Assignment 9

Problem 1 (6.1)

For a finite abelian group, one can completely specify the group by writing down the group operation table. For instance, Example 2.6 presented an addition table for $\mathbb{Z}_6$.

(a) Write down group operation tables for the following finite abelian groups: $\mathbb{Z}_5$, $\mathbb{Z}_5^*$, and $\mathbb{Z}_3 \times \mathbb{Z}_4^*$

- $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

- $\mathbb{Z}_5^* = \{1, 2, 3, 4\}$, multiplication for $\mathbb{Z}_n^*$

$\cdot$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

- $\mathbb{Z}_3 \times \mathbb{Z}_4^* = \{(a, b) : a \in \mathbb{Z}_3, b \in \mathbb{Z}_4^*\}$ $\quad \mathbb{Z}_3 = \{0, 1, 2\}$ $\mathbb{Z}_4^* = \{1, 3\}$

+ $\cdot$	(0,1)	(0,3)	(1,1)	(1,3)	(2,1)	(2,3)
(0,1)	(0,1)	(0,3)	(1,1)	(1,3)	(2,1)	(2,3)
(0,3)	(0,3)	(0,1)	(1,3)	(1,1)	(2,3)	(2,1)
(1,1)	(1,1)	(1,3)	(2,1)	(2,3)	(0,1)	(0,3)
(1,3)	(1,3)	(1,1)	(2,3)	(2,1)	(0,3)	(0,1)
(2,1)	(2,1)	(2,3)	(0,1)	(0,3)	(1,1)	(1,3)
(2,3	(2,1)	(2,1)	(0,3)	(0,1)	(1,3)	(1,1)

(b) Show that the group operation table for every finite abelian group is a Latin square; that is, each element of the group appears exactly once in each row and column.

finite abelian group has its order to be the number of elements in the underlying set group. which means in group operation table each row and column has exactly one element follow by order (it may rotate). So consequently adding each element from row and column would have a order of element too.

c) Below is an addition table for an abelian group that consists of the elements $\{a, b, c, d\}$; however, some entries are missing. Fill in the missing entries.

$y \downarrow$, $\rightarrow x$

+	a	b	c	d
a	a			
b	b	a		
c			a	
d				

(Let's say $x + y$ axis order)

From the table we know $a+a=a$, $a+b=b$, $b+b=a$, $c+c=a$

So we know $b+a=b$, we know $a+c \neq a$, $b+c \neq a$, $d+a \neq a$, $d+b \neq a$

Since a already appear in row 1, 2, 3

so $d+d=a$

row	+	a	b	c	d
1	a	a	b	c	d
2	b	b	a	d	c
3	c	c	d	a	b
	d	d	c	b	a

Similarly $b+c \neq a$ and $b+c \neq b$ and we know finite group has only one element in each row and column so we can fill the blank

Problem 2 (6.12)

Show that if G_1 and G_2 are abelian groups, and m is an integer, then $m(G_1 \times G_2) = mG_1 \times mG_2$

We have G_1 and G_2 are abelian groups and m is an integer $m \in \mathbb{Z}$

G_1 and G_2 are multiplicative abelian group

Let's say $a_1, b_1 \in G_1$ and $a_2, b_2 \in G_2$. In $G_1 \times G_2$

$(a_1, b_1) \times (a_2 \times b_2) = (a_1 a_2, b_1 b_2)$, $(a_1 a_1, b_1 b_1)$, $(a_2 a_1, b_2 b_1)$

$G_1 \times G_2$ contains 1_{G_1} and 1_{G_2} so $G_1 \times G_2$ contains $m1_{G_1}$ and $m1_{G_2}$ in mG_1. It contains $m1_{G_1}$ and mG_2 contains $m1_{G_2}$ so $mG_1 \times mG_2$ will contains $m1_{G_1}$ and $m1_{G_2}$ by closed properties $m(G_1 \times G_2) = mG_1 \times mG_2$