CS 131 – Fall 2019, Assignment 9

Problems must be submitted by Friday November 22, 2019 5:00pm, on Gradescope.

Problem 1. (10 points) Calculate a value of Euler function φ as a function of n:

a) (5 points) $\varphi(7^n)$

b) (5 points) $\varphi(5^n 2^{n+3})$

Problem 2. (10 points) Prove the following statements:

a) (5 points) Prove that for positive integer n, $n^3 + 2n$ is divisible by 3.

b) (5 points) Prove that for positive integer n, $n^3 - n$ is divisible by 6.

Problem 3. (20 points + 10 additional points) Solve the following equations in non-negative integers.

a) (5 points) $x^2 - y^2 = 221$

b) (5 points) $a + b = ab$

c) (10 points) $gcd(a, b)lcm(a, b) = b + 9$

d) (10 additional points) $x^4 + 2x^3 - y^2(1 + 2x) + x^2(1 - y^2) = 2299$

Problem 4. (35 points) How many positive integers less than 1000

a) (5 points) are divisible by 7 but not by 11?

b) (5 points) are divisible by both 7 and 11?

c) (5 points) are divisible by either 7 or 11

d) (5 points) are divisible by exactly one of 7 and 11?

e) (5 points) are divisible by neither 7 nor 11?

f) (5 points) have distinct digits?

g) (5 points) have distinct digits and are even?

Problem 5. (25 points)

a) (5 points) Find multiplicative inverse (or prove that it doesn't exist) of 3 *mod* 11 using remainders table.

b) (5 points) Find multiplicative inverse (or prove that it doesn't exist) of 6 *mod* 11 using remainders table.

c) (5 points) Find multiplicative inverse (or prove that it doesn't exist) of 2 *mod* 12 using remainders table.

d) (10 points) Find a multiplicative inverse of 247 *mod* 154 using Bezout's coefficients.