CS 124 Programming Assignment 2

30943147

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1 Caching

When multiplying two matrices $A * B = C$ using the naive matrix multiplication method, the elements of C are given as follows

$$C[i][j] = \sum_k A[i][k] * B[k][j]$$

This is naturally implemented with 3 nested for loops iterating over i, j, and k. There are $3! = 6$ possible permutations of their order. I tested all of them experimentally to determine which was best. Runtime measurements were taken by running the naive algorithm on $n \times n$ matrices with randomly generated entries for $n = 1200$.

ordering	runtime
i, j, k	5.81s
i, k, j	1.73s
j, i, k	5.54s
j, k, i	25.30s
k, i, j	2.05s
k, j, i	24.67s

The ordering i, k, j produces the best results. The caching behaviour of that ordering can be further optimized by saving the intermediate value of each $A[i][k]$ before we iterate over j. This allows the innermost loop, with iterates over j, to only access a single row of matrix B, in order, via calls to access $B[k][j]$. Since the matrices are stored in row-major order, this is very cache friendly. Saving the intermediate value of $A[i][k]$ prevents the cache from thrashing as it must repeatedly load up $A[i]$ to recalculate $A[i][k]$, which is wasted work since $A[i][k]$ isn't changing during the innermost loop over j.