

## Problem 1

Consider the following equation which describes the transverse deflection associated with a simply supported beam subject to a uniform transverse load  $q(x) = q_0$ ,

$$\frac{d^2}{dx^2} \left[ EI \frac{d^2 w(x)}{dx^2} \right] = q_0 \quad \text{for } 0 < x < L,$$

subject to boundary conditions,

$$w(0) = EI \frac{d^2 w(0)}{dx^2} = 0, \quad \text{and} \quad w(L) = EI \frac{d^2 w(L)}{dx^2} = 0.$$

### (a) 10 points

Develop and clearly indicate the weak form of this differential equation.

### (b) 5 points

Using the following approximation for  $u$ ,

$$u \approx u_h = c_1 x (x - L),$$

use the Ritz method to determine the coefficient  $c_1$ .

### (c) 5 points

Using the following approximation for  $u$ ,

$$u \approx u_h = c_1 \sin \left( \frac{\pi x}{L} \right)$$

use the Ritz method to determine the coefficient  $c_1$ .

### (d) 5 points

Compare your answers from (b) and (c) with the analytic solution at  $x = L/2$ . Which is more accurate? Why? (Hint: It has something to do with the boundary conditions.)

## Problem 2

We have seen that the following one-dimensional boundary value problem describes the physics of many interesting problems in engineering

$$-\frac{d}{dx} \left[ a \frac{du}{dx} \right] + cu - f = 0 \quad \text{for } 0 < x < L,$$

subject to the boundary conditions

$$u(0) = u_0 \quad \text{and} \quad \left[ a \frac{du}{dx} \right]_{x=L} = Q_0,$$

where the  $x$  is the independent variable,  $u = u(x)$  is the dependent variable, and  $a = a(x)$ ,  $c = c(x)$ ,  $f = f(x)$ ,  $u_0$ , and  $Q_0$  are the *data* of the problem.

### (a) 30 points

Write a general one-dimensional finite element code to solve this problem. General means that the user will input the *node* locations and specify the functions  $a(x)$ ,  $c(x)$ , and  $f(x)$  as well as the boundary conditions  $u_0$  and  $Q_0$ . The output of the code should be the value of  $u$  at the user specified nodes. You can restrict the code to using only linear interpolated elements.

You can verify your code with following data and results:

1. For  $a(x) = 1 - x/2$ ,  $c(x) = 0$ ,  $f(x) = 0$ ,  $u_0 = 0$ , and  $Q_0 = 1$ ,

$x$	$u(x)$
0.00	0.00000
0.25	0.26666
0.50	0.57435
0.75	0.93799
1.00	1.38244

2. For  $a(x) = 1 - x/2$ ,  $c(x) = x$ ,  $f(x) = x$ ,  $u_0 = 0$ , and  $Q_0 = 1$ ,

$x$	$u(x)$
0.00	0.00000
0.40	0.46695
0.60	0.73242
0.65	0.80319
0.70	0.87615
1.00	1.38355

3. For  $a(x) = 1$ ,  $c(x) = 0$ ,  $f(x) = x^2$ ,  $u_0 = 0$ , and  $Q_0 = 2$ ,

$x$	$u(x)$
1	0.000
2	22.08
3	40.00
4	48.75

**(b) 20 points**

A one-dimensional heterogeneous porous medium of length  $L = 1$  m, has a steady state pressure distribution as shown in the following table.

$x$ (m)	$p(x)$ (kPa)
0.00	100.0
0.05	100.005
0.10	100.010
0.15	100.014
0.20	100.018
0.25	100.021
0.30	100.024
0.35	100.026
0.40	100.029
0.45	100.030

$x$ (m)	$p(x)$ (kPa)
0.50	100.032
0.55	100.033
0.60	100.034
0.65	100.035
0.70	100.036
0.75	100.037
0.80	100.038
0.85	100.038
0.90	100.039
0.95	100.040
1.00	100.040

The porous media is subject to the a constant pressure a  $x = 0$  of  $p = 100$  kPa and an exit flow rate of  $Q = 100$  m/s is measured.

Use the code developed in part (a) to help you estimate what the permiablity  $\kappa(x)$  this material. You can use a consant viscosity of  $\mu = 1$  Pa·s. Clearly indicate your answer and explain your approach (preferably with plots). The total diffusivity coefficient is  $K = \frac{\kappa}{\mu}$ .

**Note:** Submit a working version of your code to [Canvas](#). Any supplemental material explaining your answer in part (b) can be turned in to me via hard copy or scanned and submitted to Canvas with your code.