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ECE 3793 – Project 2

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Introduction

In this project, our team will be deriving and applying Fourier constants across a simple step function. During this process several MATLAB calculations will be performed, as well as deriving the Fourier constant integral to be used in recreating a signal. These signals and constants will then be plotted. The code that is used to derive the Fourier constants will then be used to derive the constants of different instruments playing the C3 note.

Part I: Derivation for the Transmitter

Given a signal with a frequency of 500 Hz,

$$x(t) = \cos(2\pi \times 500t) \tag{1}$$

and an arbitrary carrier frequency (F_c), to which the carrier signal is

$$x_c(t) = \cos(2\pi F_c t) \tag{2}$$

Calculation 1: $x_{TX}(t)$ as a sum of cosines

$$x_{TX}(t) = x(t)x_c(t) \tag{3}$$

Using the known signals, $x(t)$ and $x_c(t)$, we can substitute these functions into $x_{TX}(t)$.

$$x_{TX}(t) = (\cos(2\pi \times 500t))(\cos(2\pi F_c t))$$

Using the trigonometric identity

$$\cos(u)\cos(v) = \frac{1}{2}[\cos(u+v) + \cos(u-v)] \tag{4}$$

Substituting in equation 1 for the cos(u) term, and equation 2 for the cos(v) term,

$$\cos(2\pi \times 500)\cos(2\pi F_c t) = \frac{1}{2}[\cos(2\pi \times 500t + 2\pi F_c t) + \cos(2\pi \times 500t - 2\pi F_c t)]$$

Resulting in the equation

$$x_{TX}(t) = \frac{1}{2}[\cos(2\pi \times 500t + 2\pi F_c t) + \cos(2\pi \times 500t - 2\pi F_c t)] \tag{5}$$

Then distributing the 1/2 to each cosine

$$x_{TX}(t) = \frac{1}{2}\cos(2\pi \times 500t + 2\pi F_c t) + \frac{1}{2}\cos(2\pi \times 500t - 2\pi F_c t)$$

and factoring out 2π from each cosine term

$$x_{TX}(t) = \frac{1}{2}\cos(2\pi(500t + F_c t)) + \frac{1}{2}\cos(2\pi(500t - F_c t))$$

And finally, factoring out t from each term.

$$x_{TX}(t) = \frac{1}{2}\cos(2\pi t(500 + F_c)) + \frac{1}{2}\cos(2\pi t(500 - F_c))$$

Calculation 1 Results:

As shown through using trigonometic identities, and basic algebraic techniques, $x_{TX}(t)$ can be expressed as a sum of cosines as follows.

$$x_{TX}(t) = \frac{1}{2}\cos(2\pi t(500 + F_c)) + \frac{1}{2}\cos(2\pi t(500 - F_c)) \tag{6}$$

Calculation 2: The Fourier Transform of $x_{TX}(t)$

The Fourier Transform for a continuous signal is defined as

$$X(F) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi Ft}dt \tag{7}$$

Thankfully, the function $x_{TX}(t)$ has a known Fourier Transform pair. The known pairing is

$$x(t) = \cos(2\pi F_0 t) \xleftrightarrow{\mathcal{F}} X(F) = \frac{1}{2}[\delta(F - F_0) + \delta(F + F_0)] \tag{8}$$