Research Notes

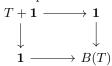
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1 Definitions of B(-)

 $B:U\to U$ can be defined as a HIT: $B(T):\equiv$

- * : B(T)
- $q: T \to (* =_{B(T)} *)$

Also as a pushout:



Or as the connected component:

 $B(\operatorname{Aut}(T)) :\equiv \Sigma_{X:U} || X = T ||_{-1}$

2 $B(\Omega T)$ embedding into $B\mathbf{Aut}(\Omega T)$

The first attempt was to $B \hookrightarrow B\mathrm{Aut}(G)$ with a map defined by $* \mapsto *$ and $q_t \mapsto q_E$ where $E: T \to T$ is an equivalence. Doesn't work because T doesn't have a group structure on it.

Modifying that you can show an embedding $BT \hookrightarrow B\mathrm{Aut}(BT)$ defined by $*\mapsto *$ and $q_t\mapsto q_E$ where $E:BT\to BT$ is defined by $*\mapsto *$ and $q_s\mapsto q_tq_sq_{t^{-1}}$

2.1 Best version

What we probably really want here is an embedding $BG \hookrightarrow B\mathrm{Aut}(G)$. More generally, for any type T, we can define the embedding $B\Omega T \hookrightarrow B\mathrm{Aut}\Omega T$ by $*\mapsto *$ and $q_g \mapsto q_{E_g}$ where $E_g : G \cong G$ is a map defined by $*\mapsto *$ and $h\mapsto ghg^{-1}$.

The special case of $S^1 \cong B(\mathbb{Z})$ mapping into $B(S^1 = S^1) \cong B(S^1 + S^1)$. $B(S^1 = S^1)$ is a point, two loops, and on each of those one 2-loop. The embedding of the kind defined here maps the star to the star and loop to one of the two loops.

3 Yoneda Embedding

 $y: X \hookrightarrow (X \to U)$ is an embedding defined by $x \mapsto \lambda y.(x =_X y)$ and $x_1 = x_2 \mapsto$ (the map from $\lambda y.(x_1 = y) \to \lambda y.(x_2 = y)$ given by path composition).

4 To do

We know that $(S^1=S^1)\cong (S^1+S^1)$. Can we get a similar formula for $(\bigvee_n S^1=\bigvee_n S^1)$?