Homework 0

This homework is *required* and consists mostly of review problems.

This homework is due at 11 PM on August 30, 2023.

Submission Format: Your homework submission should consist of a single PDF file that contains all of your answers (any handwritten answers should be scanned).

1. **Course Setup**

   Please complete the following steps to get access to all course resources.

   (a) Visit the course website at `http://www.eecs127.github.io/` and familiarize yourself with the syllabus.

   (b) Verify that you can access the class Ed site at `https://edstem.org/us/courses/42325`.

   (c) Make sure you can access the Gradescope at `https://www.gradescope.com/courses/566395`. If not, make a private Ed post to contact us.

   (d) When are self grades due for this homework? In general, when are self grades due? Where are the self-grade assignments?

   (e) How many homework drops do you get? Are there exceptions?

2. What Prerequisites Have You Taken?

The prerequisites for this course are

- EECS 16A & 16B (Designing Information Devices and Systems I & II) **OR** MATH 54 (Linear Algebra & Differential Equations),
- CS 70 (Discrete Mathematics & Probability Theory), and
- MATH 53 (Multivariable Calculus).

Please list which of these courses you have taken. If you have taken equivalent courses at a separate institution, please list them here. If you are unsure of course material overlap, please refer to the EECS 16A, EECS 16B, and CS 70 websites (`https://www.eecs16a.org/`, `https://www.eecs16b.org/`, and `http://www.sp22.eecs70.org/`, respectively) and the MATH 53 textbook (*Multivariable Calculus* by James Stewart).

The course material this semester will rely on knowledge from these prerequisite courses. If you feel shaky on this material, please use the first week to reacquaint yourself with it.

3. Determinants

Consider a unit box $\mathcal{B}$ in $\mathbb{R}^2$ — i.e., the square with corners $\begin{bmatrix}0\\0\end{bmatrix}$, $\begin{bmatrix}0\\1\end{bmatrix}$, $\begin{bmatrix}1\\0\end{bmatrix}$, and $\begin{bmatrix}1\\1\end{bmatrix}$. Define $A(\mathcal{B})$ as the parallelogram generated by applying matrix A to every point in $\mathcal{B}$.

(a) For $A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$, calculate the location of each corner of $A(\mathcal{B})$.

(b) Write the area of $A(\mathcal{B})$ as a function of $\det(A)$.

HINT: How are the basis vectors $\begin{bmatrix}0\\1\end{bmatrix}$ *and* $\begin{bmatrix}1\\0\end{bmatrix}$ *transformed by the matrix multiplication?*

(c) Calculate the area of $A(\mathcal{B})$ for each of the following values of A.

i. $A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$

ii. $A = \begin{bmatrix}2 & 1\\4 & 3\end{bmatrix}$

iii. $A = \begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}$

iv. $A = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$

4. Subspaces and Dimensions

Consider the set $\mathcal{S}$ of points $(x_1, x_2, x_3) \in \mathbb{R}^3$ such that

$$x_1 + 2x_2 + 3x_3 = 0, \quad 3x_1 + 2x_2 + x_3 = 0. \tag{1}$$

(a) Find a 2×3 matrix A for which $\mathcal{S}$ is exactly the null space of A.

(b) Determine the dimension of $\mathcal{S}$ and find a basis for it.

5. Orthogonality

Let $\vec{x}, \vec{y} \in \mathbb{R}^n$ be two linearly independent unit-norm vectors; that is, $\|\vec{x}\|_2 = \|\vec{y}\|_2 = 1$.

(a) Show that the vectors $\vec{u} = \vec{x} - \vec{y}$ and $\vec{v} = \vec{x} + \vec{y}$ are orthogonal.

(b) Find an orthonormal basis for $\mathrm{span}(\vec{x}, \vec{y})$, the subspace spanned by $\vec{x}$ and $\vec{y}$.

6. Homework Process

With whom did you work on this homework? List the names and SIDs of your group members.

NOTE: If you didn't work with anyone, you can put "none" as your answer.

