Problem 1.

a) $\neg(M \wedge K)$

b) $\neg M \wedge \neg K$ (or the same answer as (d), logically equivalent)

c) $\neg M \vee \neg K$

d) $\neg(M \vee K)$ (or the same answer as (b),logically equivalent)

Problem 2.

a) Three propositions. J: Jane wins math prize, M: Pete wins math prize, C: Pete wins chem prize. Premises: $\neg(J \wedge M)$, $M \vee C$, J. The conclusion, C, is true. Logic: When J is true, M must be false to make the first premise true. When M is false, C must be true to make the second premise true. So the only row of the truth table in which all the premises are true is $J = T, M = F, C = T$. Since the conclusion is also true for this case, it's a valid argument.

b) Three propositions: J is John is telling the truth, and similarly for B and S. Premises: $J \vee B$, $\neg S \vee \neg B$. Conclusion: $J \vee \neg S$.

J	B	S	$J \vee B$	$\neg S \vee \neg B$	$J \vee \neg S$
F	F	F	F	T	T
F	F	T	F	T	F
F	T	F	T	T	T
F	T	T	T	F	F
T	F	F	T	T	T
T	F	T	T	T	T
T	T	F	T	T	T
T	T	T	T	F	T

The premises are all true in rows 3, 5, 6, 7, and the conclusions are true for all those rows. Valid argument.

Alternatively, you can consider two cases.

Case 1: B is false. From the first premise, J must be true. Therefore, $J \vee \neg S$.

Case 2: B is true. From the second premise, S must be false. Therefore, $J \vee \neg S$.

c) Let S denote sales will go up, E is for expenses will go up, and H is happy boss. Translating the premises, we have $S \to H$ and $E \to \neg H$, and the conclusion $\neg(S \wedge E)$. If both S and E are operative, the two premises contradict each other, since the boss has to be both happy and not happy (maybe typical in the real world, but not in logic). So this argument is logically valid. One could debate the logic underlying the premises, but not the validity of the argument when the premises are true.

Problem 3.

a) Just need the columns in the truth table to match. Very easy:

P	Q	$P \leftrightarrow Q$	$P \wedge Q$	$\neg P \wedge \neg Q$	$(P \wedge Q) \vee (\neg P \wedge \neg Q)$
F	F	T	F	T	T
F	T	F	F	F	F
T	F	F	F	F	F
T	T	T	T	F	T

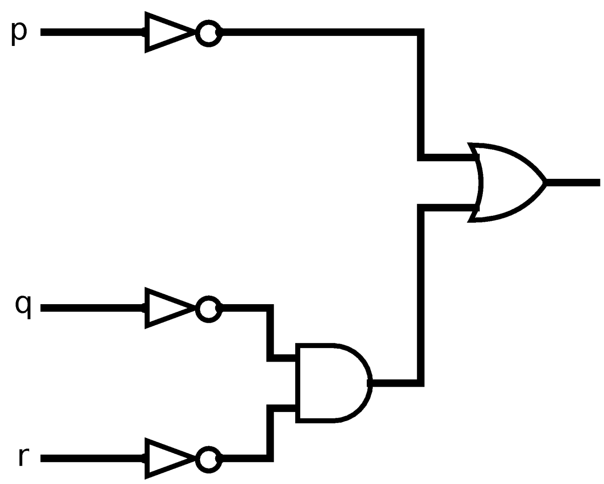

Figure 1:

b)

$$\begin{aligned}(P \to Q) \vee (P \to R) &\equiv (\neg P \vee Q) \vee (\neg P \vee R) && \text{(Conditional Law x 2)}\\ &\equiv \neg P \vee \neg P \vee Q \vee R && \text{(Associativity of OR)}\\ &\equiv \neg P \vee (Q \vee R) && \text{(Idempotence of OR)}\\ &\equiv P \to (Q \vee R) && \text{(Conditional Law)}\end{aligned}$$

c)

$$\begin{aligned}(P \to Q) \vee (Q \to R) &\equiv (\neg P \vee Q) \vee (\neg Q \vee R) && \text{(Conditional Law x 2)}\\ &\equiv \neg P \vee (Q \vee \neg Q) \vee R && \text{(Associativity)}\\ &\equiv \neg P \vee (\text{True}) \vee R && \text{(basic fact)}\\ &\equiv \text{True} && \text{(Tautology Law)}\end{aligned}$$

Problem 4.

$$\begin{aligned}A \equiv &((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))\\ \equiv &(\neg p \wedge \neg q) \vee (\neg r \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge r) && \text{(Distributive Law x 2)}\end{aligned}$$

Consider $B \equiv (\neg p \wedge \neg q) \vee (\neg p \wedge q)$. To simplify this, consider two cases.

Case 1: q is false. $B = (\neg p \wedge T) \vee (\neg p \wedge F) = \neg p \vee F = \neg p$.

Case 2: q is true. Similarly, here $B = \neg p$.

Thus, $B \equiv \neg p$ and $A \equiv (\neg r \wedge \neg q) \vee \neg p \vee (\neg p \wedge r)$.

Let's simplify $C \equiv \neg p \vee (\neg p \wedge r)$. Similarly to the above, we can consider two cases: r is false and r is true. In both cases C simplifies to $\neg p$.

Therefore, $A \equiv \neg p \vee (\neg r \wedge \neg q)$.

See Figure 1.

Problem 5. This problem illustrates how important it is to treat the problem carefully. We provide two reasonings, the first (i) makes the assumption that the lady and the tiger cannot be in the same room, and the second proceeds (ii) without any assumptions besides what is given to us strictly in the exercise. The reasoning in both cases follows.

Proof. (i) Assume that the tiger and the lady cannot be in the same room (since it could be too dangerous for the lady). In that case, can it be the case that the first sign is correct, and the second wrong? Of course not, because if the first sign is true, then the second must be true – that is, if there is a lady in Room I and a tiger in Room 2, then it is certainly the case that one of the rooms contains a lady and the other a tiger. Therefore it must be that the second sign is true, and the first false. Since the second is true, there exists a lady in a room and a tiger in the other, and since the first sign is false, the lady is in Room II, and the tiger in Room I.

(ii) In this case, unless we make the aforementioned assumption, we cannot conclude where the lady is. While we still obtain with the same reasoning that the sign 2 is true, and sign 1 false, it could be the case that the lady and the tiger are both in Room I, or the lady in room II and the tiger in room I, or even that they are both in Room II. All these scenarios make sign 2 true, and sign 1 false.

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