Jacobs University Bremen

Sensor fusion 1

Automation
Course CO23-320203

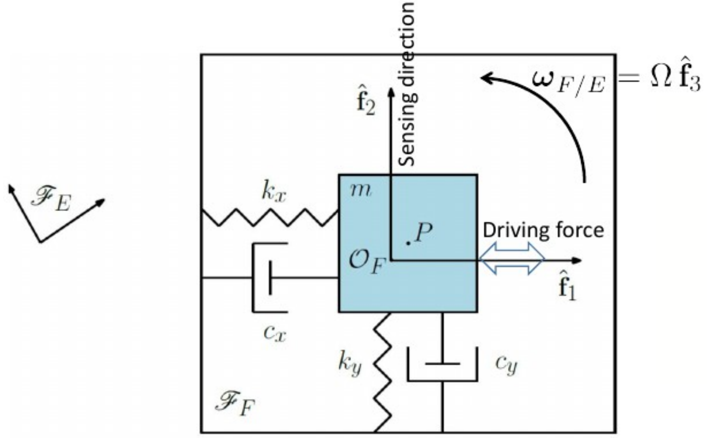
Recap: CVG analysis

- We have applied analysis of the dynamics and frequency space analysis to the idealised MEMS Coriolis Vibratory Gyroscope (one axis only)

$$\ddot{x} + \omega_n^2 x + \frac{\omega_n}{Q}\dot{x} - 2\Omega\dot{y} = \frac{1}{m}F(t)$$

$$\ddot{y} + \frac{\omega_n}{Q}\dot{y} + \omega_n^2 y = -2\Omega\dot{x}$$

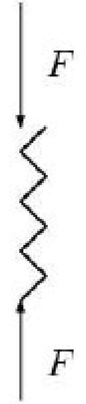
$$\frac{Y(s)}{X(s)} = \frac{-2\Omega s}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2} \qquad \frac{Y_D}{X_D} = \frac{2\,\Omega\,Q}{\omega_n}$$

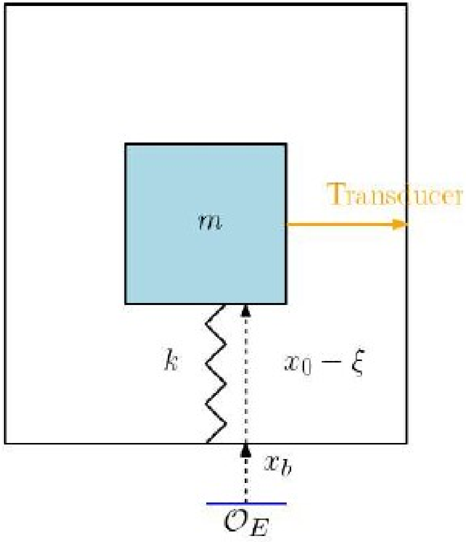


MEMS Accelerometer

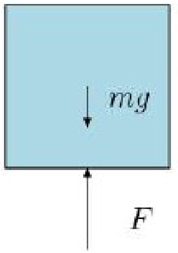
...is much less of a challenge:

$$x = x_b + x_0 - \xi$$
$$\dot{x} = \dot{x}_b - \dot{\xi}$$
$$\ddot{x} = \ddot{x}_b - \ddot{\xi} \qquad a \triangleq \ddot{x} = \ddot{x}_b - \ddot{\xi}$$





- In this idealisation, the transducer measures the deflection ξ
- Physics tells us
  - that the (massless) spring will obey the Hooke's law: $F = k\,\xi$
  - That the mass will behave according to the Newton's law: $ma = F$
  - The gravity mg will act on the mass



MEMS Accelerometer

- The transducer will thus read:

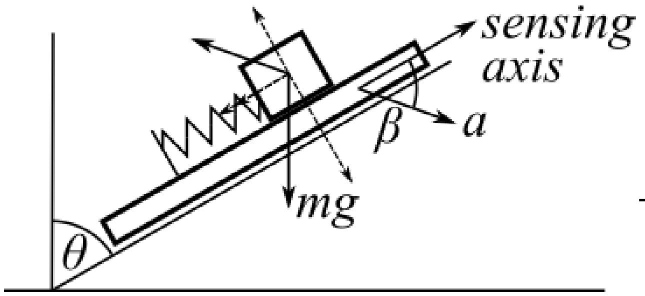
$$ma = k\,\xi - mg$$

$$\xi = \frac{m}{k}(a+g)$$

- In freefall? $\ddot{x}_b = -g,\ \ddot{\xi} = 0 \quad a = \ddot{x}_b - \ddot{\xi} = -g,\ \xi = 0$
- Just resting flat on the table? $a = 0,\ \xi = mg/k$
- On a Falcon Heavy going to space? $a = 3g,\ \xi = 4mg/k$
- Is it really so simple? → Remember, we’re just looking at one axis without taking orientation into account!

MEMS Accelerometer

- The analysis so far was for the acceleration and gravity aligned with the sensing direction
  - If we take into consideration that
    - The acceleration acting on the system is at angle β to the sensing axis s
    - the accelerometer is tilted from vertical by an angle θ

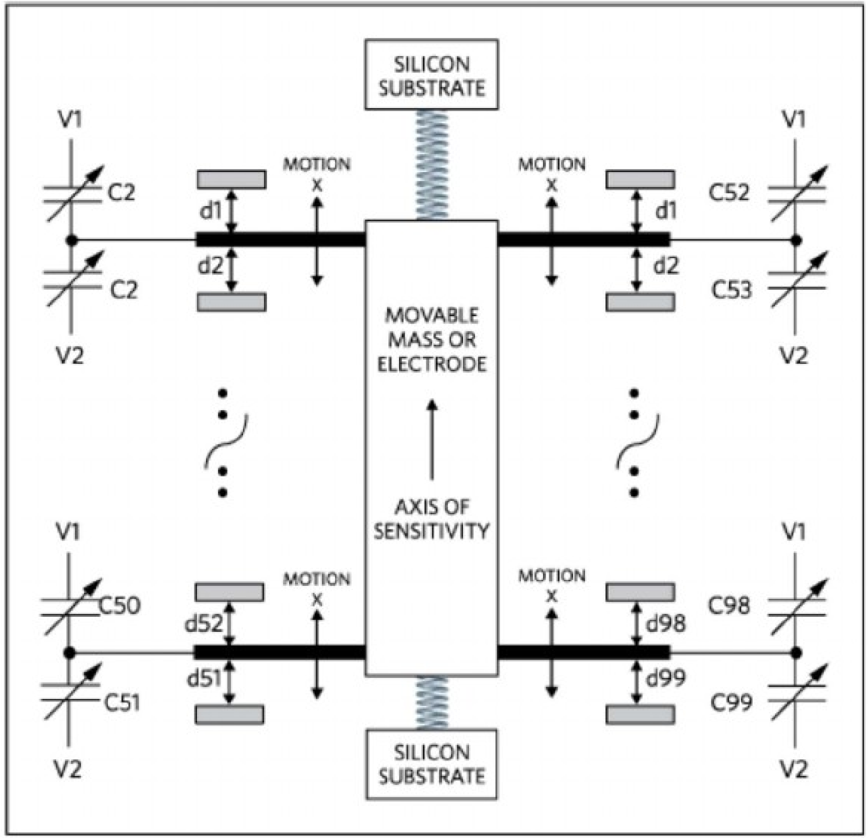


$$\xi = \frac{m}{k}(\mathbf{a}^\top \mathbf{s} + \mathbf{g}^\top \mathbf{s}) = \frac{m}{k}(a\,cos(\beta) + g\,cos(\theta))$$

→ if we have negligible proper acceleration, we can use this relationship to sense the tilt angle!

MEMS Accelerometer

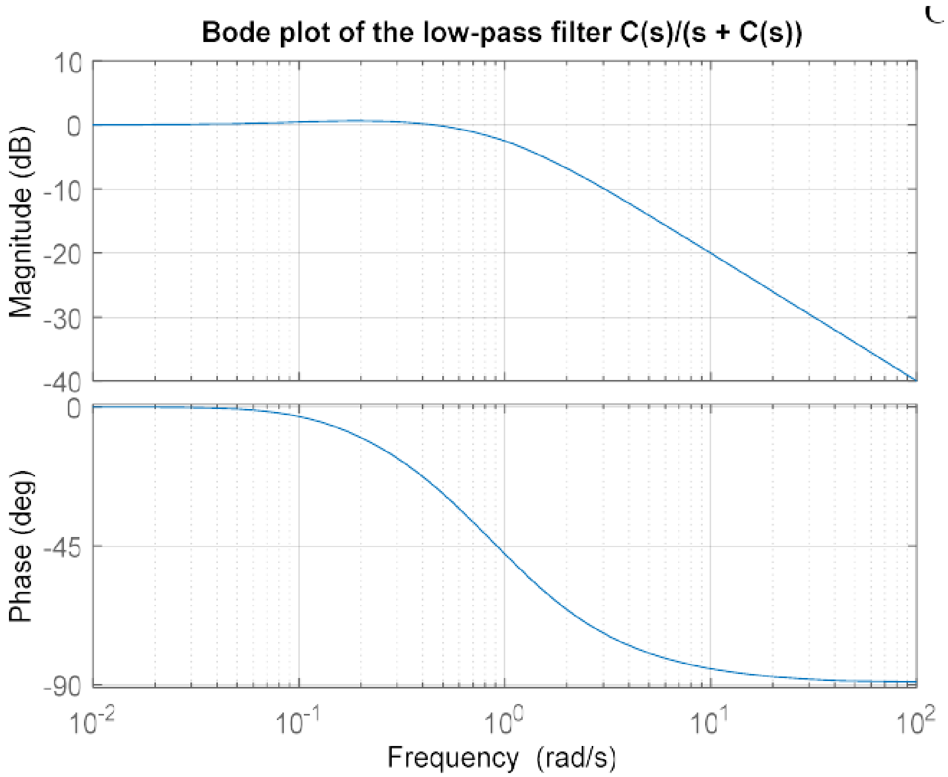
- MEMS accelerometer: an example of a practical design
  - Based on capacitive sensing, featuring multiple capacitors
  - One test mass per sensing direction
  - ξ proportional to ΔC

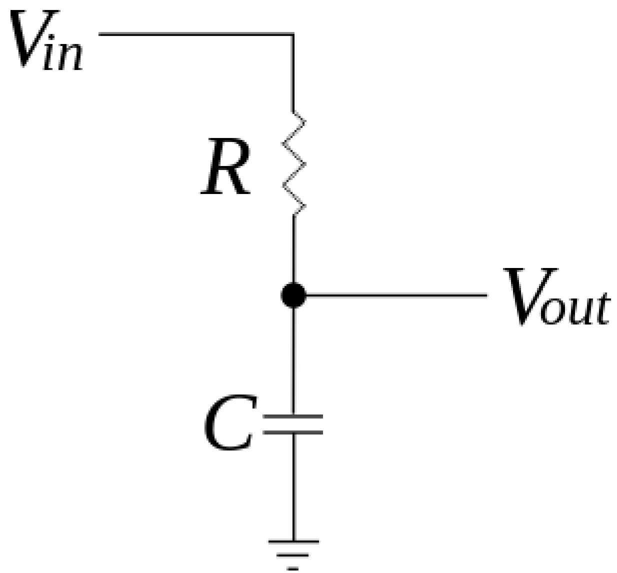


Sensor fusion

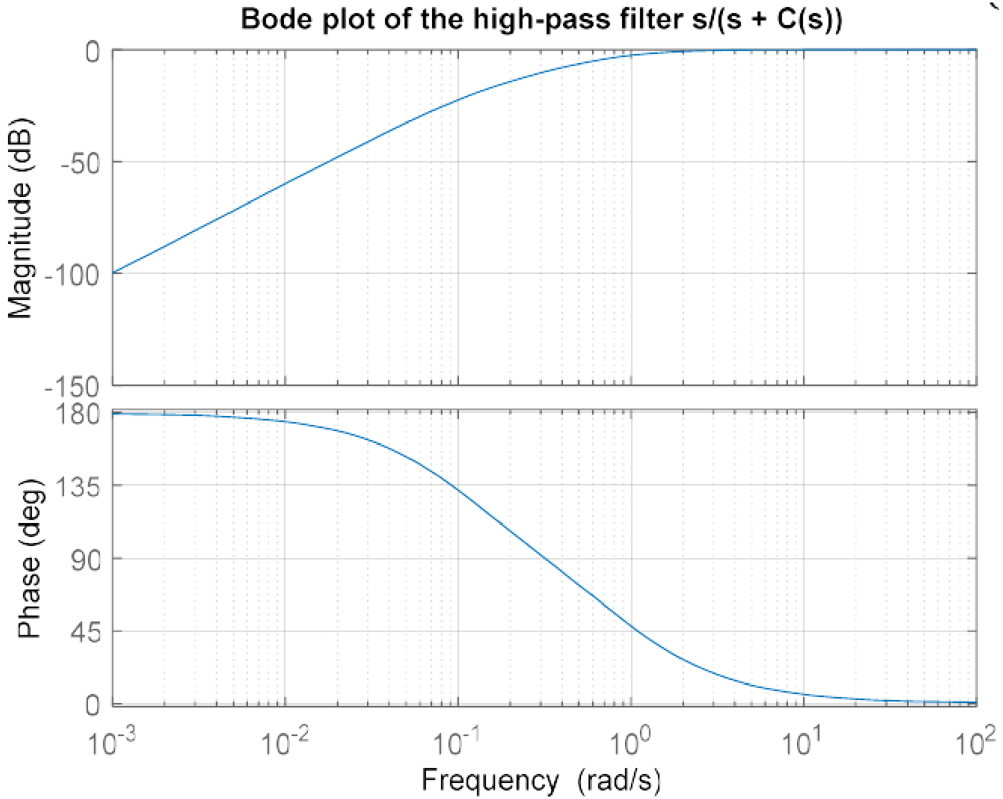
- Having taken a look into the working of an accelerometer and a gyroscope, let us reflect on advantages of using them together.
- Motivation: both sensors are burdened with multiple types of errors (as explained during the lecture on sensors)
- But: the principal types of noise and/or systematic errors can be “complementary” - exhibiting opposite characteristics for both sensors
  - Gyroscope: high frequency noise, constant or slowly varying bias (= in the absence of rotation, the gyro reads a value of $b_0 \neq 0$)
  - Accelerometer: perturbations due to accelerations when used to sense the direction of the gravitation vector

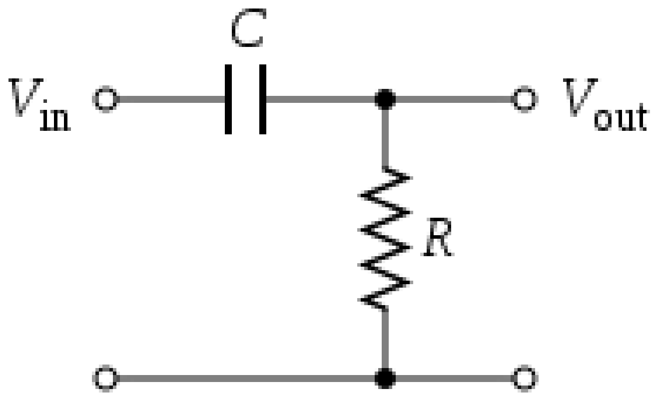
Recap: Filters – low pass filter



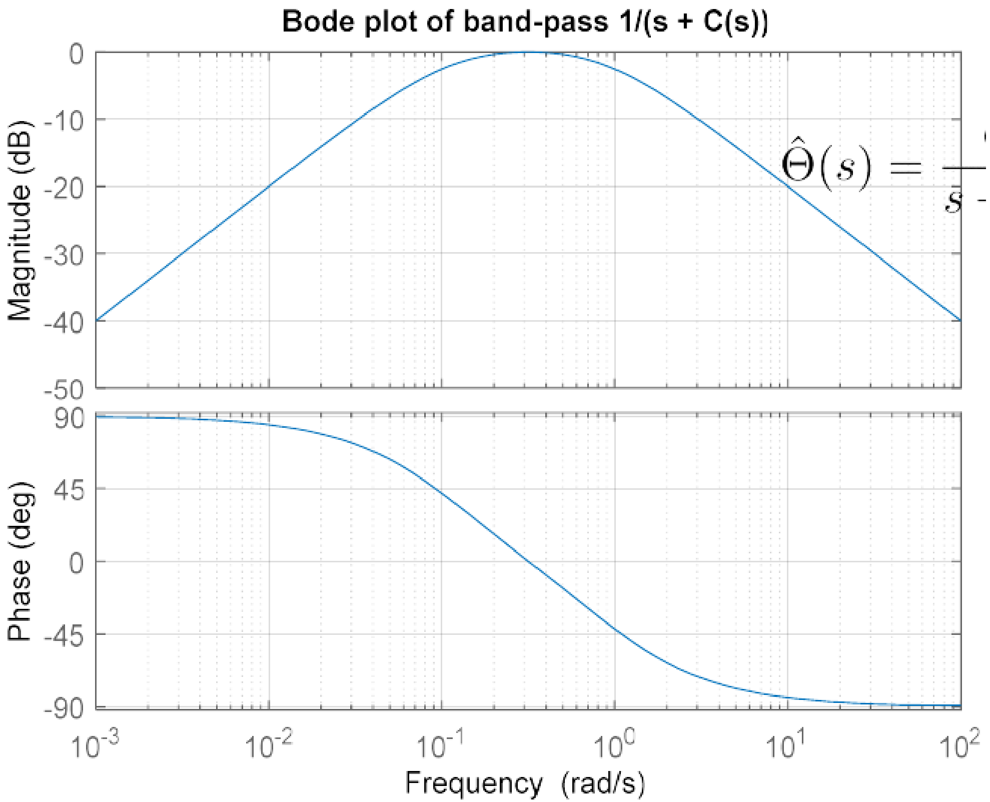


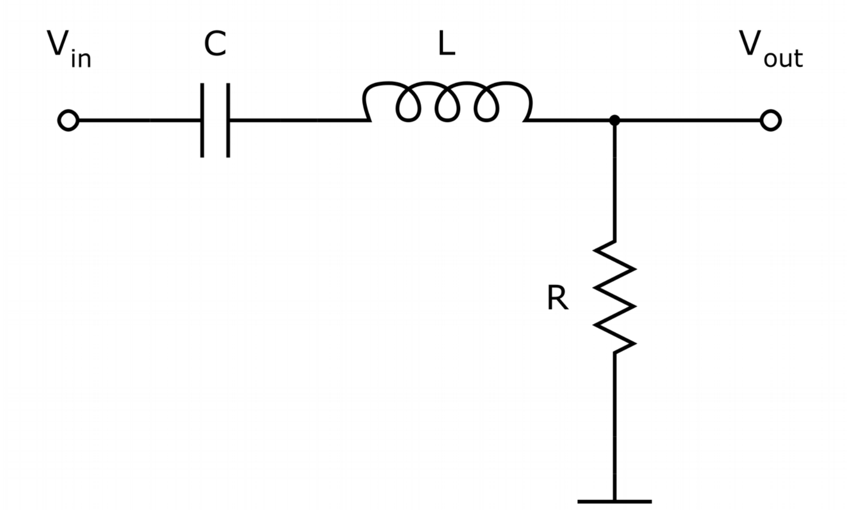
Recap: Filters – high pass filter





Recap: Filters – band pass filter





Simple complementary filter

- A situations where two or more sensors measure related quantities but show complementary characteristics in terms of noise (bias, etc)
- Let's choose such filters that satisfy $F_1(s)+F_2(s) = 1$, for example a low-pass / high-pass filter pair
- In theory, a perfect estimation (the filters' sum is 1!) of the filtered quantity can be obtained while the noises get efficiently filtered out.
- Consider two measurements of the same physical quantity $x(t)$:

$$y_1(t) = x(t) + n_1(t) \qquad \text{Noise } n_1 \text{ is high-frequency}$$

$$y_2(t) = x(t) + n_2(t) \qquad \text{Noise } n_2 \text{ is low-frequency}$$

- ...and two filters:

$F_1(s)$ is low-pass, and,

$F_2(s)$ is high-pass.

Simple complementary filter

- Let's use an unspecified (for now) all-pass filter variable *C(s)* and define the two component filters:

$$F_1(s) = \frac{C(s)}{C(s) + s} \qquad \text{Low Pass}$$

$$F_2(s) = 1 - F_1(s) = \frac{s}{C(s) + s} \qquad \text{High Pass}$$

(*C(s)* could be as simple as a multiplicative constant = constant gain)

- In frequency/Laplace domain:

$$\hat{X}(s) = F_1(s)\, Y_1(s) + F_2(s)\, Y_2(s)$$
$$\hat{X}(s) = F_1(s)\, [X(s) + N_1(s)] + F_2(s)\, [X(s) + N_2(s)]$$

$$\hat{X}(s) = X(s) + F_1(s)\, N_1(s) + F_2(s)\, N_2(s)$$

- Naive demonstration of how the noise is effectively removed

Complementary filter – heterogenous sensors

- Such characteristics can be exploited also when the sensors measure different quantities related through kinematics

For example:

<u>Acceleration due to gravity</u> (=gravity vector and its orientation) measured by an accelerometer burdened by high frequency noise

and

<u>Rate of rotation</u> measured by a gyroscope burdened by a (near)constant bias.

Sensor filtering and fusion – our motivating example

- A good discussion of the interest of sensor filtering and fusion for android devices: https://www.youtube.com/watch?v=C7JQ7Rpwn2k – compare it to the reality 5 years later when all new phones had the discussed solutions already built in…
- The availability of MEMS gyros and accelerometers makes this combination a good case study for us
- Goal: stable and accurate orientation measurement

1^{st} order complementary filter

- Consider a 1-dimensional case of a body equipped with an accelerometer and a gyroscope
- In the absence of accelerations on the body, the accelerometer with its sensing axis along the unit vector x measures a scalar value $a = |g| cos\ \theta$
- ...thus θ can be considered known (up to the sign)
- The sensor is burdened by high-frequency noise nh(t). The total signal is:

$$y_a(t) = \theta(t) + n_h(t)$$

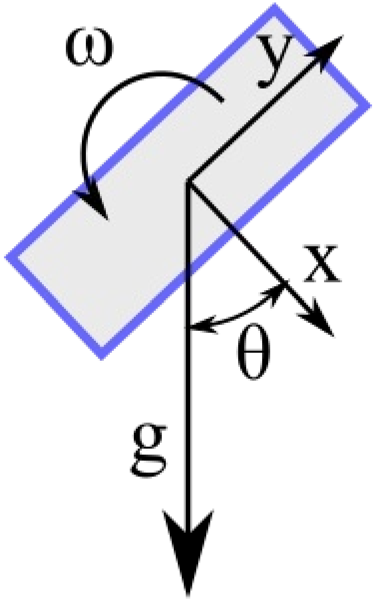
- The gyro measures the rotational velocity around the z axis with low frequency noise (=bias) nl(t):

$$y_g(t) = \omega(t) + n_\ell(t)$$

- The simple kinematics is then:

$$\dot{\theta} = \omega$$

- How can we obtain the best estimate of $\hat{\theta}(t)$?



1^{st} order complementary filter

- The same filters can be applied:

$$F_1(s) = \frac{C(s)}{C(s)+s}$$
$$F_2(s) = 1 - F_1(s) = \frac{s}{C(s)+s}$$

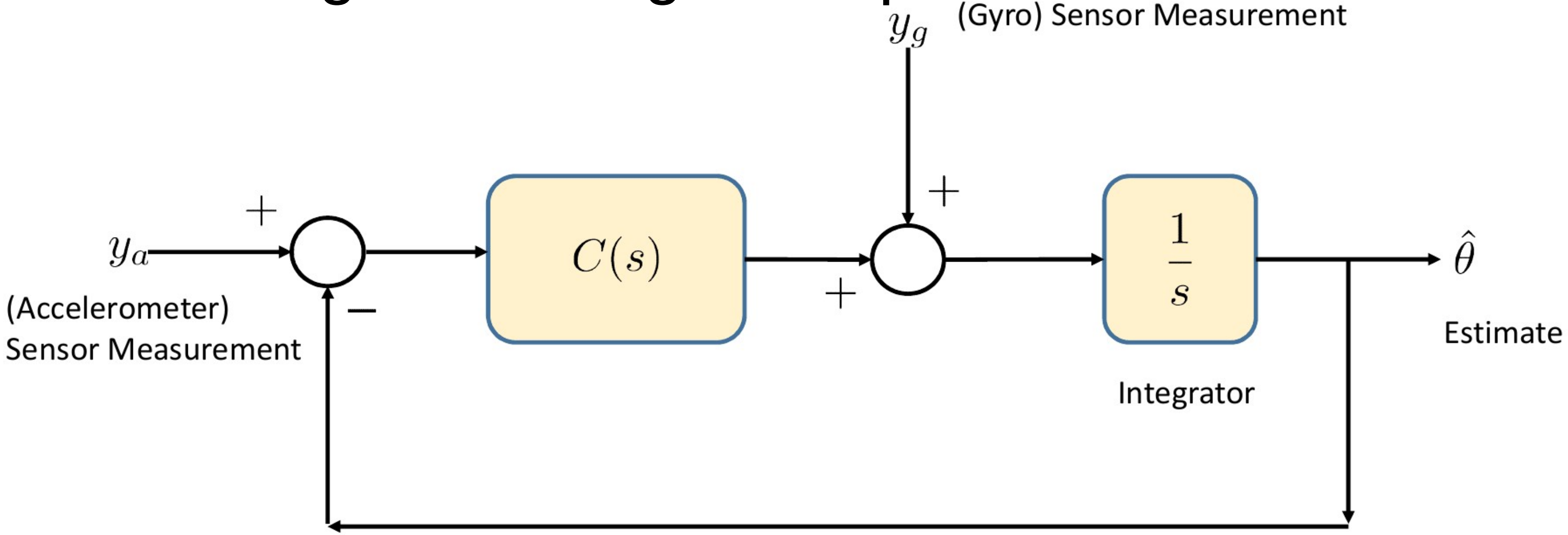
- The kinematics in frequency domain: $$\Theta(s) = \frac{1}{s}\omega(s)$$

- Both combined to form a filtered value $\hat{\Theta}(s)$:

$$\begin{aligned}\hat{\Theta}(s) &= F_1(s)(\Theta(s) + N_h(s)) + F_2(s)\frac{1}{s}(\omega(s) + N_\ell(s))\\ &= \Theta(s) + F_1(s)N_h(s) + F_2(s)\frac{N_\ell(s)}{s}\\ &= \Theta(s) + \frac{C(s)}{s+C(s)}N_h(s) + \frac{1}{s+C(s)}N_\ell(s)\end{aligned}$$

1st order complementary filter

- The following block diagram represents this filter:



$$\hat{\Theta}(s) = \frac{C(s)}{s + C(s)} Y_a(s) + \frac{1}{s + C(s)} Y_g(s)$$

1^{st} order complementary filter

- The high-pass filter in front of the gyro signal turned into a band-pass!
- The noise is still high-pass filtered.
- What choice of $C(s)$?
  - Just a proportional gain K_P is the simplest choice
  - What happens if we add another component? For example an integral element? $C(s) = K_P + K_I/s$
    - it has an effect on reducing the steady-state error of the system → bias
- We will be coming back to this example!
- Let us first recall the question of a dynamic systems' (model, filter, estimator, etc) equilibrium points and their stability