DIGGERS Internship: Payoff definition of vanilla derivative products

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1 Notations

Let's use the following notations:

- T, the maturity
- $\tau = T - t$
- K, the strike
- σ, the volatility
- S_T, the price of the stock (or any other asset) at maturity
- S_t, the price of the stock (or any other asset) at time t
- M_T, $\max_{t \in \mathcal{T}} S_t$
- m_T, $\min_{t \in \mathcal{T}} S_t$

At last, let's introduce the following function: $(x - K)^+ = max(x - K; 0)$

2 Vanilla options: Call+Put

Payoff of a European call option: $X = (S_T - K)^+$
Payoff of a European put option: $X = (K - S_T)^+$

Let's recall the expressions that can be obtained thanks to Black and Scholes model:

- European Call = S $e^{-a\tau}$ $\mathcal{N}(d_1)$ - K $e^{-r\tau}$ $\mathcal{N}(d_2)$
- European Put = K $e^{-r\tau}$ $\mathcal{N}$d(-d_2) - S $e^{-a\tau}$ $\mathcal{N}$(-d_1)

where r is the risk-free rate and a, the dividend yield. Let's also recall:

$$d_1 = \frac{ln(\frac{S}{K}) + \tau(r - a + \frac{\sigma^2}{2})}{\sigma\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

3 Asian Options with different means

Let's denote by $A_T = \sum_{t=1}^{T} \frac{S_t}{T}$, the average price over the life of the option.
Then, we define the payoff: Asian Call $X = (A_T - K)^+$, and Asian Put $X = (K - A_T)^+$. For a detailed analysis of the valuation of Asian options in continuous-time and the particular case of American-style Asian option, here is an interesting article [1].
Vorst and Kemna suggest that Asian-style options can be priced by replacing the arithmetic average of the payoff function of with geometric average. Then, the payoff can re-written as followed:

$$X = ((\prod_{t=1}^{T} S(t))^{\frac{1}{n}} - K)^+ \tag{1}$$

This leads to a log-normal distribution for A_T in Vorst's approximation.

4 Barrier Options with discrete monitoring

Barrier options are path-dependent options. Let's define H, the barrier and X, the payoff.
A knock-in option is a type of barrier option where the rights associated with that option only come into existence when the price of the underlying security reaches a specified barrier during the option's life. Once a barrier is knocked in, or comes into existence, the option remains in existence until it expires.
Contrary to knock-in barrier options, knock-out barrier options cease to exist if the underlying asset reaches a barrier during the life of the option. Knock-out barrier options may be classified as up-and-out or down-and-out.

4.1 Down-and-out call

For some H>0,

$$X = (S_T - K)^+ \mathbb{1}_{m_T > H} \tag{2}$$

4.2 Down-and-in call

For some H>0,

$$X = (S_T - K)^+ \mathbb{1}_{m_T \leq H} \tag{3}$$

4.3 Up-and-out call

For some H>0,

$$X = (S_T - K)^+ \mathbb{1}_{M_T < H} \tag{4}$$

4.4 Up-and-in call

For some H>0,

$$X = (S_T - K)^+ \mathbb{1}_{M_T \geq H} \tag{5}$$

4.5 Parity between CUO, CUI and Call

$$Call = CUO + CUI \tag{6}$$

5 Basket Options [2]

A basket option is a type of financial derivative where the underlying asset is a group, or basket, of commodities, securities, or currencies. As with other options, a basket option gives the holder the right, but not the obligation, to buy or sell the basket at a specific price, on or before a certain date.

Let's consider d assets with correspondingly different weights of the assets, denoted as $\omega_1\omega_2, ..., \omega_d$.
We can assume that: $\sum_{i=1}^{d} \omega_i = 1$.
Considering a Basket Call Option, the payoff can be defined as followed:

$$X = (\sum_{i=1}^{d} \omega_i S_i(T) - K)^+ \tag{7}$$

6 Discrete Asian Basket Options [3]

The payoff of discrete Asian Basket Options is a combination between the one of an Asian option and the the one of a Basket option.
Therefore, it can be defined as followed:

$$X = (\frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{d} \omega_i S_i(T) - K)^+ \tag{8}$$

Whenever an option payout is denominated in currency other than the underlying asset, the value of the option does not only depend on the performance of the underlying but also on the exchange rate between the payout currency and the currency of the underlying. Such options are often called quanto options.[4] There exists different analytical expressions to get the valuation of Asian Quanto-Basket Options, that are explained in the article linked above.

References

[1] Jesper Karlsson, Pricing of European- and American-style Asian Options using the Finite Element Method.
https://www.diva-portal.org/smash/get/diva2:1236468/FULLTEXT01.pdf

[2] JIN ZENG, PRICING ASIAN OPTIONS AND BASKET OPTIONS BY MONTE CARLO METHODS.
https://macsphere.mcmaster.ca/bitstream/11375/23088/2/Zeng_Jin_201712_MasterofScience.pdf

[3] DAHL and BENTH, Valuation of Asian Basket Options with Quasi Monte-Carlo Techniques and Singular Value Decomposition.
http://webdoc.sub.gwdg.de/ebook/e/2002/maphysto/publications/mps-rr/2001/7.pdf

[4] Ville Viitasaari, VValuation of Asian Quanto-Basket Options.
https://sal.aalto.fi/publications/pdf-files/tvii11_public.pdf