Assignment 10

problem 1 (6.15)

Write down the cosets of $(Z_{35}^*)^2$ in Z_{35}^*, along with the multiplication table for the quotient group $Z_{35}^* / (Z_{35}^*)^2$.

The element of $Z_{35}^* = \{1, 2, 3, 4, 6, 8, 9, 11, 12, 13, 16, 17, 18, 19, 22, 23, 24, 26, 27, 29, 31, 32, 33, 34\} \rightarrow 24$ elements

The element of $(Z_{35}^*)^2 = \{1, 4, 9, 16, 29, 11\}$, 6 elements

$|(Z_{35}^*)| / |(Z_{35}^*)^2| = 24/6 = 4$ since it is $*$ we use multiplicative notation

$[1]_H = \{1, 4, 9, 16, 29, 11\} \rightarrow 0$
$[2]_H = \{2, 8, 18, 32, 23, 22\} \rightarrow 0$
$[3]_H = \{3, 12, 27, 13, 17, 33\} \rightarrow 0$
$[4]_H = \{4, 16, 1, 29, 11, 9\} \rightarrow$ same as $[1]_H$
$[6]_H = \{6, 24, 19, 26, 34, 31\} \rightarrow 0$

Cosets are $[1]_H, [2]_H, [3]_H, [6]_H$

Multiplicative table for the quotient group $Z_{35}^* / (Z_{35}^*)^2$

$\cdot$	$[1]_H$	$[2]_H$	$[3]_H$	$[6]_H$
$[1]_H$	$[1]_H$	$[2]_H$	$[3]_H$	$[6]_H$
$[2]_H$	$[2]_H$	$4\cdot2[1]_H$	$[6]_H$	$[3]_H$
$[3]_H$	$[3]_H$	$[6]_H$	$[1]_H$	$[2]_H$
$[6]_H$	$[6]_H$	$[3]_H$	$[2]_H$	$[1]_H$

Problem 2 (6.37)

Which of the following pairs of groups are isomorphic? Why or why not?

(a) $Z_2 \times Z_2$ and Z_4

(b) 8_{12}^* and Z_8^*

(c) Z_5^* and Z_4

(d) $Z_2 \times Z$ and Z

(e) Q and Z

(f) $Z \times Z$ and Z

Isomorphic is one-to-one mapping with the same cardinality.

(a) is not isomorphic since $Z_2 \times Z_2 = Z_2$, and Z_4 does not have to same. Cardinality

(b) is isomorphic since $Z_{12}^* = \{1, 5, 7, 11\}$, $Z_8^* = \{1, 3, 5, 7\}$ they have same cardinality (11 map to 3

(c) is isomorphic since $Z_5^* = \{1, 2, 3, 4\}$, $Z_4 = \{0, 1, 2, 3\}$ corresponding one-to-one with the same cardinality

(d) is isomorphic since $Z_2 \times Z = Z$ Z and Z are isomorphic

(e) is not isomorphic since Q and Z does not have to same cardinality

(f) $Z \times Z = Z$ so it is isomorphic