Convex Optimization
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Course CO24-300491
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Prepare for the Final Exam

Task 1 Convex Sets

Which of the following sets are convex? In each case, draw the set, and prove your answer either by directly verifying the definition, by applying theorems discussed in class or by discussing a counter example.

a) $\{(x,y) \in R^2 \mid xy > 9 \wedge x \geq 0\}$

b) $\{(x,y) \in R^2 \mid xy < 9 \wedge x \geq 0\}$

c) $\{(x,y) \in R^2 \mid \log_{10}(xy) < 9 \wedge x \geq 0\}$

Task 2 Convex Functions

Which of the following functions is convex? Prove your answers directly by verifying the definition, by applying theorems discussed in class or by discussing a counter example. Make sure you check all the parts of the definition.

a) $f:\{(x,y) \in R^2 \big| x^2 + y^2 \leq 4\} \to R,\ f(x,y) = xy$

b) $f:\{(x,y) \in R^2 \big| x^2 + y^2 \leq 4\} \to R,\ f(x,y) = \log_{10}(xy)$

c) $f:\{(x,y) \in R^2 \big| x^2 + y^2 \leq 4\} \to R,\ f(x,y) = \log_{10}(e^x e^y)$

Task 3 Dual Function

Consider the following minimization problem:

minimize f_0 (x, y) = 2x + 7y
subject to f_1 (x, y) = 2- x - y ≤ 0
and f_2 (x, y) = 1 - y ≤ 0

where x, y ∈ ℝ.

(a) Find the Lagrange function $L(x, y, \lambda_1, \lambda_2)$. Use $\lambda_0 = 1$.

(b) Find the solution $(\hat{x}, \hat{y})$ and $(\hat{\lambda}_1, \hat{\lambda}_2)$ by means of the KKT theorem – check all the conditions.

(c) Find the dual function.

(d) Write the dual problem as it follows from your dual function.

(e) Solve the dual problem. (Check with the primal solution).

Task 4 Lagrange Multipliers

An object is free to move within the set $S = \{(x, y) \in R^2 \mid x + y \le \cos(y - x)\}$. It tries to come as close as possible to the point P = (1,1). Where should it go?

In order to solve the problem, sketch the set S.

[Hint: A linear co-ordinate transform might help you to understand the situation. Although the set S is NOT convex, there is only one local minimum, which is also the global one we are looking for.]

Mind: This problem is not convex. Still…
- Sketch a qualitatively correct picture of the set S and the point P.
- Explain why the constraint should be binding.
- Write the problem in standard form with Lagrange multiplier λ, and show that there is only a single solution which is then the desired minimum.

Task 5 Convex Matrix Function

Consider the function $f: S^n_{++} \to S^n_{++}$, mapping a matrix M to its inverse M^{-1}.

Use a suitable power series expansion of $(M + \delta M)^{-1}$to show that this function is convex.

Hint: Find the second order term!