CS131 - Assignment 8

Problem 1.

a) $\forall x \in R, \lfloor x^2 \rfloor = (\lfloor x \rfloor)^2$

For all real number x, It does not work!

for example $x = 2.5$

$x^2 = 6.25$

$\lfloor x^2 \rfloor = 6,$

$(\lfloor x \rfloor)^2 = (2)^2 = 4$

$\therefore \lfloor x^2 \rfloor \neq (\lfloor x \rfloor)^2$

So the statement is False

b) $\forall x \in R, \lfloor 2x \rfloor = 2\lfloor x \rfloor$

for $x = 1.5$

$\lfloor 2x \rfloor = \lfloor 3 \rfloor = 3$

$2\lfloor 1.5 \rfloor = 2 \times 1 = 2$

$3 = \lfloor 2x \rfloor \neq 2\lfloor x \rfloor = 2$

So the statement is false

c) $\forall x \in R, \lceil \lfloor x \rfloor \rceil = \lfloor x \rfloor$

Definition of floor function (x) is that the greatest integer that is less than or equal to x. And the definition of Ceiling function (x) is that the least integer that is greater than or equal to x

so the ceiling function for integer, the result is the same as input so the statement is True, $\lceil \lfloor x \rfloor \rceil = \lfloor x \rfloor$

d) $\forall x \in R, \lceil -x \rceil = -\lfloor x \rfloor$

Let x be represeted as $x = y + r$, y is the smallest integer close to x and r is the remainder, $r = x - y$ express $\lceil -x \rceil = \lceil -y - r \rceil = -y$ and $-\lfloor x \rfloor = -(\lfloor x \rfloor) = -(y) = -y$ so the statement is True

True

Problem 2.

a) Convert 101 from decimal to binary.

$$
\begin{array}{l}
2 \,|\, 101 \\
2 \,|\, 50 + 1 \\
2 \,|\, 25 + 0 \\
2 \,|\, 12 + 1 \\
2 \,|\, 6 + 0 \\
2 \,|\, 3 + 0 \\
\quad 1 + 1
\end{array}
= 1100111_{(2)}
$$

b) Convert 206 from octal to binary

$206_{(8)}$

$2_{(8)} \to 010_{(2)}$

$0 \to 000_{(2)}$

$6 \to 110_{(2)}$

$\therefore 206_{(8)} = 010/000/110$

$= 10000110_{(2)}$

$2^3 \cdot 3 \cdot 5 = 120$

$2 \cdot 2 \cdot 3$

(C) Convert CAF from hexadecimal to decimal

0 1 2 3 4 5 6 7 8 9 A B C D E F

10 11 12 13 14 15

$C(12) \rightarrow 12 \times 16^2 \rightarrow 12 \times 256$

$A(10) \rightarrow 10 \times 16^1 \rightarrow 160$

$F(15) \rightarrow 15 \times 16^0 \rightarrow 15$ $\therefore CAF_{(16)} = 3072 + 160 + 15 = (3247)_{10}$

problem 3.

a) $\forall x \in \mathbb{Z}, 6 \mid (x-1)x(x+1)$

For all x which is integers, $(x-1)x(x+1)$ are three consecutive integers. In three consecutive integers, one of three possible integers are divided by 3 with no remainder and also there we exist at least one even number in three consecutive integers. so product of three consecutive integers are divisible by 6.

b) $\forall x \in \mathbb{Z}, 120 \mid (x-2)(x-1)x(x+1)(x+2)$

express 120 with multiples of prime numbers $= 2^3 \cdot 3 \cdot 5$. so 120 is consisted with multiple of 2 and 4, one 3, and one 5. In 5 consecutive numbers, there must be at least one multiple of 5. In the same concept In 5 consecutive numbers, there must be at least one multiple of three. And when x is even number, $x-2$, x, $x+2$ are even numbers, when x is odd number, $x-1$, $x+1$ are even numbers. And every other even number is multiple of 4, One of two are multiple of 2, the other is 4, Since $120 = 2^3 \times 3 \times 5$, $(x-2)(x-1)x(x+1)(x+2)$ is divisible by 120

Problem 4.

Prove that there are infinetely many primes with remainder 3 when divided by 4.

For all prime x, $\exists x, x = q \cdot 4 + 3$

For the sake of contradiction, assume that the set of primes with remainder 3 when divided by 4 is finite. Thus we can define $G = \{p_0, p_1, p_2, \cdots p_n\}$ when P_i represents a prime in the form of $4x+3$ for some integers x.

Let $p = 4(p_0 \cdot p_1 \cdot p_2 \cdots p_{n+1})+3$. Since p is greater than p_n, and leaves remainder 3, p is both odd and composite, so its prime factorization does not include even number (value 2). Thus all of its prime factors will either be in the form of $4n+1$ or $4n+3$. Assume all of the prime factors of p are in the form $4n+1$. If this is the case, then p must also be in form $4n+1$ as any two numbers multiplied in that form will stay in that form. This is contradiction as p is in an form of $4n+3$ for some integer x.

Thus p has a prime divisor in the form of $4n+3$ must be greater than p_n. we have contradiction, therefore the set of primes with remainder 3 when divided by 4 is infinite problem 5

a) Bezout's coefficient for $x \cdot 122 + y \cdot 16 = \gcd(122, 16)$

$122 = 16(7) + 10 \longrightarrow 10 = (1)122 - (7)16$
$16 = 10(1) + 6 \longrightarrow 6 = (1)16 - (1)10$
$10 = 6(1) + 4 \longrightarrow 4 = (1)10 - (1)6$
$6 = 4(1) + 2 \longrightarrow 2 = (1)6 - (1)4$
$4 = 2(2) + 0$

$2 = (1)6 - (1)4$
$= (1)6 - 1((1)10 - (1)6))$
$= (1)6 - (1)10 + (1)6$
$= (2)6 - (1)10$
$= 2((1)16 - (1)10) - (1)10$
$= (2)16 - (2)10 - (1)10$
$= (2)16 - (3)10$
$= (2)16 - (3)((1)122 - (7)16))$
$= (2)16 - (3)122 + (21)16$
$= (23)16 - (3)122$

$\therefore x = -3 \quad y = 23$

b) $a \mid \mathrm{lcm}(a,b)$ & $b \mid \mathrm{lcm}(a,b)$

i) consider $\gcd(a,b) = m$, $a = m \cdot k$ & $b = m \cdot I$ for some integer k and $l \in \mathbb{Z}$, $\mathrm{lcm}(a,b) = m \cdot k \cdot g$ since $a \mid \mathrm{lcm}(a,b)$ and since $b \mid \mathrm{lcm}(a,b)$, $\mathrm{lcm}(a,b) = m \cdot I \cdot h$ for some integers g & h. Since $m \cdot k \cdot g = m \cdot I \cdot h$, $kg = Ih$ and refer this as R.
$\therefore \mathrm{lcm}(a,b) = m \cdot R$

$\gcd(a,b) \cdot \mathrm{lcm}(a,b) = m \cdot m \cdot R = m^2 \cdot R = (m \cdot k) \times (m \cdot I)$
$= m^2 R$ so this is True.

ii) use linear combination.

$\gcd(a,b) = s \cdot a + t \cdot b$ for Integers s, t.

Define $d = \gcd(a,b)$, $m = \frac{ab}{d}$

To prove $m = \text{lcm}(a,b)$, show:

1) m is a common multiple of a, b

$d = s \cdot a + t \cdot b$
$a = d \cdot I$, $b = d \cdot k$ for some integer I & k

$ab = d \cdot I \times d \cdot j = d^2 \cdot I \cdot j$
$\frac{ab}{d} = d \cdot I \cdot j$ so $m = d \cdot I \cdot j$

$\therefore$ m is a common multiple of a, b

2) if k is an arbitrary positive multiple of a, b. then $k \geq m$ by showing that $m \mid k$.

$a \mid k$, $b \mid k$

$K = a \cdot H = b \cdot U$ for some integer. H & U
$\therefore K = a \cdot H = d \cdot I \cdot H = d \cdot j \cdot U = b \cdot U$.
$I \cdot H = j \cdot U$
$I = \frac{j \cdot U}{H}$

$m = d \cdot I \cdot j = d \cdot \frac{k \cdot U}{H} \cdot j = \frac{j^2 \cdot U}{H} d$

$K = \cancel{d} \cdot \cancel{j} \cdot \cancel{U} = \frac{1}{\frac{j}{H}}$ $\quad k \div m = \cancel{d} \cdot \cancel{j} \cdot \cancel{U} \times \frac{H}{\cancel{j^2} \cdot \cancel{U} \cdot \cancel{d}} = \frac{H}{j}$
$m = \cancel{d} \cdot \frac{\cancel{j^2} \cdot \cancel{U}}{H}$