

Indep: $Pr[X \cap Y] = Pr[X] \cdot Pr[Y]$

$$\frac{Pr[X \cap Y]}{Pr[X]} = Pr[Y|X]$$

$$\begin{matrix} X & Y & Z \\ & & \downarrow \\ & & \frac{1}{4} \end{matrix}$$

pairwise independence $\frac{1}{4}$ is how they are independent?

Expectation

Distribution

PDF
 $Pr[X=x]$
 \downarrow

Not to continuous bc $Pr[X=x]$ will be zero

PDF of continuous random variables
 $f_X(x)$ and $\delta f(x)$.

PDF

$$Pr[X \leq x] \rightarrow \text{CDF of } X$$

$$E[(X - E[X])^2]$$

$$= \int x^2 - 2xE[X] + E[X]^2 f(x) dx$$

$$\hookrightarrow E[X^2] - 2E[X]^2$$

$$E[X^2] - 2 \cdot E[X]E[X] + E[X]^2$$

$$= 2 \cdot E[X] \cdot E[X]$$

$$= E[X^2] - 2E[X]^2 + E[X]^2$$

$$= E[X^2] - E[X]^2$$

$$E[X \cdot E[X]] = E[X] \cdot E[X]$$

Using expectation rule.

Covariance

Properties

$$\text{Cov}(X+c, Y) = \text{Cov}(X, Y)$$

$$\text{Cov}(aX+bY, Z) = a \cdot \text{Cov}(X, Z) + b \cdot \text{Cov}(Y, Z)$$

$$\text{Cov}(aX, Z)$$

$$\text{Cov}(bY, Z)$$

$$\text{Cov}(W, Z) = \text{Cov}(1+X+XY^2, 1+X) \quad N(0,1)$$

$$= \text{Cov}(X, 1+X)$$

$$= \text{Cov}(X(1+Y^2), X) = \text{Cov}(X, X) + \text{Cov}(X, XY^2)$$

$$= 1?$$

$$\hookrightarrow E[(X - E[X])(XY^2 - E[XY^2])]$$

Kull square

$$E[X \cdot Y^2] = 0$$

$$= E[X^2 Y^2]$$

$$E[X^2] = E[X] \cdot E[Y^2]$$

$$= 1$$

$$\int_0^{\infty} (-x \cdot y^2 + x \cdot y^2) dx = 0$$

$$E[XY] = E[X] \cdot E[Y]$$

$$\begin{aligned}
 &= \iint x \cdot y \cdot \underbrace{f_{xy}(x,y)}_{\text{independent}} dx dy \\
 &= \iint x \cdot y \cdot f_x(x) f_y(y) dx dy \\
 &= \iint x \cdot f_x(x) dx \cdot y \cdot f_y(y) dy \\
 &= \int x \cdot f_x(x) dx \int y \cdot f_y(y) dy \\
 &= \underbrace{E[x^2]}_1 \cdot \underbrace{E[y^2]}_1 \\
 &\hookrightarrow \text{variance} + E[x]^2 \\
 &E[x^2] - E[x]^2 \leq 0
 \end{aligned}$$

Secretary problem.

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The best of the first $i-1$ applicants
is in the first $r-1$ application

