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MATHEMATICS

A MATHEMATICAL STUDY ON MHD VISCOUS DUE TO A SHRINKING SHEET

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Abstract*:*

In this research paper, we investigate the effect of heat and mass transfer in magnetohydrodynamic flow due to a shrinking sheet with suction. We have to derive the approximate analytical expression for dimensionless velocity, dimensionless temperature and dimensionless concentration profiles. The analytical results for the dimensionless velocity, dimensionless temperature and dimensionless concentration profile for various parameters are compared with previous published work and also displayed graphically.

Keywords: Chemical reaction; Suction at the surface; Porous shrinking sheet; Magnetic effect; Homotopy analysis method.

1. Introduction

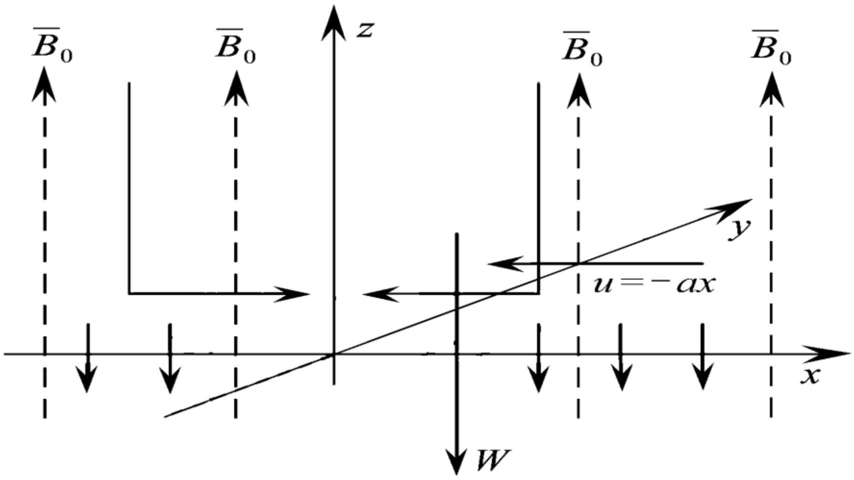
The study of the boundary layer flow of an electrically conducting fluid through a porous media has many applications in manufacturing and natural process which include cooling of electronic devices by cooling of nuclear reactors during emergency shutdown. Magnetohydrodynamics (MHD) is the study of the flow of electrically conducting fluids in a magnetic field [1]. Many experimental and theoretical studies on conventional electrically conducting fluids indicate that magnetic field markedly changes their transport and heat transfer characteristics. Sulochana [2] explained the MHD free convective flow of viscous incompressible fluid and mass transfer over a stretching sheet with chemical reaction. The presence of porous media in a boundary layer flow can significantly change the flow field and, as a consequence, affect the heat transfer rate at the surface The present trend in the field of magnetic strength analysis is to give a mathematical model for the system to predict the reactor performance [3].

The study of boundary layer viscous fluid flows and heat transfer due to stretching surface have many important applications in engineering process and industrial. Dinesh rajotia et al [4] found the boundary layer flow over a shrinking surface is encountered in several technological processes. Such situations occur in polymer processing, manufacturing of glass sheets, paper production, in textile industries and many others incited a study on the boundary layer flow of a viscous fluid towards a linear stretching sheet. An exact similarity solution for the dimensionless differential system was obtained. Rishneudu Bhattacharya [5] discussed heat transfer on a continuous stretching sheet. Afterwards, many investigations were made to examine flow over a stretching/shrinking sheet under different aspects of MHD, suction/injection, heat and mass transfer etc. The chemical reaction plays important role are agricultural fields and cooling towers.

Therefore, in the present paper, the dual solution of all MHD boundary layer flow and heat transfer of an electrically conducting fluid due to a shrinking sheet with thermal and heat source/sink, have been studied.

2 Mathematical formulation of the problem

We consider a flow of a viscous incompressible fluid past an infinite vertical porous flat plate. A MHD boundary layer flow of a viscous incompressible, electrically conducting fluid moving over the surface of a semi-infinite impermeable flat plate is considered with a uniform velocity in presence of heat source and radiation. The viscosity and thermal conductivity of the fluid are assumed to be functions of temperature. Introduce a coordinate system (x, y) with x -axis along perpendicular to the sheet and y-axis also along perpendicular to the sheet.



The chemical reactions are taking place in the flow and a constant suction is imposed at the horizontal surface. The governing boundary layer equations of momentum, energy and diffusion for the MHD flow in terms of vector notation are defined as follows:

Continuity equation:

$$div\,\vec{V} = 0 \tag{1}$$

Momentum equation:

$$\left(\vec{V}.grad\vec{V}\right) = -\frac{1}{\rho}grad\,p + v\nabla^2\vec{V} + \frac{1}{\rho}\vec{j}\times\vec{B} \tag{2}$$

Energy equation

$$\left(\vec{V}.grad\right)T = \frac{k_e}{\rho c_p}\nabla^2 T \tag{3}$$

Species concentration equation:

$$\left(\vec{V}.grad\right)C = D\nabla^2 C \pm k_1 C \tag{4}$$

where

$\vec{j} = \sigma\left(\vec{E} + \vec{V}\times\vec{B} - \frac{1}{e n_e}grad\,p_e\right)div\vec{B} = 0, curl\,\vec{H} = 0\, and\, curl\,\vec{E} = 0$ and $\vec{V}$ is the velocity vector, p is the pressure, is the kinematic coefficient of viscosity. Continuity equation in terms of vector notation (unsteady flow) is

$$\frac{\partial\rho}{\partial t} + \nabla.\left(\rho\vec{V}\right) = 0 \tag{5}$$

For steady incompressible flow: $\frac{\partial\rho}{\partial t} = 0\, and\, \rho$ constant.

Continuity equation becomes $\nabla.\left(\rho\vec{V}\right) = 0$, it implies that

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \tag{6}$$

Finally, the continuity equation (steady flow) is reduced to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{7}$$

Under these conditions, the basic governing boundary layer equations of momentum, energy and discussion for mixed convection flow neglecting Joule's viscous dissipation can be simplified to the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0(continuity) \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{\rho}\frac{\partial p}{\partial x}+v\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$-\frac{\sigma B_0{}^2}{\rho}u-\frac{v}{K}u\ (x-Momentum) \qquad (9)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\begin{bmatrix}\frac{1}{\rho}\frac{\partial p}{\partial y}+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\\ -\frac{\sigma B_0{}^2}{\rho}v-\frac{v}{K}v\ (y-Momentum)\end{bmatrix} \qquad (10)$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=\frac{1}{\rho}\frac{\partial \rho}{\partial z}+v\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)\ (z-Momentum) \qquad (11)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)\ (Energy) \qquad (12)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial y^2}\right)-k_1C\quad (Diffusion) \qquad (13)$$

Investigate the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer past a porous shrinking sheet with suction. The governing boundary layer equations of momentum, energy, and mass diffusion in terms of the velocity components , u, v and w .We investigate the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer past a porous shrinking sheet with suction. The governing boundary layer equations of momentum, energy, and mass diffusion in terms of the velocity component $u, v\, and\, w.$

The boundary conditions applicable to the present flow are

$$u=-U=-ax, v=-a(m-1)y \qquad (14)$$

$$w=-W, T=T_w, C=C_w\ at\ y=0 \qquad (15)$$

$$u\to 0, T\to T_\infty, C\to C_\infty\ as\ y\to\infty \qquad (16)$$

where $a>0$ is the shrinking constant and *W* is the suction velocity. The cases $m=1$ and $m=2$ correspond to shrinking sheets in the *x*- and *y*-directions, respectively.

Introducing the following similarity transformations

$$u=ax\,f'(\eta), v=a(m-1)y\,f'(\eta), w=-\sqrt{avm}\,f(\eta), \eta=\sqrt{\frac{a}{v}}\,z \qquad (17)$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}\ and\ \phi=\frac{C-C_\infty}{C_w-C_\infty} \qquad (18)$$

The eqn. (1) is identically satisfied and the eqn.(8) can be integrated to give

$$\frac{p}{\rho}-v\frac{\partial w}{\partial z}-\frac{w^2}{2}+constant \qquad (19)$$

The eqns.(7)-(9) reduces to the following boundary value problem

$$f'''-(M^2+Pr\lambda)f'-f'^2+m\,f\,f''=0 \qquad (20)$$

$$\theta''+m\,Pr\,f\,\theta'-Pr\,\theta\,f'=0 \qquad (21)$$

$$\phi''-Sc\,f'\phi+m\,Sc\,f\,\phi'-Sc\gamma\phi=0 \qquad (22)$$

The boundary conditions can be written a

$$\eta\to\infty, f'(\infty)=0, \theta(\infty)=0, \phi(\infty)=0 \qquad (23)$$

$$\eta=0:\ f(0)=S, f'(0)=-1, \theta(0)=1, \phi(0)=1 \qquad (24)$$

where the Prandtl number *pr*, Schmidt number *Sc*, Magnetic parameter *M*, porosity parameter λ and suction parameter *S* can be defined as follows:

$$Pr = \frac{v}{\alpha}, Sc = \frac{v}{D}, M^2 = \frac{\sigma B_0^{\ 2}}{\rho a}\ \lambda = \frac{\alpha}{a K}\ and\ S = \frac{W}{m\sqrt{a v}} \tag{25}$$

When *m=1* (sheet shrinks in *x*-direction) and *m=2* (sheet shrinking axisymmetrically).

3. Solution of the problem using the new Homotopy analysis method

In this paper, the nonlinear ordinary differential equations (20) to (22) with boundary conditions eqns. (23) and (24) were solved analytically. The approximate analytical expression for the dimensionless distance η and dimensionless velocity $f'(\eta)$.

New Homotopy analysis method is a non-perturbative analytical method for obtaining series solutions to nonlinear equations and has been successfully applied to numerous problems in science and engineering [21-25]. In comparison with other perturbative and non-perturbative analytical methods, New HAM offers the ability to adjust and control the convergence of a solution via the so-called convergence-control parameter. Because of this, New HAM has proved to be the most effective method for obtaining analytical solutions to highly non-linear differential equations. Previous applications of New HAM have mainly focused on non-linear differential equations in which the non-linearity is a polynomial in terms of the unknown function and its derivatives. As seen above, the non-linearity present in electro hydrodynamic flow takes the form of a rational function, and thus, poses a greater challenge with respect to finding approximate solutions analytically. Our results show that even in this case, New HAM yields excellent results.

Liao [25-29] proposed a powerful analytical method for non-linear problems, namely the Homotopy analysis method. This method provides an analytical solution in terms of an infinite power series. However, there is a practical need to evaluate this solution and to obtain numerical values from the infinite power series. In order to investigate the accuracy of the New Homotopy analysis method [30-31] solution with a finite number of terms, the system of differential equations were solved. The New Homotopy analysis method is a good technique comparing to another perturbation method. The New Homotopy analysis method contains the auxiliary parameter , which provides us with a simple way to adjust and control the convergence region of solution series. The approximate analytical solution of the eqns. (14)-(16) using the Homotopy analysis method is h

$$f(\eta) = \left[\begin{array}{l} S + \dfrac{e^{-A\eta}}{A} + \ c_1 + c_2\sqrt{\beta}e^{\sqrt{\beta}\eta} - c_3\sqrt{\beta}e^{-\sqrt{\beta}\eta} \\ + h\left(-\dfrac{s^2}{B} + \dfrac{\left(\dfrac{1}{A^2} - m\right)e^{-2A\eta}. - 2A}{-8A^2 + 2BA} + \dfrac{\left(mA + \dfrac{2}{A}\right)Se^{-A\eta}}{-A^3 + BA} \right) \end{array}\right] \tag{26}$$

$$\theta(\eta) = \left[\begin{array}{l} e^{-Pr\, m\eta} + C_4\eta + C_5 \\ + h\left(S\, e^{-Pr\, m\eta} + \dfrac{m^2\ Pr^2}{A(Pr\, m + A)^2} e^{-(Pr\, m + A)\eta} + \dfrac{Pr}{(Pr\, m + A)^2} e^{-(Pr\, m + A)\eta} \right) \end{array}\right] \tag{27}$$

$$\phi(\eta) = \left[\begin{array}{l} e^{-mSc\gamma\eta} + C_6\eta + C_7 \\ + h\left(\dfrac{SC\, e^{-(A+\infty)\eta}}{(A+D)^2}\left(\dfrac{mD}{A} - 1\right) + \dfrac{SC}{D^2}\left(mSD + \gamma\right) \right) \end{array}\right] \tag{28}$$

Where

$$C_1 = h\left[-C_3 - \frac{\left(\dfrac{1}{A^2} - m\right)}{-8A^2 + 2BA} - \frac{\left(-mA + \dfrac{2}{A}\right)S}{-A^3 + BA} \right] \tag{29}$$

$$C_3 = h\left[\frac{\dfrac{-s^2}{B} - \dfrac{2A\left(\dfrac{1}{A^2} - m\right)}{-8A^2 + 2BA} - \dfrac{\left(-mA + \dfrac{2}{A}\right)S + A}{-A^3 + BA}}{\sqrt{\beta}}\right] \quad (30)$$

$$C_5 = h\left[-s - \frac{m^2 Pr^2}{A(Pr\, m + A)^2} - \frac{Pr}{(Pr\, m + A)^2}\right] \quad (31)$$

$$C_7 = -h\left[\frac{Sc}{(A+D)^2}\left(\frac{mD}{A} - 1\right) + \frac{SC}{D^2}(mSD + \gamma)\right] \quad (32)$$

$$C_2 = C_4 = C_6 = 0 \quad (33)$$

$$A = Pr\,\lambda Mm \quad (34)$$

$$D = mSc\gamma \quad (35)$$

4. Results and discussion

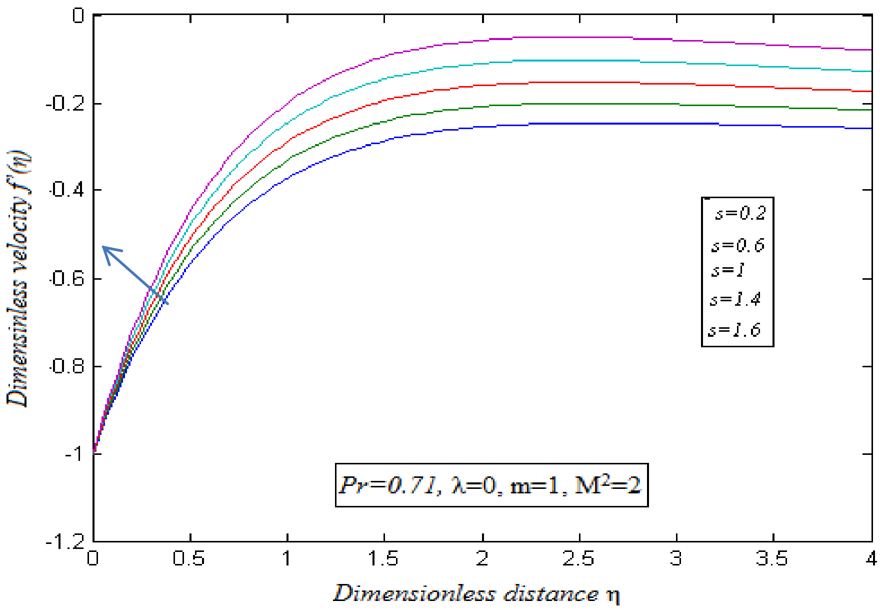
Fig (2) reprsents the dimensionless velocity $f'(\eta)$ versus the distance η. From Fig 2 (a), (b)and (c) it is clear that whean the effect of suction parameter S, shriking sheet parameter m and magnentic stength M increases the corresponding velocity profile increases in some fixed value of the other parameters. Fig (3) represents the dimensionless temperature $\theta(\eta)$ versue the distance η.

From Fig 3 (a) and (b) it is noted that when the effect of shrinking sheet parameter m and magnetic strength M increases the corresponding profile decreases in some fixed value of the other parameters.

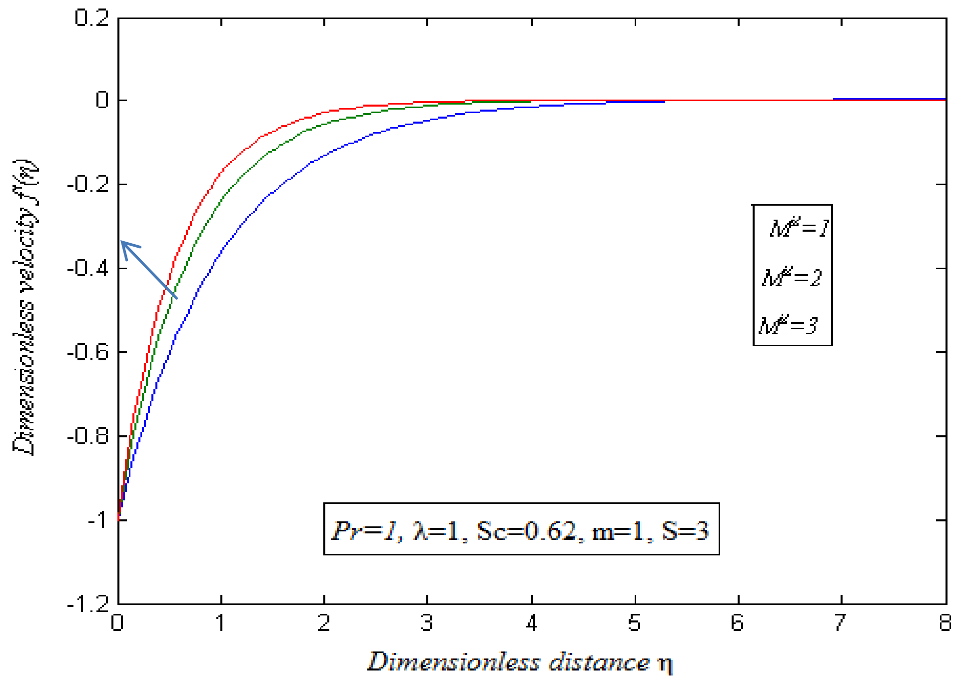
Fig (4) represents the dimensionless concentration $\phi(\eta)$ versus the distance η.

From fig 4 (a) and (b) it is clear that when the effect of shrining sheet parameter m and magnetic strength M increases the corresponding concentration profile decreases in some fixed value of the other parameters. From Table 1 it is observed that when we increases the shrinking, suction, porosity and magnetic parameters the corresponding skin friction also increases and rate of heat and mass transfer decreases .Also it is noted that heat transfer and skin friction uniform whereas the rate of mass transfer decreases with increases of chemical reaction parameter.

(a)



(b)



(c)

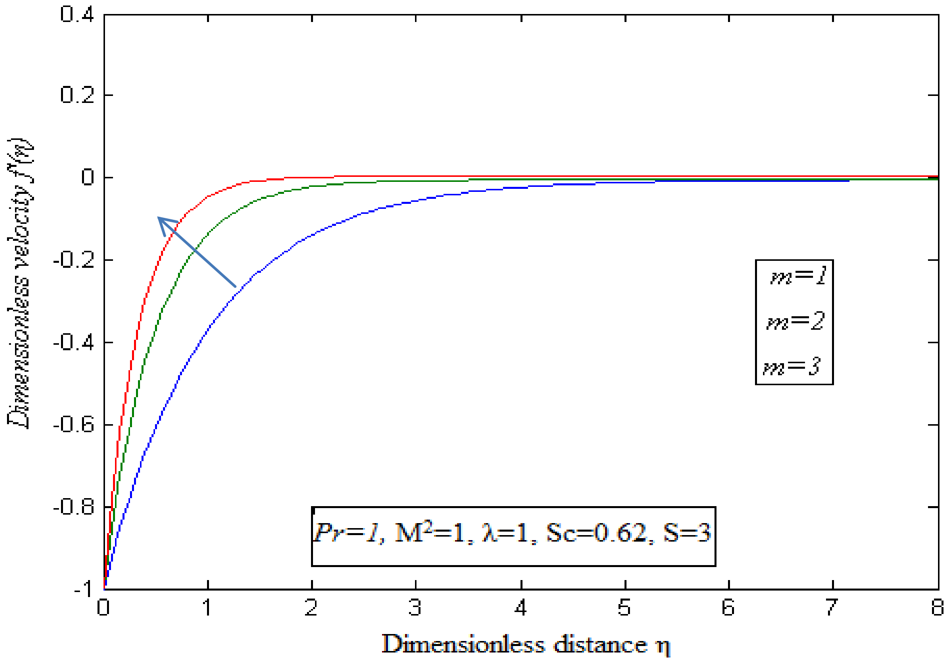
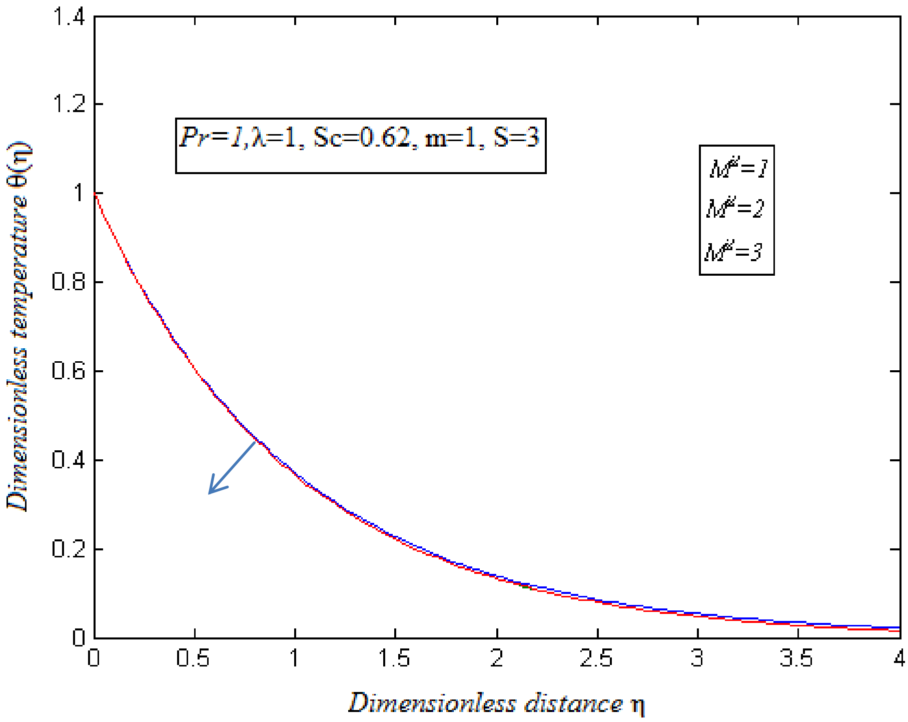


Fig.2: Dimensionless velocity $f'(\eta)$ versus the dimensionless distance η. The curves are plotted using the eqn.(26) for various values of the dimensionless parameter Pr, λ, m, M^2 and in some fixed values of the other dimensionless parameters, when

$(a) Pr = 0.71, \lambda = 0, m = 1, M^2 = 2; (b) Pr = 1, \lambda = 1, Sc = 0.62, m = 1, S = 3;$
$(c) Pr = 1, M^2 = \lambda = 1, Sc = 0.62, S = 3.$

(a)



(b)

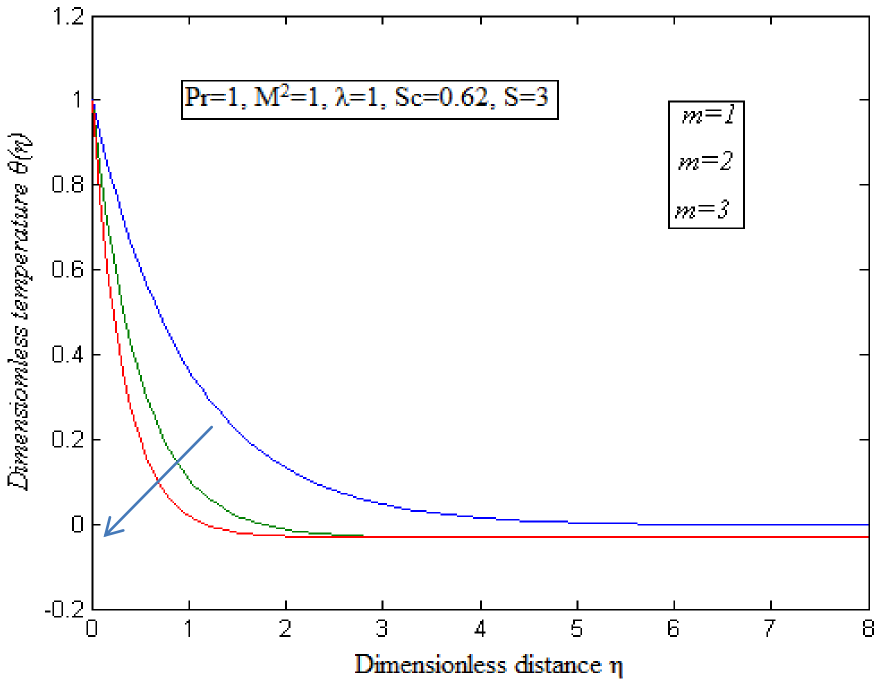


Fig.3: Dimensionless temperature $\theta(\eta)$ versus the dimensionless distance η. The curves are plotted using the eqn.(27) for various of the dimensionless parameters and in some fixed values of the other dimensionless parameters, when

$(a) Pr = \lambda = 1, Sc = 0.62, m = 1, S = 3; (b) Pr = M^2 = \lambda = 1, Sc = 0.62, S = 3.$

(a)

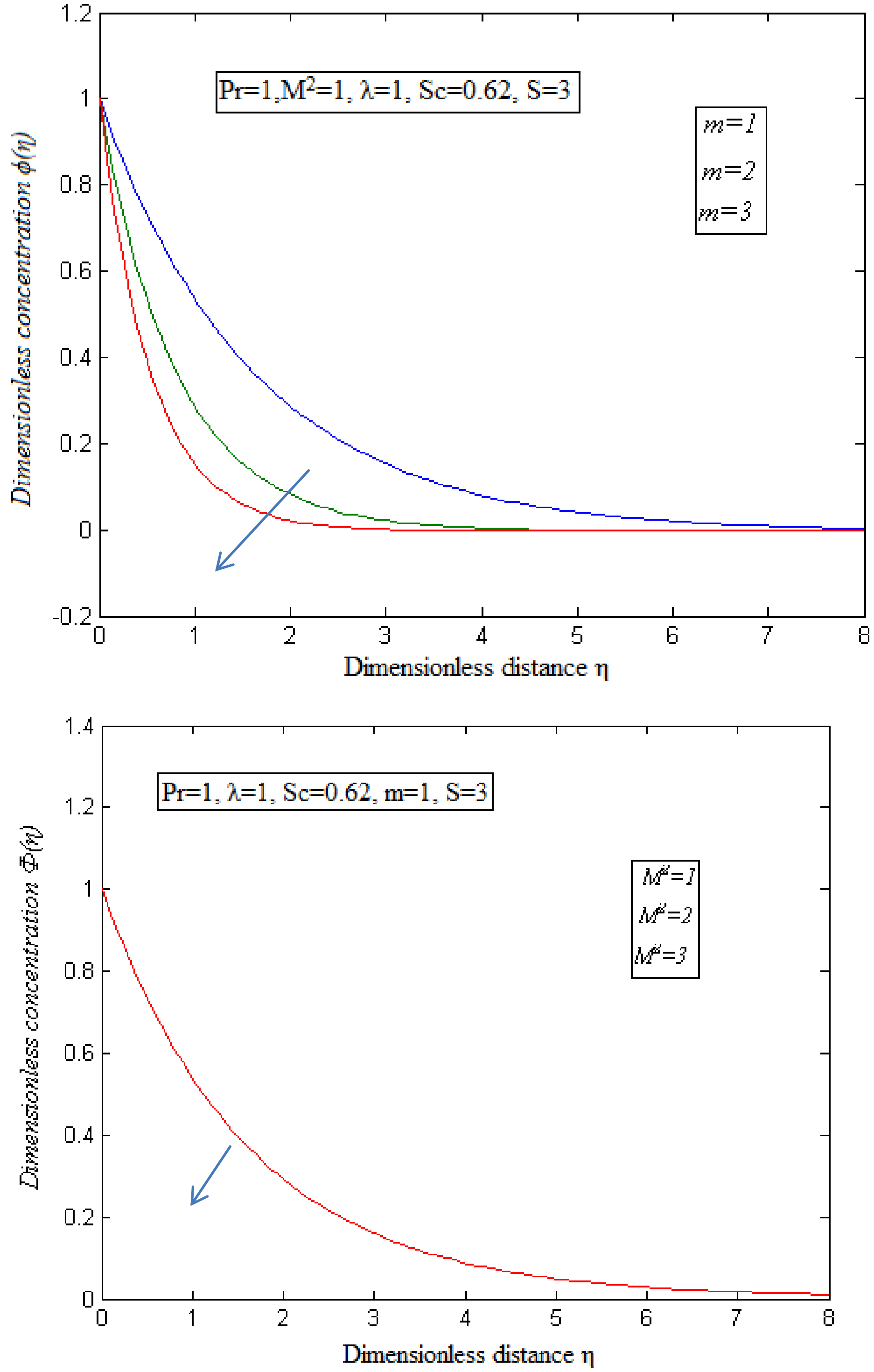


Fig.4: Dimensionless concentration $\phi(\eta)$ versus the dimensionless distancce η. The curves are plotted using the eqn.(28) for various values of the dimensionless parameters $\Pr, \lambda, Sc, m, S, M^2$ and in some fixed values of the other dimensiomless parameters.

Table 1: Analysis for skin friction and rate of heat and mass transfer

	$f''(0)$		$\theta'(0)$		$\phi'(0)$	
	Previous work	Current work	Previous work	Current work	Previous work	Current Work
$\lambda = 1$	3.302776	3.30286	-2.665537	-2.66537	-2.410283	-2.41026
$\lambda = 2$	3.561553	3.56147	-2.680315	-2.68001	-2.417000	-2.41721
$\lambda = 4$	4	4.10317	-2.702455	-2.70243	-2.427225	-2.42700
$M^2 = 1$	3.302776	3.26003	-2.665537	-2.66524	-2.410283	-2.41019
$M^2 = 2$	3.561553	3.56023	-2.680315	-2.68029	-2.417000	-2.41687
$M^2 = 3$	3.791288	3.79109	-2.692318	-2.69338	-2.422522	-2.42239
$m = 1$	2.414214	2.41420	-1.493292	-1.49347	-1.917597	-1.91749
$m = 2$	4.124816	4.12398	-3.608226	-3.60820	-2.865386	-2.86524
$m = 3$	6.001955	6.00012	-5.653797	-5.65358	-3.944462	-3.83920

5. Conclusion

In this paper the effect of chemical reaction heat and mass transfer refer MHD flow of incompressible viscous fluid over a shrinking sheet are investigated. The effect of decreasing the porosity of shrinking sheet then the rate of heat and mass transfer are increases. It is seen that the increases of the shrinking parameter dimensionless velocity, dimensionless temperature and dimensionless concentration are decreases. Also we compared the physical quantities for various parameters

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Appendix: A

Basic concept of the Homotopy analysis method [21-31]

Consider the following differential equation:

$$N[u(t)] = 0 \qquad \text{(A.1)}$$

Where N is a nonlinear operator, t denotes an independent variable, $u(t)$ is an unknown function. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of generalizing the conventional Homotopy method, Liao constructed the so-called zero-order deformation equation as:

$$(1-p)L[\varphi(t;p)-u_o(t)] = phH(t)N[\varphi(t;p)] \quad \text{(A.2)}$$

where $p \in [0,1]$ is the embedding parameter, $h \neq 0$ is a nonzero auxiliary parameter, $H(t) \neq 0$ is an auxiliary function, L an auxiliary linear operator, $u_o(t)$ is an initial guess of $u(t), \phi(t:p)$ is an unknown function. It is important to note thatone has great freedom to choose auxiliary unknowns in HAM. Obviously, when $p=0$ and $p=1,$ it holds:

$$\varphi(t;0) = u_o(t) \, textand \, \varphi(t;1) = u(t) \quad \text{(A.3)}$$

respectively. Thus, as p increases from 0 to 1, the solution $\varphi(t;p)$ varies from the initial guess $u_o(t)$ to the solution $u(t)$.

Expanding $\varphi(t;p)$ in Taylor series with respect to p, we have:

$$\varphi(t;p) = u_o(t) + \sum_{m=1}^{+\infty} u_m(t)p^m \quad \text{(A.4)}$$

$$u_m(t) = \frac{1}{m!} \frac{\partial^m \varphi(t;p)}{\partial p^m}\bigg|_{p=0} \quad \text{(A.5)}$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter h, and the auxiliary function are so properly chosen,the series (A.4) converges at $p=1$ then we have:

$$u(t) = u_o(t) + \sum_{m=1}^{+\infty} u_m(t) \quad \text{(A.6)}$$

Differentiating (A.2) for m times with respect to the embedding parameter p, and then setting $p=0$ and finally dividing them by m!, we will have the so-called mth -order deformation equation as:

$$L[u_m - \chi_m u_{m-1}] = hH(t)\Re_m(\vec{u}_{m-1}) \quad \text{(A.7)}$$

Where

$$\Re_m(\vec{u}_{m-1}) = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\varphi(t;p)]}{\partial p^{m-1}} \quad \text{(A.8)}$$

And

$$\chi_m = \begin{cases} 0, m \leq 1, \\ 1, m > 1. \end{cases} \quad \text{(A.9)}$$

Applying L^{-1} on both side of the eqn. (A7), we get

$$u_m(t) = \chi_m u_{m-1}(t) + hL^{-1}[H(t)\Re_m(\vec{u}_{m-1})] \quad \text{(A.10)}$$

In this way, it is easily to obtain u_m for $m \geq 1$, at M^{th} order, we have

$$u(t) = \sum_{m=0}^{M} u_m(t) \quad \text{(A.11)}$$

When $M \to +\infty,$ we get a

n accurate approximation of the original eqn. (A.1). For the convergence of the above method. If the eqn. (A.1) admits unique solution, then this method will produce the unique solution.

Appendix: B

Solution of the Non-linear Differential Equations(16) - (19) using the Homotopy Analysis Method

This appendix contains the derivation of the analytical expressions the eqns. (26) - (28) for $f(\eta), \theta(\eta)$ and $\phi(\eta)$ using the Homotopy analysis method. The eqns.(20)-(22) are as follows:

$$f''' - (M^2 + Pr\,\lambda) f' - f'^2 + m\, f\, f'' = 0 \quad \text{(B.1)}$$

$$\theta'' + m Pr\, f\, \theta' - Pr\, \theta\, f' = 0 \quad \text{(B.2)}$$

$$\phi'' - Sc\, f'\phi + m\,Sc\, f\, \phi' - Sc\,\gamma\phi = 0 \qquad \text{(B.3)}$$

With the following boundary conditions

$$\eta = 0 : f(0) = S,\, f'(0) = -1, \theta(0) = 1, \phi(0) = 1 \qquad \text{(B.4)}$$

$$\eta \to \infty, f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \qquad \text{(B.5)}$$

We construct the Homotopy for the equations (B.1), (B.2) and (B.3)as follows:

$$(1-p)\left(f''' - \left(M^2 + Pr\,\lambda\right) f'\right) = hp\left(f''' - \left(M^2 + Pr\,\lambda\right) f' - f'^2 + m\, ff''\right) \qquad \text{(B.6)}$$

$$(1-p)\left(\theta_0'' + p\theta_1'' + ...\right) = hp\begin{pmatrix} \left(\theta_0'' + p\theta_1'' + ..\right) + m\,Pr\left(f_0 + pf_1 + ..\right)\left(\theta_0' + p\theta_1' + ..\right) \\ -Pr\left(\theta_0 + p\theta_1\right)\left(f_0' + pf_1'\right) \end{pmatrix} \qquad \text{(B.7)}$$

$$\begin{aligned} &(1-p)\left(\phi_0'' + p\phi_1'' + ...\right) \\ &= h\,p\begin{pmatrix} \left(\phi_0'' + p\phi_1'' + ...\right) - Sc\left(f_0' + pf_1' + ...\right)\left(\phi_0 + p\phi_1\right) \\ + m\,Sc\left(f_0 + pf_1 + ..\right)\left(\phi_0' + p\phi_1' + ..\right) - Sc\,\gamma\left(\phi_0 + p\phi_1 + ...\right) \end{pmatrix} \end{aligned} \qquad \text{(B.8)}$$

The approximate solution of the eqns.(B.6)-(B.8) are as follows:

$$f = f_0 + pf_1 + p^2 f_2 + ... \qquad \text{(B.9)}$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + ... \qquad \text{(B.10)}$$

$$\phi = \phi_0 + p\phi_1 + p^2\phi_2 + ... \qquad \text{(B.11)}$$

By substituting the eqns.(B.9) to (B.11) and (B.10) respectively into the eqns. (B.6) to (B.8) we get the following eqns.:

$$\begin{aligned} &(1-p)\left[\left(f_0''' + pf_1''' + p^2 f_2''' + ...\right) - \left(M^2 + Pr\,\lambda\right)\left(f_0' + pf_1' + p^2 f_2' + ...\right)\right] \\ &= hp\begin{bmatrix} \left(f_0''' + pf_1''' + p^2 f_2''' + ...\right) - \left(M^2 + Pr\,\lambda\right)\left(f_0' + pf_1' + p^2 f_2' + ...\right) \\ -\left(f_0' + pf_1' + p^2 f_2' + ...\right)^2 + \\ m\left(f_0 + pf_1 + p^2 f_2 + ...\right)\left(f_0'' + pf_1'' + p^2 f_2'' + ...\right) \end{bmatrix} \end{aligned} \qquad \text{(B.12)}$$

$$\begin{aligned} &(1-p)\left[\left(\theta_0'' + p\theta_1'' + p^2\theta_2'' + ...\right)\right] \\ &= hp\begin{bmatrix} \left(\theta_0'' + p\theta_1'' + p^2\theta_2'' + ...\right) \\ + m\Pr\left(f_0 + pf_1 + p^2 f_2 + ...\right)\left(\theta_0' + p\theta_1' + p^2\theta_2' + ...\right) \\ -\Pr\left(\theta_0 + p\theta_1 + p^2\theta_2 + ...\right)\left(f_0' + pf_1' + p^2 f_2' + ...\right) \end{bmatrix} \end{aligned} \qquad \text{(B.13)}$$

$$\begin{aligned} &(1-p)\left[\left(\phi_0'' + p\phi_1'' + p^2\phi_2'' + ...\right)\right] \\ &= hp\begin{bmatrix} \left(\phi_0'' + p\phi_1'' + p^2\phi_2'' + ...\right) \\ -Sc\left(f_0' + pf_1' + p^2 f_2' + ...\right)\left(\phi_0 + p\phi_1 + p^2\phi_2 + ...\right) \\ + mSc\left(f_0 + pf_1 + p^2 f_2 + ...\right)\left(\phi_0 + p\phi_1 + p^2\phi_2 + ...\right) \\ -Sc\gamma\left(\phi_0 + p\phi_1 + p^2\phi_2 + ...\right) \end{bmatrix} \end{aligned} \qquad \text{(B.14)}$$

Equating the coefficients of p^0 and p^1 for the equations (B.12) ,(B.13),(B.14) we get the following eqns.:

$$p^0 : f_0''' - \left(M^2 + Pr\,\lambda\right) f_0' = 0 \qquad \text{(B.15)}$$

$$p^0 : \theta_0'' = 0 \qquad \text{(B.16)}$$

$$p^0 : \phi_0'' = 0 \qquad \text{(B.17)}$$

$$p^1 : f_1''' - \left(M^2 + Pr\,\lambda\right) f_1' = h\left(-f_0'^2 + m\, f_0\, f_0''\right) \qquad \text{(B.18)}$$

$$p^1 : \theta_1'' = h\left(m\,Pr\, f_0\theta_0' - Pr\,\theta_0\, f_0'\right) \qquad \text{(B.19)}$$

$$p^1 : \phi_1'' = h\left(-Sc\, f_0'\phi_0 + m\,Sc\, f_0\phi_0' - Sc\,\gamma\,\phi_0\right) \qquad \text{(B.20)}$$

Consider the initial solutions are as follows:

$$f_0 = S + \frac{e^{-A\eta}}{A} \tag{B.21}$$

$$\theta_0 = e^{-Pr\,m\,\eta} \tag{B.22}$$

$$\phi_0 = e^{-m\,Sc\,\gamma\,\eta} \tag{B.23}$$

Solving (B.17) , (B.18) and (B.19) using the boundary conditions (B.4), we get

$$f_1 = C_1 + C_2 e^{\sqrt{\beta}\eta} h\left[-\frac{S^2\eta}{B} + \frac{\left(\frac{1}{A^2} - m\right)e^{-2A\eta}}{-8A^2 + 2AB} + \frac{\left(-mA + \frac{2}{A}\right)Se^{-A\eta}}{-A^3 + BA}\right] \tag{B.24}$$

$$\theta_1 = C_4\eta + C_5 + h\left[\begin{array}{l} S\,e^{-Pr\,m\eta} + \dfrac{m^2\ Pr}{A\left(Pr\,m + A\right)^2} \quad e^{-(Pr\,m+A)\eta} \\ + \dfrac{Pr}{\left(Pr\,m + A\right)^2} e^{-(Pr\,m+A)\eta} \end{array}\right] \tag{B.25}$$

$$\phi_1 = C_6\eta + C_7 + h\left(\frac{SCe^{-(A+\infty)\eta}}{(A+D)^2}\left(\frac{mD}{A} - 1\right) + \frac{SC}{D^2}\left(mSD + \gamma\right)\right) \tag{B.26}$$

where the constants C_1 *to* C_7, A, *and* D are defined in the text eqns. (29)-(35).

According to HAM we have

$$f = \lim_{p\to 1} f(\eta) = f_0 + f_1 \tag{B.27}$$

$$\theta = \lim_{p\to 1} \theta(\eta) = \theta_0 + \theta_1 \tag{B.28}$$

$$\phi = \lim_{p\to 1} \phi(\eta) = \phi_0 + \phi_1 \tag{B.29}$$

Substituting the eqns.(B.21) and (B.24) into an eqn. (B.27), (B.22) and (B.25) into an eqn. (B.28), (B.23) and (B.26) into an eqn. (B.29) we get the solutions in the text eqns.(26) to (30).

Appendix C:
Nomenclature

Symbol	Meaning
f	Dimensionless velocity
θ	Dimensionless temperature
ϕ	Dimensionless concentration
η	Dimensionless distance
Pr	Prandtl number
Sc	Schmidt number
M^2	Magnetic parameter
λ	Porosity parameter
S	Suction parameter