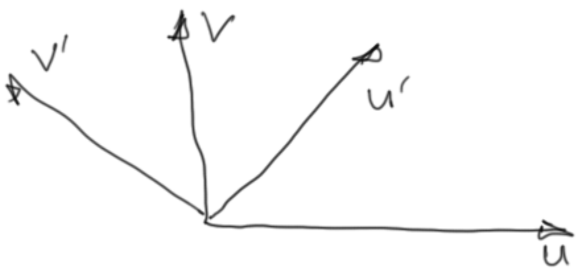
$$[t_1\ \ t_2\ \ t_3] = [n_1\ \ n_2\ \ n_3]\begin{bmatrix}\sigma_{11} & \sigma_{12} & \\ & \sigma_{22} & \\ & & \ddots\end{bmatrix}$$

$$(\vec{t}^{\,T})^T = (\hat{n}^T \sigma)^T$$

$$\boxed{\vec{t} = \sigma^T \hat{n}} \qquad \sigma^T = \sigma$$

Cauchy stress equation

$$t_i = \sigma_{ji}\,\hat{n}_i$$



$$u' = Ru$$
$$v' = Rv$$

$$\underline{\vec{v} = T\vec{u}} \rightarrow \vec{v}' = T'\vec{u}' \rightarrow \overset{I}{\cancel{R^{-1}R}}\vec{v} = R^{-1}T'R\vec{u}$$

$$\vec{v} = R^{-1}T'R\vec{u}$$

$$\vec{v} = \underbrace{R^T T' R}\vec{u}$$

$$T = R^T T' R$$
$$T' = R T R^T$$

$T' = R T R^T$

$$\sigma' = R \sigma R^T = \begin{bmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix} \qquad \sigma_I > \sigma_{II} > \sigma_{III}$$

$$Q \Lambda Q^T = D \qquad \det(\sigma - \lambda I) = 0$$

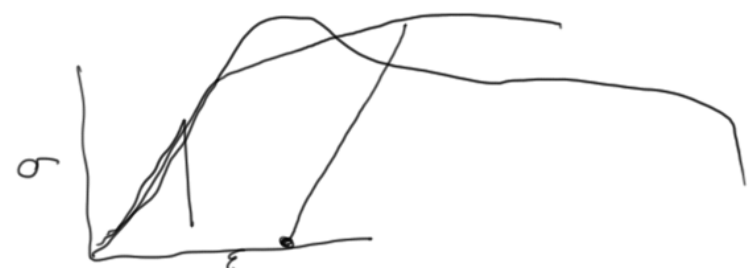
$$-\lambda^3 + I_1 \lambda^2 + I_2 \lambda + I_3 = 0$$

Invariants of stress tensor

$$I_1 = tr(\sigma) = \sigma_{ii} = \sigma_I + \sigma_{II} + \sigma_{III} = -3p$$

$$\boxed{I_2 = \frac{1}{2} \sigma_{ij} \sigma_{ij} - \frac{1}{2} I_1^2} = -(\sigma_I \sigma_{II} + \sigma_I \sigma_{III} + \sigma_{II} \sigma_{III})$$

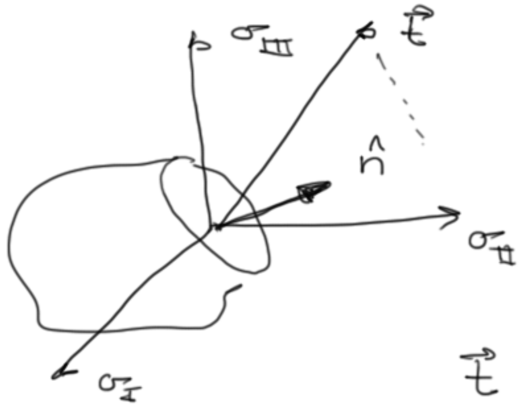
$$I_3 = \det(\sigma) = \sigma_I \sigma_{II} \sigma_{III}$$



$$\sigma_{ij} = \underbrace{\frac{1}{3} \sigma_{kk} \delta_{ij}}_{\text{spherical}} + S_{ij} \rightarrow \text{deviatoric}$$

spherical
hydrostatic
dilatational stress

$J_1 = 0$, $J_2 = \frac{1}{2} S_{ij} S_{ij}$, $J_3 = \det(S)$



$$\vec{t} = \sigma^T \hat{n} = \begin{bmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix} \begin{Bmatrix} \hat{n}_1 \\ \hat{n}_2 \\ \hat{n}_3 \end{Bmatrix} = \begin{Bmatrix} \sigma_I \hat{n}_1 \\ \sigma_{II} \hat{n}_2 \\ \sigma_{III} \hat{n}_3 \end{Bmatrix}$$

$$t_n = \vec{t} \cdot \vec{n} = \qquad = \sigma_I \hat{n}_1^2 + \sigma_{II} \hat{n}_2^2 + \sigma_{III} \hat{n}_3^2 \quad \text{—} \ 1$$

$$|\vec{t}|^2 = \cancel{\sqrt{t_s^2 + t_n^2}}^{\,2}$$

$$t^2 = t_n^2 + t_s^2 = (\sigma_I \hat{n}_1)^2 + (\sigma_{II} \hat{n}_2)^2 + (\sigma_{III} \hat{n}_3)^2 \quad \text{—} \ 2$$

$$|\hat{n}| = 1 = \hat{n}_1^2 + \hat{n}_2^2 + \hat{n}_3^2 \quad \text{—} \ 3$$

$$\begin{bmatrix} 1 & 1 & 1 \\ \sigma_I & \sigma_{II} & \sigma_{III} \\ \sigma_I^2 & \sigma_{II}^2 & \sigma_{III}^3 \end{bmatrix} \begin{Bmatrix} \hat{n}_1^2 \\ \hat{n}_2^2 \\ \hat{n}_3^2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ t_n \\ t_s^2 + t_n^2 \end{Bmatrix}$$

Solve

$$\hat{n}_1^2 = \frac{t_s^2 + (t_n - \sigma_{II})(t_n - \sigma_{III})}{\underbrace{(\sigma_I - \sigma_{II})(\sigma_I - \sigma_{III})}_{\geq 0}} \geq 0$$

$$\hat{n}_2^2 = \frac{t_s^2 + (t_n - \sigma_{III})(t_n - \sigma_I)}{\underbrace{(\sigma_{II} - \sigma_{III})(\sigma_{II} - \sigma_I)}_{\leq 0}} \leq 0$$

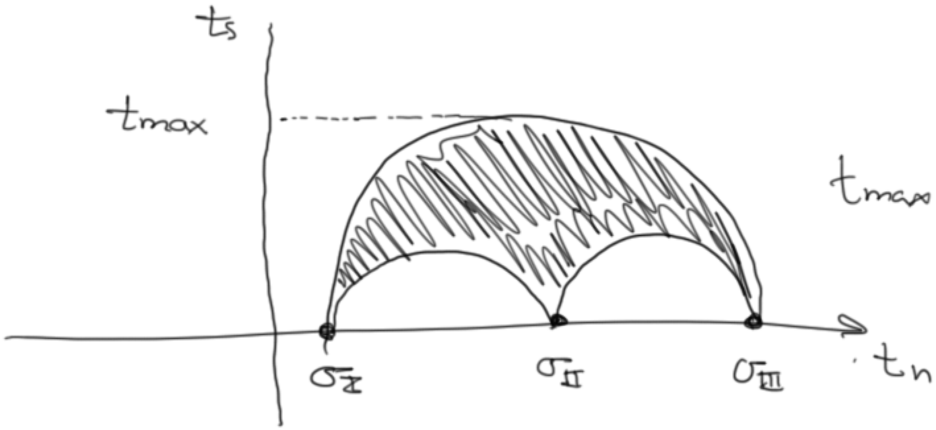
$$\hat{n}_3 = \frac{t_s^2 + (t_n - \sigma_I)(t_n - \sigma_{II})}{\underbrace{(\sigma_{III} - \sigma_I)(\sigma_{III} - \sigma_{II})}_{\geq 0}} \geq 0$$

$$\underbrace{\left[t_n - \tfrac{1}{2}(\sigma_{II} + \sigma_{III})\right]^2}_{x^2} + \underbrace{t_s^2}_{y^2} \geq \underbrace{\left(\tfrac{1}{2}(\sigma_{II} - \sigma_{III})\right)^2}_{r^2}$$

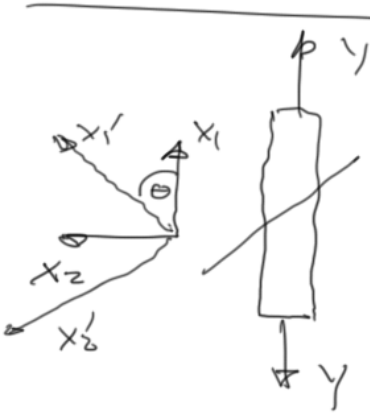
$$x^2 + y^2 \geq r^2$$

$$\left[t_n - \frac{1}{2}(\sigma_I + \sigma_{III})\right]^2 + t_s^2 \leq \left(\frac{1}{2}(\sigma_I - \sigma_{III})\right)^2$$

$$\left[t_n - \frac{1}{2}(\sigma_I + \sigma_{III})\right]^2 + t_s^2 \leq \left(\frac{1}{2}(\sigma_I - \sigma_{II})\right)^2$$



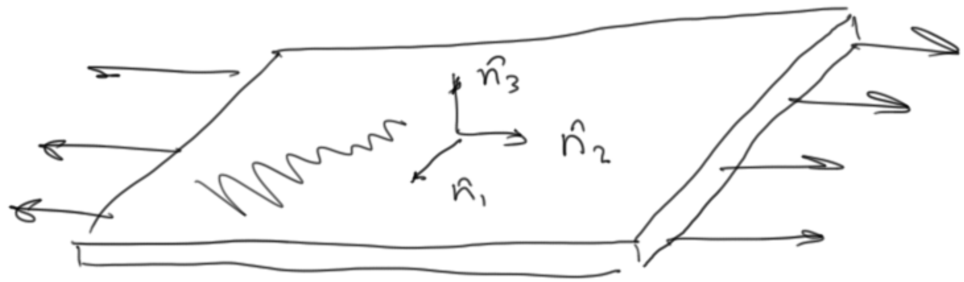
$$t_{max} = \frac{1}{2}(\sigma_I + \sigma_{III})$$



$$\sigma = \begin{bmatrix} Y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sigma' = R \sigma R^T$$

$$\sigma_{11}' = Y\cos\theta \qquad \sigma_{12}' = \frac{Y}{2}\sin\theta\cos\theta = \frac{Y}{2}\sin 2\theta \quad {}^{max} \text{ at } 45^\circ$$

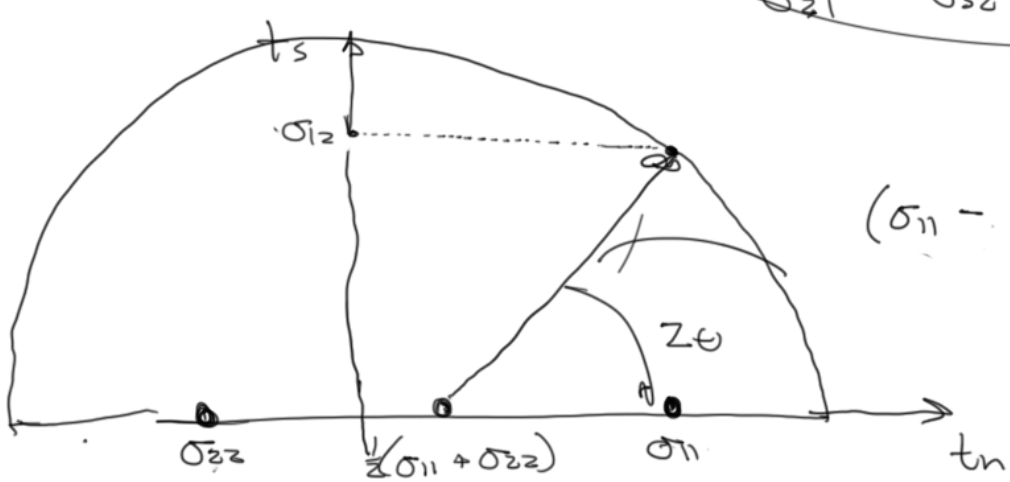


Plane stress

$$\vec{t} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$$

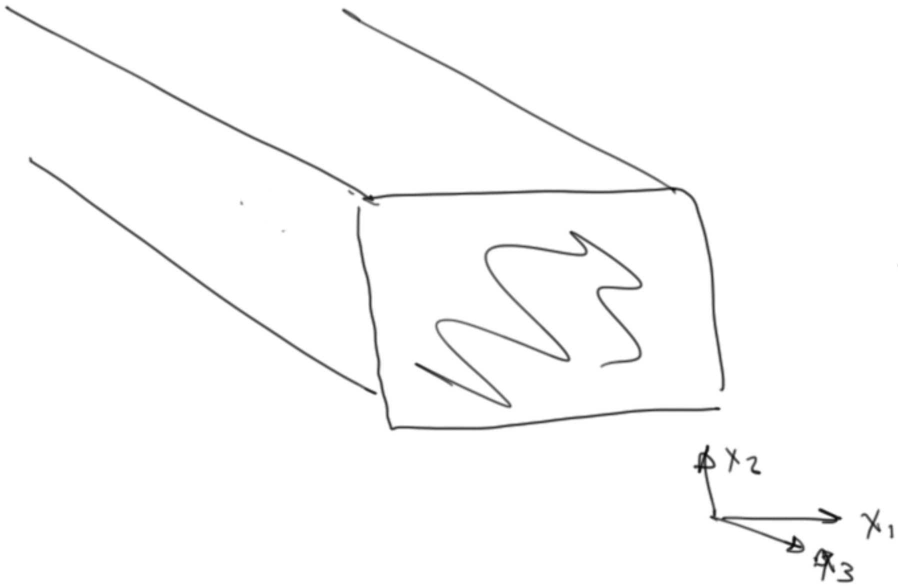
$$\sigma_{31} = \sigma_{32} = 0$$



$$\left(\sigma_{11} - \frac{\sigma_{11} + \sigma_{22}}{2}\right)^2 + \sigma_{12}^2$$

$$\tan(2\theta) = \left(\frac{2\sigma_{12}}{\sigma_{11} - \sigma_{22}}\right)$$

Plane strain



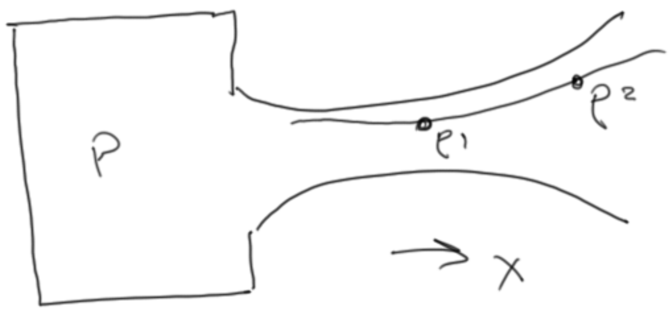
$$\varepsilon_{33} = \frac{\Delta L}{L} = 0$$

$$\varepsilon_{13} = \varepsilon_{23} = \varepsilon_{33} = 0$$

$$\varepsilon_{31} = \varepsilon_{32} = 0$$

$$\varepsilon = \frac{1}{2}\left(\nabla u + \nabla u^T\right)$$

$$\text{symm.}\left(\nabla u\right)$$



$$\frac{\partial \rho}{\partial t} = 0 \qquad \frac{\partial \rho}{\partial x} \neq 0$$

$$\vec{v} = \vec{v}\left(\vec{x}(X,t),t\right)$$

$$\frac{D}{Dt}(\vec{v}) = \frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{v}}{\partial x_k} v_k$$

$$= \underline{\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}}$$

$$\frac{D}{Dt}(\vec{v})$$

$$\frac{\partial v}{\partial t}$$