0
1          46.5              0.0
2          47.1              0.0
3          50.8              0.0
4          50.3              0.0
5          49.2              0.0
6          48.8              0.0
7          51.5              0.0
8          54.0              0.0
9          55.3              0.0
10         50.1              0.0
11         50.5              0.0
12         47.7              0.0

[13 rows x 26 columns]
```

Each row of our data frame represents the average daily ridership over a 28/29 day period for various types of transport and tickets (bus, tube etc.). We have used the `.head()` command to display the top 13 rows of the data frame (corresponding to one year). Focusing on the "Tube

Total" column, notice the dip in ridership in row 9 (presumably due to Christmas/New Year's), and also the slight dip during the summer (rows 4,5).

[4]:
```
#df_tfl.sample(3)  #random sample of 3 rows
df_tfl.tail(3)  #last 3 rows
```

[4]:
```
        Year Period       Start         End Days  Bus cash (000s)  \
242  2018/19   P 09  11 Nov '18  08 Dec '18  28d                0
243  2018/19   P 10  09 Dec '18  05 Jan '19  28d                0
244  2018/19   P 11  06 Jan '19  02 Feb '19  28d                0

     Bus Oyster PAYG (000s)  Bus Contactless (000s)  \
242                    1110                    1089
243                    1001                     949
244                    1036                    1075

     Bus One Day Bus Pass (000s)  Bus Day Travelcard (000s)  …  \
242                            0                         41  …
243                            0                         38  …
244                            0                         30  …

     Tube Contactless (000s)  Tube Day Travelcard (000s)  \
242                     1399                         249
243                     1110                         242
244                     1310                         204

     Tube Season Travelcard (000s)  Tube Other incl free (000s)  \
242                           1017                          334
243                            632                          259
244                            924                          305

     Tube Total (000s)  TfL Rail (000s)  Overground (000s)  DLR (000s)  \
242               4221              996                557         355
243               3279              750                414         270
244               3809              929                517         333

     Tram (000s)  Air Line (000s)
242         84.1              2.6
243         66.3              3.2
244         79.3              2.3

[3 rows x 26 columns]
```

The dataframe contains $N = 245$ counting periods (of 28/29 days each) from 1 April 2000 to 2 Feb 2019. We now define a numpy array consisting of the values in the ' Tube Total (000s)' column:

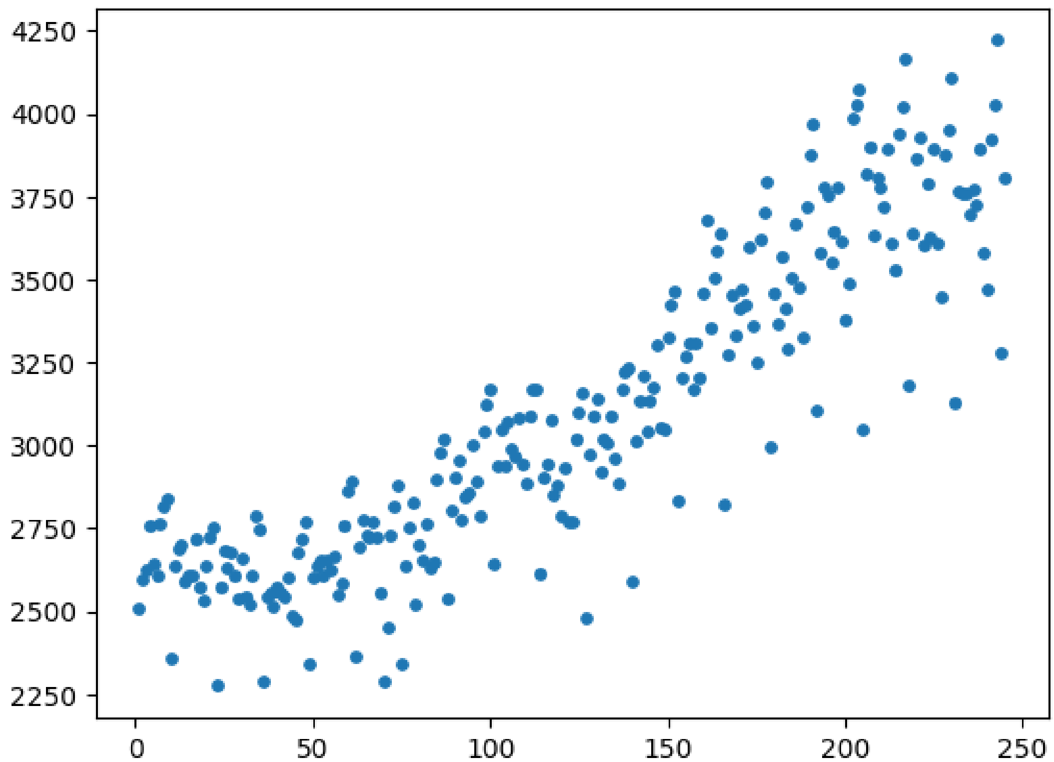
[5]:
```
yvals = np.array(df_tfl['Tube Total (000s)'])
N = np.size(yvals)
```

```
xvals = np.linspace(1,N,N) #an array containing the values 1,2....,N
```

We now have a time series consisting of points (x_i, y_i), for $i = 1, \ldots, N$, where y_i is the average daily tube ridership in counting period $x_i = i$.

1.1 3a) Plot the data in a scatterplot

[6]:
```
#Your code for scatterplot here

plt.scatter(xvals, yvals, s=15)
plt.show()
```



1.2 3b) Fit a linear model $f(x) = \beta_0 + \beta_1 x$ to the data

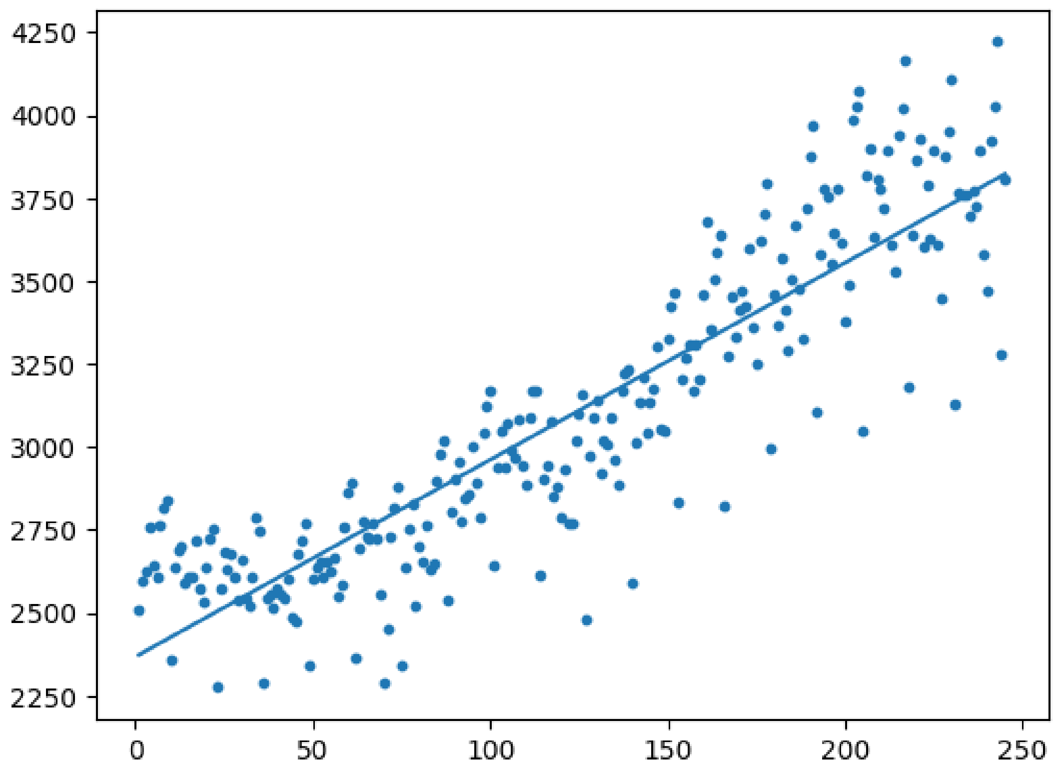
- Print the values of the regression coefficients β_0, β_1 determined using least-squares.
- Plot the fitted model and the scatterplot on the same plot.
- Compute and print the **MSE** and the R^2 coefficient for the fitted model.

All numerical outputs should be displayed to three decimal places.

[7]:
```
#Your code here

X = np.column_stack(([ xvals**0, xvals ]))
```

```python
beta = np.linalg.lstsq(X, yvals, rcond=None)[0]
fit = X.dot(beta)

print(f"beta_0 = {beta[0]:.3f}, beta_1 = {beta[1]:.3f}")

plt.plot(xvals, fit)
plt.scatter(xvals, yvals, s=10)
plt.show()

SSE = np.linalg.norm(yvals - fit) ** 2
MSE = SSE / N

fit_0 = np.array([np.mean(yvals) for _ in range(N)])
SSE_0 = np.linalg.norm(yvals - fit_0) ** 2
MSE_0 = SSE_0 / N

print(f"MSE = {MSE:.3f}, R^2 = {(1 - SSE/SSE_0):.3f}")
```

```
beta_0 = 2367.382, beta_1 = 5.939
```

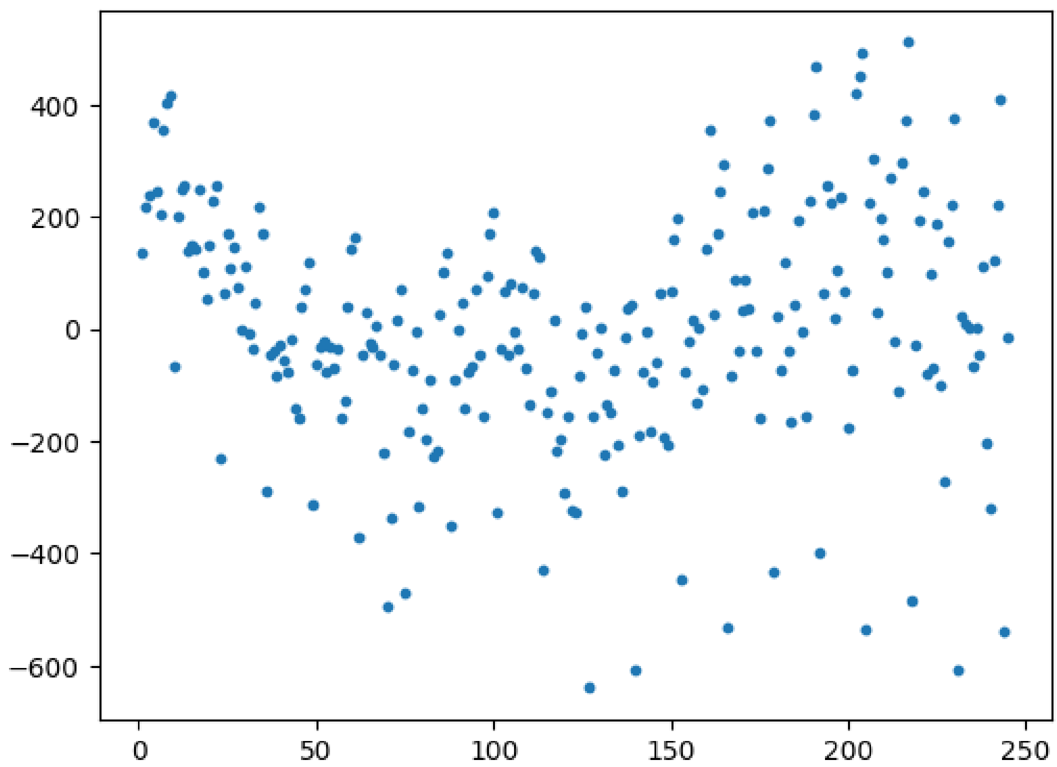


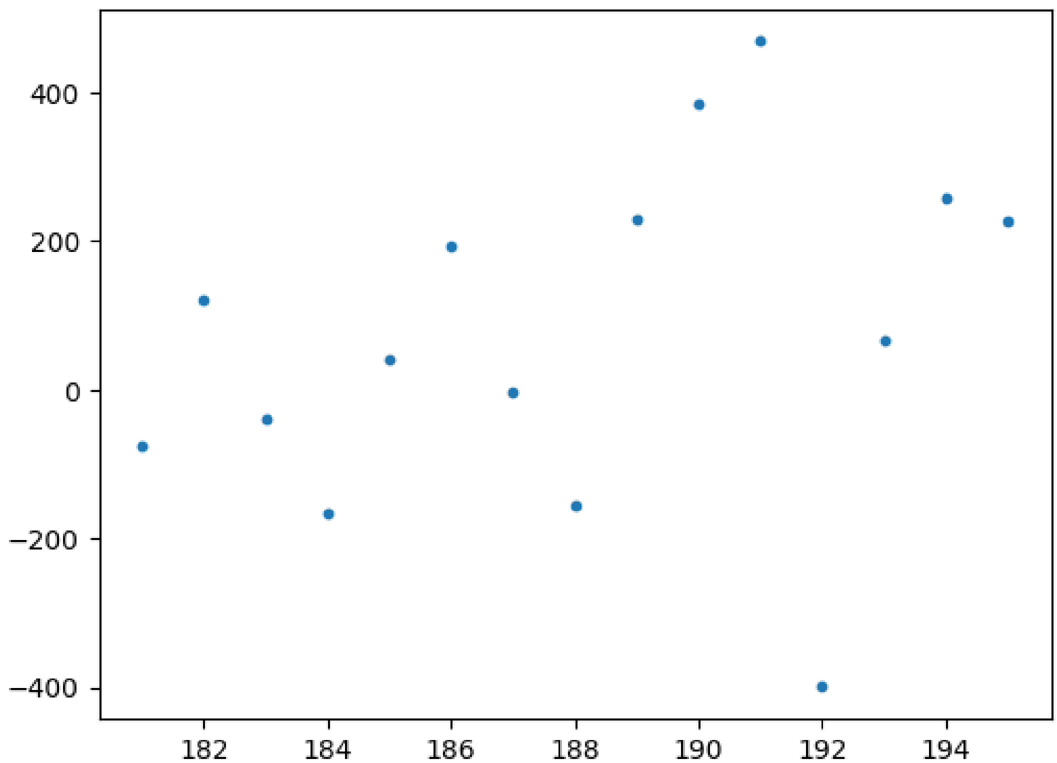

```
MSE = 45323.636, R^2 = 0.796
```

1.3 3c) Plotting the residuals

- Plot the residuals on a scatterplot
- Also plot the residuals over a short duration and comment on whether you can discern any periodic components.

[8]:
```
# Your code here

resids = yvals - fit

plt.scatter(xvals, resids, s=10)
plt.show()

plt.scatter(xvals[180:195], resids[180:195], s=10)
plt.show()
```





< Comment on periodic components here >

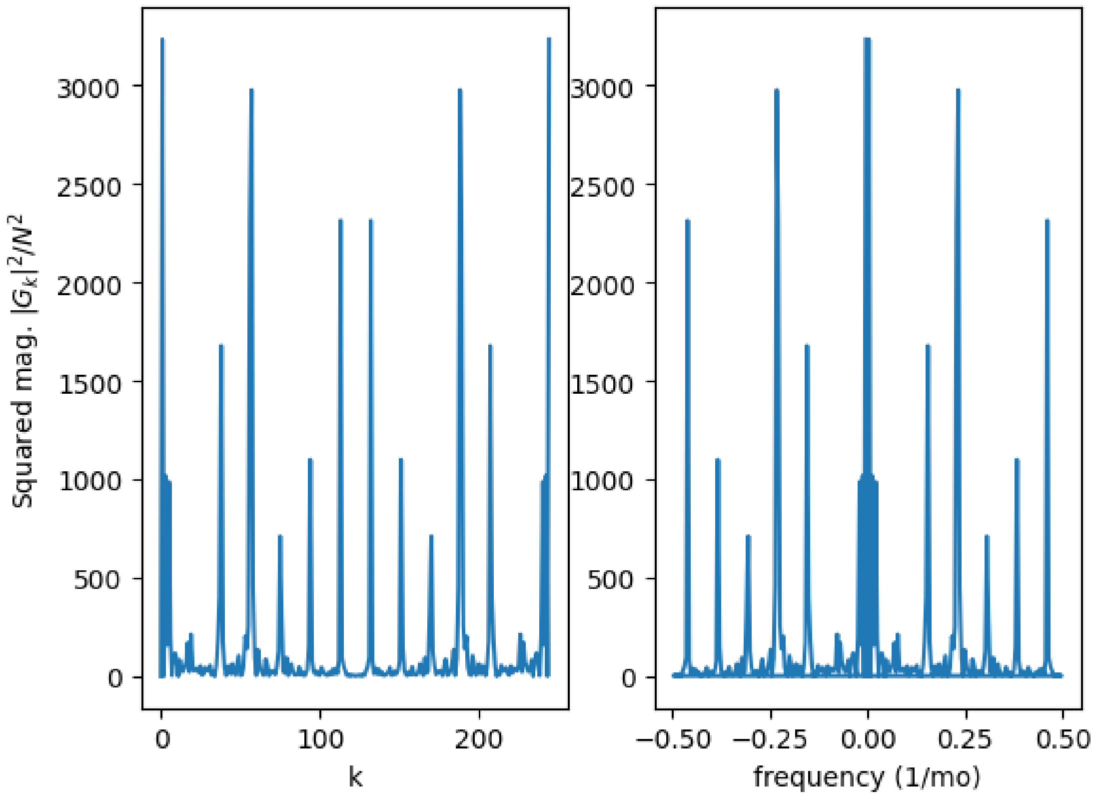
There doesn't appear to be any obvious periodic trends inside the residuals. Zooming out or zooming in has no significant difference in this regard.

1.3.1 3d) Periodogram

- Compute and plot the peridogram of the residuals. (Recall that the periodogram is the squared-magnitude of the DFT coefficients.)
- Identify the indices/frequencies for which the periogram value exceeds **50%** of the maximum.

[9]:
```
# Your code to compute and plot the periodogram

T = 1

pgram = np.abs(np.fft.fft(resids, N)/N)**2
indices = np.linspace(0, (N-1), num = N)
freqs_in_hz = np.fft.fftfreq(N)/T
freqs_in_rads = freqs_in_hz*2*math.pi

plt.subplot(121)
plt.plot(indices, pgram)
plt.xlabel('k')
```

```
plt.ylabel('Squared mag. $|G_k|^2/N^2$')
plt.subplot(122)
plt.plot(freqs_in_hz, pgram)
plt.xlabel('frequency (1/mo)')
plt.show()
```



[10]:
```
# Your code to identify the indices for which the periodogram value exceeds 50%
↪of the maximum

top_indices = indices[(pgram > 0.5*np.max(pgram))]
print(top_indices)
```

```
[  1.  38.  56.  57. 113. 132. 188. 189. 207. 244.]
```

1.4 3e) To the residuals, fit a model of the form

$$\beta_{1s}\sin(\omega_1 x) + \beta_{1c}\cos(\omega_1 x) + \beta_{2s}\sin(\omega_2 x) + \beta_{2c}\cos(\omega_2 x) + ... + \beta_{Ks}\sin(\omega_K x) + \beta_{Kc}\cos(\omega_K x).$$

The frequencies $\omega_1, \ldots, \omega_K$ in the model are those corresponding to the indices identified in Part 2c. (Hint: Each of the sines and cosines will correspond to one column in your X-matrix.)

- Print the values of the regression coefficients obtained using least-squares.

All numerical outputs should be displayed to three decimal places.

[11]:
```
# Your code here

top_ws = 2 * np.pi * freqs_in_hz[(pgram > 0.5*np.max(pgram))]

XT = np.vstack([np.sin(w * xvals) for w in top_ws if w > 0] + [np.cos(w *␣
 ↪xvals) for w in top_ws if w > 0])
X = np.transpose(XT)

beta_sc = np.linalg.inv(XT.dot(X)).dot(XT).dot(resids)

print(f"beta_s = {beta_sc[0]:.3f}, beta_c = {beta_sc[1]:.3f}")
```

```
beta_s = -51.253, beta_c = 61.628
```

1.4.1 3f) The combined fit

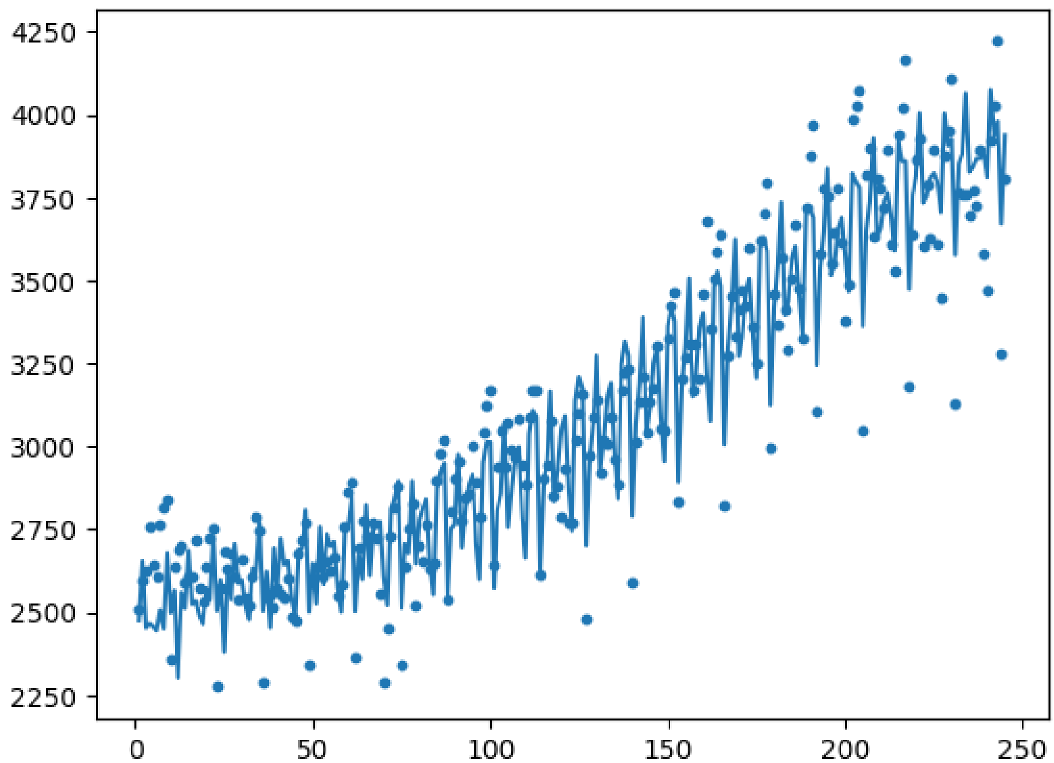
- Plot the combined fit together with a scatterplot of the data
- Compute and print the final **MSE** and R^2 coefficient. Comment on the improvement over the linear fit.

The combined fit, which corresponds to the full model

$$f(x) = \beta_0 + \beta_1 x + \beta_{s1}\sin(\omega_1 x) + \beta_{c1}\cos(\omega_1 x) + ... + \beta_{sk}\sin(\omega_k x) + \beta_{ck}\cos(\omega_k x),$$

can be obtained by adding the fits in parts 2b) and 2e).

[12]:
```
# Your code here

fit_sc = X.dot(beta_sc)
fit_tot = fit + fit_sc

plt.plot(xvals, fit_tot)
plt.scatter(xvals, yvals, s=10)
plt.show()

SSE_tot = np.linalg.norm(yvals - fit_tot) ** 2
MSE_tot = SSE_tot / N

print(f"MSE = {MSE_tot:.3f}, R^2 = {(1 - SSE_tot/SSE_0):.3f}")
```



`MSE = 20297.501, R^2 = 0.908`

< Add comment on the improvement over the linear fit. >

Significant improvement of R^2 and significant reduction in MSE, showing that this fit is a lot better than the simple linear fit.