Research Notes

February 11, 2020

1 Definitions of B(-)

$B : U \to U$ can be defined as a HIT:
$B(T) :\equiv$

- $* : B(T)$
- $q : T \to (* =_{B(T)} *)$

Also as a pushout:

$$\begin{tikzcd} T + \mathbf{1} \arrow[r] \arrow[d] & \mathbf{1} \arrow[d] \\ \mathbf{1} \arrow[r] & B(T) \end{tikzcd}$$

Or as the connected component:
$B(\mathrm{Aut}(T)) :\equiv \Sigma_{X:U} ||X = T||_{-1}$

2 $B(\Omega T)$ embedding into $B\mathbf{Aut}(\Omega T)$

The first attempt was to $B \hookrightarrow B\mathrm{Aut}(G)$ with a map defined by $* \mapsto *$ and $q_t \mapsto q_E$ where $E : T \to T$ is an equivalence. Doesn't work because T doesn't have a group structure on it.

Modifying that you can show an embedding $BT \hookrightarrow B\mathrm{Aut}(BT)$ defined by $* \mapsto *$ and $q_t \mapsto q_E$ where $E : BT \to BT$ is defined by $* \mapsto *$ and $q_s \mapsto q_t q_s q_{t^{-1}}$

2.1 Best version

What we probably really want here is an embedding $BG \hookrightarrow B\mathrm{Aut}(G)$. More generally, for any type T, we can define the embedding $B\Omega T \hookrightarrow B\mathrm{Aut}\Omega T$ by $* \mapsto *$ and $q_g \mapsto q_{E_g}$ where $E_g : G \cong G$ is a map defined by $* \mapsto *$ and $h \mapsto ghg^{-1}$.

The special case of $S^1 \cong B(\mathbb{Z})$ mapping into $B(S^1 = S^1) \cong B(S^1 + S^1)$. $B(S^1 = S^1)$ is a point, two loops, and on each of those one 2-loop. The embedding of the kind defined here maps the star to the star and loop to one of the two loops.

3 Yoneda Embedding

$y : X \hookrightarrow (X \to U)$ is an embedding defined by $x \mapsto \lambda y.(x =_X y)$ and $x_1 = x_2 \mapsto$ (the map from $\lambda y.(x_1 = y) \to \lambda y.(x_2 = y)$ given by path composition).

4 To do

We know that $(S^1 = S^1) \cong (S^1 + S^1)$. Can we get a similar formula for $(\bigvee_n S^1 = \bigvee_n S^1)$?