CS 131 – Fall 2019, Assignment 3

Problems must be submitted by Friday, September 27, on Gradescope.

Problem 1. Let predicate $C(x, y)$ mean that student x is enrolled in class y, where the domain for x consists of all students in BU, and the domain for y consists of all classes being given at BU. Express each of the following statements by a simple English sentence.

a) (2 pt) $C(Randy\ Goldberg, CS131)$

Solution. Randy Goldberg attends CS 131.

b) (3 pt) $\exists x C(x, CS111)$

Solution. There is a student who attends CS 111 ("Introduction to computer science").

c) (5 pt) $\exists y C(Carol\ Sitea, y)$

Solution. Carol Sitea attends a class.

d) (5 pt) $\exists x(C(x, CS111) \wedge C(x, CS131))$

Solution. There is a student who attends both CS 111 and CS 131.

e) (5 pt) $\exists x \exists y \forall z((x \neq y) \wedge (C(x, z) \rightarrow C(y, z)))$

Solution. There are two students such that the second one attends any class the first one attends.

f) (5 pt) $\exists x \exists y \forall z((x \neq y) \wedge (C(x, z) \leftrightarrow C(y, z)))$

Solution. There are two students that attends the same classes.

Problem 2. Argue whether each of the following arguments is valid or not. For the valid arguments, which rule of inference is used? For the invalid arguments, explain why they are invalid.

a) (5 pt) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

Solution. Simplification.

b) (5 pt) It is either hotter than 100 degrees today or pollution is dangerous. It is less than 100 degrees today. Therefore, pollution is dangerous.

Solution. Disjunctive Syllogism.

c) (5 pt) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

Solution. Modus ponens.

d) (5 pt) Steve will work in the computer company this summer. Therefore, this summer Steve will work at a computer company or he will be beach bum.

Solution. Addition.

e) (5 pt) If I exercise every day, I will become an athlete. I am an athlete. Therefore, I exercise every day.

Solution. This statement is not correct, the implication is given, but it is applied in the wrong direction.

f) (5 pt) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

Solution. Hypothetical syllogism.

Problem 3. Prove the following statements:

a) (10 pt) Let n be an arbitrary integer. If $3n + 2$ is even, then n is even.

i) Identify what is given and what is asked in this problem.

Solution.
Given: $N(n)$, where $N(x)$ - "x is an integer".
Asked: $P(3n + 2) \to P(n)$, where $P(x)$ - "x is even".

ii) Outline what is given and asked in a direct proof, a proof by contraposition and in a proof by contradiction.

Solution.
Direct proof:
Given: $N(n)$, $P(3n + 2)$.
Asked: $P(n)$.
Proof by contraposition:
Given: $N(n)$, $\neg P(n)$.
Asked: $\neg P(3n + 2)$.
Proof by contradiction:
Given: $N(n)$, $\neg P(n)$, $P(3n + 2)$.
Asked: False.

iii) Prove the statement by contraposition and contradiction.

Solution.
Proof by contraposition:
$\neg P(n)$ means that we can write n as $n = 2k + 1$, where k is an integer. Thus, $3n + 2 = 3 * (2k + 1) + 2 = 6k + 5 = 2 * (3k + 2) + 1$. Since, k is an integer, $3k + 2$ is an integer too, thus $3n + 2$ is odd number, or $\neg P(3n + 2)$ is True. QED.
Proof by contradiction:
Applying the same logic as in proof by contraposition we can get a correct implication $\neg P(n) \to \neg P(3n+2)$. Since $\neg P(n)$ is given by applying *Modus Ponens* we have $\neg P(3n+2)$ is true. Combining it with $P(3n + 2)$, which is given, we have $P(3n + 2) \wedge \neg P(3n + 2)$, which evaluates to False. QED.

b) (10 pt) Let n, m be arbitrary integers. If mn is even, then n is even or m is even.

iv) Identify what is given and what is asked in this problem.

Solution.
Given: $N(n), N(m)$, where $N(x)$ - "x is an integer".
Asked: $P(mn) \to (P(n) \vee P(m))$, where $P(x)$ - "x is even".

v) Outline what is given and asked in a direct proof, a proof by contraposition and in a proof by contradiction.

Solution. Direct proof:
Given: $N(n), N(m)$, $P(nm)$.
Asked: $P(n) \vee P(m)$.
Proof by contraposition:
Given: $N(n), N(m)$, $\neg(P(n) \vee P(m))$.
Asked: $\neg P(nm)$.
Proof by contradiction:
Given: $N(n), N(m)$, $\neg((P(n) \vee P(m))$, $P(nm)$.
Asked: False.

vi) Prove the statement by contraposition and contradiction.

Solution.
Proof by contraposition:
$\neg(P(n) \wedge P(m)) = \neg P(n) \wedge \neg P(m)$ means that n and m are both odd and we can write them as $n = 2k_1 + 1$ and $m = 2k_2 + 1$, where k_1, k_2 are integers. Thus, $mn = (2k_1 + 1) * (2k_2 + 1) = 4k_1k_2 + 2k_1 + 2k_2 + 1 = 2(2k_1k_2 + k_1 + k_2) + 1$. Since, k is an integer, $2k_1k_2 + k_1 + k_2$ is an integer too, thus mn is odd number, or $\neg P(mn)$ is True. QED.
Proof by contradiction:
Applying the same logic as in proof by contraposition we can get a correct implication $\neg(P(n) \vee P(m)) \to \neg P(nm)$. Since $\neg(P(n) \vee P(m))$ is given by applying *Modus Ponens* we have $\neg P(mn)$ is true. Combining it with $P(mn)$, which is given, we have $P(mn) \wedge \neg P(mn)$, which evaluates to False. QED.

Problem 4. Let a, b be arbitrary integers. Prove if a is multiple of 3, and b is even, then ab is a multiple of 6.

a) (5 pt) Express the statement and its negation in predicate logic.

Solution.
Defining predicates:
$N(x)$ - "x is an integer"
$A(x)$ - "x is multiple of 3"
$B(x)$ - "x is even"
$C(x)$ - "x is multiple of 6"
Statement:

$$\forall a, b N(a), N(b), (A(a) \wedge B(b)) \to C(ab)$$

$\forall a \forall b\ N(a) \to (N(b) \to ((A(a) \wedge B(b)) \to C(ab))) \equiv \forall a \forall b\ (N(a) \wedge N(b) \wedge (A(a) \wedge B(b))) \to C(ab)$
Negation:

$$\exists a, b N(a), N(b), A(a) \wedge B(b) \wedge \neg C(ab)$$

b) (10 pt) Explain as in class, what is given to you (logical premises), and the goal (conclusion).

Solution.
Given: $N(a), N(b)$.
Asked: $(A(a) \wedge B(b)) \rightarrow C(ab)$.

c) (10 pt) Outline a proof strategy (e.g., direct, contrapositive, contradiction) that you can use to prove the theorem. In your outline, you should explain what the logical premises are and the conclusion. Use your proof strategy to prove the statement.

Solution.
Direct proof:
Given: $N(a), N(b), (A(a) \wedge B(b))$.
Asked: $C(ab)$.
$A(a) \wedge B(b))$ means that a and b could be written as $a = 3k_1$ and $b = 2k_2$, where k_1, k_2 are both integers. Thus, $ab = 3k_1 * 2k_2 = 6(k_1k_2)$, where $k_1 * k_2$ is integer, thus ab is a multiplier of 6, thus $C(ab)$ is True. QED.
Proof by contraposition:
Given: $N(a), N(b), \neg C(Ab)$.
Asked: $A(a) \wedge B(b)$.
This way of proof is not applicable here. The statement itself could be proven by contraposition, but this is become similar to direct proof.
Proof by contradiction:
Given: $N(a), N(b), (A(a) \wedge B(b)), \neg C(ab)$.
Asked: False.
Applying the same logic as in direct proof we can get a correct implication $(A(a) \wedge B(b)) \rightarrow C(ab)$. Since $A(a) \wedge B(b)$ is given by applying *Modus Ponens* we have $C(ab)$ is true. Combining it with $\neg C(ab)$, which is given, we have $(ab)) \wedge \neg C(ab)$, which evaluates to False. QED.