CS131 - Homework #3

Problem 1

$C(x,y)$ = Student $x$ is enrolled in Class $y$

Domain = $x$ = All students in BU

$y$ = All classes being given at BU

a) $C$(Randy Goldberg, CS131)

→ Randy Goldberg is enrolled in Class CS131

b) $\exists x\, C(x,$ CS111)

→ There is some students who is enrolled in Class CS111

c) $\exists y\, C$(Carol Siteu, $y$)

→ Carol Siteu is enrolled in some classes $y$

d) $\exists x\,(C(x,$ CS111) $\wedge\, C(x,$ CS131))

→ There is some students who is enrolled in CS111 class and CS131 class.

e) $\exists x \exists y \forall z((x \neq y) \wedge (C(x,z) \rightarrow Y(y,z)))$

If some student $x$ takes $z$ classes then there is some student $y$ takes $z$ classes

f) $\exists x \exists y \forall z((x \neq y) \wedge (C(x,z) \leftrightarrow C(y,z)))$

If some student $x$ takes $z$ classes only if some student $y$ takes $z$ classes.

Rosen 1.6

Problem 2

a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

p

p = kangaroos live in Australia
q = kangaroos are marsupials.

$p \wedge q \rightarrow q$

It is valid, and it used Simplication.

b) It is either hotter than 100 degrees today or pollution is dangerous. It is less than 100 degrees today. Therefore, pollution is dangerous.

p = hotter than 100 degrees today
q = pollution is dangerous.

$(p \vee q) \wedge \neg p \rightarrow q$

$$\begin{array}{c} (p \vee q) \\ \neg p \\ \hline \therefore q \end{array}$$

By disjunctive Syllogism (rule of inference), Conclusion should be pollution is not dangerous, so it is invalid.

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

p = Linda is an excellent swimmer
q = Linda can work as a lifeguard.

$(p \wedge (p \rightarrow q)) \rightarrow q$

It is valid, It is Modus ponens (rule of inference).

d) Steve will work in the computer company this summer. Therefore, this Summer steve will work at a computer company or he will be beach bum.

p: steve will work in the computer company this Summer.
q = steve will be beach bum this summer

$\boxed{p \to (p \vee q)}$

It is valid, Addition (Rue of inference)

✓ e) If I exercise everyday, I will become an athlete. I am an athlete. Therefore, I exercise everyday.

p: I exercise everyday
q: I am an athlete

$$((\underset{F}{p} \overset{T}{\to} \underset{T}{q}) \wedge \underset{T}{q}) \to \underset{F}{p}$$

It is invalid, when p is false and q is true, $((\underset{F}{p} \overset{T}{\to} \underset{T}{q}) \wedge \overset{T}{q}) \to \underset{F}{p}$

Even if He is an athlete, he doesn't have to exercise everyday. $T \to F$

F) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

$p$ = I work all night on this homework
$q$ = I can answer all the exercises
$R$ = I understand the material

$$[(p \to q) \wedge (q \to R)] \to (p \to R)$$

It is valid, it is Hypothetical Syllogism.

Problem 3

a) Let $n$ be an arbitrary integer. If $3n+2$ is even, then $n$ is even

i)

Given: $n$ is an arbitrary integer.

Goal: If $3n+2$ is even, then $n$ is even.

ii) Direct proof $p \to q$

Given: $n$ is an arbitrary integer, $3n+2$ is even
Goal: $n$ is even

Contrapositive

$p \to q \equiv \neg q \to \neg p$

Given: $n$ is an arbitrary integer, $n$ is odd
Goal: $3n+2$ is odd

proof by contradiction.

Let $n$ be an arbitrary integer. If $3n+2$ is even, then $n$ is even

$n$ be an arbitrary integer
$p = 3n+2$ is even
$q = n$ is even

$p \to q \equiv \neg p \vee q$

negation: $p \wedge \neg q$

$p =$ True, $q =$ false, $\neg q$: True

given: $3n+2$ is even and $n$ is odd.

Goal: To find contradiction.

iii) Contraposition:

Suppose $n$ is an arbitrary integer and $n$ is odd. We can write $n = 2k+1$ (for some integer $k$). by multiplying by 3 and adding 2, we get $3n+2 = 6k+5$. $6k+5 = 2(3k+2)+1$. Thus, we have expressed $3n+2$ as $2(3k+2)+1$, where $3k+2$ (integer). Therefore, $3n+2$ is odd. QED

Contradiction

Suppose for the sake of contradiction that there exists some integer $n$ such that $3n+2$ is even and $n$ is odd. From this equation we get $n = 2k+1$ (for some integer $k$) and $3n+2 = 2l$ (for some integer $l$). By plugging in $2k+1$ for $n$ in equation $3n+2$, we obtain $3(2k+1)+2 = 6k+3+2 = 6k+5 = 2(3k+2)+1$ so $n$ is both even and odd, contradiction (even $\wedge$ odd) or ($2l \wedge 2k+1$) QED.

b) Let n, m be arbitrary integers, If mn is even, then n is even or m is even.

iv) Given: n, m is arbitrary integers.

Goal: If mn is even, then n is even or m is even

v) Direct proof

Given: n, m is arbitrary integers, mn is even

Goal: n is even or m is even

Contraposition

p: mn is even
q: n is even
r: m is even

$p \rightarrow (q \vee r)$

contraposition

$\equiv \neg(q \vee r) \rightarrow \neg p$

$\equiv (\neg q \wedge \neg r) \rightarrow \neg p$

Given: n, m is arbitrary integers, n is odd and m is odd

Goal: mn is odd

Contradiction

$p \rightarrow (q \vee r) \equiv \neg p \vee (q \vee r)$

negation: $p \wedge \neg(q \vee r)$

True $p \wedge (\neg q \wedge \neg r)$ True

$(q \vee r)$ False

Given: mn is even, and n is odd and r is odd

Goal: find the contradiction.

vi) Contraposition.

Suppose $n, m$ are arbitrary integers and $n$ is odd and $m$ is odd. So we can write $n = 2k+1$ (for some integer $k$) and $m = 2l+1$ (for some integer $l$) By multiply $n$ and $m$ we get $mn = (2k+1)(2l+1)$ $= 4kl + 2k + 2l + 1 = 2(2kl + k + l) + 1$. Thus, we have expessed $mn = 4kl + 2k + 2l + 1$ as $2(2kl + k + l) + 1$, where $(2kl + k + l)$ is some integer. Therefore, $mn$ is odd QED.

Contradiction.

Suppose for the sake of contradiction, that there exists some integers $m, n$ such that $mn$ is even, and $n$ is odd and $m$ is odd. From this we can get $n = 2k+1$ (for some integer $k$) and $m = 2l+1$ (for some integer $l$). By pluggin in $2k+1$ for $n$, $2l+1$ for $m$ in $mn$, we obtain $mn = (2k+1)(2l+1) = 4kl + 2k + 2l + 1$ $= 2(2kl + k + l) + 1$ so $mn$ is $2(E) + 1$ (for some integer $E$), $E = 2kl + k + l$. $mn$ is odd, which is $\neg p$. becuse Both $p$ and $\neg p$ is true, we have a pruof that contradition, so that if $mn$ is even, and $n$ is even or $m$ is even QED.

Problem 4

Let a, b be arbitrary integers. prove if a is multiple of 3, and b is even, the ab is a multiple of 6.

a) $p(x)$ = $x$ is multiple of 3
$q(y)$ = $y$ is even
$r(z)$ = $z$ is multiple of 6

$(p(a) \vee q(b)) \rightarrow r(ab)$

$\equiv (p(a) \vee q(b)) \vee \neg r(ab)$
$\neg((p(a) \vee q(b))) \vee \neg r(ab)$
$(\neg p(a) \wedge \neg q(b)) \wedge r(ab)$

b) Given: a,b are arbitrary integers

Goal: if a is multiple of 3, and b is even, the ab is a multiple of 6.

c) Direct proof.

Given: a, b are arbitrary integers, a is multiple of 3, and b is even

Goal: ab is a multiple of 6.

Suppose a, b are arbitrary integers, and a is multiple of 3, and b is even so we can write $a = 3k$ (for some integer k) and $b = 2l$ (for some integer $l$). By multiplying a and b we get $ab = (3k)(2l) = 6kl = 6kl = 6t$ (for some integer $t = kl$), Thus we have expressed $ab = 6kl = 6t$ where $(kl)$ is some integers. Therefore ab is a multiple of 6 QED.