Jacobs University Bremen

Sensors 4

Hall effect
Fiber Bragg grating
Photoelastic effect
Strain gauge

Automation
CO23-320203

Recap

- Until now, we have seen some sensors using
  - Piezoelectric effect
  - Thermoelectric effect
  - Capacitance
  - Wave propagation
- Today:
  - Electromagnetism
  - Light (a subgroup of EM waves)

Hall effect

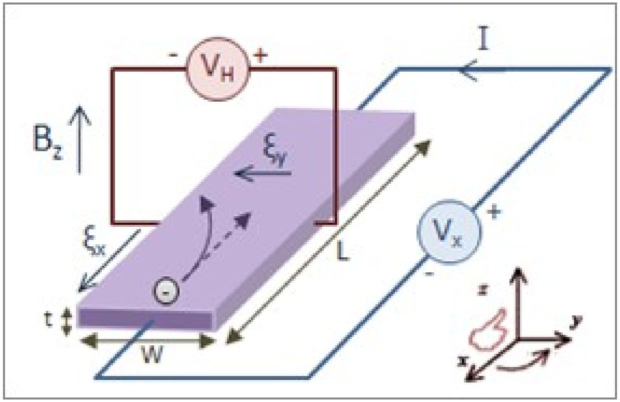
- Hall effect causes a voltage to appear across two sides of a current-carrying conductor subjected to a magnetic field
- The voltage is produced because electrons flowing in a wide conductor experience Lorenz force:

$$F = qE + qv \times B$$

where: F – force [N], q – charge [C], E – electric field [N/C], v – velocity [m/s], B – magnetic field [T = N/(Am)]

- It can be used to measure magnetic field direction and strength in one plane
- Principle of operation explained in video: https://www.youtube.com/watch?v=wpAA3qeOYiI
- It can be found integrated in many everyday items

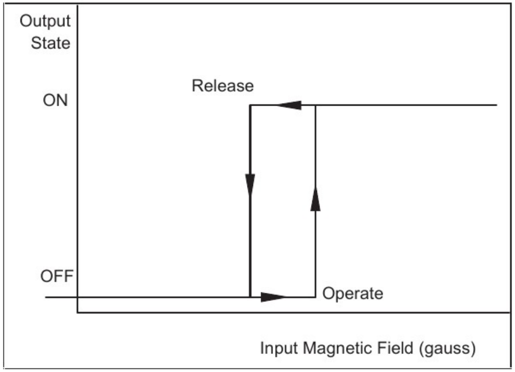
  For example, a CPU fan motor: https://www.youtube.com/watch?v=_i0rNIoo5Zk

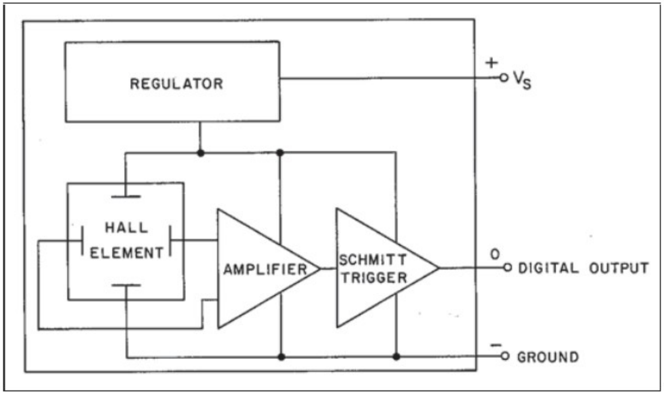


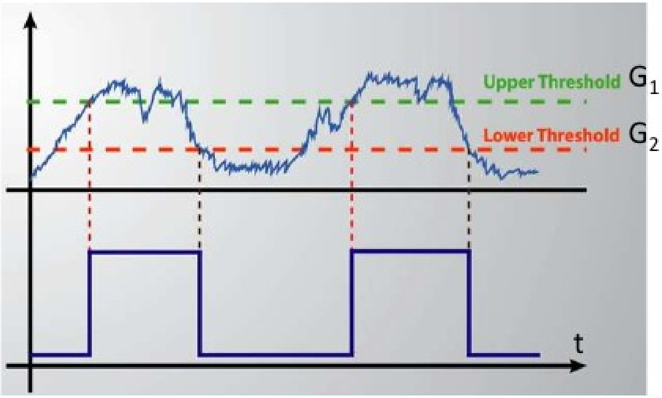
Hall effect

Practical implementation for a useful binary signal

- Use of Schmitt trigger to introduce hysteresis
- Upper-threshold only triggers when crossing from below: It changes the output from OFF to ON.
- Lower-threshold only triggers when crossing from above: It changes the output from ON to OFF.







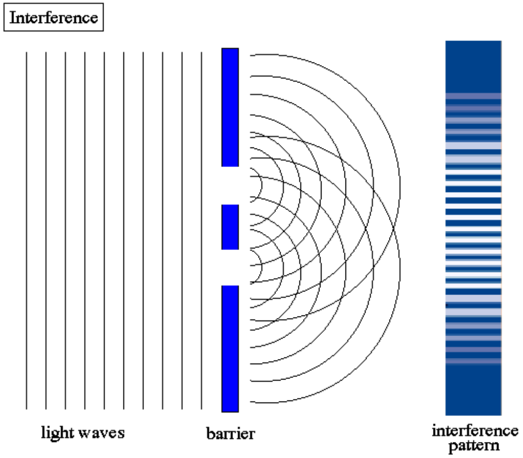
Fiber Bragg grating

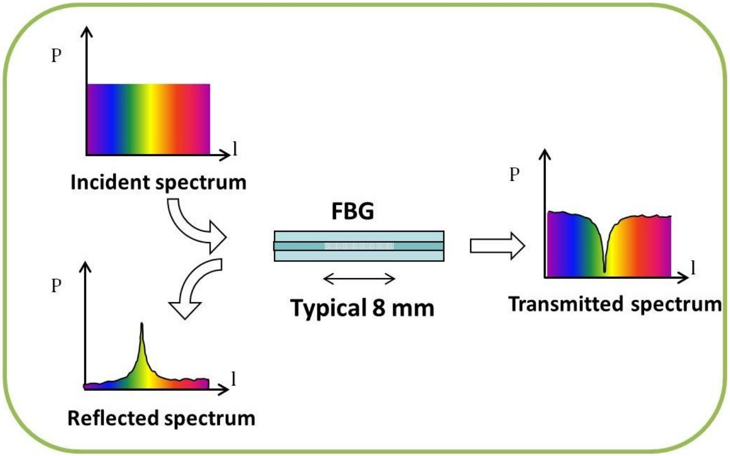
Bragg grating uses light interference for sensing strain and temperature

- Fabrication process and how they work: https://www.youtube.com/watch?v=CJGYVw8WpuQ
- A Bragg grating will reflect light of specific wavelength λ_B, regulated by the grating period

$$\lambda_B = 2 n_e \Lambda$$

where: n_e – effective refractive index of the fiber, Λ – grating constant





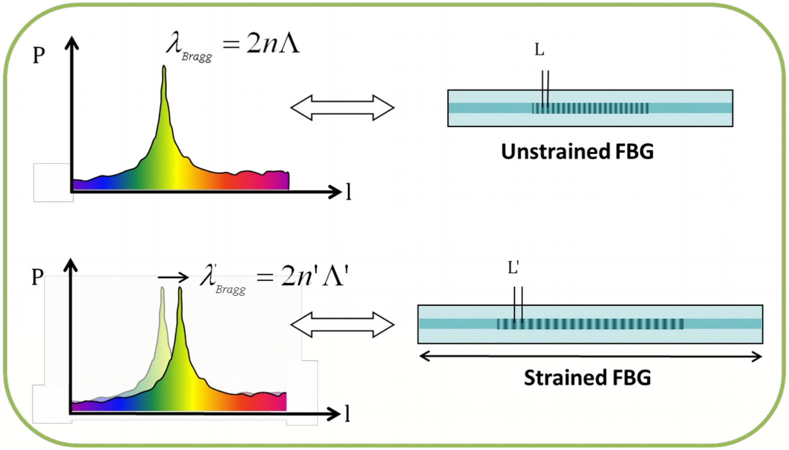
Fiber Bragg grating

- Sensing principle: this characteristic frequency will experience a relative shift if the grating constant is affected by strain or temperature expansion:

$$\left[\frac{\Delta\lambda_B}{\lambda_B}\right] = C_S\epsilon + C_T\Delta T$$

where C_S and C_T are coefficient of strain and temperature respectively

- It means we sense strain and temperature through the same variable!

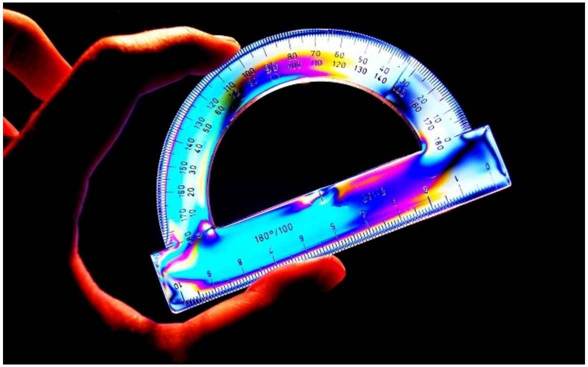


Fiber Bragg grating

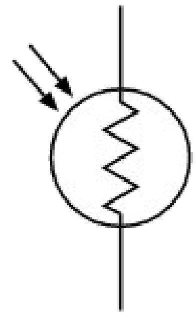
- Many Bragg gratings can be etched onto one fiber
- They can be distinguished by
  - interrogation timing
  - different reflected frequency of light
- No problem connecting this sensor: it’s already on a fiber optic link!

Photoelastic effect

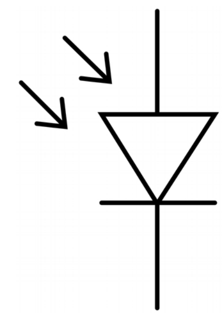
- Changes in the optical properties of a material under mechanical deformation
- Exists in any dielectric material
- Must be viewed in a setup involving two polarising filters
  - One to select the polarisation plane
  - One to analyse the plane of resulting light
- Can serve in analysis of engineering problems
  - https://www.youtube.com/watch?v=vDZ5yISiADM
- Natural application: pressure sensors using fiber optic



Light detectors

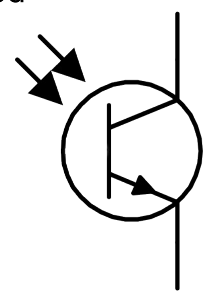


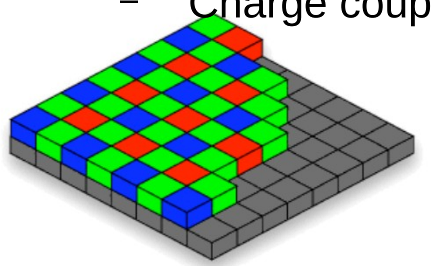
- For measuring the presence or intensity of light
  - Photoelectric effect, photomultipliers (first generation cameras)
  - Photoresistor (semiconductor)

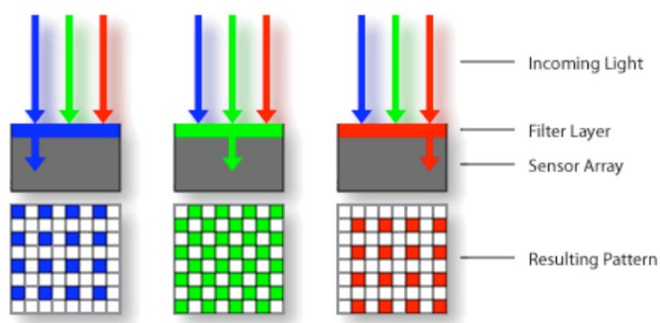


  - Photodiode (semiconduction junction, light → current)
  - Phototransistor (2 semiconductor junctions)
  - Active pixel sensor (CMOS)
  - Charge coupled device (CCD)

Diffence how signal is transported and multiplied

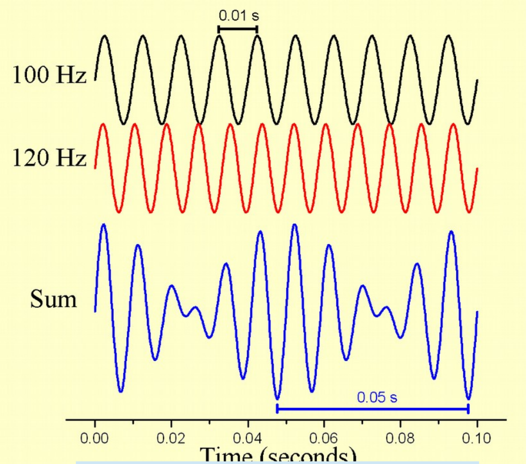




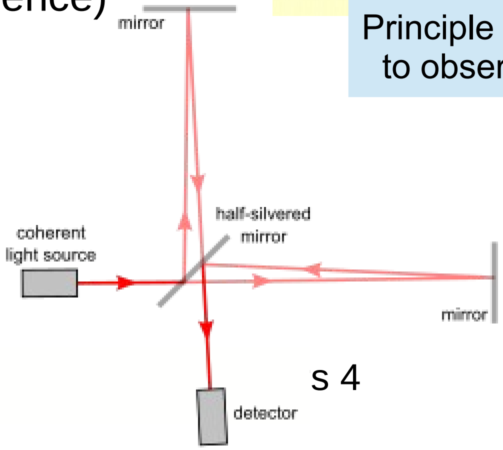


Wave measurement principles

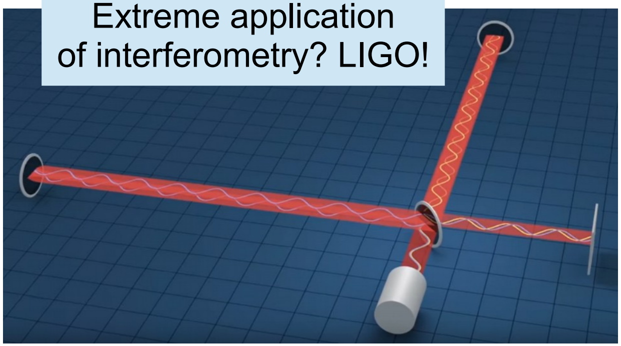
- Light is just another type of wave phenomenon, like sound
- We can use a set of principles
  - Time of flight
  - Beat frequency (frequency difference)
  - Inteferometry (phase difference)



Principle of beats – easy to observe with sound



Extreme application of interferometry? LIGO!



Piezoresistive effect

- Electric resistance changes as the resistor undergoes deformation
- Resistance as a function of the geometry of the resistor:

$$R = \rho \frac{\ell}{A}$$

where: R – resistance [Ω], ρ – resistivity [Ω·m], l – characteristic length [m], A – cross-section area [m^2]

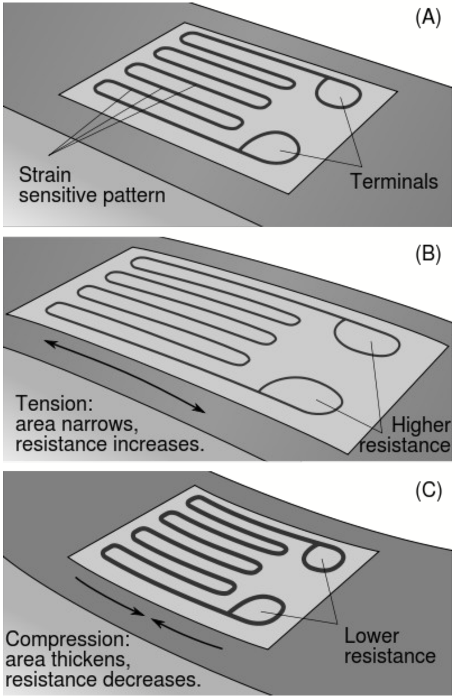
The effect of the strain ε [%] on resistance:

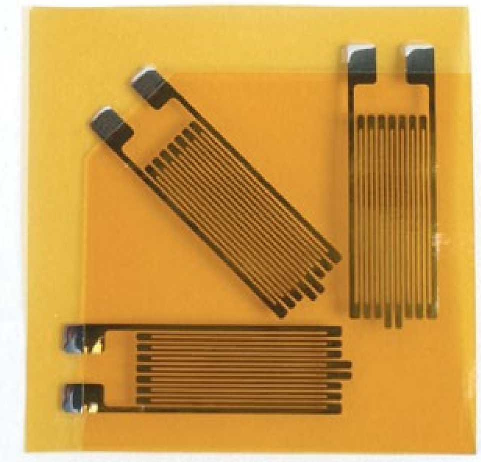
$$\frac{\Delta R}{R_0} = G\,\epsilon$$

where: G – gauge factor (in the order of ~2), R_0 – initial resistance [Ω], ΔR – resistance change [Ω]

Strain gauge

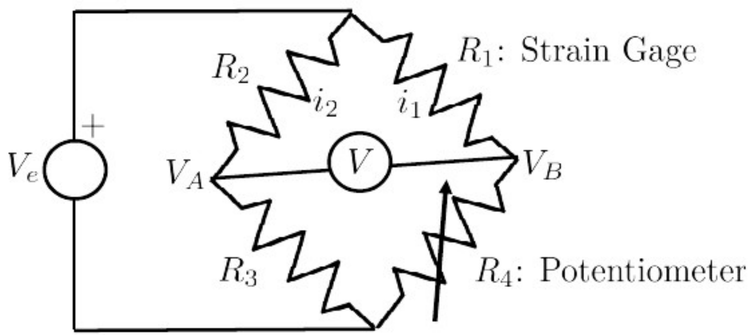
- Let's use this effect to measure the strain:
  - Many folds of a thin resistor
    can be arranged in a "rosette"
  - Simple and cheap production: metal (constantan) on plastic (polyamide)
  - In theory, we could just pick up our favourite ohm-meter and start measuring the changes
- Resistance measurement is not very easy to get right
  - Temperature (also due to Joule heating)
  - Unstable power supply
  - ...
- A simple setup, called a Wheatstone bridge will help us improve it!





Wheatstone bridge

- The idea: we cannot precisely measure current but we can fairly precisely determine when there is current, using a sensitive galvanometer
- We can use precision resistors (= of stable and well measured resistances) of R_2/R_3 ratio and a precise potentiometer R_4. By changing R_4 in the Wheatstone bridge, we cancel the current flowing between its branches, thus we know that the potential difference between V_A and V_B is zero



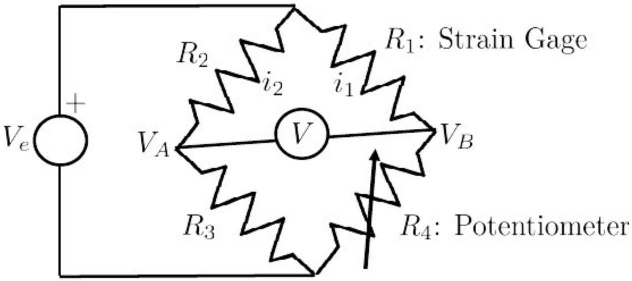
Wheatstone bridge

- Unbalanced:

$$\frac{V_A - V_B}{V_e} = \frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4}$$

- R_1 estimation (balanced bridge $\equiv V_A - V_B = 0$):

$$\hat{R}_1 = R_4 \frac{R_2}{R_3}$$



- After loading the strain gauge ($R_1 \rightarrow R_1 + \Delta R$):

$$\frac{\Delta V}{V_e} = \frac{R_3}{R_2 + R_3} - \frac{R_4}{(\hat{R}_1 + \Delta R_1) + R_4}$$

Wheatstone bridge

$$\frac{\Delta V}{V_e} = \frac{R_3}{R_2 + R_3} - \frac{R_4}{(\hat{R}_1 + \Delta R_1) + R_4}$$

- In this equation we are linking ΔR with ΔV
- Analysing of the rate of change of change of signal is important (recall the discussion of the capacitive sensing geometry choice)
- Sensitivity (formally)
  - Each sensor has a **transfer function** converting its input (what we measure) to its output (typically voltage)
  - Sensitivity is the derivative of this transfer function S at some particular value of the input s_0

$$Sens. = \frac{dS}{ds}\Big|_{s_o}$$

  - By maximising a ratio such as the one above, we increase sensitivity

Wheatstone bridge

- Wheatstone bridge is a great way to improve measurement of resistance
- Resistance is used in many different sensing technologies
  - Piezoresistive effect
  - Photoresistive effect (and thermocouple)
  - Thermoresistive effect
  - ...*resistive effect – you name it
- The bridge is used to maintain the variable resistance element in its optimal and well controlled sensing zone

Linearisation

- The transfer function of ΔR to ΔV is not linear. The following linearisation of allows to do precise sensing using this relation

$$f(R_1 + \Delta R_1, \ldots, R_4 + \Delta R_4) = \frac{\eta}{(1+\eta)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right]$$

with η being the ratio of resistance of a balanced bridge:

$$\eta = \frac{R_1}{R_4} \equiv \frac{R_2}{R_3}$$

Linearisation

- Linearisation using function's Taylor's expansion is a very useful trick. Every function f(x) can be represented around some point a as an infinite sum of terms:

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots,$$

- This way, we can obtain the linear approximation

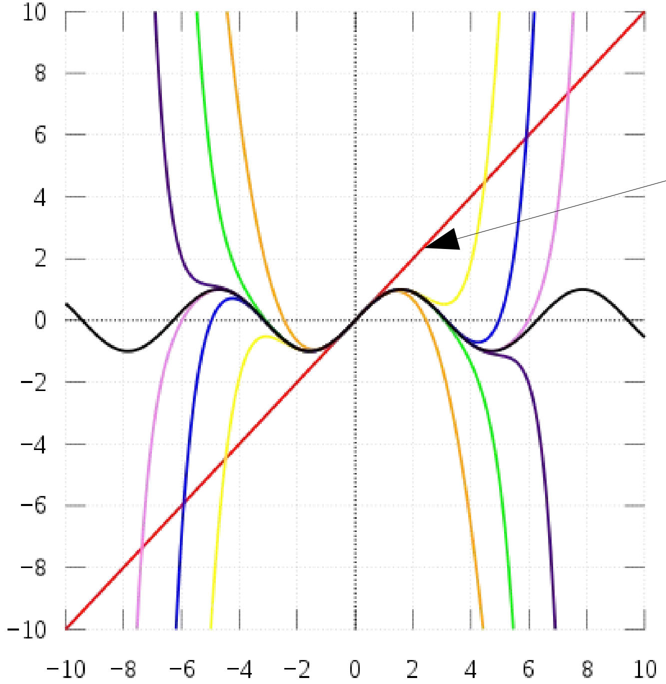
$$f(x) \approx f(a)+\frac{f'(a)}{1!}(x-a)$$

- But we musn't forget that in reality it works only if the higher order terms are negligible

$$f(x) \approx = Ax + b + O(\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\dots)$$

Linearisation using Taylor series

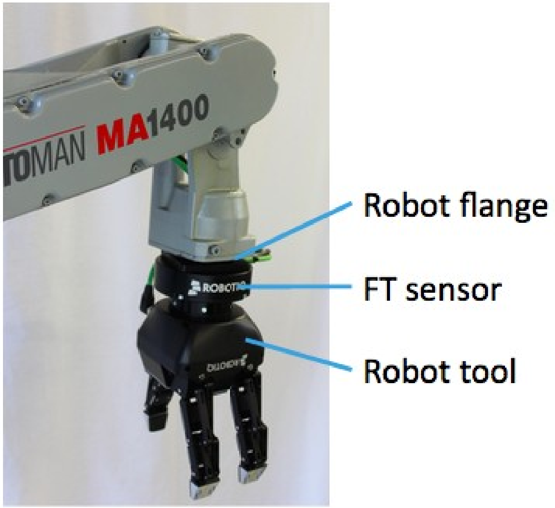
- This is how the series deals with a trigonometric function:



$$f(a) + \frac{f'(a)}{1!}(x-a)$$

Strain gauges

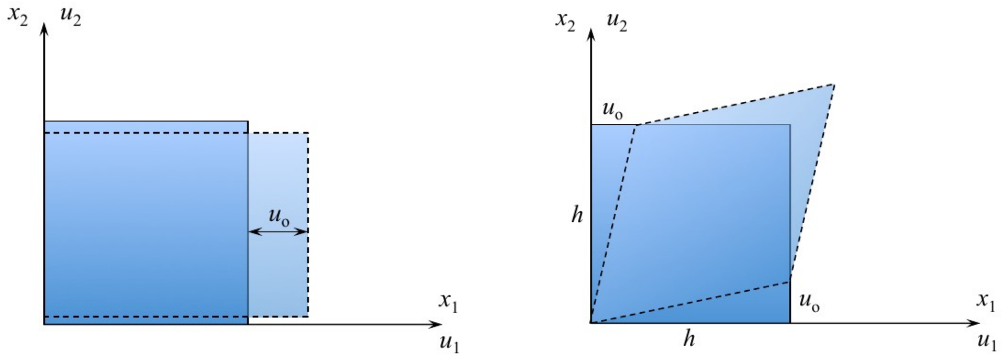
- Here is a couple of examples:



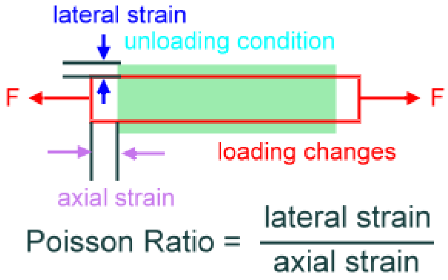
https://www.youtube.com/watch?v=b4nz_hAh7qs

Another look at strain

- How does it work for more than 1 dimension?



- Even the 2-d stretching scenario gets complicated because every material can have different Poisson ratio:



- Our naive definition in 1-D:

$$\epsilon_x = \frac{\Delta L}{L}$$

Another look at strain

- How does it work for more than 1 dimension?

$$\epsilon_x$$

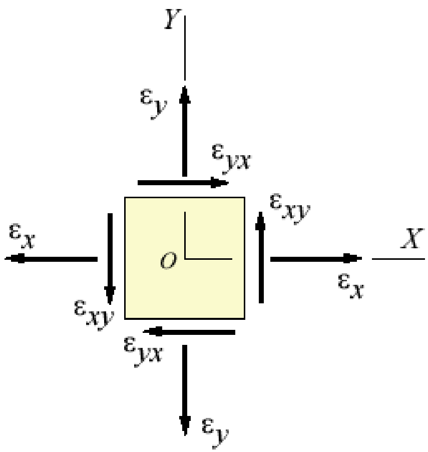
Linear strain

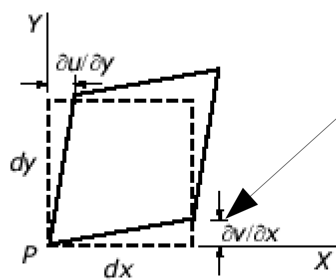
$$\begin{bmatrix} \varepsilon_x & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_y \end{bmatrix}$$

Plane strain

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

Full strain tensor

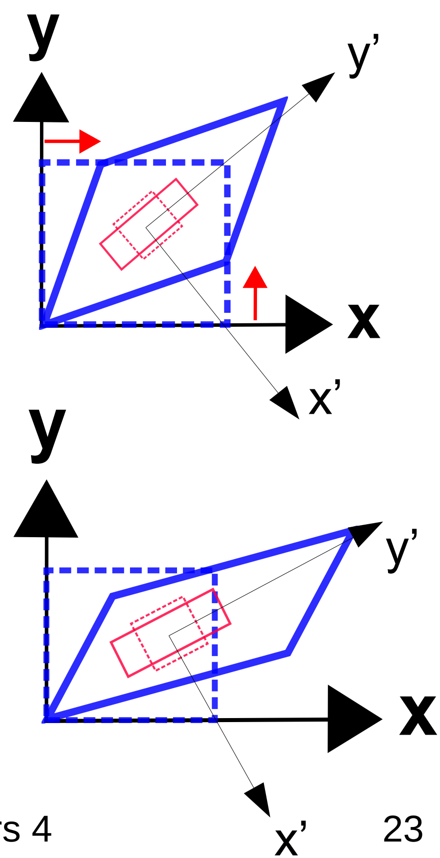




Off-diagonal strain components

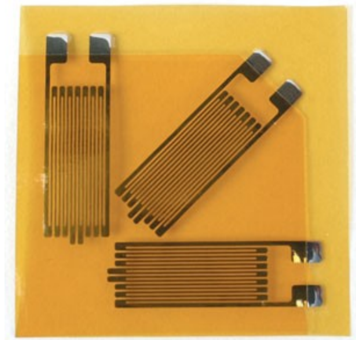
Strain in higher dimensions

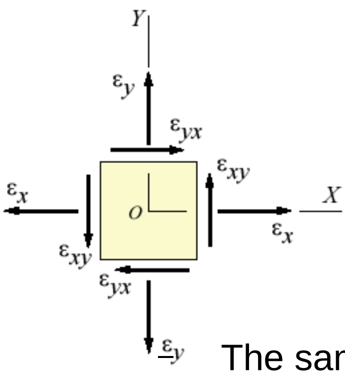
- While looking at the strain tensor, it seems that we have to know 4 components (planar stress) or 9 components (3-D case)
  - The strain tensor is symmetric! → reduction to 3/6 components respectively
  - Principal strains: we can always rotate the basis of the tensor in such way that only the terms on the diagonal will remain! (maths of rotation identical to change of basis of a matrix)
  - It does not mean that there are only two real components of strain: the angle by which we have to turn the basis is also a necessary information!



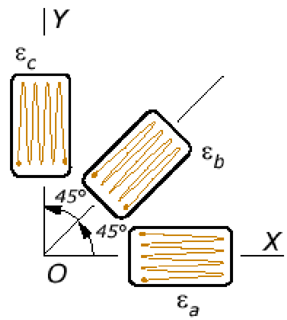
Strain gauges

- Another look at a strain rosette
  - The previous slide explains why we need 3 components: the tensor nature of strain!
  - The calculations necessary to recover the whole tensor:

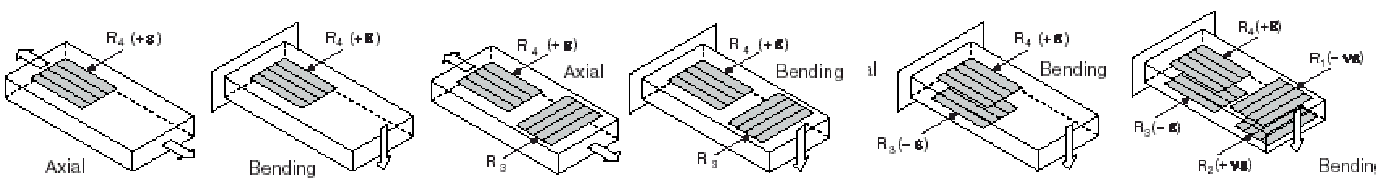




$$\begin{cases} \varepsilon_x = \varepsilon_a \\ \varepsilon_y = \varepsilon_c \\ \varepsilon_{xy} = \varepsilon_b - \dfrac{\varepsilon_a + \varepsilon_c}{2} \end{cases}$$

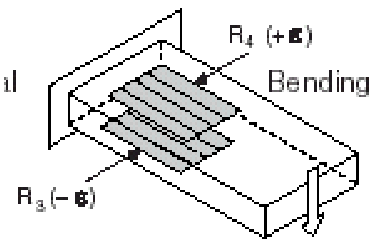


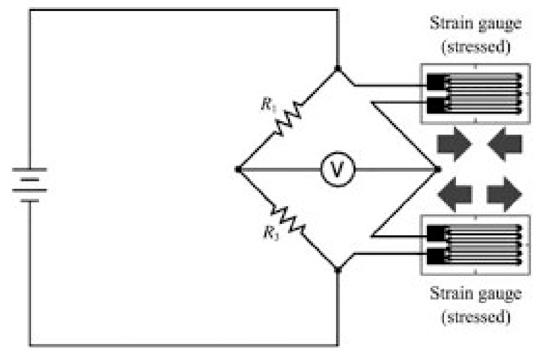
The same principles can serve to design cases for bending and axial strains:



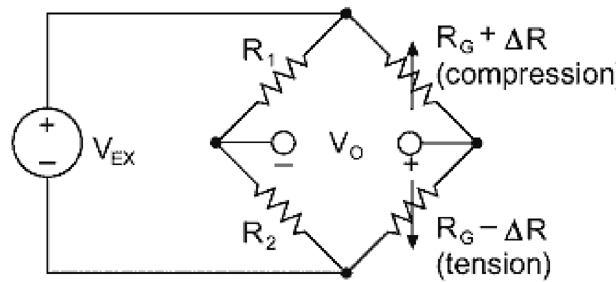
Strain gauges

- Instead of using a quarter Wheatstone bridge, we can use a half bridge:





- With opposite signs for ΔR_1 and ΔR_4 the voltage difference at the output increases!



$$f(R_1 + \Delta R_1, \ldots, R_4 + \Delta R_4) =$$
$$= \frac{\eta}{(1+\eta)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right]$$