Exercise 5.16

use stirling's approximation (previous exercise) to show that

$$\binom{2m}{m} = \Theta(4^m/\sqrt{m})$$

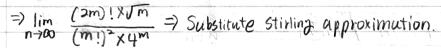
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In the book, stirling approximation from 5.16 is that n!~ (n/e) \(\sqrt{2\pi n} \)

Big Theta's definition is that f is bounded both above and below by g asymptotically f(n) = O(g(n)) so $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 10$ n stank, which constant is bigger than O

So From the question, $\lim_{n\to\infty} {2m \choose m} / {4^n / \sqrt{m}} = \lim_{n\to\infty} \frac{{2m \choose m} \times \sqrt{m}}{4^m}$

Since $\begin{pmatrix} \chi \\ \gamma \end{pmatrix} = \frac{\chi!}{\chi! \times (\ell-\gamma)!}$



=>
$$\lim_{n\to\infty} (2m/e)^{2m} \sqrt{4\pi m} \times \sqrt{m}$$
 => $\lim_{n\to\infty} \frac{2^{2m}}{m} \sqrt{4\pi m^2} \times e^{2m}$ => $\lim_{n\to\infty} \frac{2m}{(m/e)^m} \sqrt{2\pi m}^2 \times 4^m$ => $\lim_{n\to\infty} \frac{2m}{(2\pi m)^2} \times 4^m$ => $\lim_{n\to\infty} \frac{2m}{(2\pi m)^2$

$$= \lim_{n \to \infty} \frac{\sqrt{n}}{n} = \lim_{n \to \infty} \sqrt{\frac{1}{n}}$$

Since lim fin) hus a constant value II, (2m) = 0 (4m/vm)

Exercise 5-20 Using the prime number theorem, show that UCY NX.

From chebyshev's theta function he have

Use theorem 5.5 yal = Zlog P & log X Z 1 = Tal) log x

also from prime number theorem, TO()~2/1002

V(X) = Z 100 & T(X) 100 X = (X/100x) · 100x = X

50 Y(2) -2.