Homework Assignment 1

1. The motion of a certain continuous medium is defined by the equations

$$
\begin{aligned}
x_1 &= \frac{1}{2}\left(X_1 + X_2\right) e^t + \frac{1}{2}\left(X_1 - X_2\right) e^{-t}, \\
x_2 &= \frac{1}{2}\left(X_1 + X_2\right) e^t - \frac{1}{2}\left(X_1 - X_2\right) e^{-t}, \\
x_3 &= X_3
\end{aligned}
$$

   a. Compute the following

      i. The Green-Lagrange strain tensor E
      ii. The linear (small) strain tensor ε

      Plot the 11, 22, and 12 components of E and ε on the same figure from time $t = 0$ to $t = 0.05$.

   b. Compute the following

      i. The rate-of-deformation tensor D
      ii. The rate-of-change of the small strain tensor $\dot{\varepsilon} = \frac{d\varepsilon}{dt}$

      Plot the 11, 22, and 12 components of D and $\dot{\varepsilon}$ on the same figure from time $t = 0$ to $t = 0.05$.

2. Given the following stress tensor

$$
\sigma = \begin{bmatrix} 36 & 27 & 0 \\ 27 & -36 & 0 \\ 0 & 0 & 18 \end{bmatrix}
$$

   Find:

   a. the components of the traction vector acting on a plane with unit normal vector $\hat{n}^T = [2/3, -2/3, 1/3]$
   b. the magnitude of the traction vector found in (a)
   c. its component in the direction of the normal
   d. the angle between the traction vector and the normal

3. Given the following stress tensor

$$
\sigma = \begin{bmatrix} 18 & 0 & 24 \\ 0 & -50 & 0 \\ 24 & 0 & 32 \end{bmatrix}
$$

   Find:

   a. the principle stresses $\sigma_I, \sigma_{II}, \sigma_{III}$

b. the three invariants I_1, I_2, I_3

c. the deviatoric stress

d. the two nonzero invariants of the deviatoric stress, i.e. J_2, J_3

4. Show that
$$\frac{\partial J_2}{\partial \sigma_{ij}} = S_{ij}$$
where J_2 is the second invariant of the deviatoric stress tensor, S_{ij}.

5. For each of the following stress states (values not given are zero), plot the three Mohr's circles and determine the maximum shear stress.

   a. Uniaxial tension $\sigma_{11} = 40$

   b. Biaxial stress $\sigma_{11} = -10, \sigma_{22} = 30$

   c. Hydrostatic tension of magnitude 100 psi

   d. $\sigma_{11} = -60, \sigma_{22} = 100, \sigma_{33} = 40$

   e. $\sigma_{11} = 10, \sigma_{22} = 40, \sigma_{21} = \sigma_{12} = 20$