Multinomial

$X \sim Bin(n, p)$ — success

↳ coin Tosses

Central limit theorem

Variance: How spread out

Standard deviation: Require Assumption

$(60H, 40T)$

$Pr(X=60) = \binom{100}{60} p^{60} (1-p)^{40}$

Now (die) there are more than 2 outcomes

합치기

$\{H, T, C\}$

$p, \; q, \; 1-p-q$

$\Rightarrow (10, 50, 40)$

Induction way

$$\frac{100!}{10!\,50!\,40!}\left(\frac{1}{3}\right)^{100}$$

→ ordering does not matter

같은 적용

$n=5 \quad (3H, 2T)$

T_1, T_2 2가지 종류의 Tail 이 있다고 하자

H_1, H_3, H_2 3가지 Head의 종류가 있다고하자

$\underline{H_1}\ \underline{H_2}\ \underline{T_2}\ \underline{H_3}\ \underline{T_1}$

$\underline{H_2}\ \underline{H_3}\ \underline{T_1}\ \underline{H_1}\ \underline{T_2}$ 위자리의 종류 (H인지 T인지)는 같고, H 사이에서 변환, T사이에서의 변환이 같은건 $3! \times 2!$ 이다.

$$\frac{5!}{3!\,2!}$$

Example

$X = (1, 2, 3, 4, 5, 6)$ die

$p = (0.1, 0.2, 0.5, 0.05, 0.05, 0.1)$

Pr (10 1s, 20개의 2, 3개의 3, 100개의 4, 5개의 5, 8개의 6)

$$\frac{146!}{10!\,20!\,3!\,100!\,5!\,8!}(0.1)^{10}(0.2)^{20}(0.5)^{3}(0.05)^{100}(0.05)^{5}(0.1)^{8}$$

poisson distribution

λ: Lamda : rate

why poisson is PMF

$X \in \mathbb{N} = \{0, 1, 2, \dots\}$

$$Pr(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

$$\sum_{k}^{\infty} \cancel{k}\frac{e^{-\lambda}\lambda^k}{k!} = 1$$

Taylor series

$$e^{\lambda} = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = ($$

$$e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^{-\lambda} \times e^{\lambda} = 1$$

Approximation

$$\frac{1}{0.99} = 1.01 \qquad \frac{1}{1-x} \doteq 1+x$$

1.2.1 Taylor approximation

잘모르는 복잡한 함수를 다항함수로 바꿀수 있다.

Definition 1.2 Taylor approximation

무한히 미분가능

Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function with infinite derivatives Let $a \in \mathbb{R}$ be a fixed constant. The Taylor approximation of f at $x=a$ is

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

($f^{(n)}$ → n번미분)

where $f^{(n)}$ denotes the n^{th}-order derivative of f

$$E[X] = \sum_{k=0}^{\infty} k \frac{e^{-\lambda}\lambda^k}{k!} = \lambda e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)}$$

$$e^{-\lambda} \sum_{k=1}^{\infty-1} \frac{\lambda^{k-1 \to k'}}{(k-1)!} = e^{-\lambda} \cdot e^{-\lambda} = 1$$

$k' = k-1$

Geometric random variable

only need 1 probability

($\quad$ n-1

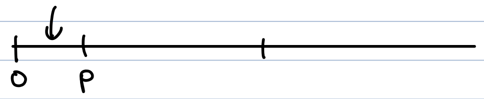
$E[\cdot] = \sum_{N=N}$ $p(1-p)$

Continous Random Variable

$X \sim Bin(10^{12}, \frac{15}{10^{12}})$

$rand() \sim U(0,1)$

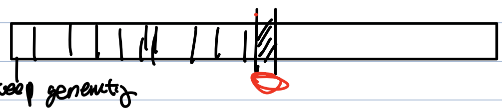
rand( )



Mark

Office Hour ☆

$\cancel{X} = \cancel{X_1} + \cdots + X_{10^{12}}$



Donald Knuth

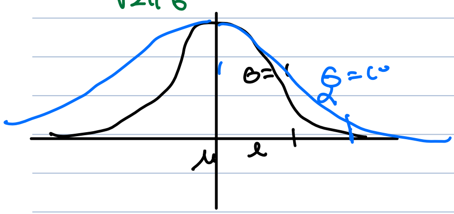
keep generating

p × n coins

∞

Gaussian distribution

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$



☆ 수식

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

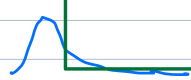
$x-\mu$: deviation

σ^2 : variance

or $N(\mu, \sigma^2)$

$N(\mu, \sigma)$

Heavy tail : Extreme low . Salaries .



billionaire

$Pr(X=k) \propto \frac{1}{k^2}$

☆ $Bin(n,p) \sim N(np, np(1-p))$

variance

mean

Expectation

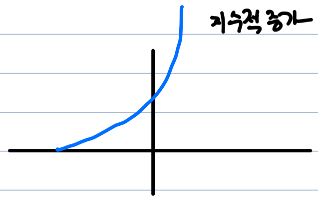
when varience ↓ more stiper

Narrow . Higher point of max

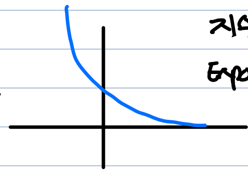
Variance ↑ more wider

wider, Heighest point is lower

e^x



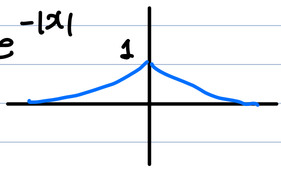
e^{-x}



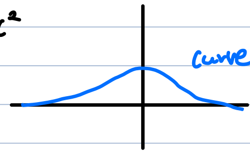
지수적감소

Exponential decay $\sqrt{e}$ every time

$e^{-|x|}$

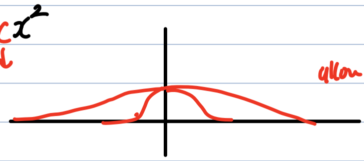


e^{-x^2}



curve 매끄럽게 바꾸어줌 다.

e^{-cx^2}



allow to strech and squeeze

$e^{-cx^2} = (e^c)^{-x^2} = (2.718)^{-x^2}$

잘은것 ↙

$e^{-\frac{1}{2}(x/6)^2} \; -\frac{1}{2}\frac{x^2}{6^2} = e^{-\frac{1}{2 6^2}x^2}$

e^{-x^2} 밑의 넓이는 $\sqrt{\pi}$ (확률) $\therefore \frac{1}{\sqrt{\pi}} e^{-x^2} \approx 1$

$\frac{1}{\sqrt{\pi}} e^{-\frac{1}{2}(x/6)^2} = \frac{1}{\sqrt{\pi}} e^{-\frac{1}{2 6^2} x^2} = \frac{1}{\sqrt{\pi}} e^{-x^2} \times e^{\frac{1}{2 6^2}} = 1 \times 6\sqrt{2}$ $\therefore \frac{1}{6\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x}{6^2}\right)^2}$

여기에 mean이 붙는다

$\frac{1}{6\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{2 6^2}\right)^2}$

Research

Brian Plumer PhD student 오피스

Deep learning. Kullis [office]

How binomial + Gaussian is Laplace distribution

↑

Laplace $(x|\mu, b) =$

Also distance. but Not square like Gausian

$\uparrow$

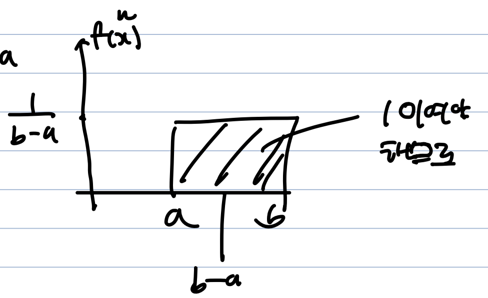
$\frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right)$

$\mu=0 \quad x=10$

$(x-\mu)^2=100$ 가우시안 $e^{-\frac{100}{2\sigma^2}}$

laplace e^{-10}

Beta



$f(x)=C \qquad a \le x \le b$

$C(b-a)=\int_a^b f(x)\,dx$

뾰족한 산처럼 생겼다.

$f(x)=\frac{1}{2b}\exp\left[\frac{-|x-\mu|}{b}\right]$

범위 $-\inf < x < \inf$ Gaussian과 동일.

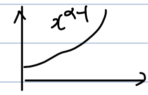
mean (Expectation) $E(x)=\mu$

Variance $= 2b^2$

CLT (Central limit Theorem)을 Binomial distribution 에 적용시켜보기

확률변수 X가 이항분포 $B(n,p)$ n이 충분히 클때 $n \ge 50$ 변수 X는 Gaussian(np, npq)를 따른다.

Beta distribution



$$B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \text{ for } \alpha,\beta>0$$

$\Gamma(\alpha)$ is a gamma function,

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\,dx \text{ for } \alpha>0$$

$\Gamma(n) = (n-1)!$

$\Gamma(1) = 1$

$$Beta(x|a,b) = \frac{1}{B(a,b)}\, x^{a-1}(1-x)^{b-1}$$

$$E[x] = \int_{-\infty}^{\infty} \frac{x}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$\underline{\sigma z + \mu} \left(e^{-\frac{z^2}{2}}\right)$$

$$z = \frac{x-\mu}{\sigma}$$

$$x^2 e^{-\frac{z^2}{2}}$$

$$\frac{1}{\sqrt{2\pi\sigma}} \int (\sigma z + \mu) e^{-\frac{z^2}{2}}$$

$$\underline{\int z e^{-\frac{z^2}{2}}}$$

$$-e^{-\frac{z^2}{2}}$$