1.1 Characteristics of Time Series Data

a) **Time dependence**: Data points are ordered in time and have a natural temporal sequence, which means that prior observations frequently influence the value of each observation.

b) **Autocorrelation**: Statistical measure that describes the relationship between an observation in time series and its own past values.

c) **Stationarity**: Statistical properties of time series do not change over time.

d) **Nonstationarity**: Statistical properties, like mean and variance, change over time, indicating that values at time point (t) can be influenced by preceding values at times like $t - 1$ or $t - 2$.

e) **Seasonality**: Recurrent fluctuations at fixed intervals (e.g., daily, monthly, yearly), influenced by factors like time of year, month, or day which are predictable and repetitious. Examples are retail sales increasing during popular holidays.

f) **Trends**: Long-term movement in the data indicates direction and movement over time. Examples are rising global temperatures and housing prices post pandemic.

g) **Cyclic patterns**: Recurrent phenomena without fixed periods, attributed to complex circumstances that are unpredictable and challenging to identify. Examples are forest growth and fire cycles.

h) **Irregularity or noise (irregular component)**: Random variations without a recurring pattern, attributed to unforeseen events or anomalies. Examples are rapid stock market fluctuations before and after a political event.

i) **Frequency**: Data is sampled at regular time intervals (e.g., hourly, daily, monthly).

j) **Duration**: Length of time between observations.

1.2 Time Series Forecasting Methods

Various techniques and algorithms are available to perform time series forecasting based on the data characteristics learned in the above section. They can be "broadly" classified into two categories – univariate and multivariate.

1.2.1 Univariate

Univariate time series analysis focuses on the study of a single time series to understand its underlying patterns and make forecasts. Let's understand some popular techniques:

a) **Moving Average (MA)**: The Moving Average model computes the average of a fixed number of previous observations to predict future values.

b) **Autoregressive (AR)**: Autoregressive models are a class of models that describe a linear relationship between an observation at a particular time and a certain number of lagged observations (i.e., past values) of the same series.

c) **Autoregressive Moving Average (ARMA)**: This model is a combination of AR (Autoregressive) and MA (Moving Average), and this combination is done to improve the approximation.

d) **Autoregressive Integrated Moving Average (ARIMA)**: This model is a combination of three models – AR (Autoregressive), MA (Moving Average), and Integrated (the number of times differencing is done to make data stationary).

e) **Seasonal Autoregressive Integrated Moving Average (SARIMA)**: SARIMA is an extension of ARIMA that can handle seasonal effects present in the data.

f) **Exponential Smoothing**: Exponential smoothing methods forecast future values by weighting past observations exponentially.

g) **SES**: Suitable for data without trend or seasonality.

h) **Holt's Linear Trend Model**: Extends SES to capture linear trends.

i) **Holt-Winters Seasonal Model**: Extends Holt's model to capture seasonality.

j) **Fourier Analysis**: Fourier Analysis decomposes a time series into sinusoidal components. It is useful for identifying cyclical patterns.

k) **Kalman Filter**: The Kalman filter is an algorithm that uses a series of measurements over time, containing statistical noise and other inaccuracies, to estimate unknown variables.

l) **Hidden Markov Models**: Models time series data as sequences of hidden states with observable outcomes, useful for sequential data with unknown state transitions.

1.2.2 Multivariate

Multivariate time series analysis extends the techniques used in univariate time series to multiple interrelated time series. Exogenous variables which are external factors affecting the target variable are included to make models robust. Examples are sales of the book impacted by exogenous variables such as target audience, reviews, and current topics in trend.

a) **Seasonal Autoregressive Integrated Moving Average with Exogenous Regressors (SARIMAX)**: SARIMAX is an extension of ARIMA which can handle seasonal effects and also include external influencing factors into the model.

b) **Vector Autoregression (VAR)**: VAR models generalize the univariate autoregressive model to capture the linear interdependencies among multiple time series.

c) **Vector Autoregressive Moving Average (VARMA)**: VARMA models extend VAR models by including moving average terms.

d) **Vector Autoregression Moving Average with Exogenous Regressors (VARMAX)**: This model is an extended version of VAR and VARMA models by incorporating exogenous variables.

e) **Vector Error Correction Model (VECM)**: VECM is used for nonstationary time series that are cointegrated. It extends the VAR model to include error correction terms, capturing long-term equilibrium relationships.

f) **Generalized Autoregressive Conditional Heteroskedasticity Models (GARCH)**: GARCH models are designed to capture the changing variances over time, especially useful for modeling financial time series data which often exhibit volatility clustering which are periods of oscillation followed by a period of relative calm.

g) **Convolutional Neural Networks (CNNs)**: CNNs can be adapted to capture spatial dependencies in multivariate time series by treating time series data as images or sequences.

h) **Recurrent Neural Network (RNN), Gated Recurrent Unit (GRU), Long Short-Term Memory (LSTM)**: A type of neural network that is well suited for sequence prediction problems. These neural networks can capture long-term dependencies in multivariate time series.

i) **Transformers**: Originally developed for natural language processing, transformers can be adapted for multivariate time series by capturing relationships between different variables and leveraging attention mechanisms.