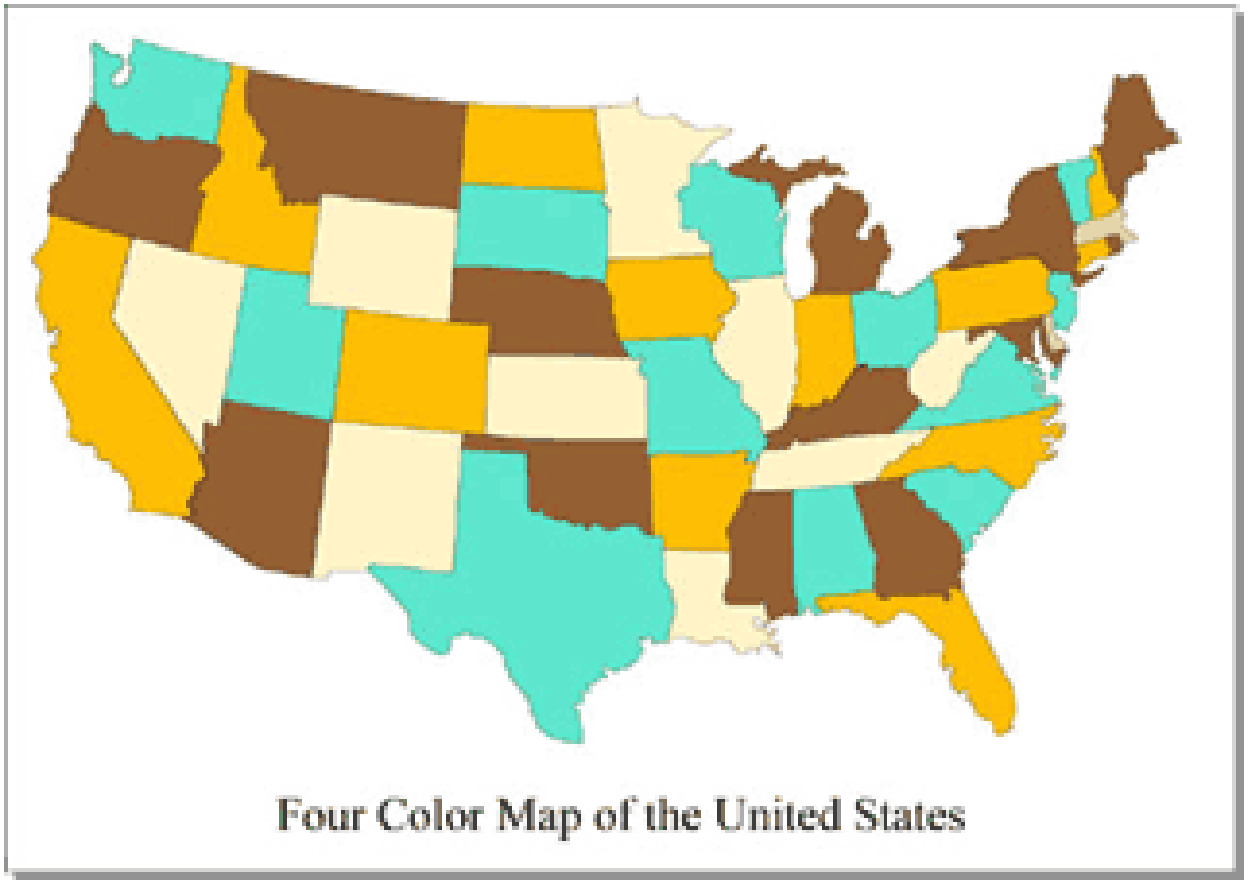
Recursive Backtracking

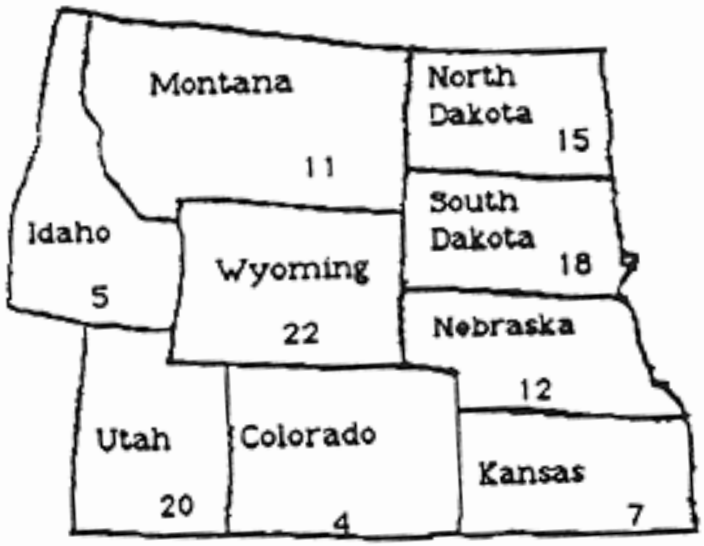
another problem…



Four Color Map of the United States

Recursive Backtracking II: Map Coloring

- Using just four colors (e.g., red, orange, green, and blue), we want color a map so that no two bordering states or countries have the same color.
- Sample map (numbers show alphabetical order in full list of state names):



- This is another example of a problem that can be solved using recursive backtracking.

General Template for Recursive Backtracking

```
// n is the number of the variable that the current
// call of the method is responsible for
boolean findSolution(int n, possibly other params) {
    if (found a solution) {
        this.displaySolution();
        return true;
    }
    // loop over possible values for the nth variable
    for (val = first to last) {
        if (this.isValid(val, n)) {
            this.applyValue(val, n);
            if (this.findSolution(n + 1, other params)) {
                return true;
            }
            this.removeValue(val, n);
        }
    }
    return false;
}
```

Applying the Template to Map Coloring

```
boolean findSolution(n, perhaps other params) {
    if (found a solution) {
        this.displaySolution();
        return true;
    }
    for (val = first to last) {
        if (this.isValid(val, n)) {
            this.applyValue(val, n);
            if (this.findSolution(n + 1, other params)) {
                return true;
            }
            this.removeValue(val, n);
        }
    }
    return false;
}
```

template element	***meaning in map coloring***
n	
found a solution	
val	
isValid(val, n)	
applyValue(val, n)	
removeValue(val, n)	

Applying the Template to Map Coloring

```
boolean findSolution(n, perhaps other params) {
    if (found a solution) {
        this.displaySolution();
        return true;
    }
    for (val = first to last) {
        if (this.isValid(val, n)) {
            this.applyValue(val, n);
            if (this.findSolution(n + 1, other params)) {
                return true;
            }
            this.removeValue(val, n);
        }
    }
    return false;
}
```

template element	***meaning in map coloring***
n	state number
found a solution	state number > num of last state
val	color (iterates over the four colors)
isValid(val, n)	no bordering states have the color
applyValue(val, n)	apply the color to the state
removeValue(val, n)	remove the color from the state

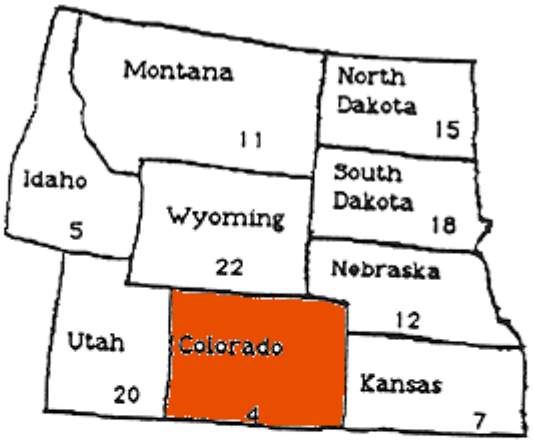
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

We color Colorado through Utah without a problem.

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

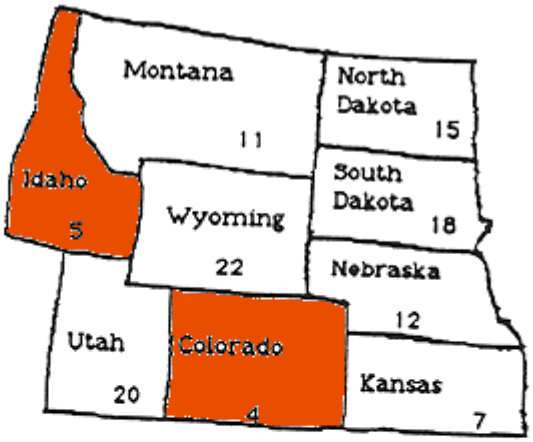


We color Colorado through Utah without a problem.

Colorado: red

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



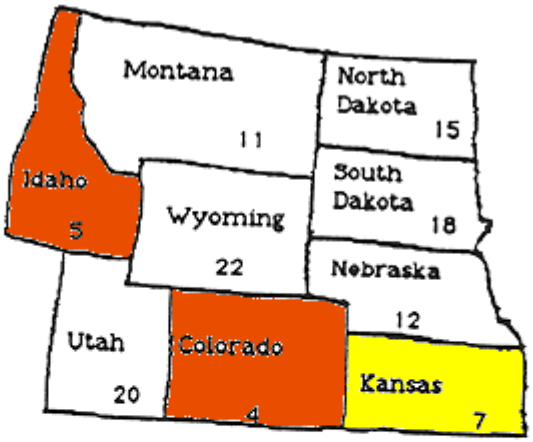
We color Colorado through Utah without a problem.

Colorado: red

Idaho: red

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

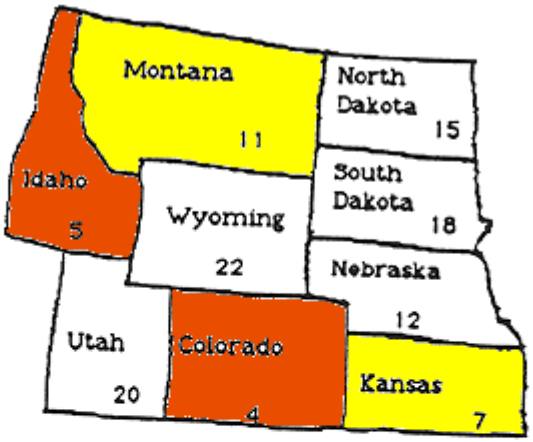


We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

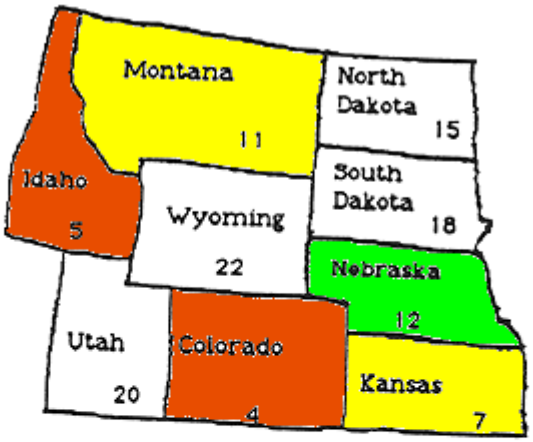


We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

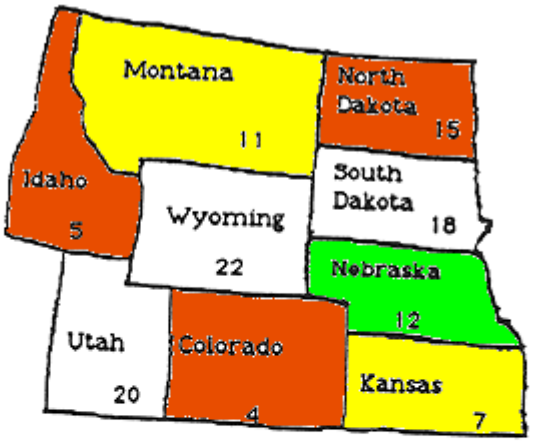


We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

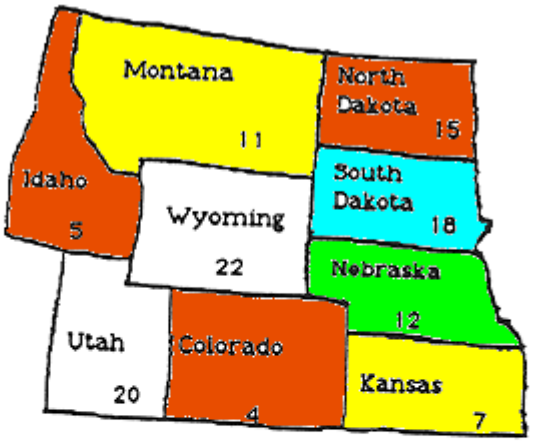


We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

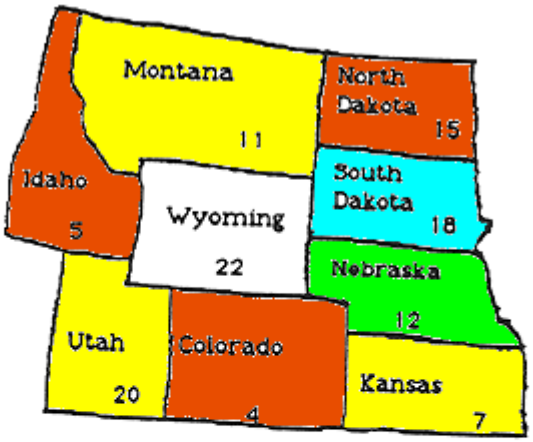


We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

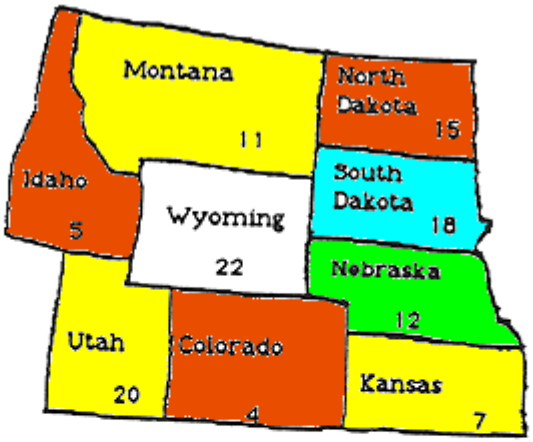


We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow

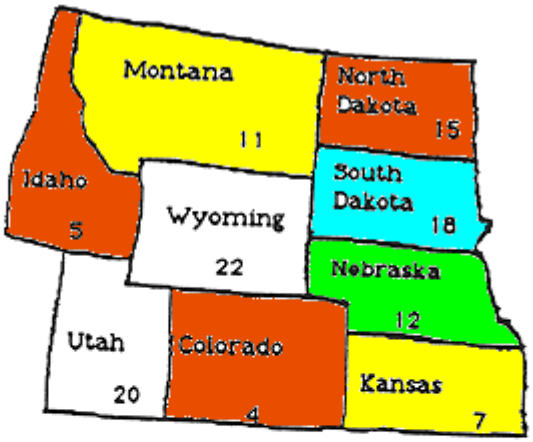
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

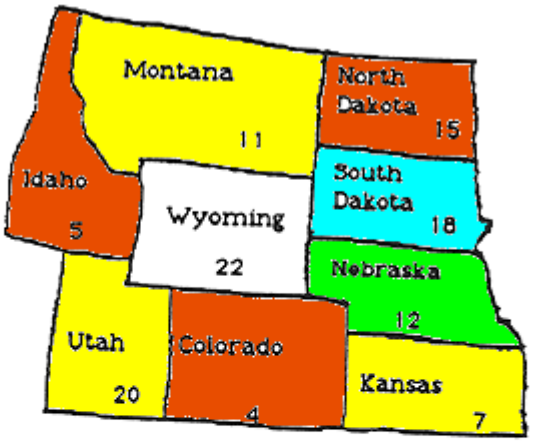
- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow



No color works for Wyoming, so we backtrack…

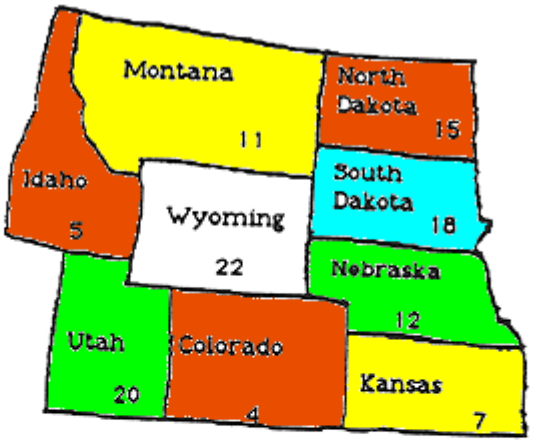
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow

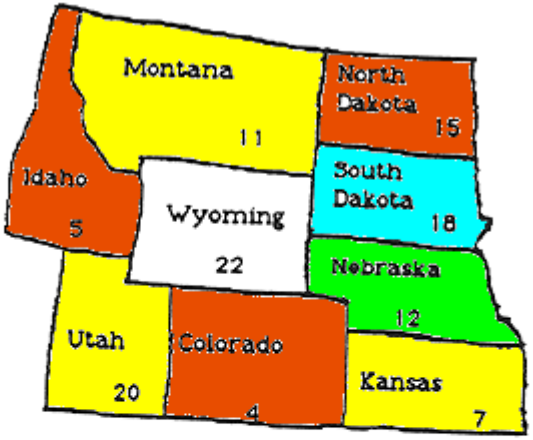


No color works for Wyoming, so we backtrack…

- Color Utah green.

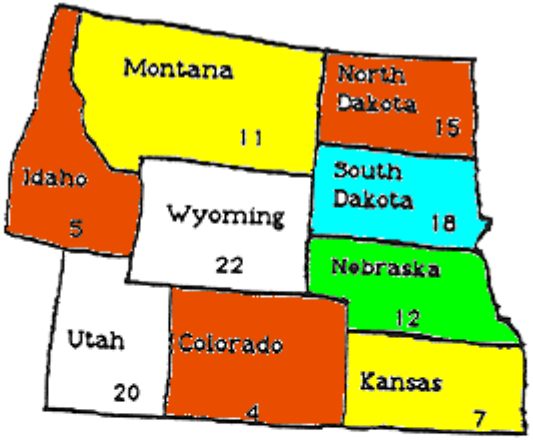
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow

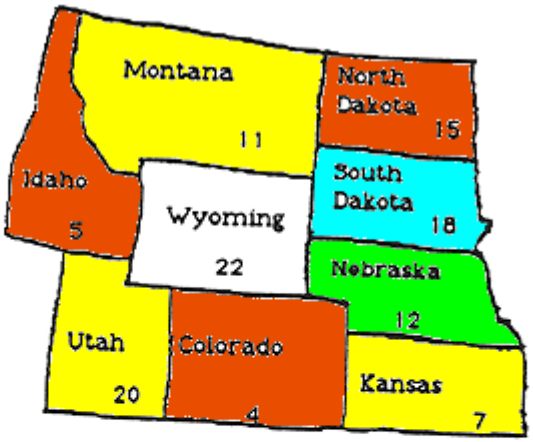


No color works for Wyoming, so we backtrack…

- Color Utah green.
- No color works for Wyoming.
- Backtrack to Utah.

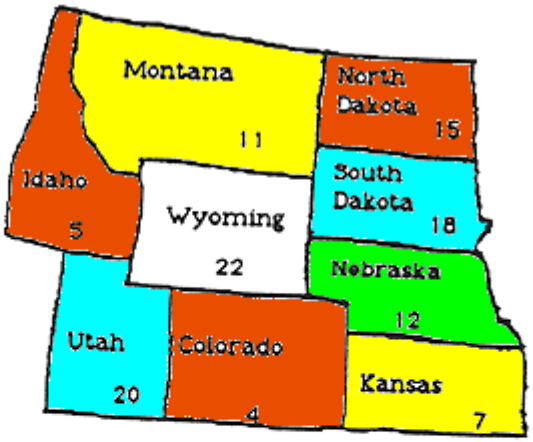
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow

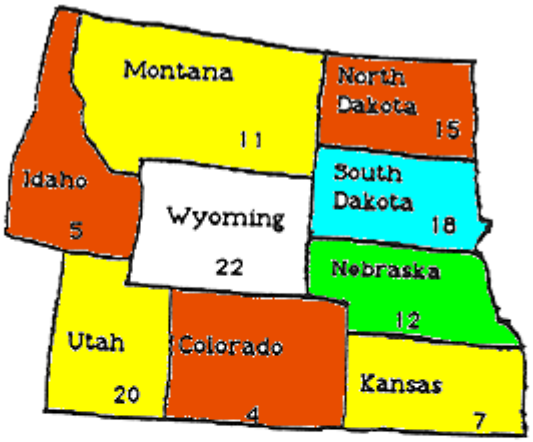


No color works for Wyoming, so we backtrack…

- Color Utah green.
- No color works for Wyoming.
- Backtrack to Utah.
- Color Utah blue.

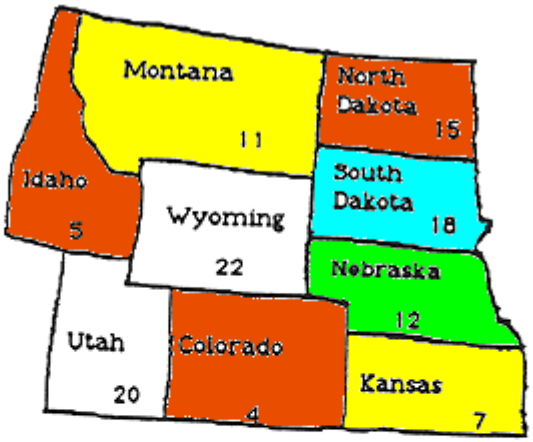
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow

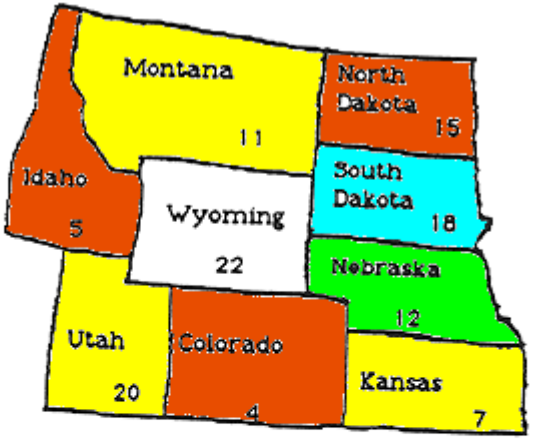


No color works for Wyoming, so we backtrack…

- Color Utah green.
- No color works for Wyoming.
- Backtrack to Utah.
- Color Utah blue.
- No color works for Wyoming.
- Backtrack to Utah.

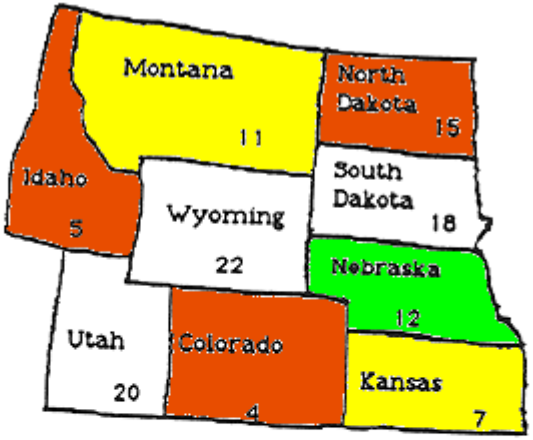
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow

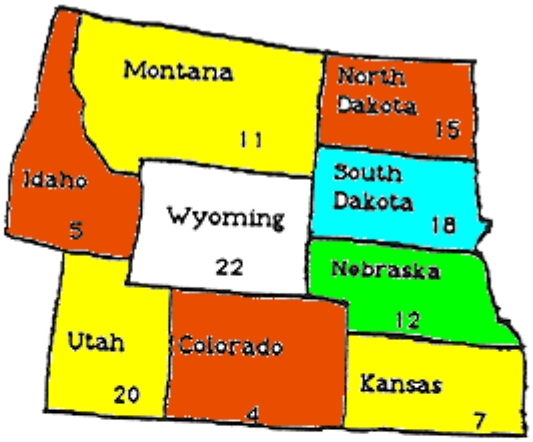


No color works for Wyoming, so we backtrack…

- Color Utah green.
- No color works for Wyoming.
- Backtrack to Utah.
- Color Utah blue.
- No color works for Wyoming.
- Backtrack to Utah.
- No colors left to try for Utah.
- Backtrack to South Dakota.

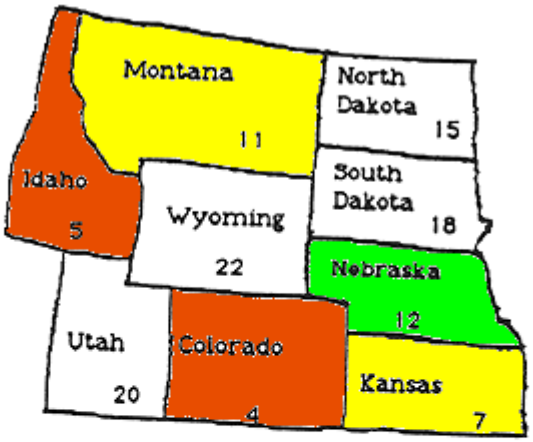
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

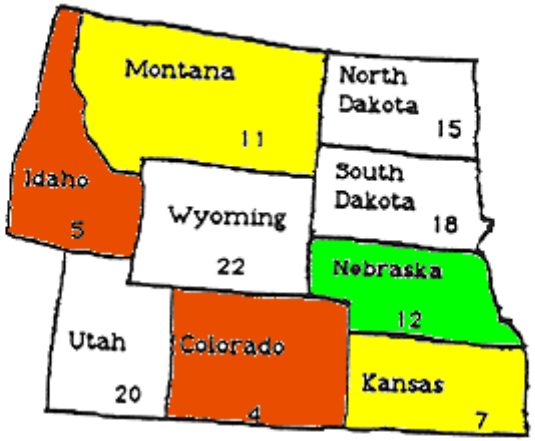
- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow



No color works for Wyoming, so we backtrack…

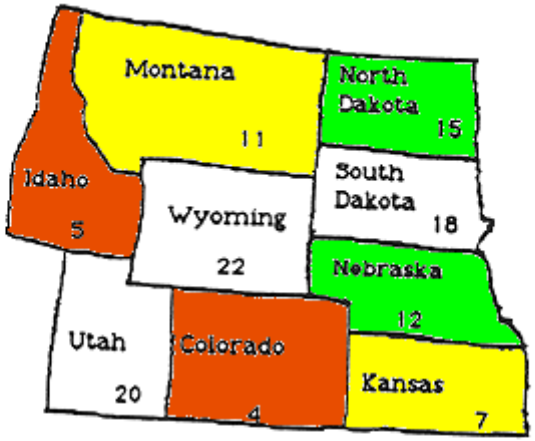
- Color Utah green.
- No color works for Wyoming.
- Backtrack to Utah.
- Color Utah blue.
- No color works for Wyoming.
- Backtrack to Utah.
- No colors left to try for Utah.
- Backtrack to South Dakota.
- No colors left to try for SD.
- Backtrack to North Dakota.

Map Coloring Example (cont.)



Now we can complete the coloring:

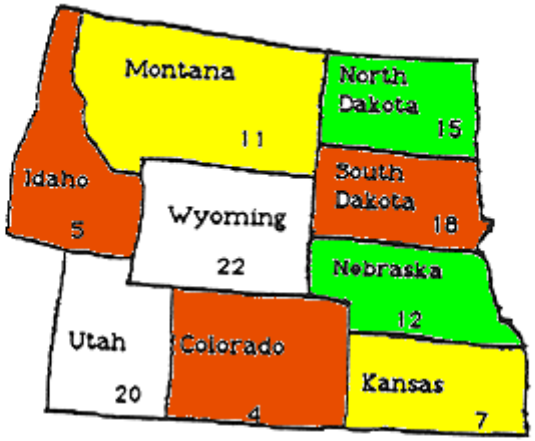
Map Coloring Example (cont.)



Now we can complete the coloring:

North Dakota: green

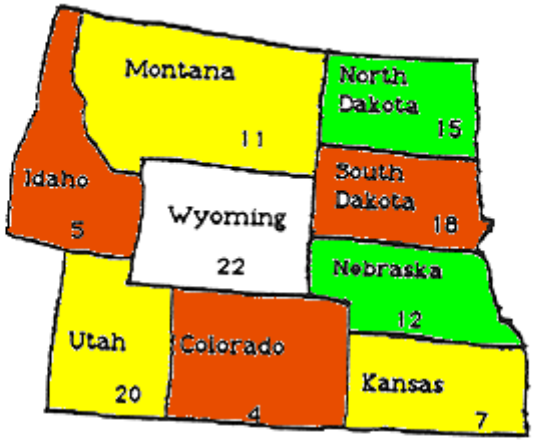
Map Coloring Example (cont.)



Now we can complete the coloring:

North Dakota: green
South Dakota: red

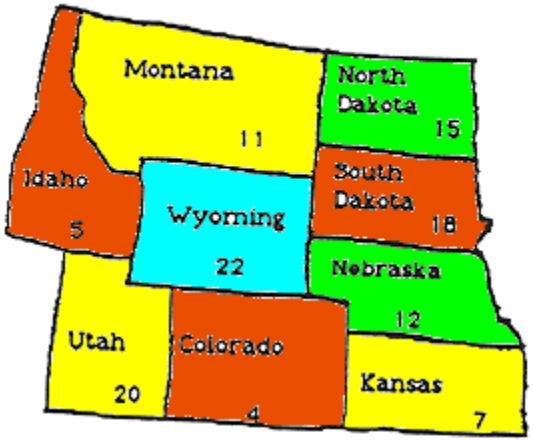
Map Coloring Example (cont.)



Now we can complete the coloring:

North Dakota: green
South Dakota: red
Utah: yellow

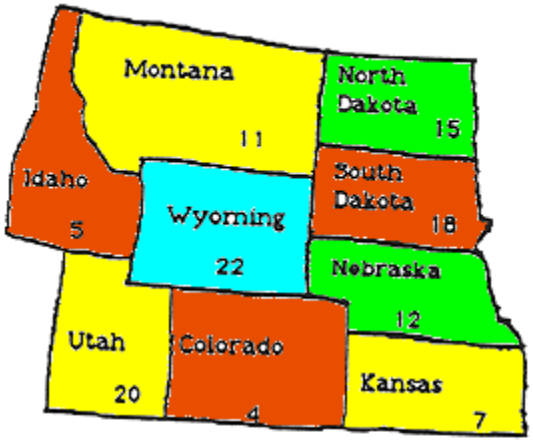
Map Coloring Example (cont.)



Now we can complete the coloring:

North Dakota: green
South Dakota: red
Utah: yellow
Wyoming: blue

Map Coloring Example (cont.)



Now we can complete the coloring:

North Dakota: green
South Dakota: red
Utah: yellow
Wyoming: blue
done!

POWER OF THE STACK