

Homework 2

Keep digesting ... solve on your own ... write down your solutions explicitly ... give explanations ... so: be prepared for the final exam ☺

Task 1 Rectangular Cuboids (*lecture 5*)

Consider an n-dimensional rectangular cuboid with edges of non-negative lengths x_1, x_2, \dots, x_n .

a) (*not sooo difficult*) Goal: Maximize the cuboid's volume

$$V = \prod_{i=1}^n x_i$$

subject to the constraint that the total length of all the edges involved

$$L = 2^{n-1} \sum_{i=1}^n x_i$$

is a constant. Solve this problem both, by elimination, and by means of the Lagrange multiplier trick. Do not forget to compare the two results.

b) (*somewhat challenging...technically*) Repeat part a) with a different constraint. Here, the total surface

$$S = 2 \sum_{i=1}^n \prod_{\substack{j=1 \\ j \neq i}}^n x_j$$

is kept constant. Example: For $n=3$, we have $S = 2(x_1x_2 + x_1x_3 + x_2x_3)$. Again, solve the problem by elimination, and by means of the Lagrange multiplier trick.

Hint: If you feel that the general-n case is too hard for a start, try with $n = 2, 3$ before you consider the general problems.

Task 2 Understanding the Lagrange Trick (*lecture 5*)

Explain: Why is the Lagrange multiplier trick is useful? Why is it usually easier to apply than the direct elimination approach? Consider the steps to take...