



# Chapter 7: Sparse Arrays, Cell Arrays, and Structures

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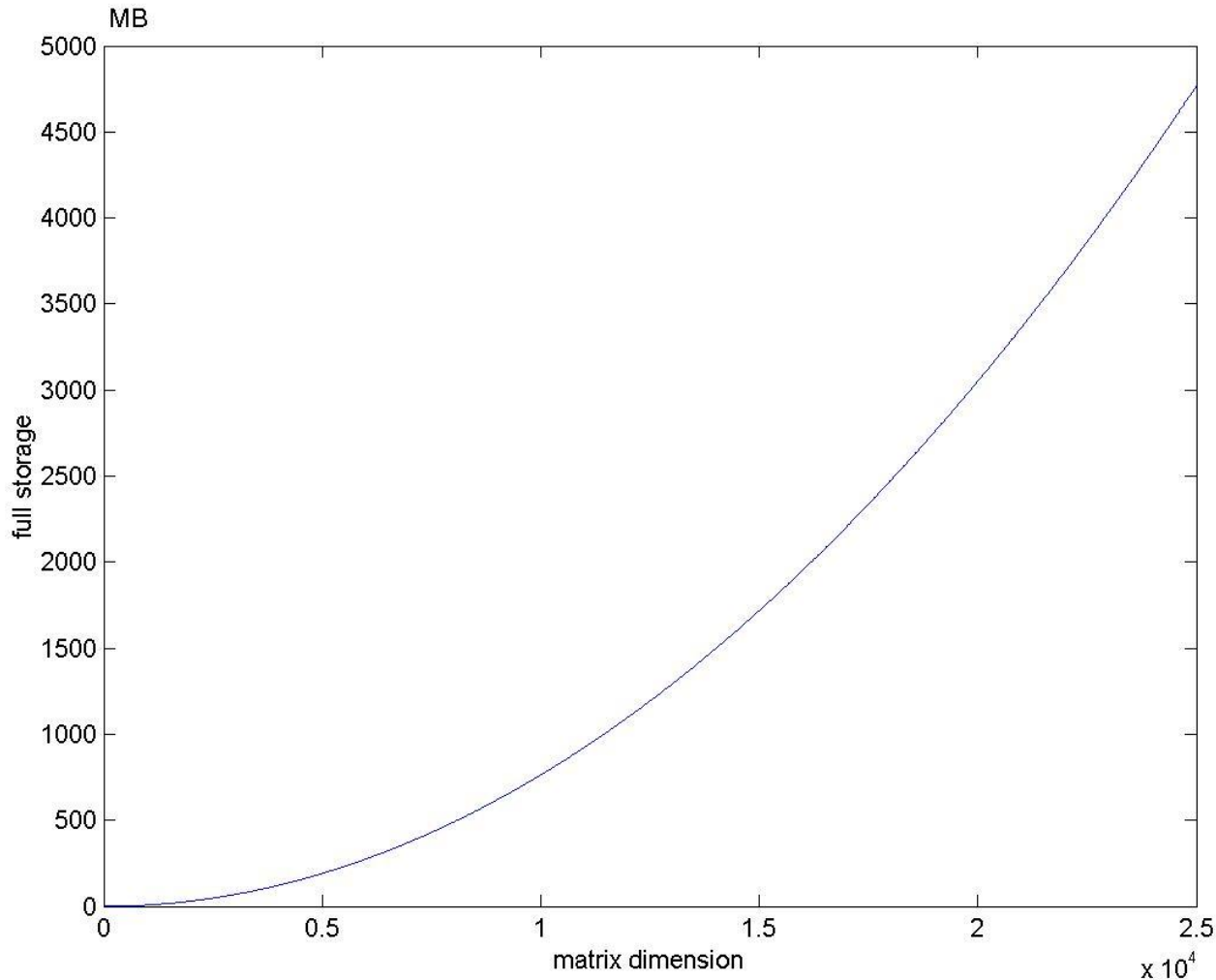


# Why introduce Sparse Arrays?

---

```
>> x=1:10:25000;  
>> y=((x.^2*8)/1024)/1024;% mb  
>> plot(x,y)  
>> text(0,5000,'MB')  
>> ylabel('full storage')  
>> xlabel('matrix dimension')  
>> y(end)           ➔  ans = 4764.9  
>> y(end)/1024      ➔  ans = 4.6533
```

# Full matrix storage



memory:  
 $25000 \times 25000$   
 $\times 8$  bytes  
 $= 4765\text{MB}$   
 $= 4.65\text{GB}$



# Matrix / Array Storage

In standard matrix implementations,

- the **computational complexity is  $O(N^3)$  operations**,
- the **storage complexity is  $O(N^2)$  operations**.

Table 1: Memory requirement of a matrix in double precision versus its dimension  $N$

Dimension $N$	1000	3000	5000	7000
Storage (MB)	7.7	68.7	190.8	373.9

9000	11000	13000	15000	17000
618.0	923.2	1289.4	1716.6	2204.9



# Matrix / Array Storage (Cont.)

---

- 1) What happens if we don't have enough memory storage?
- 2) **Virtual memory**?
- 3) What could we do?



# Sparse Arrays

- ```
[0 1 0 0 0 0 0 0 0 0 0;
 1 0 0 0 1 0 0 0 0 0 0;
 0 0 0 1 0 0 0 0 0 0 0;
 0 0 1 0 0 0 0 0 0 1 0;
 0 1 0 0 0 1 0 0 0 0 0;
 0 0 0 0 1 0 0 0 0 0 0;
 0 0 0 0 0 0 0 0 1 0 1;
 0 0 0 0 0 0 1 0 0 0 0;
 0 0 0 1 0 0 0 0 0 0 0;
 0 0 0 0 0 0 1 0 0 0 0]
```

$10 \times 10 = 100$  elements

value 1: 14 elements

value 0: 86 elements

Sparse data type:

**three** values for each nonzero element are saved:

value of the element;  
the row and column numbers  
where the element is located



# Sparse Arrays (Cont.)

---

```
>> a=eye(10)
```

```
a =
```

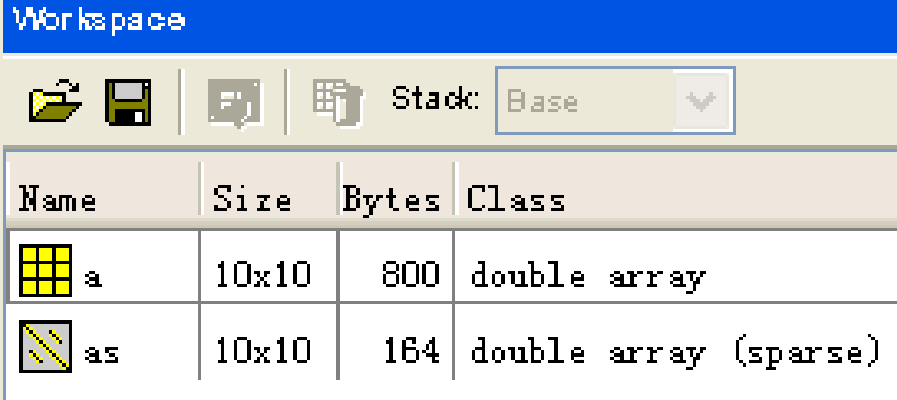
|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

# Sparse Arrays (Cont.)



```
>> as=sparse(a)
```

```
as =
```

```
(1,1)    1  
(2,2)    1  
(3,3)    1  
(4,4)    1  
(5,5)    1  
(6,6)    1  
(7,7)    1  
(8,8)    1  
(9,9)    1  
(10,10)   1
```



The image shows a screenshot of the MATLAB Workspace window. It has a blue title bar labeled 'Workspace'. Below the title bar is a toolbar with icons for opening, saving, and deleting files, and a 'Stack' dropdown menu currently set to 'Base'. The main area contains a table with the following data:

| Name                                                                                 | Size  | Bytes | Class                 |
|--------------------------------------------------------------------------------------|-------|-------|-----------------------|
|  a  | 10x10 | 800   | double array          |
|  as | 10x10 | 164   | double array (sparse) |

**压缩了5倍：原空间的20%就够了**





# Generating Sparse Matrices

```
>> a=speye(5)
```

a =

|       |   |
|-------|---|
| (1,1) | 1 |
| (2,2) | 1 |
| (3,3) | 1 |
| (4,4) | 1 |
| (5,5) | 1 |

```
>> b=full(a)
```

b =

|   |   |   |   |   |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 |



# Working with Sparse Matrices

```
>> a=speye(5)
```

a =

|       |   |
|-------|---|
| (1,1) | 1 |
| (2,2) | 1 |
| (3,3) | 1 |
| (4,4) | 1 |
| (5,5) | 1 |

```
>> b=full(a)
```

b =

|   |   |   |   |   |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 |



## Working with Sparse Matrices (Cont.)

```
>> a(2,1)=2
```

a =

|       |   |
|-------|---|
| (1,1) | 1 |
| (2,1) | 2 |
| (2,2) | 1 |
| (3,3) | 1 |
| (4,4) | 1 |
| (5,5) | 1 |

```
>> c=a+2*b
```

c =

|   |   |   |   |   |
|---|---|---|---|---|
| 3 | 0 | 0 | 0 | 0 |
| 2 | 3 | 0 | 0 | 0 |
| 0 | 0 | 3 | 0 | 0 |
| 0 | 0 | 0 | 3 | 0 |
| 0 | 0 | 0 | 0 | 3 |

full matrix



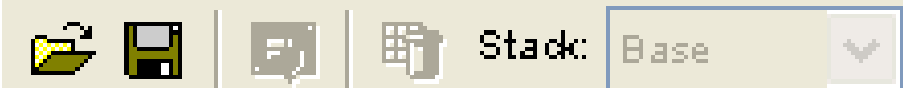

# Cell Arrays

- Each element of a cell array is a pointer to another data structure, and those data structures can be of different types.

|                                                     |                                     |
|-----------------------------------------------------|-------------------------------------|
| cell 1,1<br>'project name:<br>Computer networks'    | cell 1,2<br>'PI: Lisa'              |
| cell 2,1<br>['J.F.Mends';<br>'A. L. Albert'<br>...] | cell 2,2<br>[2000;<br>3000;<br>...] |

# Creating Cell Arrays

`a=cell(2,2)`

| Workspace                                                                           |      |       |            |
|-------------------------------------------------------------------------------------|------|-------|------------|
|   |      |       |            |
| Name                                                                                | Size | Bytes | Class      |
|  a | 2x2  | 16    | cell array |

```
>> a=cell(2,2)
```

```
a =
```

```
    []    []  
    []    []
```



## Creating Cell Arrays (Cont.)

```
>> a{1,1}=[1 2 3; 4 5 6; 7 8 9];  
>> a{1,2}='Matlab Programming';  
>> a{2,1}=[3+4i 5; 4+6i 7];  
>> a{2,2}=[];
```

**a{m,n}**: set/display the **values** of cell  
a at location (m,n)

**a(m,n)**: set/display the data **structure** of cell  
a at location (m,n)

# Parentheses ( ), Brackets [ ], Braces { }

>> help .

Operators and special characters.

## Arithmetic operators.

|          |                                   |      |
|----------|-----------------------------------|------|
| plus     | - Plus                            | +    |
| uplus    | - Unary plus                      | +    |
| minus    | - Minus                           | -    |
| uminus   | - Unary minus                     | -    |
| mtimes   | - Matrix multiply                 | *    |
| times    | - Array multiply                  | .*   |
| mpower   | - Matrix power                    | ^    |
| power    | - Array power                     | .^   |
| mldivide | - Backslash or left matrix divide | \    |
| mrdivide | - Slash or right matrix divide    | /    |
| ldivide  | - Left array divide               | .\   |
| rdivide  | - Right array divide              | ./   |
| kron     | - Kronecker tensor product        | kron |

## Relational operators.

|    |                         |    |
|----|-------------------------|----|
| eq | - Equal                 | == |
| ne | - Not equal             | ~= |
| lt | - Less than             | <  |
| gt | - Greater than          | >  |
| le | - Less than or equal    | <= |
| ge | - Greater than or equal | >= |

## Logical operators.

|     |                                              |   |
|-----|----------------------------------------------|---|
| and | - Logical AND                                | & |
| or  | - Logical OR                                 |   |
| not | - Logical NOT                                | ~ |
| xor | - Logical EXCLUSIVE OR                       |   |
| any | - True if any element of vector is nonzero   |   |
| all | - True if all elements of vector are nonzero |   |

## Special characters.

|       |         |   |
|-------|---------|---|
| colon | - Colon | : |
|-------|---------|---|

Parentheses ( )

Brackets [ ]

Braces { }

|       |                            |     |
|-------|----------------------------|-----|
| punct | - Function handle creation | @   |
| punct | - Decimal point            | .   |
| punct | - Structure field access   | .   |
| punct | - Parent directory         | ..  |
| punct | - Continuation             | ... |
| punct | - Separator                | ,   |
| punct | - Semicolon                | ;   |
| punct | - Comment                  | %   |

|            |                                   |             |
|------------|-----------------------------------|-------------|
| punct      | - Invoke operating system command | !           |
| punct      | - Assignment                      | =           |
| punct      | - Quote                           | '           |
| transpose  | - Transpose                       | .'          |
| ctranspose | - Complex conjugate transpose     | '           |
| horzcat    | - Horizontal concatenation        | [,]         |
| vertcat    | - Vertical concatenation          | [;]         |
| subsasgn   | - Subscripted assignment          | ( ), { }, . |
| subsref    | - Subscripted reference           | ( ), { }, . |

## Bitwise operators.

|          |                                   |
|----------|-----------------------------------|
| bitand   | - Bit-wise AND.                   |
| bitcmp   | - Complement bits.                |
| bitor    | - Bit-wise OR.                    |
| bitmax   | - Maximum floating point integer. |
| bitxor   | - Bit-wise XOR.                   |
| bitset   | - Set bit.                        |
| bitget   | - Get bit.                        |
| bitshift | - Bit-wise shift.                 |

## Set operators.

|           |                        |
|-----------|------------------------|
| union     | - Set union.           |
| unique    | - Set unique.          |
| intersect | - Set intersection.    |
| setdiff   | - Set difference.      |
| setxor    | - Set exclusive-or.    |
| ismember  | - True for set member. |



# Viewing the Cell Arrays

---

```
>> a{1,1}
```

```
ans =
```

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

```
>> a(1,1)
```

```
ans =
```

[3x3 double]





# Viewing the Cell Arrays(Cont.)

---

```
>> a
```

```
a =
```

```
[3x3 double]    'Matlab Programming'  
[2x2 double]    []
```

Displays the data structures in each element of a cell array in a condensed form that limits each data structure to a single line.

输入: condensed juice

简明英汉词典

索引

● **condensed juice**  
浓缩(果)汁

- ≡ condense
- ≡ condensed
- ≡ condensed aromatic
- ≡ condensed balance sheet
- ≡ condensed buttermilk
- ≡ condensed film
- ≡ condensed fish soluble
- ≡ condensed fluid
- ≡ condensed gas
- ≡ condensed income statement
- ≡ condensed instruction deck
- ≡ **condensed juice**
- ≡ condensed milk
- ≡ condensed monolayer
- ≡ condensed nucleus

- ≡ condensed phase
- ≡ condensed phosphate
- ≡ condensed profile
- ≡ condensed section
- ≡ condensed spark
- ≡ condensed specifications
- ≡ condensed steam
- ≡ condensed tannin extracts
- ≡ condensed type
- ≡ condensed water outlet
- ≡ condensed water removal
- ≡ condensed whey
- ≡ condenser
- ≡ condenser armature
- ≡ condenser auxiliary
- ≡ condenser bank
- ≡ condenser block
- ≡ condenser bolometer
- ≡ condenser box
- ≡ condenser bracket

condenser capacity

简明英汉词典

● **condenser capacity**  
(1) 电容器电容量  
(2) 冷凝器容量

- condenser coil
- condenser component
- condenser coupling
- condenser current
- condenser current compens
- condenser damping
- condenser discharge
- condenser divider
- condenser duty
- condenser excitation
- condenser expansion joint
- condenser flange
- condenser foil
- condenser gasket



# Viewing the Cell Arrays(Cont.)

---

```
>> celldisp(a)
```

```
a{1,1} =
```

```
1    2    3
4    5    6
7    8    9
```

```
a{2,1} =
```

```
3 + 4i    5
4 + 6i    7
```

```
a{1,2} =
```

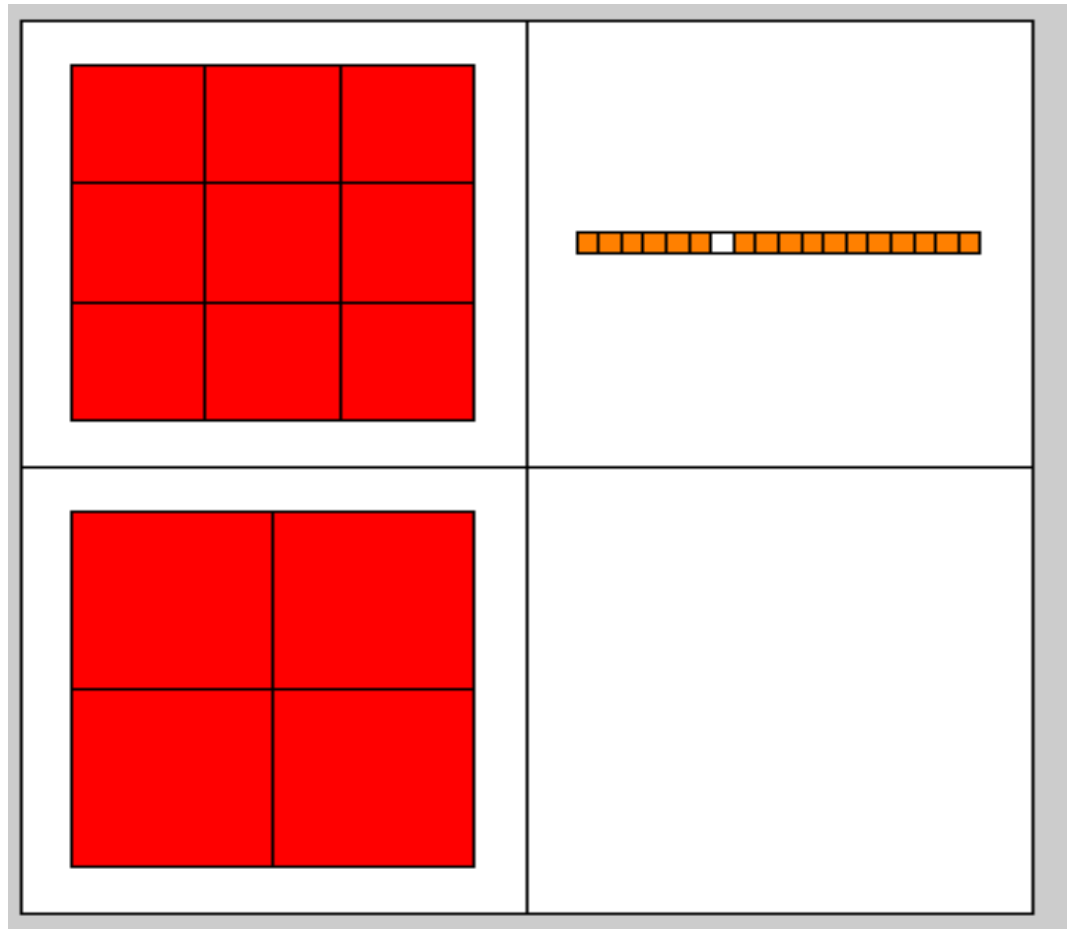
```
Matlab Programming
```

```
a{2,2} =
```

```
[]
```

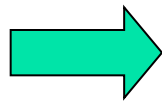
# Viewing the Cell Arrays(Cont.)

>> cellplot(a)



# Extending Cell Arrays

|            |            |
|------------|------------|
| cell (1,1) | cell (1,2) |
| cell(2,1)  | cell(2,2)  |



|           |           |                |
|-----------|-----------|----------------|
| cell(1,1) | cell(1,2) | []             |
| cell(2,1) | cell(2,2) | []             |
| []        | []        | cell(3,3)<br>1 |

cell(3,3)=1



# Extending Cell Arrays (Cont.)

```
a=cell(3,3);  
for i=1:3  
    for j=1:3  
        a{i,j}=[i,j];  
    end  
end
```

(a)

```
for i=1:3  
    for j=1:3  
        a{i,j}=[i,j];  
    end  
end
```

(b)



# Extending Cell Arrays (Cont.)

Recall the learned knowledge!

CASE 1:

```
square=zeros(1,100);  
for i=1:100  
    square(i)=i^2;  
end
```

CASE 2:

```
for i=1:100  
    square(i)=i^2;  
end
```

In CASE 2, the vector square has different size at different time. At each time, Matlab has to

- 1) **create** a new vector;
- 2) **copy** the contents of the old array to the new longer array;
- 3) **add** the new value to the array;
- 4) **delete** the old array.



# Using Data in Cell Arrays

cell(1,1)

```
[1 2 3;  
4 5 6;  
7 8 9]
```

cell(1,2)

```
'Matlab  
Programming'
```

cell(2,1)

```
[3 + 4i 5  
4 + 6i 7]
```

cell(2,2)

```
[]
```

```
>> a{1,1}
```

```
ans =
```

```
1    2    3  
4    5    6  
7    8    9
```

```
>> a{1,1}(2,3)
```

```
ans =
```

```
6
```



# Significance of Cell Arrays

## Application example:

---

```
x=0:pi/100:2*pi;  
y=sin(2*x);
```

```
plot(x,y);
```

```
plot(x,y,'-ro');
```

```
plot(x,y,'-ro','LineWidth',3.0,'MarkerSize',  
8,'MarkerEdgeColor','b','MarkerFaceColor','g');
```



## Significance of Cell Arrays (Cont.)

- A variable number of input arguments
- Arguments may have different data types

```
plot(x,y);
```

```
cell a <== x y
```

```
plot(x,y,'-ro');
```

```
cell a <== x y 'String'
```

```
plot(x,y,'-ro','LineWidth',3.0,'MarkerSize',  
8,'MarkerEdgeColor','b','MarkerFaceColor','g');
```

```
cell a <== x y 'string 1' 'string2' value 1 ....
```



# Significance of Cell Arrays (Cont.)

- **varargin**

This argument returns a cell array with any number of actual arguments.

- **varargout**

cf. nargin  
nargout  
nargchk  
error  
warning  
inputname

```
>> help varargin
VARARGIN Variable length input argument list.
    Allows any number of arguments to a function. The variable
    VARARGIN is a cell array containing the optional arguments to the
    function. VARARGIN must be declared as the last input argument
    and collects all the inputs from that point onwards. In the
    declaration, VARARGIN must be lowercase (i.e., varargin).
    For example, the function,
        function myplot(x,varargin)
            plot(x,varargin{:})
    collects all the inputs starting with the second input into the
    variable "varargin". MYPLOT uses the comma-separated list syntax
    varargin{:} to pass the optional parameters to plot. The call,
        myplot(sin(0:.1:1),'color',[.5 .7 .3],'linestyle',':')
    results in varargin being a 1-by-4 cell array containing the
    values 'color', [.5 .7 .3], 'linestyle', and ':'.
```



## Significance of Cell Arrays (Cont.)

---

```
plot(x,y);
```

```
plot(x,y,'-ro');
```

```
function plotline(varargin)
```



## Significance of Cell Arrays (Cont.)

---

```
function plotline(varargin)
```

```
msg=nargchk(2,inf,nargin);  
error(msg);
```

```
j=0;  
string="";
```



## Significance of Cell Arrays (Cont.)

---

```
for i=1:nargin
    if ischar(varargin{i})
        string=varargin{i};
    else
        j=j+1;
        x(j)=varargin{i}(1);
        y(j)=varargin{i}(2);
    end
end
```



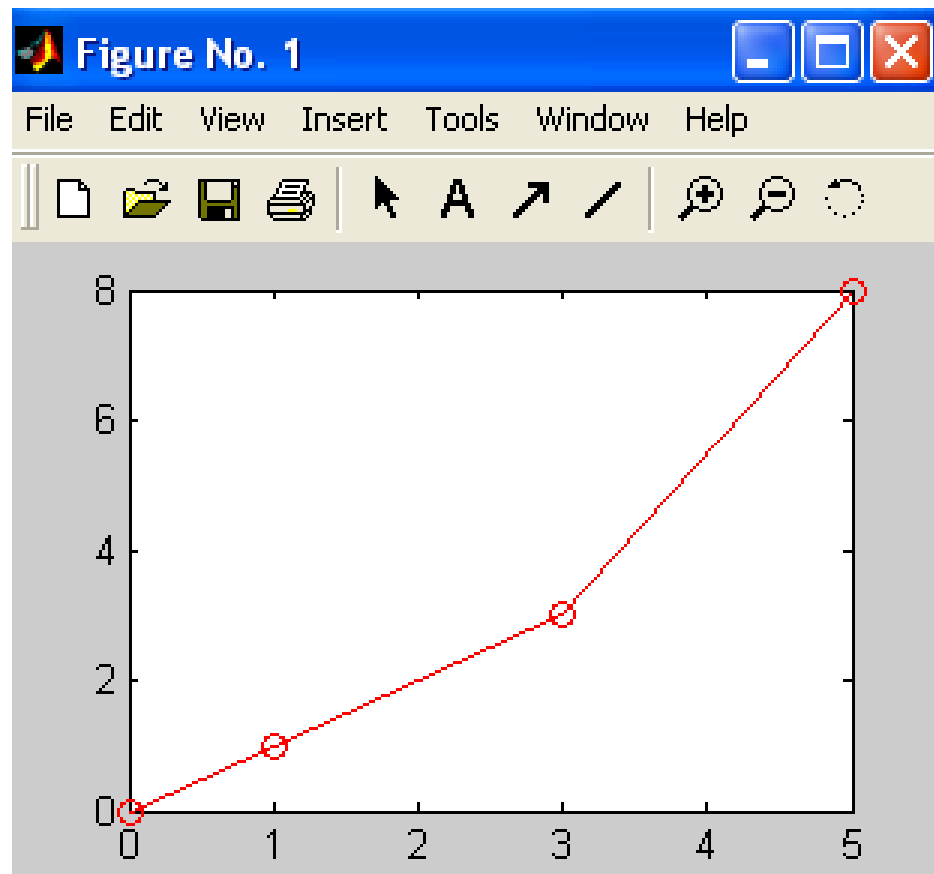
## Significance of Cell Arrays (Cont.)

---

```
if isempty(string)
    plot(x,y);
else
    plot(x,y,string);
end
```

# Significance of Cell Arrays (Cont.)

```
>> plotline([0 0], [1 1], [3 3], [5 8], '-ro');
```







# About varargout

>> help varargout      VARARGOUT Variable length output argument list.

Allows any number of output arguments from a function. The variable VARARGOUT is a **cell array** containing the optional output arguments from the function. VARARGOUT must be declared as the last output argument and must contain all the outputs after that point onwards. In the declaration, VARARGOUT must be lowercase (i.e., varargout).

VARARGOUT is not initialized when the function is invoked. You must create it before your function returns. Use NARGOUT to determine the number of outputs to produce.

For example, the function,

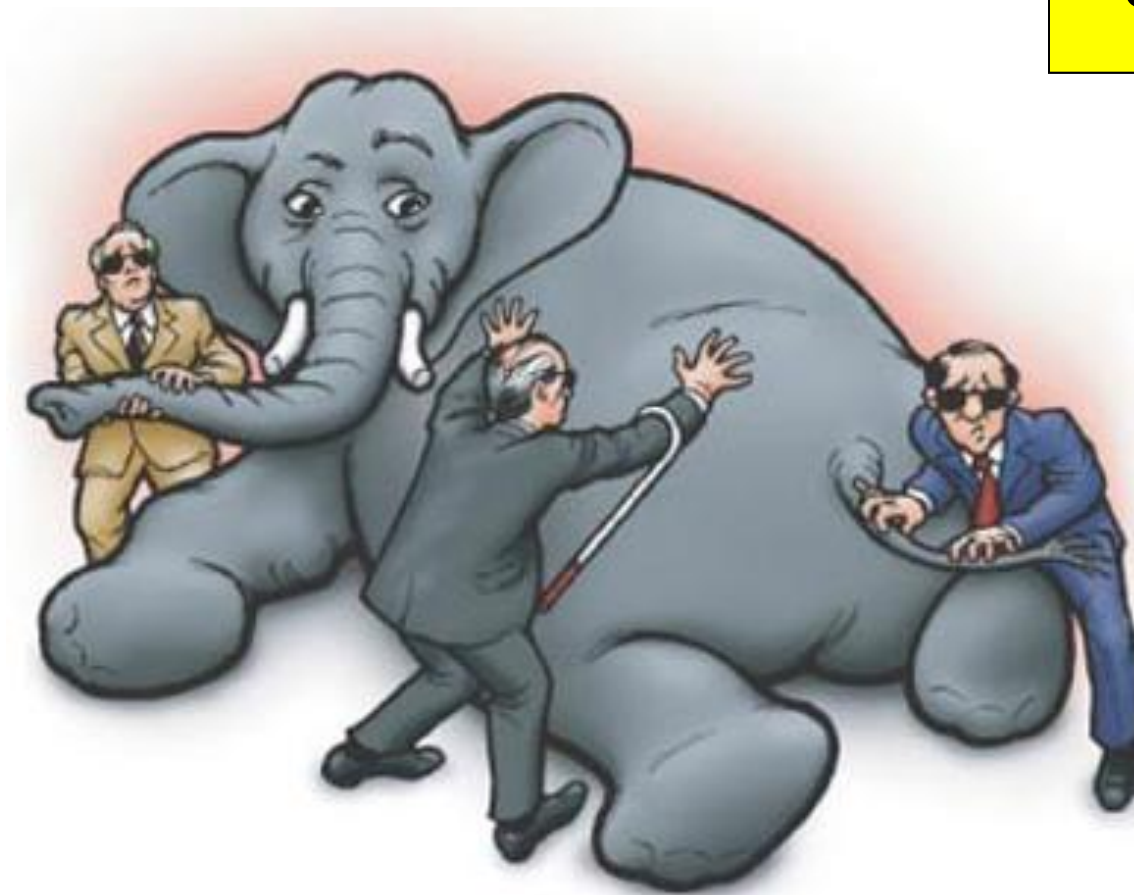
```
function [s,varargout] = mysize(x)
nout = max(nargout,1)-1;
s = size(x);
for i=1:nout, varargout(i) = {s(i)}; % varargout{i} = s(i), in some new version
end
```

returns the size vector and optionally individual sizes. So,

```
[s,rows,cols] = mysize(rand(4,5));
```

returns `s = [4 5]`, `rows = 4`, `cols = 5`.

# Structure Arrays



elephant

trunk

tusk长牙

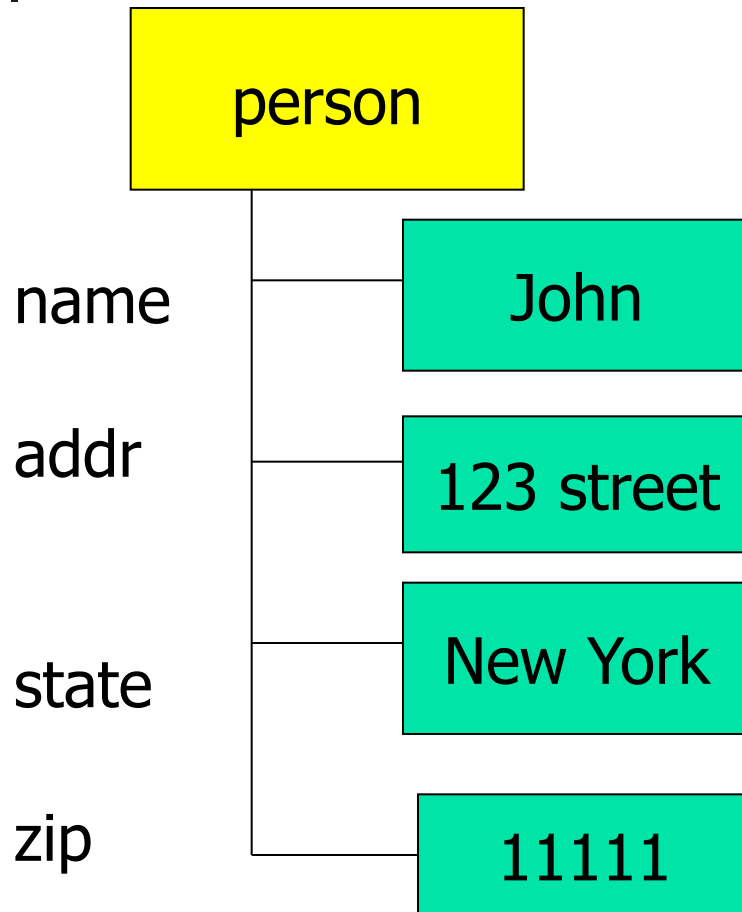
ear

body

tail

读老子感言：道不可口，可口者乃其似也。

# Structure Arrays (Cont.)



```
person.name='John';  
person.addr='123 street';  
person.state='New York';  
person.zip=11111;
```

A **structure** is a **data type** in which each individual property is given a name

**field**: an individual property of a structure

cf. the situation in C<sub>35</sub>



# Creating Structures

---

```
var=struct(fields)
```

```
a=struct('name','addr','state','zip');
```

```
a(1).name='John';  
a(1).addr='123 street';  
a(1).state='New York';  
a(1).zip=111111;
```

```
a(2).name='...';  
a(2).addr='...';  
a(2).state='...';  
a(2).zip=...;
```

```
a(3).name=...  
...
```

# Be careful: Different Versions!!

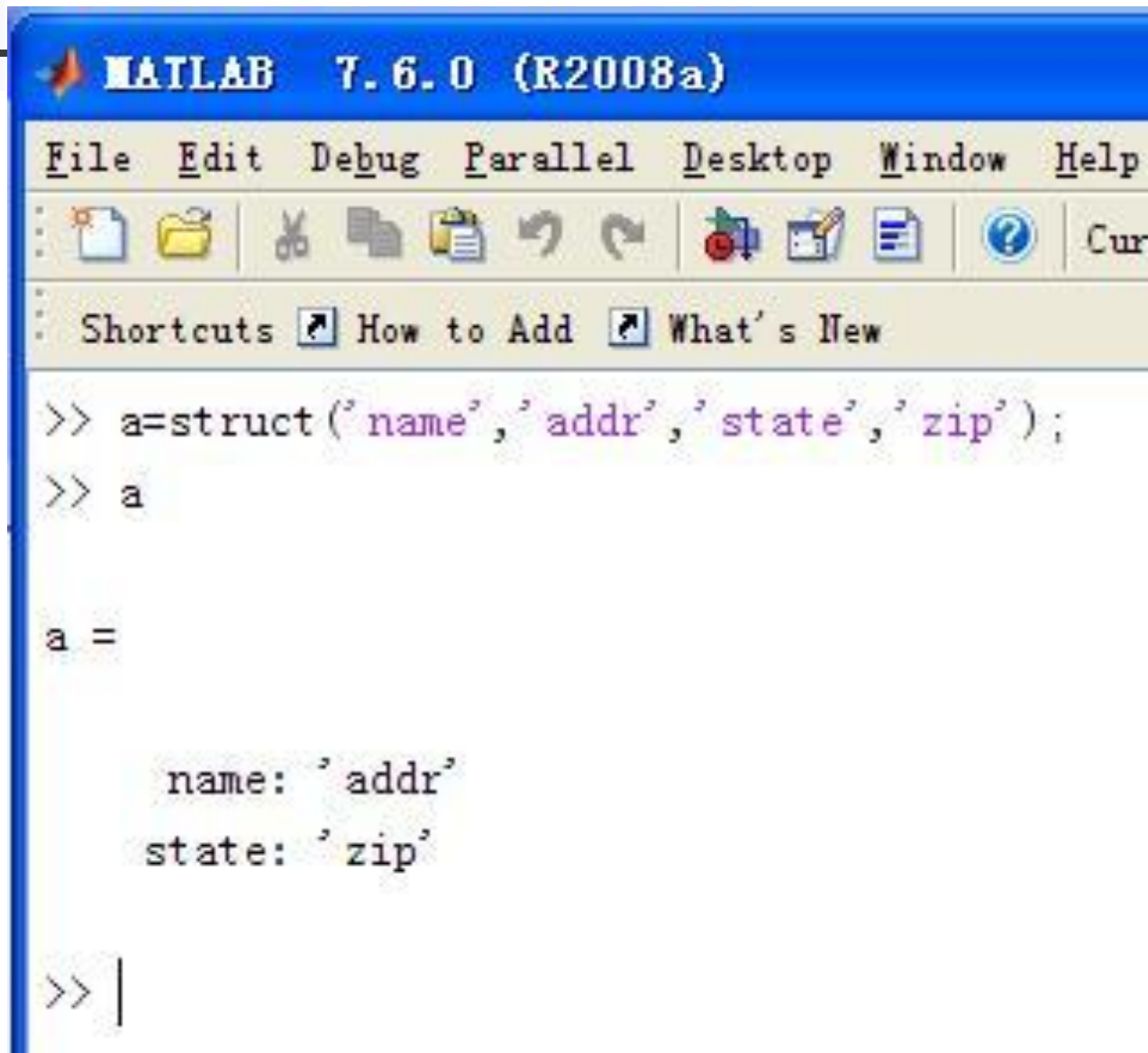
```
>> help struct      STRUCT Create or convert to structure array.  
    S = STRUCT('field1',VALUES1,'field2',VALUES2,...) creates a  
    structure array with the specified fields and values. The value  
    arrays VALUES1, VALUES2, etc. must be cell arrays of the same size,  
    scalar cells or single values. Corresponding elements of the value  
    arrays are placed into corresponding structure array elements.  
    The size of the resulting structure is the same size as the  
    value cell arrays or 1-by-1 if none of the values is a cell.  
    STRUCT(OBJ) converts the object OBJ into its equivalent  
    structure. The class information is lost.  
    STRUCT([]) creates an empty structure.  
    To create fields that contain cell arrays, place the cell  
    arrays within a VALUE cell array. For instance,  
    s = struct('strings',{'hello','yes'},'lengths',[5 3])  
    creates the 1-by-1 structure  
    s = strings: {'hello' 'yes'}  
        lengths: [5 3]  
    Example  
    s = struct('type',{'big','little'},'color','red','x',{3 4})
```

# Be careful: Different Versions!!

```
>> a=struct('name','addr','state','zip')
a = name: 'addr'
     state: 'zip'
>> a(1)
ans = name: 'addr'
     state: 'zip'
>> a(2)
??? Index exceeds matrix dimensions.
>> a(2)=a(1);
>> a
a = 1x2 struct array with fields:
    name
    state
>> a(2)
ans = name: 'addr'
     state: 'zip'
```

证实→勿轻言否定→证伪

# MATLAB 7.6 version:

A screenshot of the MATLAB 7.6.0 (R2008a) software interface. The window has a blue title bar with the MATLAB logo and version text. Below the title bar is a menu bar with options: File, Edit, Debug, Parallel, Desktop, Window, and Help. Under the menu bar is a toolbar with various icons for file operations (new, open, save, delete, copy, paste, undo, redo), a search icon, and a 'Cur' button. Below the toolbar is a panel with 'Shortcuts', 'How to Add', and 'What's New' links. The main workspace displays MATLAB commands and their output. The commands entered are: >> a=struct('name','addr','state','zip'); and >> a. The output shows 'a =' followed by a struct array with fields 'name: 'addr'' and 'state: 'zip''. The prompt >> | is visible at the bottom.

```
MATLAB 7.6.0 (R2008a)
File Edit Debug Parallel Desktop Window Help
: [new] [open] [save] [delete] [copy] [paste] [undo] [redo] [search] [Cur]
: Shortcuts [?] How to Add [?] What's New
>> a=struct('name','addr','state','zip');
>> a

a =

    name: 'addr'
   state: 'zip'

>> |
```





# Adding Fields to Structures

- By adding a new field to any **element** in a structure array, this field is automatically added to all of the elements in the array

```
a(1).grade=[2 3];
```

```
a(1).name='John';  
a(1).addr='123 street';  
a(1).state='New York';  
a(1).zip=111111;  
a(1).grade=[2 3];
```





# Removing Fields from Structures

---

- `new_struct=rmfield(old_array,'field')`

```
b=rmfield(a,'zip');
```



# getfield and setfield functions

---

- `var=getfield(array,{array_idx},'field',{field_idx})`
- `var=array(array_index).field(field_index)`

```
a(1).name='John';  
a(1).addr='123 street';  
a(1).state='New York';  
a(1).zip=111111;  
a(1).grade=[2 3];
```

```
b=getfield(a,{1},'grade',{2});
```

```
b=a(1).grade(2);
```



## getfield and setfield functions (Cont.)

- `f=setfield(array,{array_idx},'field',{field_idx},val)`
- `array(array_index).field(field_index)=value`

```
a(1).name='John';  
a(1).addr='123 street';  
a(1).state='New York';  
a(1).zip=111111;  
a(1).grade=[2 3];
```

```
f=setfield(a,{1},'grade',{1},4);
```

```
a(1).name='John';  
a(1).addr='123 street';  
a(1).state='New York';  
a(1).zip=111111;  
a(1).grade=[4 3];
```



# Sincere Thanks!

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- Using this group of PPTs, please read
- [1] Yunong Zhang, Weimu Ma, Xiao-Dong Li, Hong-Zhou Tan, Ke Chen, MATLAB Simulink modeling and simulation of LVI-based primal-dual neural network for solving linear and quadratic programs, Neurocomputing 72 (2009) 1679-1687
- [2] Yunong Zhang, Chenfu Yi, Weimu Ma, Simulation and verification of Zhang neural network for online time-varying matrix inversion, Simulation Modelling Practice and Theory 17 (2009) 1603-1617