Problem 2 (6.37)

Which of the following pairs of groups are isomorphic? Why or why not?

(a) $Z_2 \times Z_2$ and Z_4

(b) Z_{12}^* and Z_8^*

(c) Z_5^* and Z_4

(d) $Z_2 \times Z$ and Z

(e) Q and Z

(f) $Z \times Z$ and Z

Isomorphic is one-to-one mapping with the same cardinality.

(a) is not isomorphic since $Z_2 \times Z_2 = Z_2$, and Z_4 does not have the same cardinality

(b) is isomorphic since $Z_{12}^* = \{1, 5, 7, 11\}$, $Z_8^* = \{1, 3, 5, 7\}$ they have same cardinality (11 map to 3

(c) is isomorphic since $Z_5^* = \{1, 2, 3, 4\}$, $Z_4 = \{0, 1, 2, 3\}$ corresponding one-to-one with the same cardinality

(d) is isomorphic since $Z_2 \times Z = Z$ · Z and Z are isomorphic

(e) is not isomorphic since Q and Z does not have the same cardinality

(f) $Z \times Z = Z$ so it is isomorphic

Assignment 11

$\langle 33 b$
$\langle 32 b$
11

problem 1 (6.38)
Find $\alpha_1, \alpha_2 \in \mathbb{Z}_{15}^*$ such that $\mathbb{Z}_{15}^* = \langle \alpha_1, \alpha_2 \rangle$

$\mathbb{Z}_{15}^* = \{1, 2, 4, 7, 8, 11, 13, 14\}$, $|\mathbb{Z}_{15}^*| = 8$

from these elements of $\mathbb{Z}_{15}^*$ we can find subgroups generated by elements

$1^1 = 1$

$2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16 = 1 \pmod{15}$, $2^5 = 32 = 2 \pmod{15}$

$4^1 = 4$, $4^2 = 1 \pmod{15}$ $4^3 = 64 = 4 \pmod{15}$

$7^1 = 7$, $7^2 = 49 = 4 \pmod{15}$ $7^3 = 343 = 13 \pmod{15}$, $7^4 = 2401 = 1 \pmod{15}$

$8^1 = 8$, $8^2 = 4 \pmod{15}$, $8^3 = 2 \pmod{15}$, $8^4 = 1 \pmod{15}$

$11^1 = 11 \pmod{15}$ $11^2 = 1 \pmod{15}$

$13^1 = 13$, $13^2 = 4 \pmod{15}$, $13^3 = 7 \pmod{5}$, $13^4 = 1 \pmod{15}$

$14^1 = 14$, $14^2 = 1 \bmod 15$

so generator of $\mathbb{Z}_{15}^* = \{2, 7, 11, 13\}$
select 2, 7 therefore $\langle 2, 7 \rangle$ This means the cyclic group $\mathbb{Z}_{15}^*$ can be generated with power of 2 and 7 so

$\langle 2, 7 \rangle$ and $\langle 7, 11 \rangle$, $\langle 11, 13 \rangle$
etc.