

Coursework 2 - "Optimise"

Circuit 1 - Boolean Algebra

- $X = A' + ((A + 1) \cdot B') + ((B + 1) \cdot C') + (C \cdot A')$
- $Y = ((A + B' + C') \cdot A') + ((B' \cdot C) \cdot B)'$
- $Z = (A' + ((A + 1) \cdot B') + ((B + 1) \cdot C') + (C \cdot A'))' + (((A + B' + C') \cdot A') + ((B' \cdot C) \cdot B))'$
- **Final:** $Z = AB + AC + BC$

Simplifying X:

1. **Rule 2:**
 $\Rightarrow A' + (1 \cdot B) + (1 \cdot C') + (C \cdot A')$
2. **Rule 4:**
 $\Rightarrow A' + B' + C' + (C \cdot A')$
3. **Rule 11:**
 $\Rightarrow A' + B' + C' + A'$
4. **Rule 5:**
 $\Rightarrow A' + B' + C'$
5. **Simplified:** $X = A' + B' + C'$

Simplifying Y:

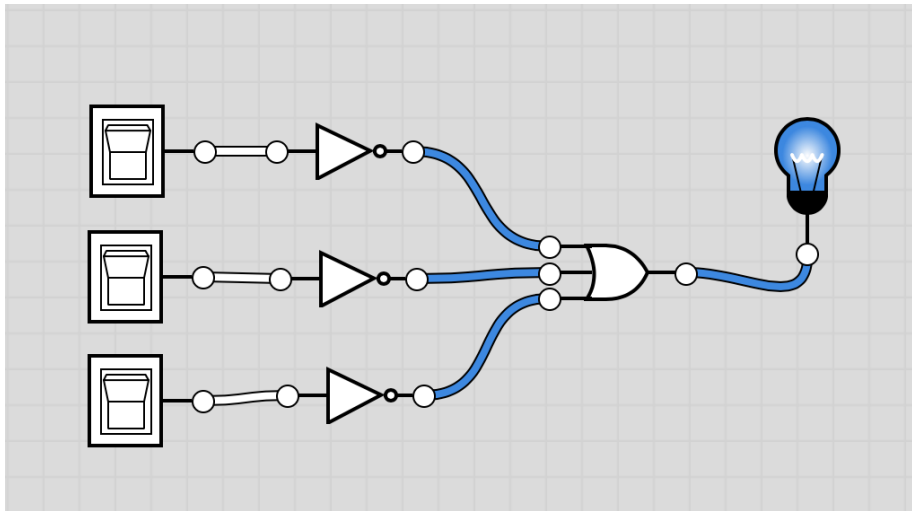
1. **Rule 11:**
 $\Rightarrow ((A + B' + C') \cdot A') + (B + C)'$
2. **DeMorgan's Theorem:**
 $\Rightarrow ((A + B' + C') \cdot A') + (B' \cdot C')$
3. **Rule 8:**
 $\Rightarrow ((0 + B' + C') \cdot A') + (B' \cdot C')$
4. **Rule 1:**
 $\Rightarrow ((B' + C') \cdot A') + (B' \cdot C')$
5. **Distributive Law:**
 $\Rightarrow (A' \cdot B') + (A' \cdot C') + (B' \cdot C')$
6. **Simplified:** $Y = (A' \cdot B') + (A' \cdot C') + (B' \cdot C')$

Simplifying Z:

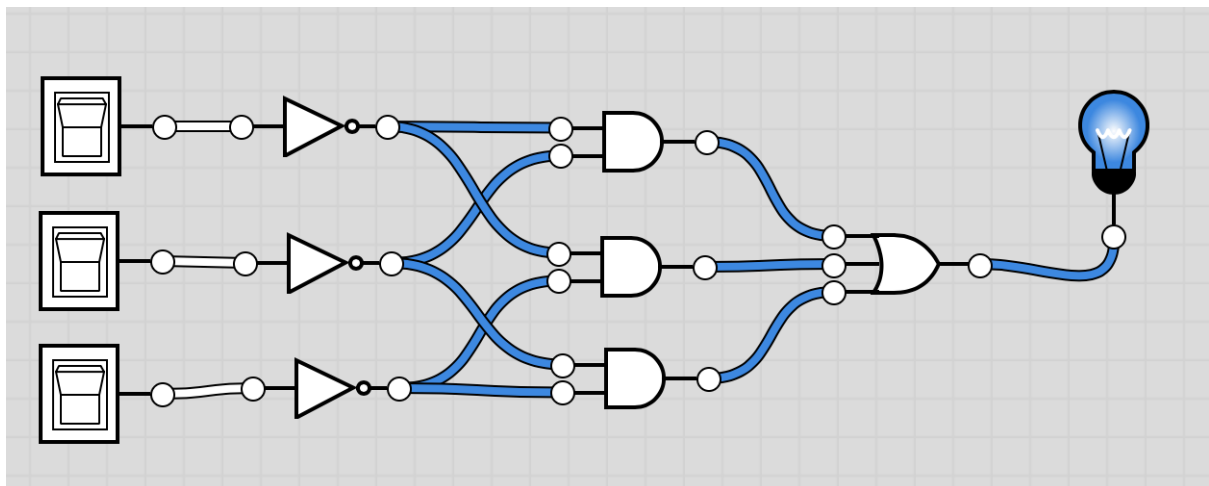
1. $Z = (A' + B' + C')' + ((A' \cdot B') + (A' \cdot C') + (B' \cdot C'))'$
2. **DeMorgan's Theorem:**
 $\Rightarrow (A \cdot B \cdot C) + ((A' \cdot B') + (A' \cdot C') + (B' \cdot C'))'$
3. **Rule 9:**
 $\Rightarrow (A \cdot B \cdot C) + A \cdot B + A \cdot C + B \cdot C$
4. **Rule 10:**
 $\Rightarrow A \cdot B + A \cdot C + B \cdot C$
5. **Simplified:** $Z = AB + AC + BC$

Circuit 1 – Optimised Circuits & Karnaugh Map

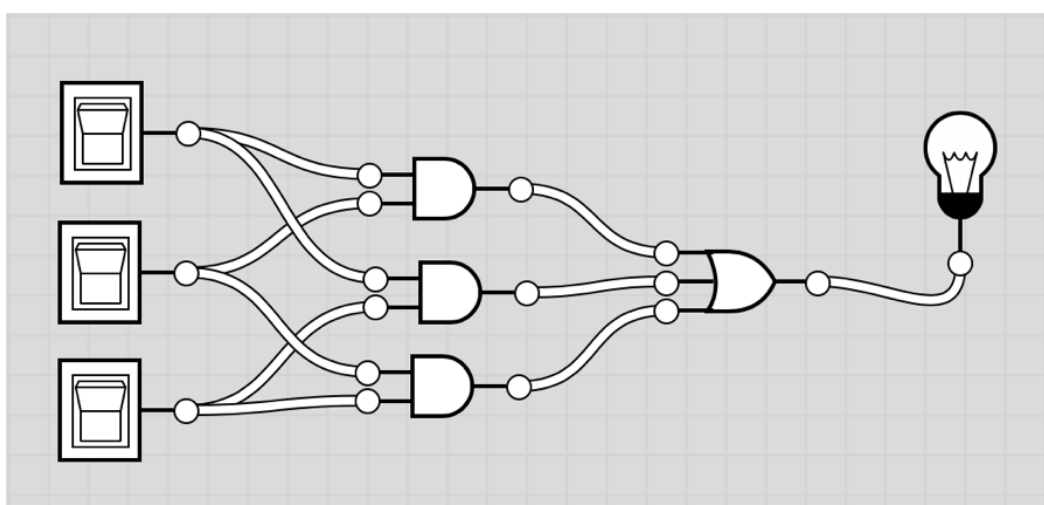
Optimised X



Optimised Y



Optimised Z



Karnaugh Map

$$X = A' + B' + C'$$

C \ AB				
	00	01	11	10
0	1	1	1	1
1	1	1	0	1

$$Y = (A' \cdot B') + (A' \cdot C') + (B' \cdot C')$$

C \ AB				
	00	01	11	10
0	1	1	0	1
1	1	0	0	0

$$Z = A \cdot B + A \cdot C + B \cdot C$$

C \ AB				
	00	01	11	10
0	0	0	1	0
1	0	1	1	1

Circuit 2 - Boolean Algebra

- $X = (A' \cdot (A + B' + C' + D')) + (B' \cdot (B + C')) + (C + (C'D))'$
- $Y = ((((((A + C') \cdot (AC')) + (A'C)) \cdot B') + (A + C')' \cdot D') + (A'B'C'))$
- $Z = (A' \cdot (A + B' + C' + D')) + (B' \cdot (B + C')) + (C + (C'D))' + ((((((A + C') \cdot (AC')) + (A'C)) \cdot B') + (A + C')' \cdot D') + (A'B'C'))$
- **Final:** $Z = (AB + AC + AD + BC + C' + D') + (B'A'CD' + B'AC'D' + A'C'D' + A'B'C')$

Simplifying X

1. **Rule 11**
 $\Rightarrow (A' \cdot (A + B' + C' + D')) + (B' \cdot (B + C')) + (C + D))'$
2. **Distributive Law**
 $\Rightarrow (A' \cdot (A + B' + C' + D')) + (B'C') + (C + D))'$
3. **Distributive Law**
 $\Rightarrow (A'B' + A'C' + A'D' + B'C' + C + D)'$
4. **Rule 9**
 $\Rightarrow AB + AC + AD + BC + C' + D'$
5. **Simplified:** $X = AB + AC + AD + BC + C' + D'$

Simplifying Y

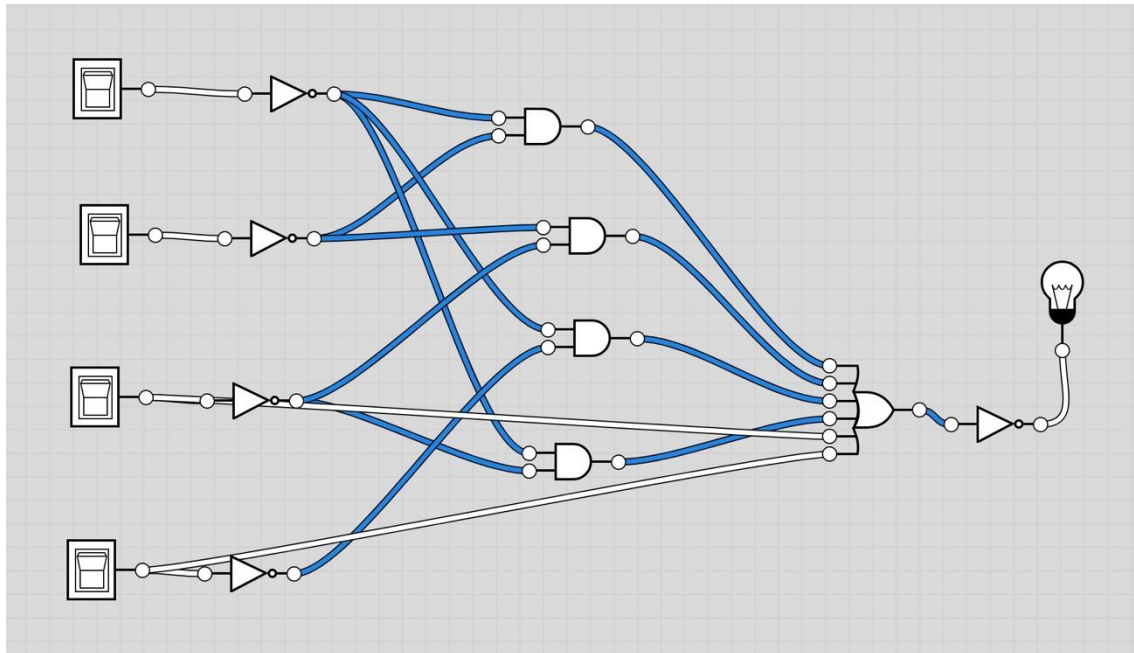
1. **Rule 12**
 $\Rightarrow ((((((AC') + (A'C)) \cdot B') + (A + C')' \cdot D') + (A'B'C'))$
2. **Rule 12**
 $\Rightarrow (((((A'C + AC') \cdot B') + (A + C')' \cdot D') + (A'B'C'))$
3. **Distributive Law**
 $\Rightarrow (((B'A'C + B'AC') + (A + C')' \cdot D') + (A'B'C'))$
4. **DeMorgan's Theorem**
 $\Rightarrow ((B'A'C + B'AC' + A'C') \cdot D') + (A'B'C')$
5. **Distributive Law**
 $\Rightarrow B'A'CD' + B'AC'D' + A'C'D' + A'B'C'$
6. **Simplified:** $Y = B'A'CD' + B'AC'D' + A'C'D' + A'B'C'$

Simplifying Z

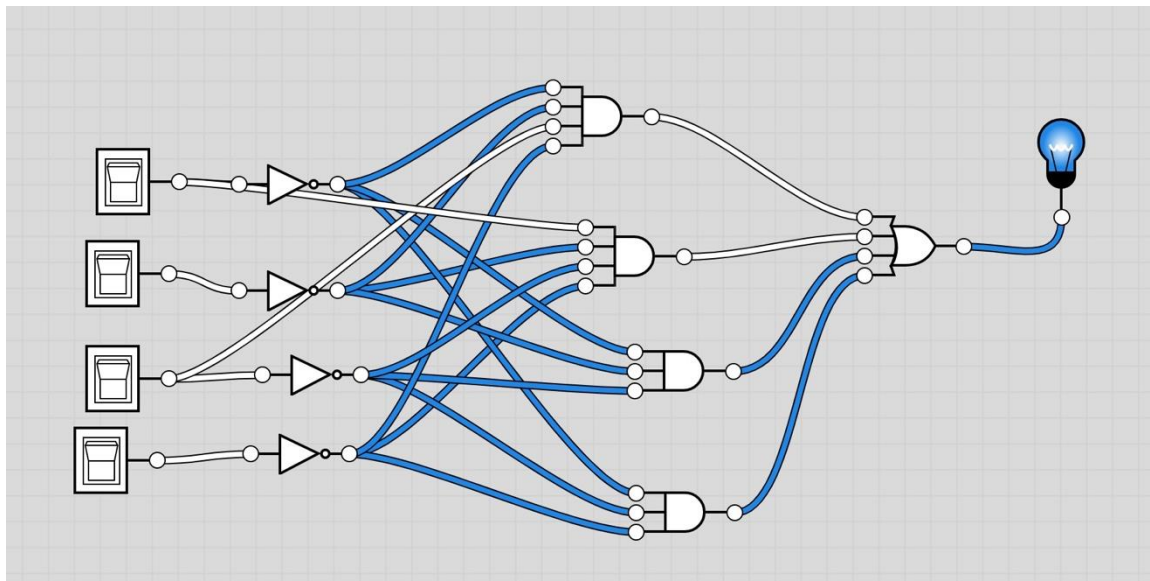
1. **Simplified:** $Z = (AB + AC + AD + BC + C' + D') + (B'A'CD' + B'AC'D' + A'C'D' + A'B'C')$

Circuit 2 – Optimised Z & Karnaugh Map

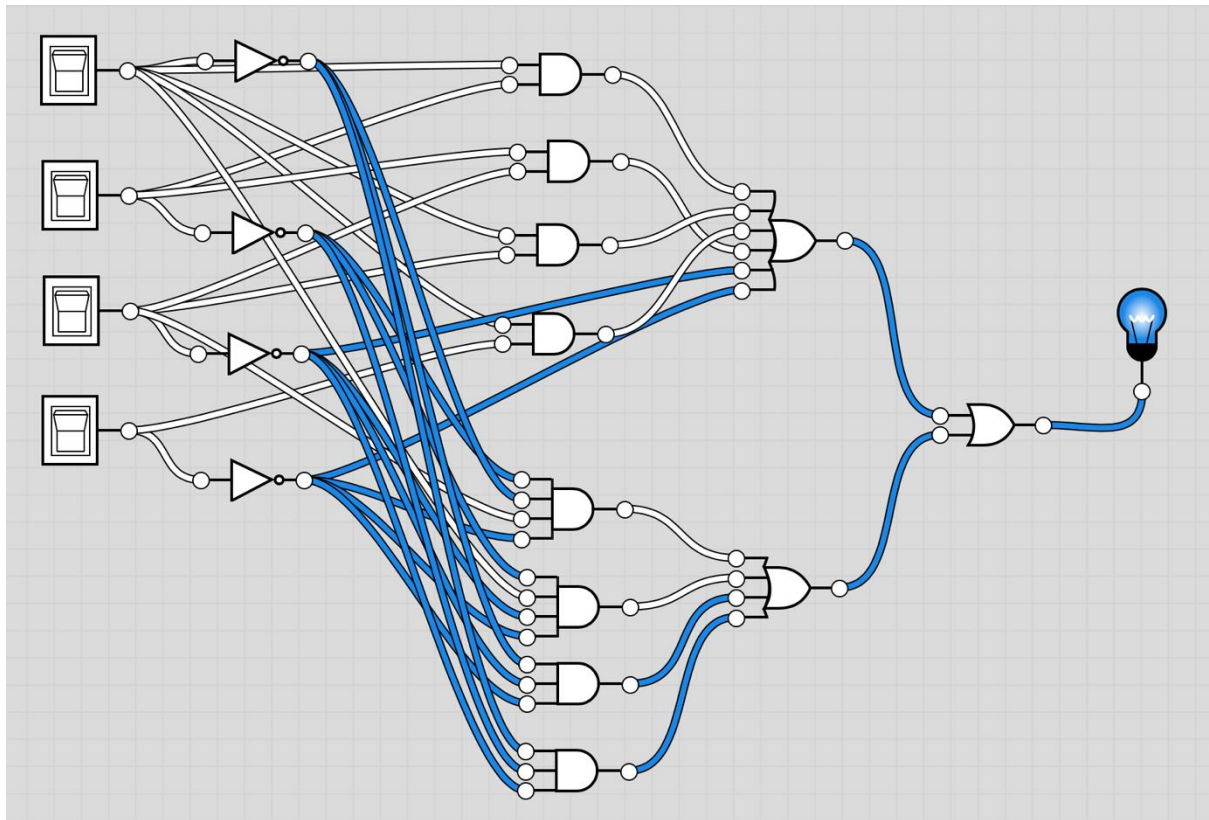
Optimised X



Optimised Y



Optimised Z



Karnaugh Map

$$X = AB + AC + AD + BC + C' + D'$$

AB \ CD	00	01	11	10
00	1	1		1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$A'B'C' + A'B + A + D'$$

$$Y = B'A'CD' + B'AC'D' + A'C'D' + A'B'C'$$

AB \ CD	00	01	11	10
00	1	1		1
01	1			
11				
10	1			

$$A'B'D' + A'B'C' + C'D'A' + C'D'B'$$

$$Z = (AB + AC + AD + BC + C' + D') + (B'A'CD' + B'AC'D' + A'C'D' + A'B'C')$$

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$A'B'C' + A'B + A + D'$$