Homework Assignment 3

Problem 1

Derive the associated flow rule for a viscoplastic solid with rate-dependent with yield criterion given by

$$\sqrt{J_2} = k + \eta \left(2\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}\right)^{\frac{1}{2m}}$$

Problem 2

A work-hardening plastic solid is assumed to obey the Mises-like yield criterion with isotropic hardening, that is, $f(\boldsymbol{\sigma}, \boldsymbol{\Lambda}) = \sqrt{J_2} - k(\varepsilon^p)$ and an associated flow rule. Show that

$$\dot{\varepsilon}^p_{ij} = \frac{\sqrt{3}S_{ij}\left(S_{kl}\dot{S}_{kl}\right)}{4k^2 k'(\varepsilon^p)}$$

with the definition of $\dot{\varepsilon}^p$

$$\dot{\varepsilon}^p = -\frac{\lambda}{\sqrt{3}}\frac{\partial f}{\partial k}$$

Problem 3

We typically draw yield surfaces in the Π-plane view, i.e. looking down the $\sigma_I = \sigma_{II} = \sigma_{III}$ line; however, we can also take a meridional "side" view by projecting the three dimensional yield surface onto a 2 dimensional plane. Perform this projection and plot both the von Mises and Tresca yield surfaces with an ordinate axis that cooresponds to the $\sigma_{II} - \sigma_{III}$ plane and an abscissa that corresponds to the $\sigma_I - \sigma_{III}$ plane. Label the points that cross the axes with values that correspond to a yield stress Y.