CS 131 - Spring, 2019, Assignment 2

problem 1

a) Every human has a stomach

Domain : Every human.

$P(x)$: x has a stomach

Answer $\forall x\, P(x)$

b) Everyone is a friend of someone

Domain : Everyone

$P(x,y)$: x is a friend of y

$\forall x \exists y\, P(x,y)$

c) Nobody likes everybody

Domain: Everybody

$P(x,y) = x$ likes y

$\forall x \exists y\, \neg P(x,y)$

T
F
T
T

$\neg(P \to Q) \equiv P \land \neg Q$

problem 2

a) $\forall x \exists y\, P(x,y)$

$\neg \forall x \exists y\, P(x,y) \equiv \exists x \neg \exists y\, P(x,y) \equiv \exists x \forall y\, \neg P(x,y)$

b) $\exists x (F(x) \to \forall y (\neg P(y) \land \forall z\, Q(z)))$

$\exists x \neg H(x) \equiv \neg \forall x\, H(x)$

$(p \lor \neg q) \equiv \neg p \to \neg q$

$p \lor \neg q \equiv q \to p$

$\neg(p \to Q) \equiv p \land \neg Q$

$\neg \exists x (F(x) \to \forall y (\neg P(y) \land \forall z\, Q(z)))$

$\equiv \forall x (F(x) \land \neg \forall y (\neg P(y) \land \forall z\, Q(z)))$

$\equiv \forall x (F(x) \land \exists y \neg (\neg P(y) \land \forall z\, Q(z)))$

$\equiv \forall x (F(x) \land \exists y (P(y) \lor \neg \forall z\, Q(z)))$ $(p \lor \neg q) \equiv \neg p \to \neg q$

$\equiv \forall x (F(x) \land \exists y (\neg P(y) \to \neg \forall z\, Q(z)))$

$\boxed{\equiv \forall x (F(x) \land \exists y (\forall z\, Q(z) \to P(y)))}$ $\neg p \to \neg q \equiv q \to p$

problem 3

a) x = a real number, $\forall x\, 0 \le \frac{1}{x}$ is always true.

This logical expression is false, because every negative number from a real number x is always false.

b) x = real numbers, $\forall x\; x^2+4x+5 > 0$

$x^2+4x+5 = (x+2)^2+1$ since it is a vertex form and the x^2 is positive polynomial, we know the lowest point of the graph is 1 which is bigger than 0, so the sentence is true.

$A \to C \equiv \neg A \vee C$

$\neg p \wedge Q \equiv P \vee \neg Q \equiv \neg(P \to q)$

problem 4

$A \wedge B = P$ $\quad \boxed{P \to C}$

$\varphi := \neg(A \to C) \wedge (((A \wedge B) \to C)) \wedge (A \to B))$

- $\neg(A \to C) = A \wedge \neg C$
- $((A \wedge B) \to C) \equiv \neg(A \wedge B) \vee C \equiv \neg A \vee \neg B \vee C$
- $A \to B \equiv \neg A \vee B$

$(A \wedge \neg C) \wedge ((\neg A \vee \neg B \vee C) \wedge (\neg A \vee B))$

$\equiv (A \wedge \neg C) \wedge (\neg A \vee ((\neg B \vee C) \wedge B)) \to$ Distributive laws.

$\equiv (A \wedge \neg C) \wedge (\neg A \vee \underbrace{(\neg B \wedge B)}_{\text{false}} \vee (C \wedge B))$

$\equiv (A \wedge \neg C) \wedge (\neg A \vee (B \wedge C))$

$\equiv ((A \wedge \neg C) \wedge (\neg A \vee B)) \wedge (\neg A \vee C)$

$(A \wedge \neg C) = X$

$\equiv ((X \wedge \neg A) \vee (X \wedge C)) \wedge (\neg A \vee B)$

$\equiv (\underbrace{A \wedge \neg A \wedge \neg C}_{\text{False}}) \vee (A \wedge \underbrace{\neg C \wedge C}_{\text{false}}) \wedge (\neg A \vee B)$

$\equiv$ (false) $\vee$ (false) $\wedge$ $(\neg A \vee B)$

$\equiv$ false $\wedge$ $(\neg A \vee B)$

$\equiv$ false.

problem 5

a) $Cup(x) = x$ caps

i) There are at most two cups.

$Cup(x) =$ "x is a cup"

at most two cups $\to$ ① 0 cup ② 1 cup ③ 2 cups No 3 cups

No more than 2 cups.

$\forall x \forall y \forall z ((Cup(x) \wedge Cup(y) \wedge Cup(z)) \to (x = y \vee y = z \vee x = z))$

ii) There are exactly two cups.
So ~~there are~~ exist two cups and no more than three cups.

$\exists x \exists y (Cup(x) \wedge Cup(y)) \wedge (x \neq y)$: there are two cups, but could be more than two

$\wedge$

No more than three cups.

$\exists x \exists y (Cup(x) \wedge Cup(y) \wedge (x \neq y)) \wedge \forall z (Cup(z) \rightarrow (z = y \vee x = z))$

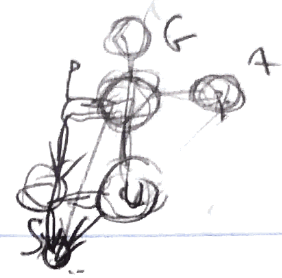
b)
i) $G(a,b)$ which is true if and only if a is a grandparent of b
$P(x,y) = x$ is a parent of y
$G(a,b) \leftrightarrow (P(a,c) \wedge P(c,b))$

ii) $S(a,b)$ which is true if and only if a is a sibling of b
$S(a,b) \leftrightarrow (P(c,a) \wedge P(c,b) \wedge a \neq b)$

iii) $A(a,b)$ which is true if and only if a is an aunt or uncle of b.
$A(a,b) \leftrightarrow (P(c,b) \wedge S(c,a))$

iv) $C(a,b)$ which is true if and only if a is a first cousin of b
$C(a,b) \leftrightarrow (A(c,b) \wedge P(c,a))$

v) $R(a,b)$ which is true if and only if a is a first cousin one removed of b
$R(a,b) \leftrightarrow \big((C(c,a) \wedge P(c,b)) \vee (P(D,b) \wedge C(a,D)) \big)$



vi) a = man, b = woman

$I(a,b)$ which is true if and only if the marriage between a and b is prohibited.

(prohibited)
No marrige: Mother, grandmother, daughter, granddaughter, sister, brother's daughter, sister's daughter, father's sister, mother's sister
(nephew) (Aunt)

$$\boxed{I(a,b) \longleftrightarrow \left(G(a,b) \vee P(a,b) \vee A(a,b) \vee S(a,b) \vee \left(S(c,b) \vee P(c,[illegible])\right)\right)}$$

b = You
a = You cannot marry
C = Your man
D = Sibling

$a \to b \to c$

$G(a,b)$ $\exists x\, P(a,x) \wedge P(x,c)$

$G(b,a)$

$I(a,b) \longleftrightarrow (G(a,b) \vee P(a,b) \vee$