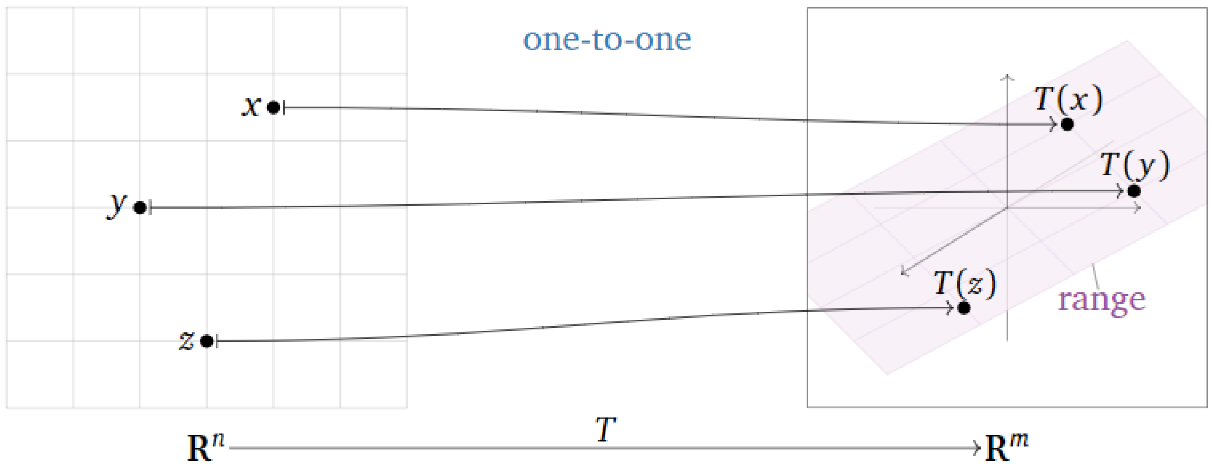
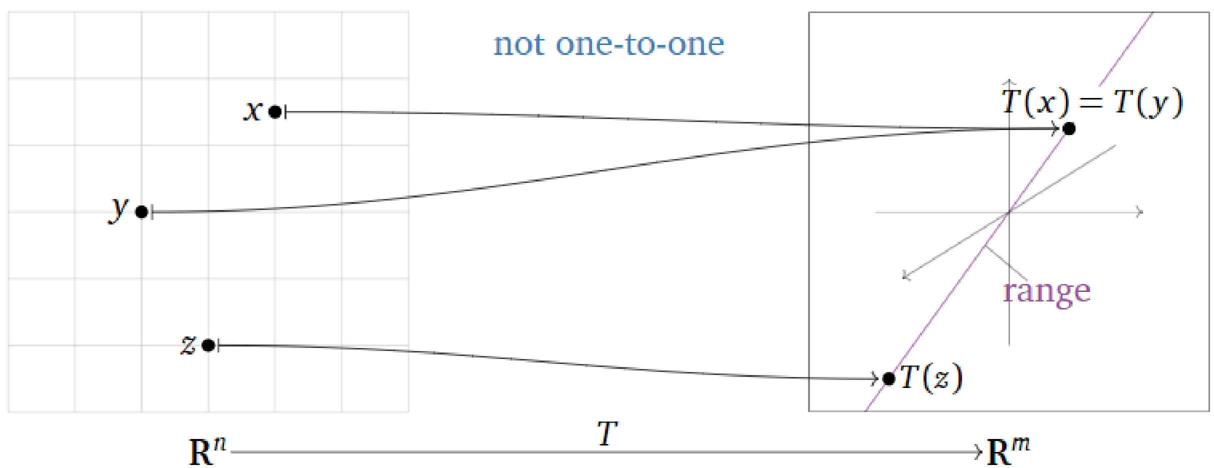
One-to-one Transformations 单射(也译作injective)

Definition (One-to-one transformations). A transformation $T: \mathbf{R}^n \to \mathbf{R}^m$ is *one-to-one* if, for every vector b in $\mathbf{R}^m$, the equation $T(x) = b$ has *at most one* solution x in $\mathbf{R}^n$.

定义：映射T：$\mathbb{R}^n \to \mathbb{R}^m$是单射，则对于任意$\mathbf{b} \in \mathbb{R}^m$，方程$T(\mathbf{x}) = \mathbf{b}$在$\mathbf{x} \in \mathbb{R}^n$中至多有一个解。



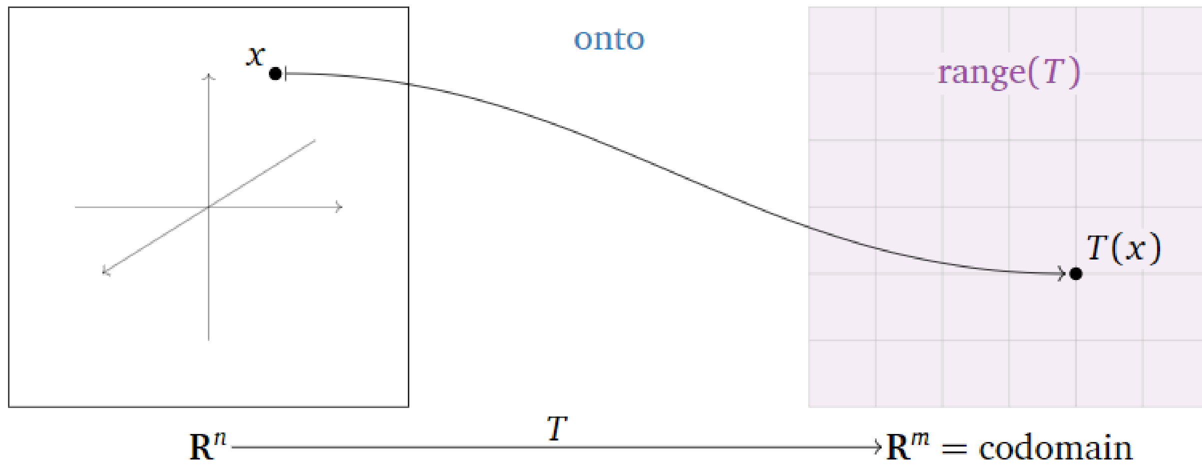


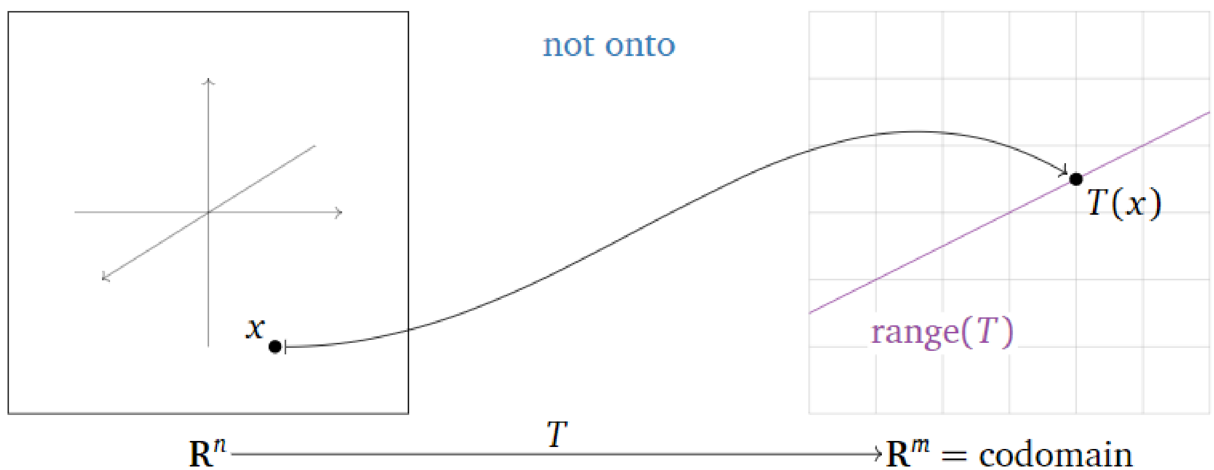
Onto Transformations

满射(也译作surjective)

Definition (Onto transformations). A transformation $T : \mathbf{R}^n \to \mathbf{R}^m$ is *onto* if, for every vector b in $\mathbf{R}^m$, the equation $T(x) = b$ has *at least one* solution x in $\mathbf{R}^n$.

定义：映射T：$\mathbb{R}^n \to \mathbb{R}^m$是满射，则对于任意$\mathbf{b} \in \mathbb{R}^m$，方程$T(\mathbf{x}) = \mathbf{b}$在$\mathbf{x} \in \mathbb{R}^n$中至少有一个解。



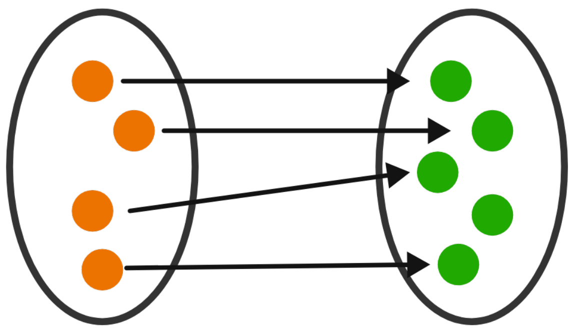


Bijective (双射)

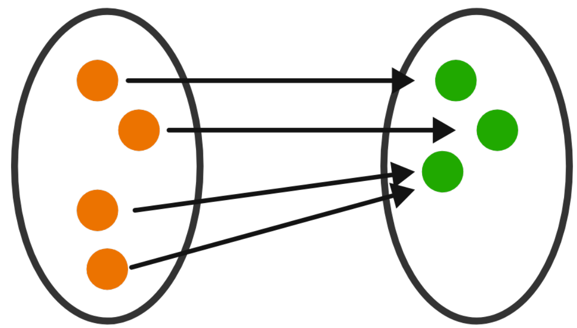
Definition： A transformation T： $\mathbb{R}^n \rightarrow \mathbb{R}^m$ is bijective if it is injective and surjective; that is, every element $\mathbf{b} \in \mathbb{R}^m$ is the image of exactly one element $\mathbf{x} \in \mathbb{R}^\mathrm{n}$.

定义：映射T：$\mathbb{R}^n \rightarrow \mathbb{R}^m$是双射，则映射$T$既为单射也为满射，即对于任意$\mathbf{b} \in \mathbb{R}^m$，方程$T(\mathbf{x}) = \mathbf{b}$在$\mathbf{x} \in \mathbb{R}^\mathrm{n}$中有唯一解。

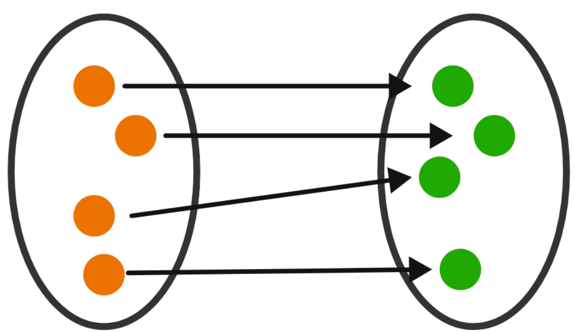
Injection (One-to-One)



Surjection (Onto)



Bijection (One-to-One and Onto)



Comparison

The above expositions of one-to-one and onto transformations were written to mirror each other. However, “one-to-one” and “onto” are complementary notions: neither one implies the other. Below we have provided a chart for comparing the two. In the chart, A is an $m \times n$ matrix, and $T : \mathrm{R}^n \to \mathrm{R}^m$ is the matrix transformation $T(x) = Ax$.

T is one-to-one	T is onto
$T(\mathbf{x}) = \mathbf{b}$ has at most one solution for every $\mathbf{b}$. 对每个$\mathbf{b}$,方程$T(\mathbf{x}) = \mathbf{b}$在至多有一个解.	$T(\mathbf{x}) = \mathbf{b}$ has at least one solution for every $\mathbf{b}$. 对每个$\mathbf{b}$,方程$T(\mathbf{x}) = \mathbf{b}$至少有一个解.
The columns of A are linearly independent. A的列向量线性独立.	The columns of A span $\mathbb{R}^m$. A的列向量张成$\mathbb{R}^m$空间.
A has a pivot in every column. A的每列都有主元.	A has a pivot in every row. A的每行都有主元.
The range of T has dimension n. T的陪域是n维的.	The range of T has dimension m. T的陪域是m维的.

Note that in general, a transformation T is bijective if it is injective and surjective; that is, every element $\mathbf{b} \in \mathbb{R}^m$ is the image of exactly one element $\mathbf{x} \in \mathbb{R}^n$.
注意：映射T是双射，则映射T既为单射也为满射，即对于任意$\mathbf{b} \in \mathbb{R}^m$，方程$T(\mathbf{x}) = \mathbf{b}$在$\mathbf{x} \in \mathbb{R}^n$中有唯一解。