Truth Table & Minterm Expansion

Computer Science 111
Boston University

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Constructive Proof!

① Specify a **truth table** defining ***any*** function yo

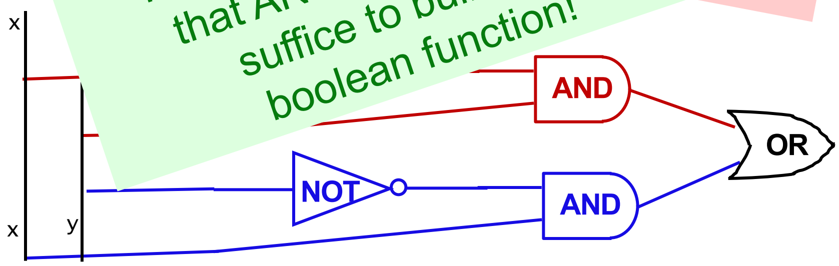
input		
x	y	fn
0	0	
0	1	1
1	0	1
1	1	

② For each input row whose output needs to be 1, build an **AND** circuit t outputs 1 *c input!*

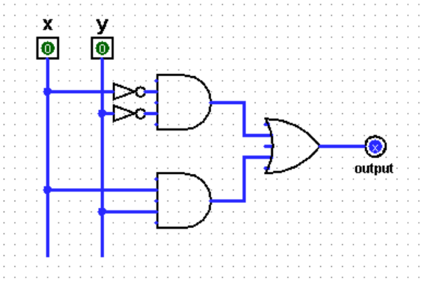
But... **ALL** functions are just boolean functions: because everything is in binary!

This is a co that AND, OR, suffice to build a boolean function!

?



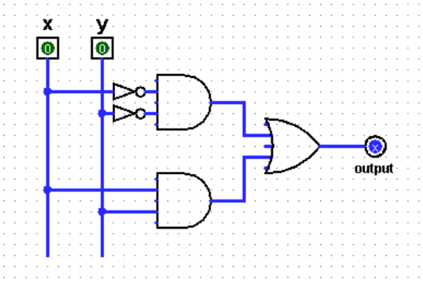
Revisiting the Circuit from Last Class ...



inputs		output
x	y	
0	0	1
0	1	0
1	0	0
1	**1**	**1**

- The top AND gate implements which row of the truth table?
  the top row
- The bottom AND gate implements which row?
  the bottom row

Boolean Expressions for Truth Tables



inputs		output
x	y	
0	0	1
0	1	0
1	0	0
1	1	1

- This truth table/circuit can be summarized by the expression:

$$\bar{x}\bar{y} + xy$$

- This expression is the *minterm expansion* of this truth table.
  - one *minterm* for each row that has an output of 1
  - combined using OR

Building a Minterm Expansion for a Boolean Function

1. If you don't have it, create the truth table.

2. Delete the rows with an output of 0.

3. Create a minterm for each remaining row (the ones with an output of 1):
   - AND the input variables together
   - if a variable has a 0 in that row, negate it
   - example: minterm for the 2nd row
     $\overline{x}\overline{y}z$

4. OR the minterms together.

inputs			output
x	y	z	
~~0~~	~~0~~	~~0~~	~~0~~
0	0	1	1
0	1	0	1
~~0~~	~~1~~	~~1~~	~~0~~
1	0	0	1
~~1~~	~~0~~	~~1~~	~~0~~
~~1~~	~~1~~	~~0~~	~~0~~
1	1	1	1

$$\overline{x}\overline{y}z + \overline{x}y\overline{z} + x\overline{y}\overline{z} + xyz$$

Minterm Expansion → Circuit

minterm expansion =

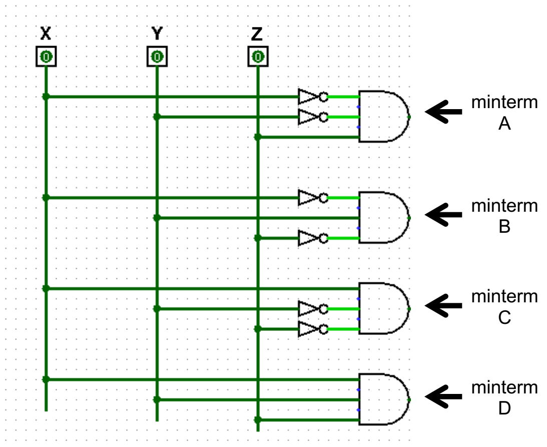
$$\overline{x}\overline{y}z + \overline{x}y\overline{z} + x\overline{y}\overline{z} + xyz$$

minterm A

minterm B

minterm C

minterm D

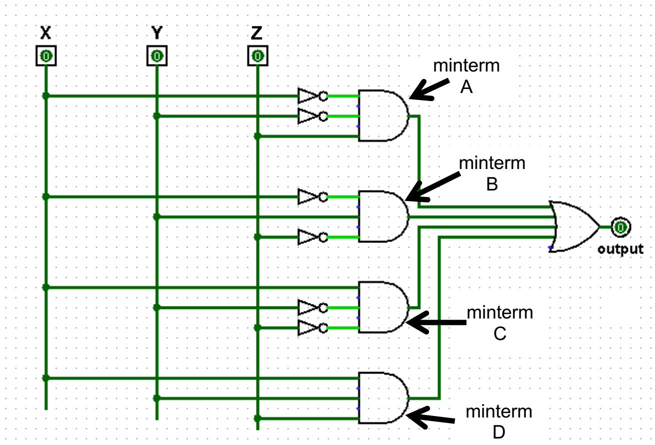


Minterm Expansion → Circuit

minterm expansion =

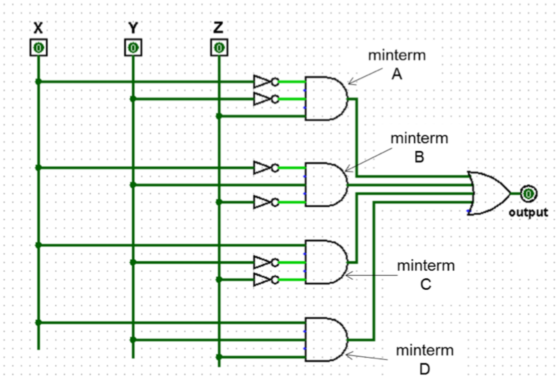
$$\overline{x}\,\overline{y}z + \overline{x}y\overline{z} + x\overline{y}\,\overline{z} + xyz$$

minterm A   minterm B   minterm C   minterm D



Odd Parity

inputs			output
x	y	z	
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



- This truth table and circuit are for the *odd parity* function.
  - outputs 1 when the number of 1 inputs is odd

What is the minterm expansion of this truth table?

inputs			output
x	y	z	
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

A. $yz + xz + xy$

B. $xyz + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}z$

C. $\bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$

D. $\bar{x}\bar{y}\bar{z} + \bar{x}yz + x\bar{y}z + \bar{x}\bar{y}z$

E. none of the above

What is the minterm expansion of this truth table?

inputs			output
x	y	z	
0	0	0	1
~~0~~	~~0~~	~~1~~	~~0~~
~~0~~	~~1~~	~~0~~	~~0~~
0	1	1	1
~~1~~	~~0~~	~~0~~	~~0~~
1	0	1	1
1	1	0	1
~~1~~	~~1~~	~~1~~	~~0~~

$\bar{x}\bar{y}\bar{z} + \bar{x}yz + x\bar{y}z + xy\bar{z}$

A. $yz + xz + xy$

B. $xyz + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}z$

C. $\bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$

D. $\bar{x}\bar{y}\bar{z} + \bar{x}yz + x\bar{y}z + \bar{x}\bar{y}z$

E. none of the above

Building a Minterm Expansion for a Boolean Function

ex: `greater_than_4(x, y, z)`

→ 1 if the 3-digit binary number xyz > 4

→ 0 otherwise

for example:

- `greater_than_4(1, 1, 0)` → 1 (True)
  because $110_2 = 6_{10}$, and 6 > 4
- `greater_than_4(0, 1, 1)` → 0 (False)
  because $011_2 = 3_{10}$, and 3 is *not* > 4

Building a Minterm Expansion for a Boolean Function

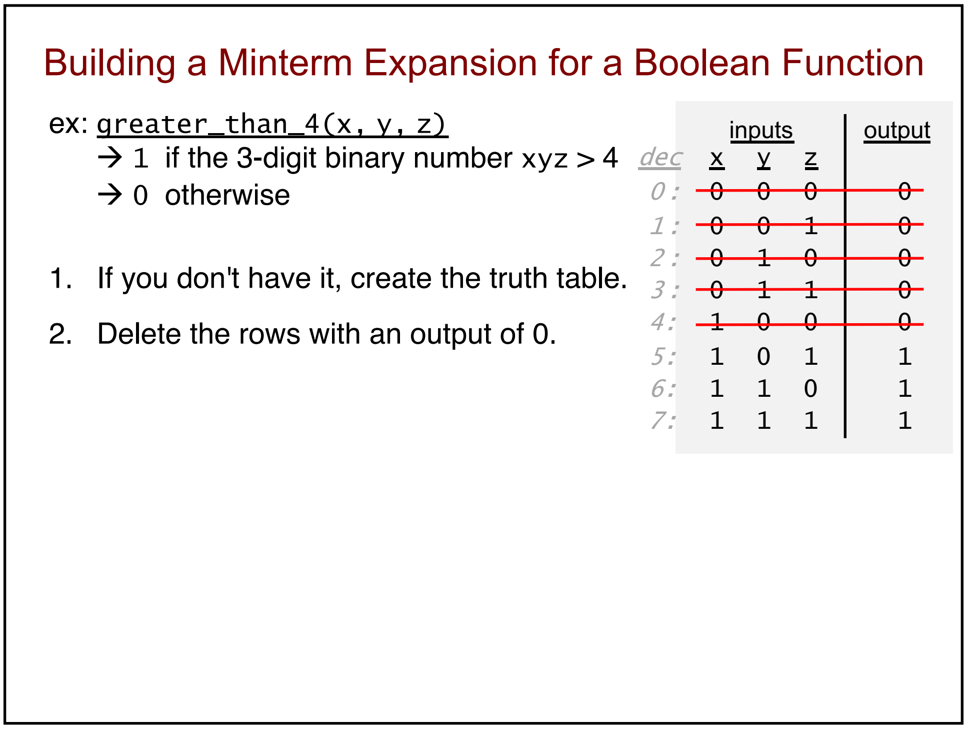
ex: `greater_than_4(x, y, z)`

→ 1 if the 3-digit binary number xyz > 4

→ 0 otherwise

1. If you don't have it, create the truth table.

	inputs			output
dec	x	y	z	
0 :	0	0	0	0
1 :	0	0	1	0
2 :	0	1	0	0
3 :	0	1	1	0
4:	1	0	0	0
5:	1	0	1	1
6:	1	1	0	1
7:	1	1	1	1



Building a Minterm Expansion for a Boolean Function

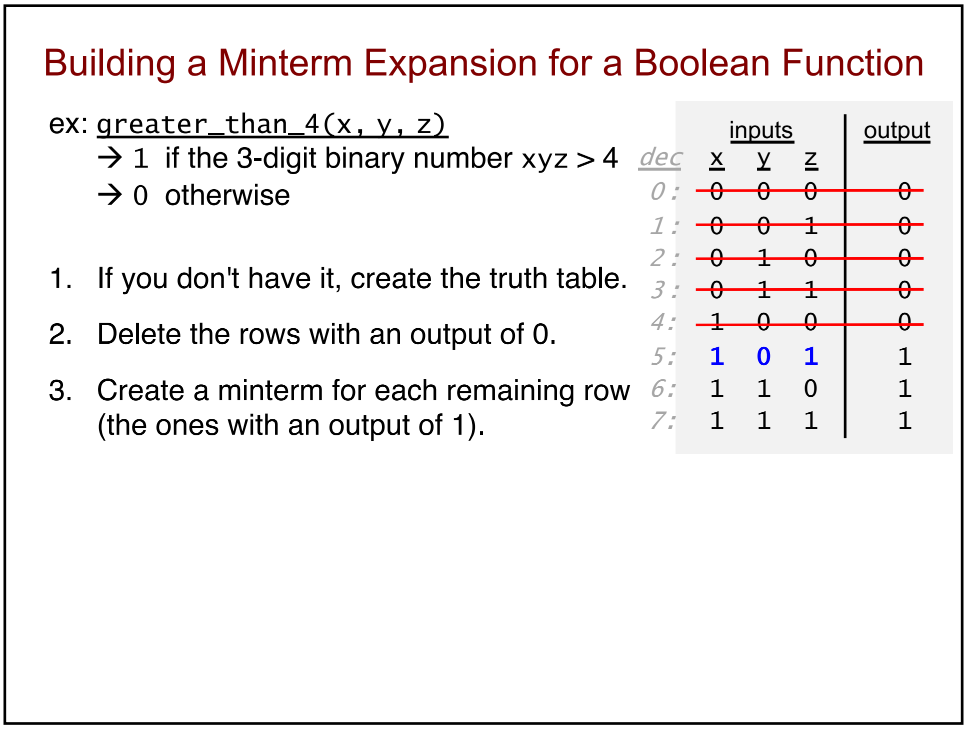
ex: `greater_than_4(x, y, z)`

→ 1 if the 3-digit binary number xyz > 4

→ 0 otherwise

1. If you don't have it, create the truth table.
2. Delete the rows with an output of 0.

	inputs			output
dec	x	y	z	
0:	~~0~~	~~0~~	~~0~~	~~0~~
1:	~~0~~	~~0~~	~~1~~	~~0~~
2:	~~0~~	~~1~~	~~0~~	~~0~~
3:	~~0~~	~~1~~	~~1~~	~~0~~
4:	~~1~~	~~0~~	~~0~~	~~0~~
5:	1	0	1	1
6:	1	1	0	1
7:	1	1	1	1



Building a Minterm Expansion for a Boolean Function

ex: `greater_than_4(x, y, z)`

→ 1 if the 3-digit binary number xyz > 4

→ 0 otherwise

1. If you don't have it, create the truth table.
2. Delete the rows with an output of 0.
3. Create a minterm for each remaining row (the ones with an output of 1).

	inputs			output
dec	x	y	z	
0:	~~0~~	~~0~~	~~0~~	~~0~~
1:	~~0~~	~~0~~	~~1~~	~~0~~
2:	~~0~~	~~1~~	~~0~~	~~0~~
3:	~~0~~	~~1~~	~~1~~	~~0~~
4:	~~1~~	~~0~~	~~0~~	~~0~~
5:	**1**	**0**	**1**	1
6:	1	1	0	1
7:	1	1	1	1

What Is the Minterm for the **BLUE** Row?

	inputs			output
dec	x	y	z	
0:	~~0~~	~~0~~	~~0~~	~~0~~
1:	~~0~~	~~0~~	~~1~~	~~0~~
2:	~~0~~	~~1~~	~~0~~	~~0~~
3:	~~0~~	~~1~~	~~1~~	~~0~~
4:	~~1~~	~~0~~	~~0~~	~~0~~
5:	1	0	1	1
6:	1	1	0	1
7:	1	1	1	1

A. $x + \bar{y} + z$

B. $\bar{x} + y + \bar{z}$

C. $\bar{x}y\bar{z}$

D. $x\bar{y}z$

E. none of the above

What Is the Minterm for the **BLUE** Row?

	inputs			output
dec	x	y	z	
0:	~~0~~	~~0~~	~~0~~	~~0~~
1:	~~0~~	~~0~~	~~1~~	~~0~~
2:	~~0~~	~~1~~	~~0~~	~~0~~
3:	~~0~~	~~1~~	~~1~~	~~0~~
4:	~~1~~	~~0~~	~~0~~	~~0~~
5:	1	0	1	1
6:	1	1	0	1
7:	1	1	1	1

A. $x + \bar{y} + z$

B. $\bar{x} + y + \bar{z}$

C. $\bar{x}y\bar{z}$

D. $\mathbf{x\bar{y}z}$

E. none of the above

Building a Minterm Expansion for a Boolean Function

ex: `greater_than_4(x, y, z)`

→ 1 if the 3-digit binary number xyz > 4

→ 0 otherwise

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	inputs			output
dec	x	y	z	
0:	~~0~~	~~0~~	~~0~~	~~0~~
1:	~~0~~	~~0~~	~~1~~	~~0~~
2:	~~0~~	~~1~~	~~0~~	~~0~~
3:	~~0~~	~~1~~	~~1~~	~~0~~
4:	~~1~~	~~0~~	~~0~~	~~0~~
5:	1	0	1	1
6:	1	1	0	1
7:	1	1	1	1

$x\bar{y}z$ $\quad$ $xy\bar{z}$ $\quad$ xyz

Building a Minterm Expansion for a Boolean Function

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   - AND the input variables together
   - if a variable has a 0 in that row, negate it
4. OR the minterms together.

	inputs			output
dec	x	y	z	
0:	~~0~~	~~0~~	~~0~~	~~0~~
1:	~~0~~	~~0~~	~~1~~	~~0~~
2:	~~0~~	~~1~~	~~0~~	~~0~~
3:	~~0~~	~~1~~	~~1~~	~~0~~
4:	~~1~~	~~0~~	~~0~~	~~0~~
5:	1	0	1	1
6:	1	1	0	1
7:	1	1	1	1

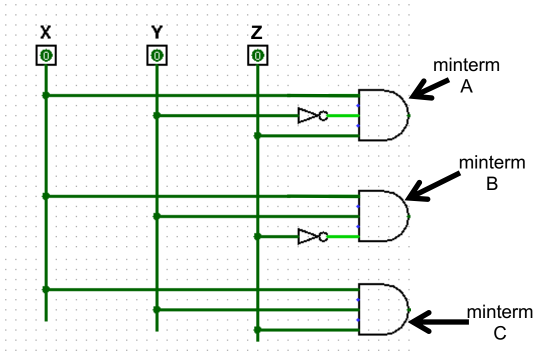
$$x\bar{y}z + xy\bar{z} + xyz$$

Minterm Expansion → Circuit

minterm expansion =

$x\bar{y}z + xy\bar{z} + xyz$

minterm A minterm B minterm C



minterm A

minterm B

minterm C

Minterm Expansion → Circuit

minterm expansion =

$x\bar{y}z + xy\bar{z} + xyz$

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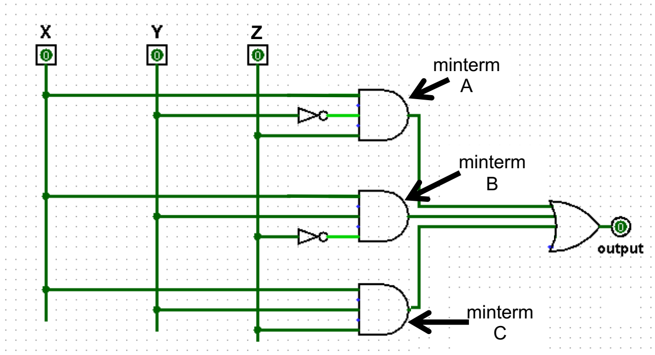
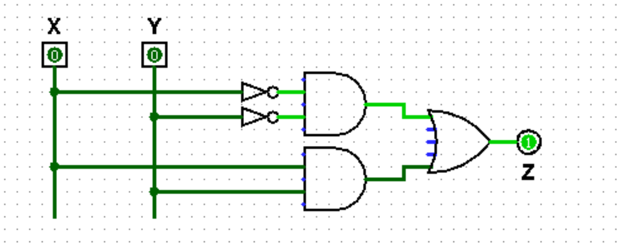


minterm A

minterm B

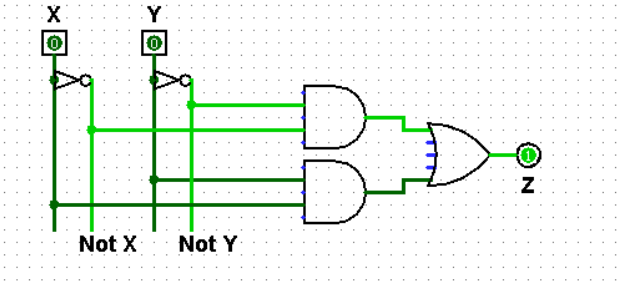
minterm C

Adding "Rails" for the NOT of Each Input



Here's a circuit that we looked at earlier.

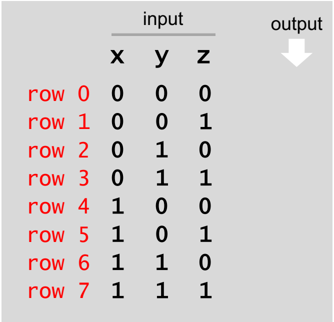
It tests whether x == y.



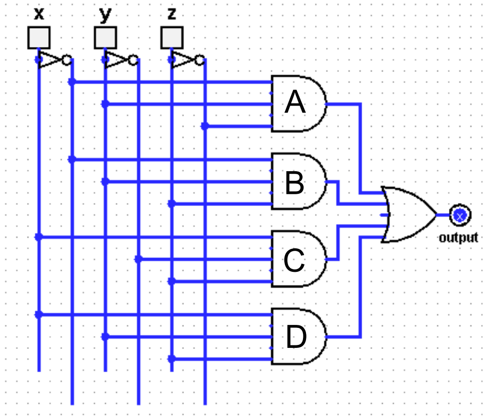
Here's an alternative version that adds "rails" for Not X and Not Y.

In some cases (but not this one!), doing so can reduce the number of NOT gates.

Which AND gate corresponds to row 3 of the table?

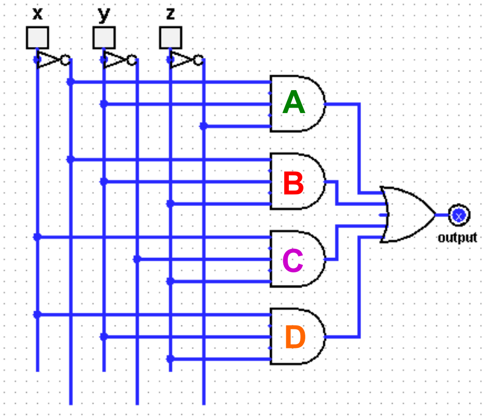


	input			output
	x	y	z	
row 0	0	0	0	
row 1	0	0	1	
row 2	0	1	0	
row 3	0	1	1	
row 4	1	0	0	
row 5	1	0	1	
row 6	1	1	0	
row 7	1	1	1	



Which AND gate corresponds to row 3 of the table?

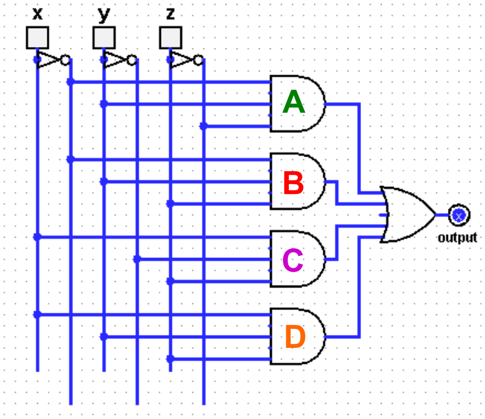
	input			output
	x	y	z	
row 0	0	0	0	0
row 1	0	0	1	0
row 2	0	1	0	1
row 3	0	1	1	1
row 4	1	0	0	0
row 5	1	0	1	1
row 6	1	1	0	0
row 7	1	1	1	1



- Complete the rest of the truth table.
- What is its minterm expansion as a formula/expression?
- If the inputs represent a three-bit integer, what property of integers does the circuit compute?

Which AND gate corresponds to row 3 of the table?

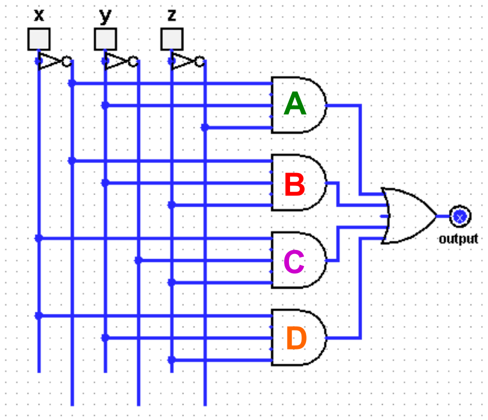
	input			output
	x	y	z	
row 0	0	0	0	0
row 1	0	0	1	0
row 2	0	1	0	1
row 3	0	1	1	1
row 4	1	0	0	0
row 5	1	0	1	1
row 6	1	1	0	0
row 7	1	1	1	1



- Complete the rest of the truth table.
- What is its minterm expansion as a formula/expression?
  $\bar{x}y\bar{z} + \bar{x}yz + x\bar{y}z + xyz$
- If the inputs represent a three-bit integer, what property of integers does the circuit compute?

Which AND gate corresponds to row 3 of the table?

	input			output
	x	y	z	
row 0	0	0	0	0
row 1	0	0	1	0
row 2	0	1	0	1
row 3	0	1	1	1
row 4	1	0	0	0
row 5	1	0	1	1
row 6	1	1	0	0
row 7	1	1	1	1

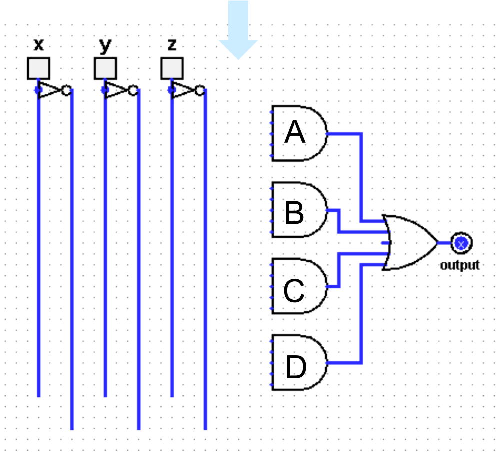


- Complete the rest of the truth table.
- What is its minterm expansion as a formula/expression?
  $\overline{x}y\overline{z} + \overline{x}yz + x\overline{y}z + xyz$
- If the inputs represent a three-bit integer, what property of integers does the circuit compute? whether the integer is prime

DIY!

Add the wires needed to build a circuit for the truth table at left…

input			output
x	y	z	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

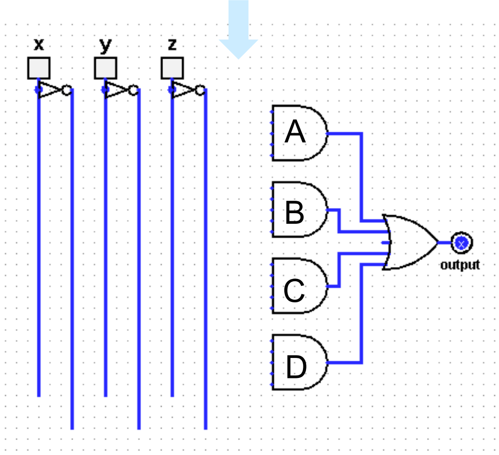


First make a minterm expansion!
Then complete the circuit!

DIY!

Add the wires needed to build a circuit for the truth table at left…

input			output
x	y	z	
~~0~~	~~0~~	~~0~~	~~0~~
~~0~~	~~0~~	~~1~~	~~0~~
~~0~~	~~1~~	~~0~~	~~0~~
0	1	1	1
~~1~~	~~0~~	~~0~~	~~0~~
1	0	1	1
1	1	0	1
1	1	1	1

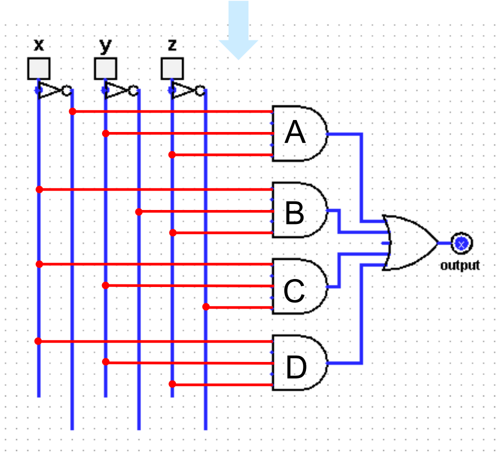


$\bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$

DIY!

Add the wires needed to build a circuit for the truth table at left…

input			output
x	y	z	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

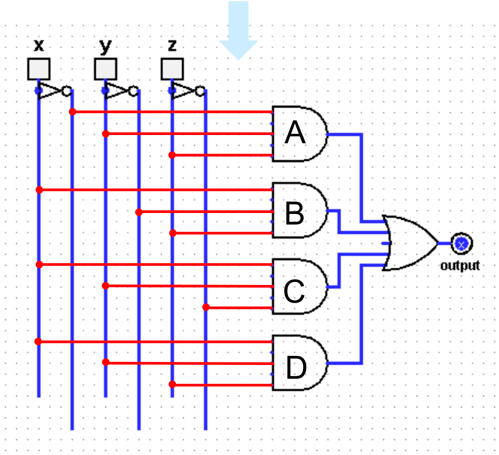


$\bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$

DIY!

Add the wires needed to build a circuit
for the truth table at left…

input			output
x	y	z	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



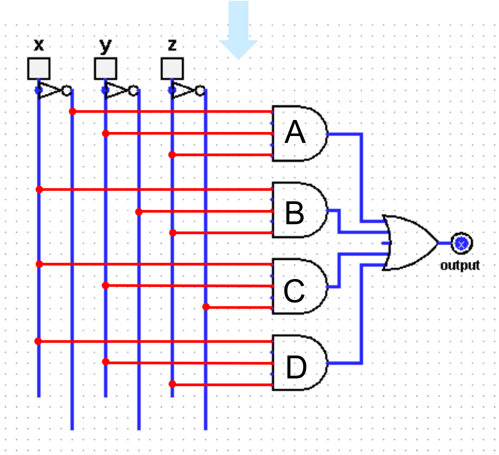
$\bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$

What does this circuit compute?

DIY!

Add the wires needed to build a circuit
for the truth table at left…

input			output
x	y	z	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

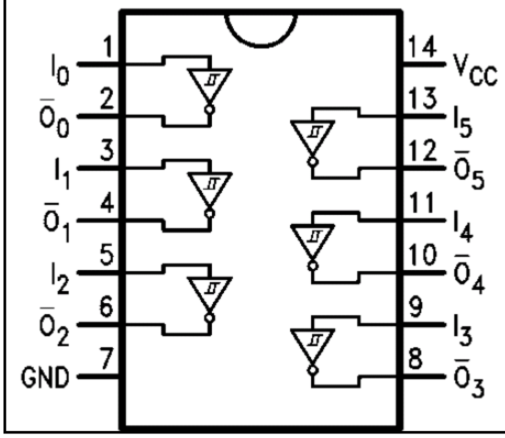


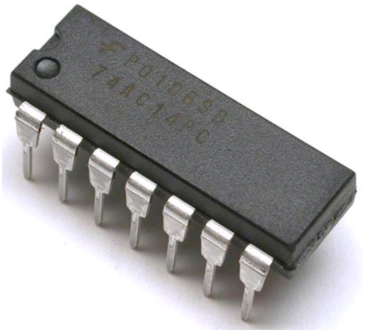
$\bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$

the majority function – outputs 1 if a majority of the inputs is 1

Physical Logic Gates!

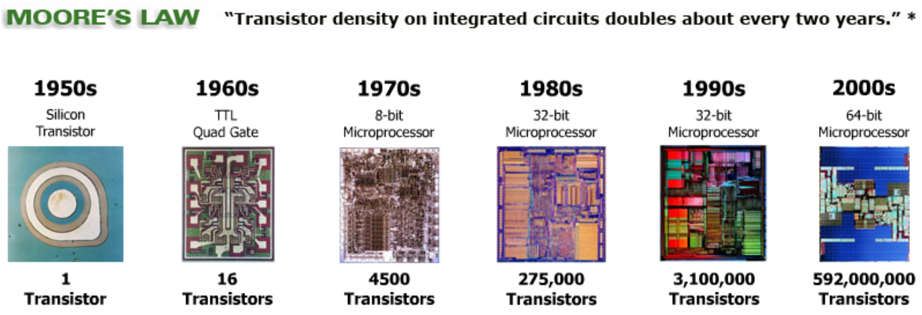
A transistor





6 NOT gates in one chip!

Advances in the Number of Transistor in IC



MOORE'S LAW **"Transistor density on integrated circuits doubles about every two years." ***

1950s	**1960s**	**1970s**	**1980s**	**1990s**	**2000s**
Silicon Transistor	TTL Quad Gate	8-bit Microprocessor	32-bit Microprocessor	32-bit Microprocessor	64-bit Microprocessor
1 Transistor	**16 Transistors**	**4500 Transistors**	**275,000 Transistors**	**3,100,000 Transistors**	**592,000,000 Transistors**