Técnicas de Concepção de Algoritmos: **B&B: The Knapsack Problem**

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Exemplo de Aplicação: B&B

- A typical example of a knapsack problem is the following:

  Consider a hiker who is going bushwalking carrying a backpack (knapsack). He can carry a maximum weight of 40 kg, and he has available a total of six different items (e.g. sleeping bag, tins of food, etc.) which he would like to take if possible. Assume that he can assign a value to each of the items as shown in the following table.

Item	Value	Weight
1	$76	16 Kg
2	$40	10 Kg
3	$20	6 Kg
4	$30	6 Kg
5	$24	8 Kg
6	$96	16 Kg

Exemplo de Aplicação: B&B

- **Problem:**
  His problem is then to select items to pack so that the value carried in his knapsack is the maximum possible!

  Let:
  - *n* be the total number of items
  - *W* be the maximum weight that can be carried
  - *v_i* be the value of item *i*
  - *w_i* be the weight of item *i*
  - *x_i* be the numbr of units *i* which are packed in a load (*x* = {0, 1})

  - $V = \sum_{i=1}^{n} v_i x_i$
  - $\sum_{i=1}^{n} w_i x_i \leq W \; and \; x_i = 0 \; or \; 1 \; for \; i = 1, 2, 3, \ldots, n$
- In this case: n = 6, W = 40, v_4 = 30, w_4 = 6 (Value: $210)

Exemplo de Aplicação: B&B

- What if fractions of items can be packed?

$$\sum_{i=1}^{n} w_i x_i \leq W \; and \; 0 \leq x_i \leq 1 \; for \; i = 1, 2, 3, \ldots, n$$

Item	Value	Weight	Value Density
6	$96	16 Kg	6 $/Kg
4	$30	6 Kg	5 $/Kg
1	$76	16 Kg	4.75 $/Kg
2	$40	10 Kg	4 $/Kg
3	$20	6 Kg	3.33 $/Kg
5	$24	8 Kg	3 $/Kg

- Solution: x_6 = x_4 = x_1= 1, x_2 = 0.2, x_3 = x_5 = 0

Referências e mais informação

- Elvin J. Moore (1982) “The Knapsack Problem”, Parabola Online, Vol. 18, Issue 1.