Distinct Element estimation using min

$E[z_k]$

$= \frac{k}{n+1}$

pdf

n uniform samples $(0,1)$

$z = \min(x_1, \dots)$

cdf

$E[z] = \int_0^1 t\, f_z(t)\, dt$

$= n\int_0^1 \underline{t(1-t)^{n-1}}\, dt$

integration by part

$$\boxed{\begin{aligned} p_z(t) &= n \cdot (1-t)^{n-1} \\ F_z(t) &= Pr[z \le t] = 1 - Pr[z > t] \\ &= 1 - (1-t)^n \\ p_z(t) &= n \cdot (1-t)^n \end{aligned}}$$

$\frac{d}{dt}\, t(1-t)^n$

$= \rightarrow (-n \cdot t(1-t)^{n-1} + (1-t)^n) \times \frac{-1}{n} \quad - \quad (1-t)^{n+1} \cdot \frac{1}{n+1} \cdot \frac{1}{n}$

$= \quad t(1-t)^{n-1} - \frac{1}{n}(1-t)^n$

$\frac{-t \cdot (1-t)^n}{n}$

$n \times \left(t(1-t)^n \cdot \frac{-1}{n} - (1-t)^{n+1} \cdot \frac{1}{n+1}\frac{1}{n}\right)$

$= (t(1-t)^n \cdot -1 - (1-t)^{n+1}\frac{1}{n+1})$

0

$- \left(-(1)^{n+1}\frac{1}{n+1}\right) = \frac{1}{n+1}$

min in Ex..

$V_1 = \frac{1}{n+1}$

$\Rightarrow n = \frac{1}{V_1} - 1$

$V_n = \frac{n}{n+1}$

$= 1 - \frac{1}{n+1} \qquad \frac{-1}{V_n - 1} - 1$

Morris algorithm

How to save 1 bit?

n increment

$\log_2 n/2 = \log_2 n - 1$

dividing by 2, saving 1 bit

$$x - (m^2+2m+1) = \frac{m^2-m}{2}$$

$$\frac{3}{2}m^2 + \frac{3}{2}m + 1 \qquad - m^2 - 2m - 1$$

$$2x - 2m^2 - 4m - 2 = m^2 - m$$

$$\frac{3m^2+3m+2}{2}$$

$$\frac{3}{2}m^2 + \frac{3}{2}m + 1$$

$E[Z_{n+1}^2]$

$= E[4^{X_{n+1}}]$

$= \sum_{j=0}^{\infty} P[X_n = j]\left[4^j\left(1-\frac{1}{2^j}\right) + 4^{j+1}\frac{1}{2^j}\right]$

$4^j - 2^j + 2^j = 4^j$

2^{2j+2-j}

2^{j+2}

$2^j \times 4$

$4^j - 2^j + \frac{2^j \times 4}{2 \cdot 2^j}$

Counter value does not change

$4^j + 3 \cdot 2^j$

$= E[Z_n^2] + 3E[Z_n]$

$x = n \times 1$

$n \times 1 \times 1 \times n = \boxed{n \times n}$

$x \cdot x^T$

$(x^T x)' = 2x$ or $2x^T$