Optimization
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Course CO24-300491
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Homework 4

Keep digesting … solve on your own … write down your solutions explicitly … give explanations … so: be prepared for the final exam ☺.

Task 1 Convex Sets

Which of the sets below are convex? – Prove your statement!

a) $S_1 = \{x \in \mathbb{R}^n \mid \sum_{i=1}^{n} x_i^2 \leq 10\}$ … read as the set of all vectors $x \in \mathbb{R}^n$ such that $\sum_{i=1}^{n} x_i^2 \leq 10$.

b) $S_2 = \{x \in \mathbb{R}^n \mid \|x\| \leq 10\}$ … read as the set of all vectors $x \in \mathbb{R}^n$ such that the norm of x is ≤ 10. Notice, that we do not specify which norm we have in mind, here.

c) $S_3 = S_1 \cap \{x \in \mathbb{R}^n \mid \sum_{i=1}^{n} (x_i - 2)^2 \leq 10\}$

Task 2 KKT Theorem

Use the KKT theorem to (efficiently) solve the following optimization problem:

$$f_0(\boldsymbol{x}) = (x_1 - 3)^2 + (x_2 - 4)^2 \rightarrow \min$$

subject to the constraints
$$f_1(\boldsymbol{x}) = x_1^2 + x_2^2 - 4 \leq 0$$
$$f_2(\boldsymbol{x}) = 1 - x_1 \leq 0$$

In order to fully benefit from the theorem, start by illustrating the situation.

Task 3 Dual Function – Dual Problem

minimize $f_0(x_1, x_2) = x_1 + x_2$

$$\text{subject to} \quad \begin{array}{rcl} 2x_1 + x_2 & \geq & 1 \\ x_1 + 3x_2 & \geq & 1 \\ x_1 & \geq & 0 \\ x_2 & \geq & 0 \end{array}$$

Start by making a sketch of the feasible set, and some sets of constant cost.
Then solve the (primal) problem directly, i.e., guess the solution from your picture, write the Lagrange function $\mathcal{L}(x, \lambda)$ and confirm your guess via the KKT theorem.
In a second approach, find the dual function
$$g(\lambda) = \inf_x\{\mathcal{L}\}$$
Then write the dual problem (maximize function g). Also solve that problem.
Finally, compare your two solutions.