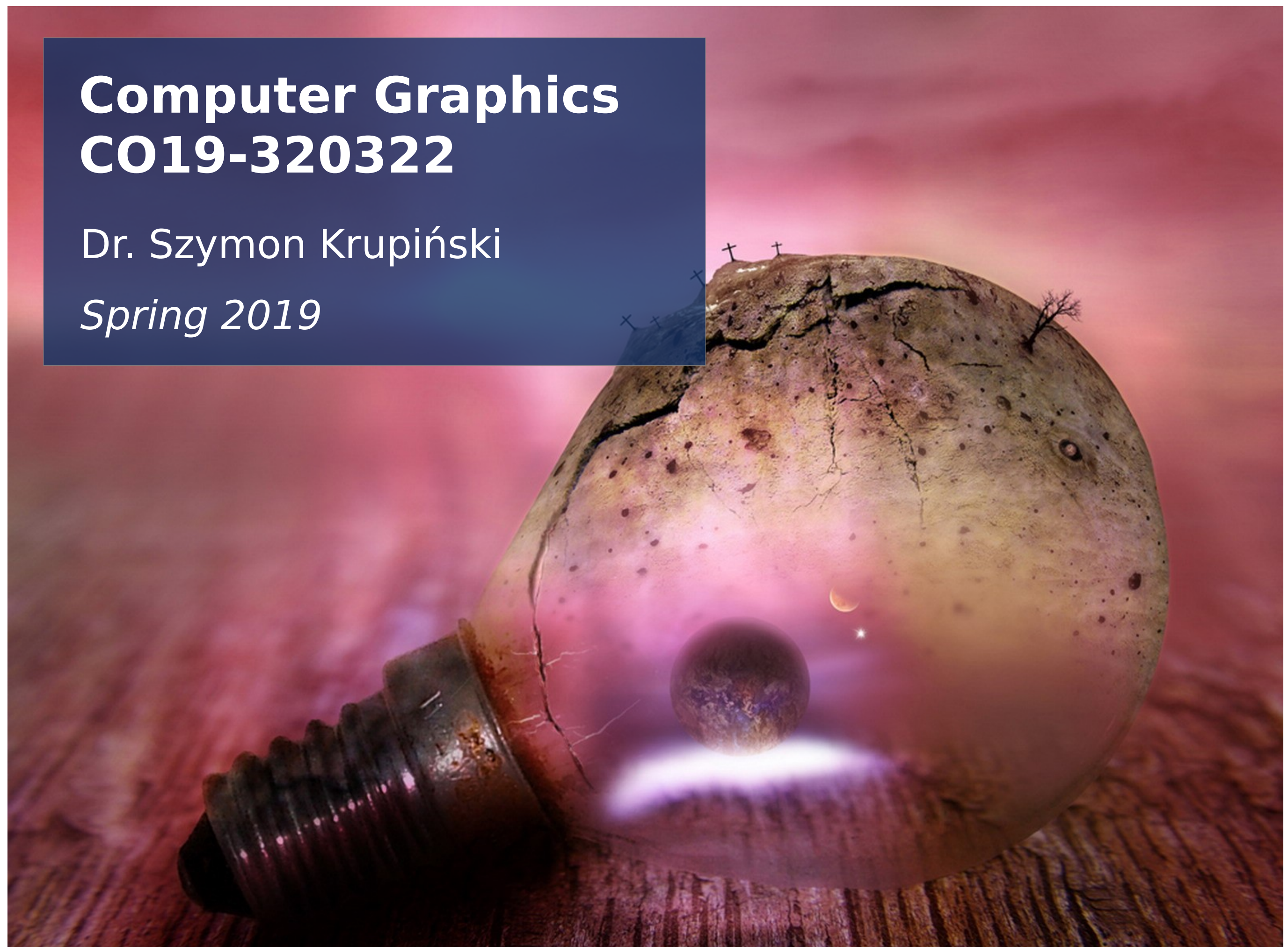
Lecture 21: Geometry 2

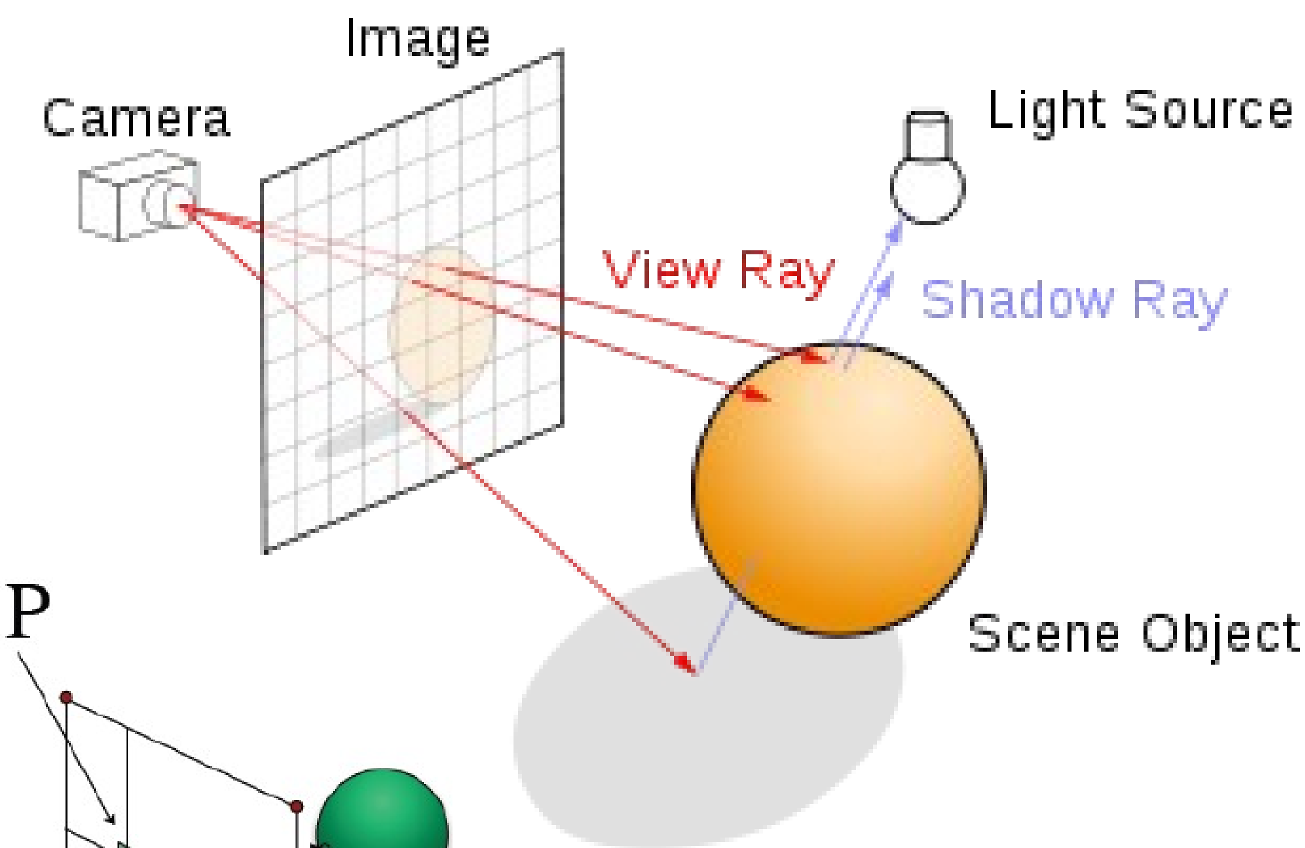


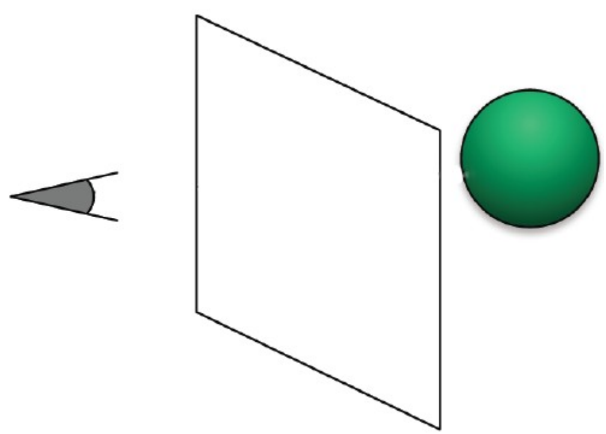
Combination Geometry + Ray tracing

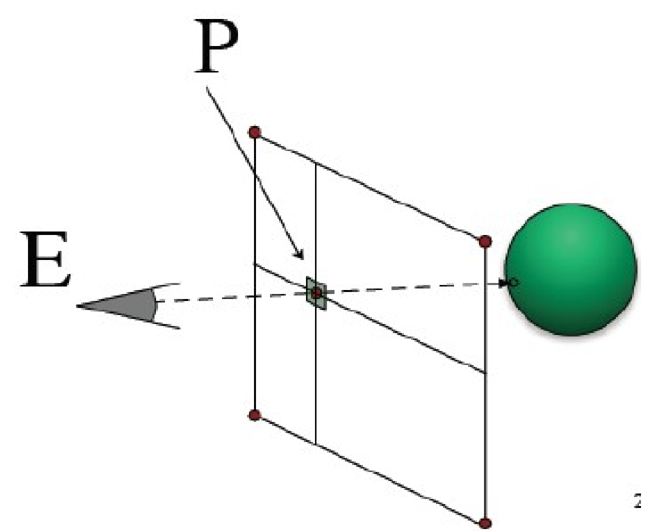
- Recall:
  - ray geometry:

$$R(t) = E + t(P - E)$$

$$t \in [0, +\infty)$$

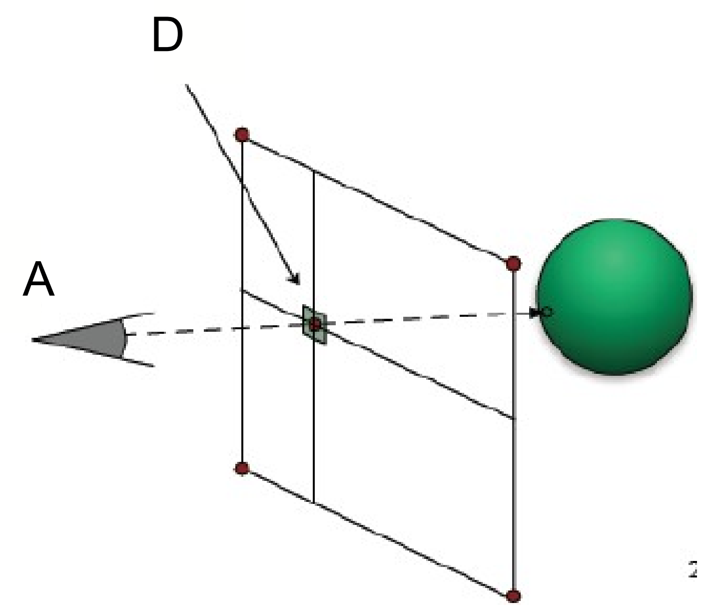






Combination: Geometry + Ray tracing

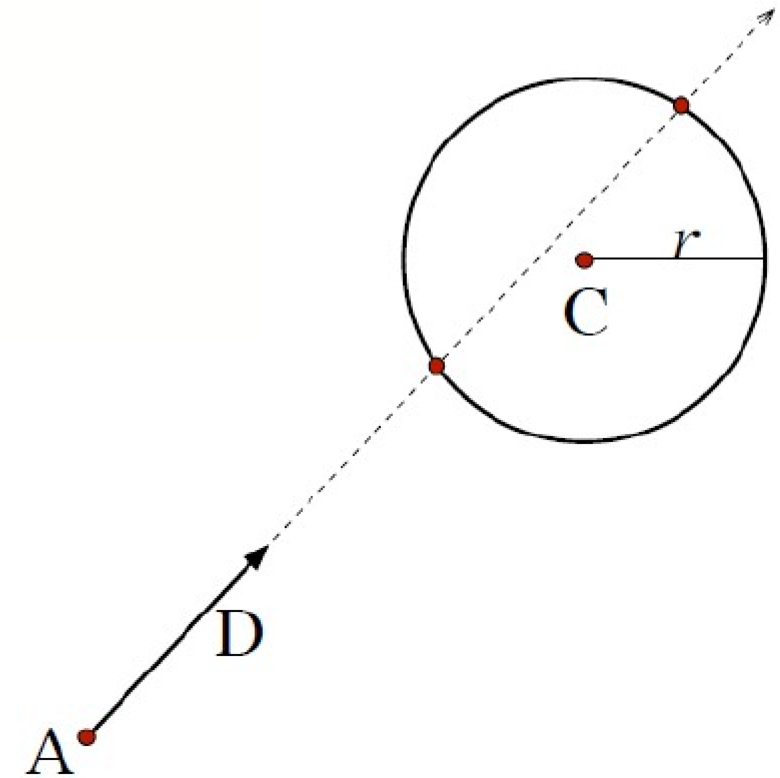
- Given a ray $R(t)=A+tD$, find the first intersection with any object where $t \geq t_{min}$ and $t \leq t_{max}$
- The object geometry can be a polygon mesh, an implicit surface, a parametric surface, etc.
- Doesn't have to be decomposed into triangles (as was required by OpenGL)
- Goal: write intersection code for all kinds of different geometries



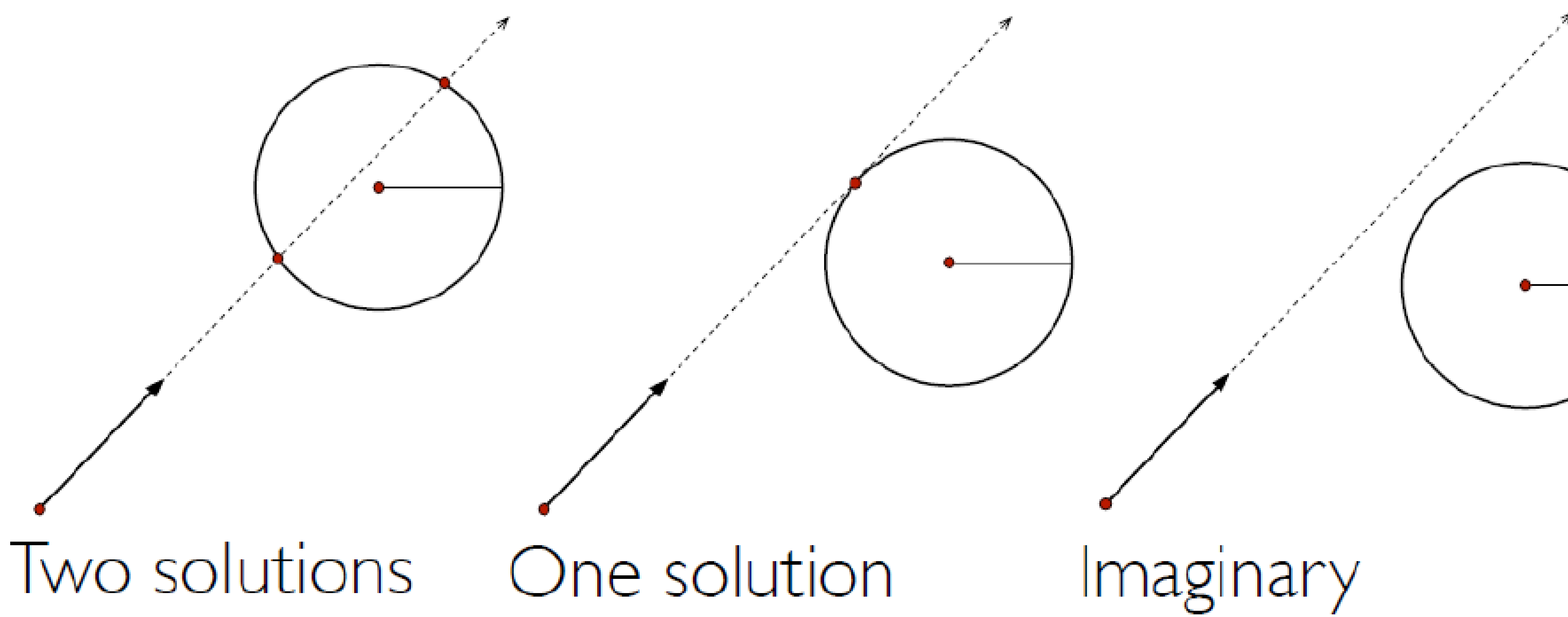
Ray - Sphere

- Ray equation: $R(t) = A + tD$
- Implicit equation for a sphere:
  $(X - C)^2 = r^2$
- Combine them together:
  $(A + tD - C)^2 = r^2$
- Quadratic equation in t :
  $t^2 + 2(A - C) \bullet Dt + (A - C)^2 - r^2 = 0$
- With discriminant:

$$\Delta = 4\,[(A - C) \bullet D]^2 - 4(A - C)^2 + r^2$$



Ray - Sphere



$\Delta > 0$ $\Delta = 0$ $\Delta < 0$

- For the case with two solutions, choose the first intersection (smaller t)

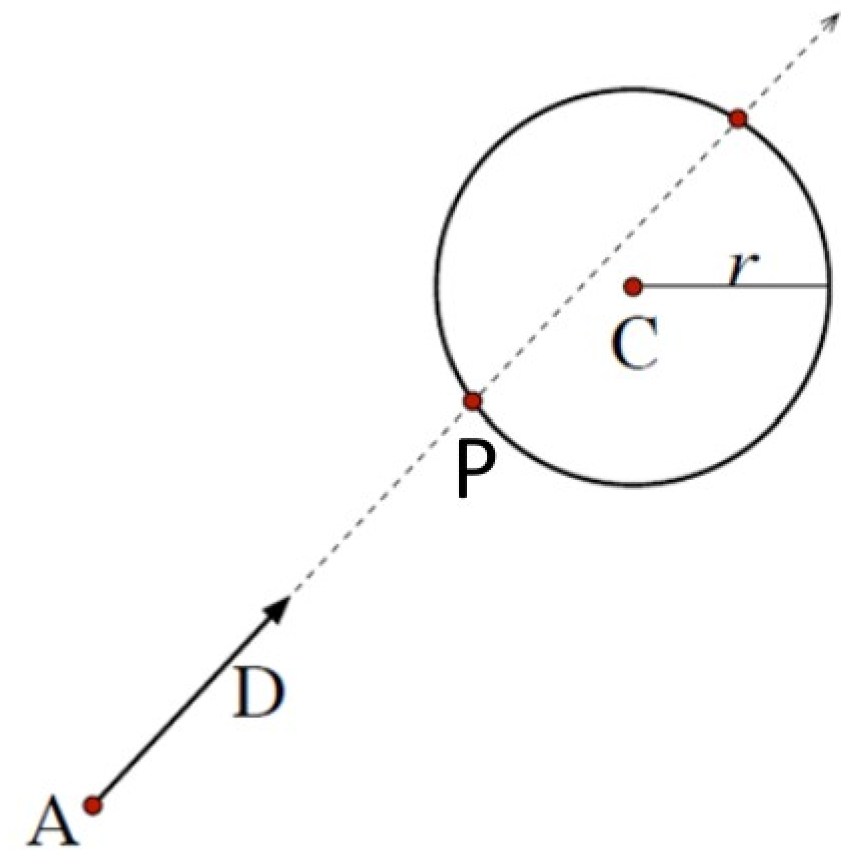
Ray - Sphere

- Intersection Point:

$$P = A + t_{\text{int}} D$$

- Intersection Normal:

$$N = \frac{P - C}{|P - C|}$$



Implicit surfaces

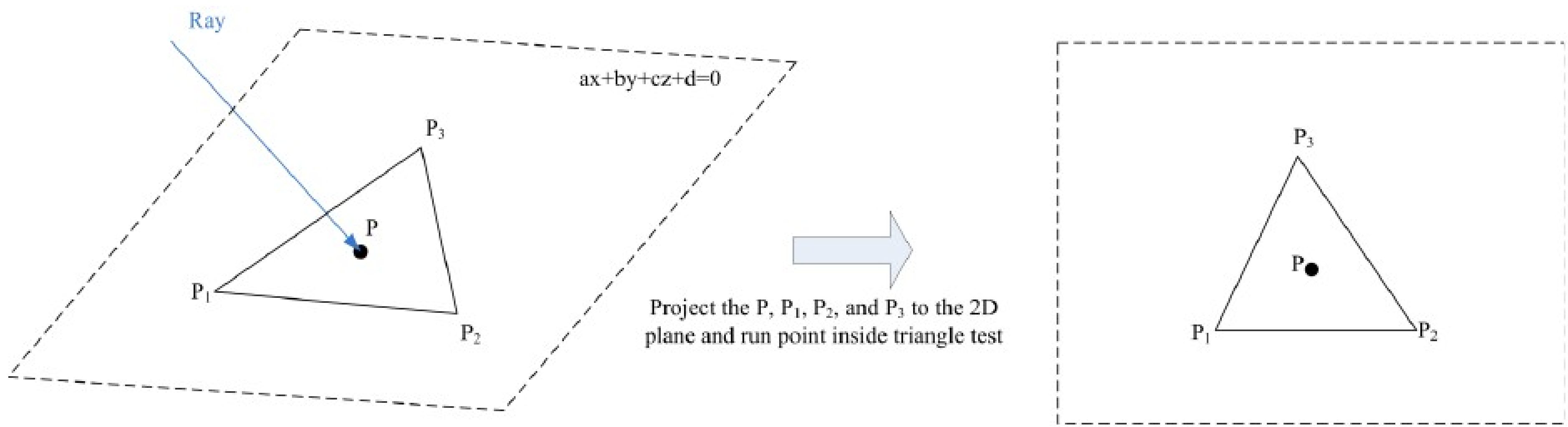
- General rule for finding an intersection of a ray with an implicit surface:
  - Substitute ray equation into the implicit surface equation $f(x,y,z) = 0$
  - Search for the first root

Ray - plane

- Just another example!
- Ray equation: $R(t) = A + tD$
- Implicit equation for a plane: $ax + by + cz + d = 0$
- Combine them together and solve for t to find the point of intersection:
  $a(x_A + tx_D) + b(y_A + ty_D) + c(z_A + tz_D) + d = 0$

Ray - triangle

- First, find the ray-plane intersection with the plane containing the triangle
- Then, project the 3 triangle vertices and the ray-plane intersection point to a truly 2D plane
- Finally, run the point-inside-triangle test in 2D as done during rasterization...

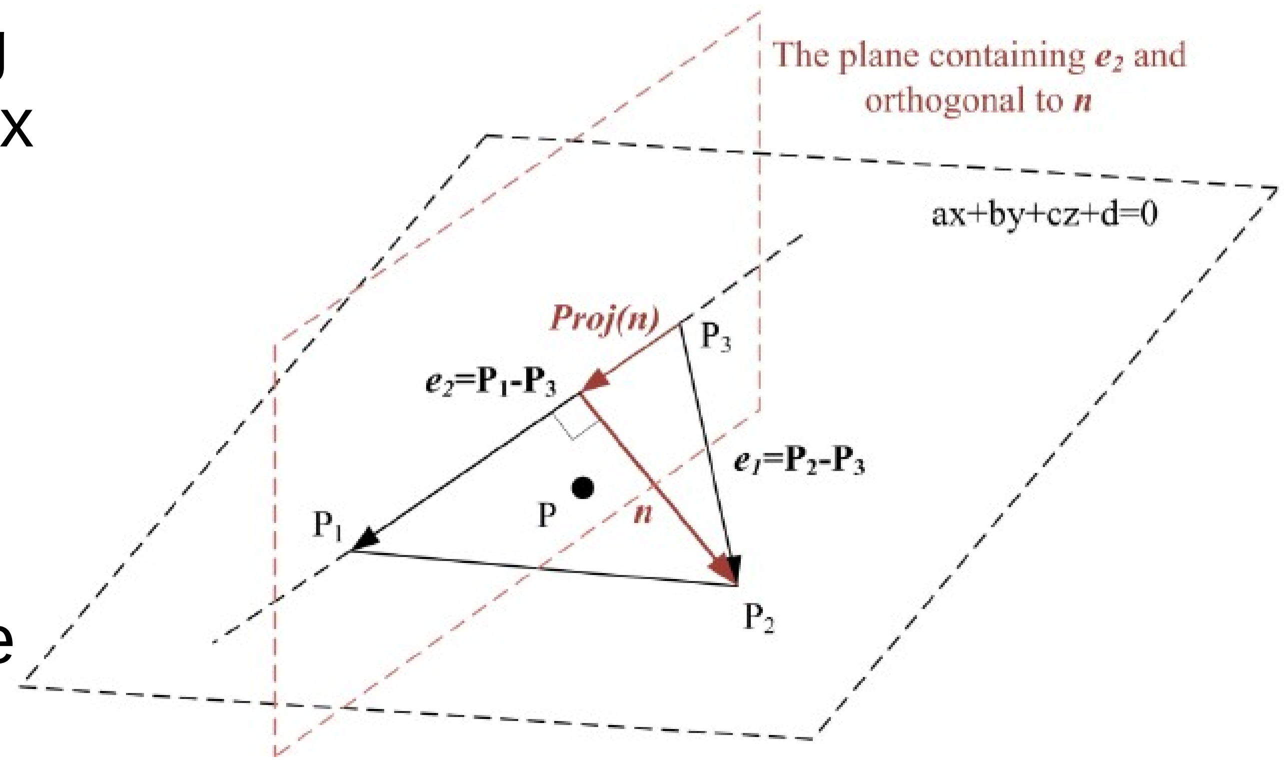


Ray - triangle

- Compute the normal direction n orthogonal to edge e, and pointing towards the edge's opposite vertex in the plane of the triangle:

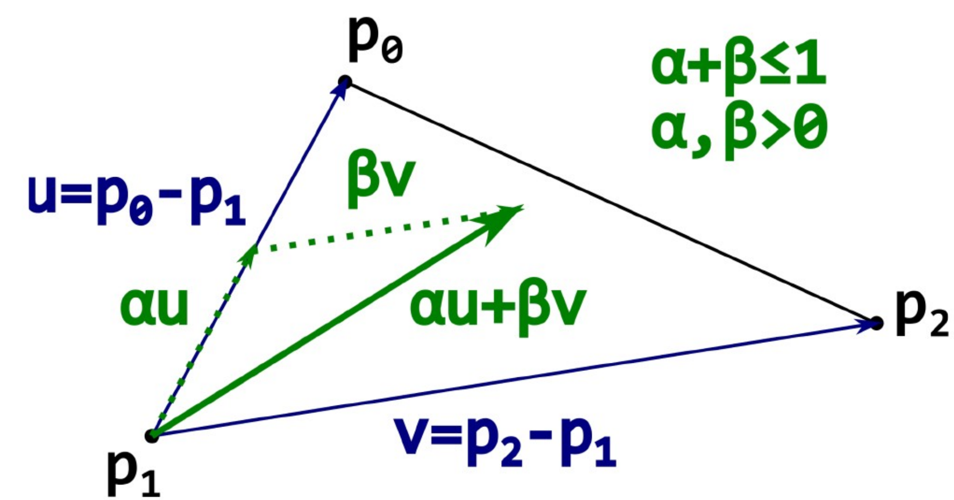
$$\mathbf{n} = \mathbf{e}_1 - \frac{\mathbf{e}_1 \bullet \mathbf{e}_2}{\|\mathbf{e}_2\|^2}\mathbf{e}_2$$

- Given an endpoint P_1 of e, test whether $(P - P_1) \bullet n \geq 0$ to see if the intersection point is "inside" that edge



Ray - triangle

- Using barycentric coordinates:
- Ray equation: $R(t) = A + tD$
- Parametric equation for triangle:
  $X = P_1 + a(P_2 - P_1) + b(P_3 - P_1)$
- Combine:
  $A + t_D = P_1 + a(P_2 - P_1) + b(P_3 - P_1)$



- $P_1 = (x_1, y_1, z_1)$,
  $P_2 = (x_2, y_2, z_2)$,
  $P_3 = (x_3, y_3, z_3)$

$$\begin{cases} x_A + tx_D = x_1 + \alpha(x_2 - x_1) + \beta(x_3 - x_1) \\ y_A + ty_D = y_1 + \alpha(y_2 - y_1) + \beta(y_3 - y_1) \\ z_A + tz_D = z_1 + \alpha(z_2 - z_1) + \beta(z_3 - z_1) \end{cases}$$

3 equations with 3 unknowns...

Ray - triangle

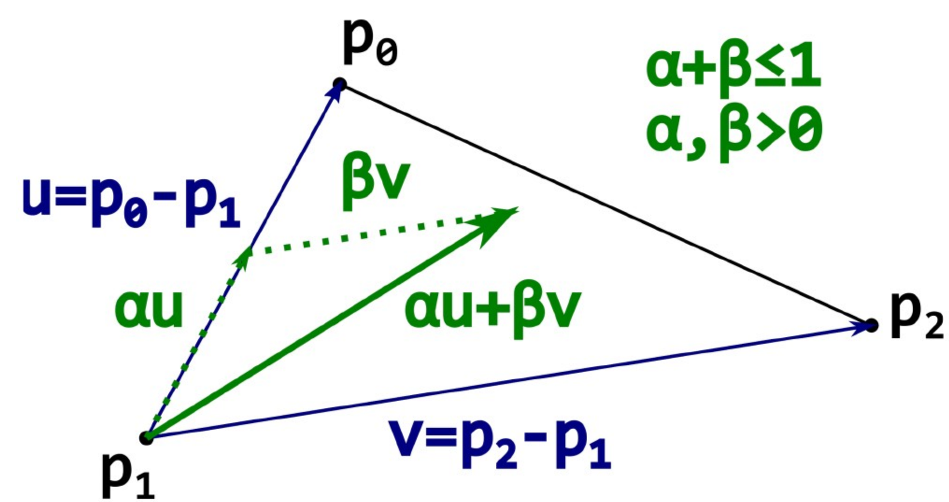
- Matrix form:

$$\begin{bmatrix} x_2 - x_1 & x_3 - x_1 & -x_D \\ y_2 - y_1 & y_3 - y_1 & -y_D \\ z_2 - z_1 & z_3 - z_1 & -z_D \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ t \end{bmatrix} = \begin{bmatrix} x_A - x_1 \\ y_A - y_1 \\ z_A - z_1 \end{bmatrix}$$

$$\begin{cases} x_A + tx_D = x_1 + \alpha(x_2 - x_1) + \beta(x_3 - x_1) \\ y_A + ty_D = y_1 + \alpha(y_2 - y_1) + \beta(y_3 - y_1) \\ z_A + tz_D = z_1 + \alpha(z_2 - z_1) + \beta(z_3 - z_1) \end{cases}$$

- Satisfying

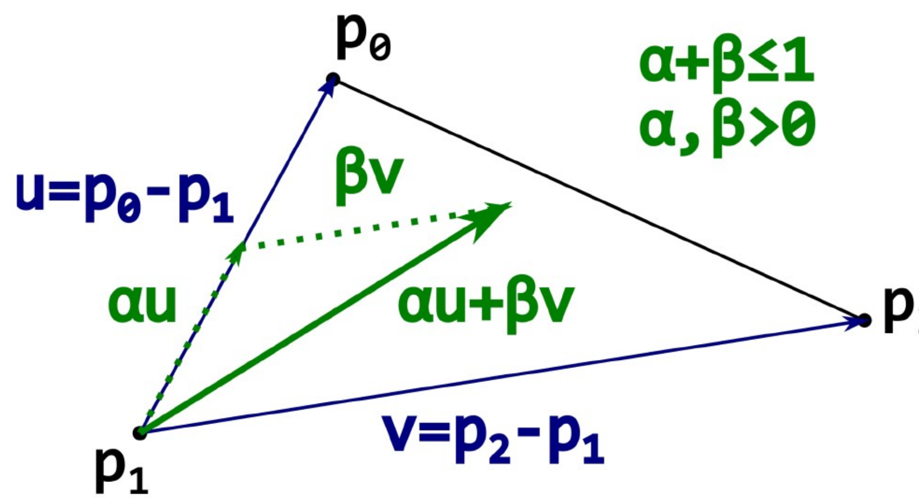
$$\begin{cases} t_{\min} \le t \le t_{\max} \\ 0 \le \alpha \le 1 \\ 0 \le \beta \le 1 - \alpha \end{cases}$$



Ray - triangle

- Intersection algorithm:

```
bool RayTriangle(Ray R, Vec3 V1, Vec3 V2,
                 Vec3 V3, Interval [tmin, tmax])
{
     compute t;
     if(t < t min or t > tmax) return false;
     compute α;
     if(α < 0 or α > 1) return false;
     compute β;
     if(β < 0 or β > 1- α) return false;
     return true;
}
```



The calculations are gradual – each time an “if” statement helps avoid unnecessary computation due to one of the conditions excluding a possibility of intersection

Thank you!

- Questions?