#1) 8 bars of low-fat cereal

(a) the means : $\frac{0.65+0.72+0.45+0.55+0.58+0.39+0.68+0.52}{8}$

$= \frac{4.54}{8} = \boxed{0.5675}$

(b) the variance $= \{(0.65-0.5675)^2+(0.72-0.5675)^2+(0.45-0.5675)^2+(0.55-0.5675)^2+(0.58-0.5675)^2$
$+(0.39-0.5675)^2+(0.68-0.5675)^2+(0.52-0.5675)^2\} \times \frac{1}{8-1}$

$(-0.1175)^2 \quad (-0.0175)^2 \quad (0.0125)^2$

$= \{0.00680625 + 0.02325625 + 0.01380625 + 0.00030625 + 0.00015625$
$+ 0.03150625 + 0.01265625) \times (0.00225625)\} \times \frac{1}{7}$

$(-0.1775)^2 \quad (0.1125)^2 \quad (-0.0475)^2$

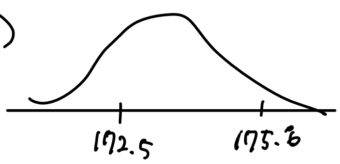
$= 0.09075 \times \frac{1}{7} = 0.01296428571 \quad \boxed{\therefore\ 0.01296}$

#2) $n = 1000$ students

$X \sim N(174.5, 6.9^2)$

(a) mean of $\bar{X}$ = mean of X when $n = 1000$, $\therefore \mu_{\bar{x}} = \mu_x = 174.5$
Standard deviation of $\bar{x}$ = $\sigma/\sqrt{n} = 6.9/\sqrt{25} = 6.9 \div 5 = \boxed{1.38}$

(b)



$P(172.5 < x < 175.8)$

$= P\left(\frac{172.5-0.05-174.5}{1.38} < Z < \frac{175.8+0.05-174.5}{1.38}\right)$

$= P(-1.49 < z < 0.98)$

$= 0.8365 - 0.0681 = 0.7684$

$\therefore 0.7684 \times 200 = 153.68 = \boxed{154}$

(c) $P(X < 172.0)$

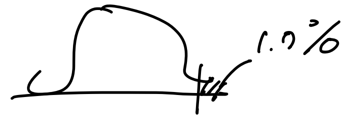
$P\left(\frac{X-174.5}{1.38} < \frac{172.0-0.05-174.5}{1.38}\right) = P(Z < -1.85) = 0.322$

$(0.322) \times (200) = 6.44 \quad \therefore$ $\boxed{\text{6 sample means}}$

3) $n = 50$, $\bar{x} = 0.23$ $\sigma = 0.1$

$$z = \frac{0.23-0.2}{0.1/\sqrt{50}} = \frac{0.03}{0.01414} \approx 2.12$$

$$P(\bar{x} \geq 0.23) = P(z \geq 2.12) = 0.0170$$



n>30 so 0.20 should be close to $\bar{X}$ so 0.20 is too small

4) $n=5$

$$\bar{X} = \frac{(305+312+296+304+307)}{5} = \frac{1524}{5} = 304.8$$

$$\sigma^2 = \frac{1}{5-1}\left((305-304.8)^2 + (312-304.8)^2 + (296-304.8)^2 + (304-304.8)^2 + (307-304.8)^2\right)$$

$$= \frac{1}{4}(0.04 + 51.84 + 77.44 + 0.64 + 4.84) = \frac{1}{4}(134.8)$$

$$= 33.7$$

5) $\bar{X} = 5000 \quad \sigma_x = 400 \quad n=36 \qquad \sigma/\sqrt{n} = 400/6 = 66$

(a) $P(4800 < \mu < 5200) = P\left(\frac{4800-5000}{400/6} < z < \frac{5200-5000}{400/6}\right)$

$$= P\left(\frac{-200}{400/6} < z < \frac{200}{400/6}\right) = P(-3 < z < 3) = 0.9987 - 0.0013$$

$$= 0.9974$$

(b) $P(4900 < \bar{X} < 5100) = 0.99 \quad P(0.005 < z < 0.995)$

$$\downarrow \qquad \downarrow$$

$$-2.575 \qquad 2.575$$

$$\frac{5100-5000}{400/\sqrt{n}} = 2.575 \qquad \frac{100}{400/\sqrt{n}} = 2.575$$

$$\frac{4900-5000}{400/\sqrt{n}} = -2.575 = \frac{-100}{400/\sqrt{n}} = -2.575$$

$*\ \frac{\cancel{100}}{\cancel{400}}\ /4\ \sqrt{n} = 2.575 \quad \sqrt{n} = 2.575 \times 4 = 10.3 \quad (\sqrt{n})^2 = (10.3)^2 = 106.09$

$\boxed{\therefore\ n = 107 \text{ or bigger}}$

6)

$\mu = 53{,}000$

$$\frac{(48000 + 53000 + 45000 + 61000 + 59000 + 56000 + 63000 + 49000 + 53000 + 54000)}{10}$$

$= 54{,}100$

$\bar{x} = 54{,}100$

$$\frac{1}{10-1}\Big((48000-54100)^2 + (53000-54100)^2 + (45000-5400)^2 + (61000-54100)^2 + (59000-54100)^2 + (56000-5400)^2 + (63000-5400)^2 + (49000-54100)^2 + (53000-54100)^2 + (54000-54100)^2\Big)$$

$$\sqrt{\frac{1}{9}\Big((6100)^2 + (-1100)^2 + (-9100)^2 + (6900)^2 + (4900)^2 + (1900)^2 + (8900)^2 + (-5100)^2 + (-1100)^2 + (-100)^2\Big)}$$

$s = 5801.34$

$n = 10$

$$t = \frac{54100 - 53{,}000}{5801.34\sqrt{10}} = 0.60$$

$P(T \geq 0.60)$

$\hookrightarrow\ \boxed{0.2 \sim 0.3 \text{ not a rare event}}$

7) $\sigma = 0.0015$, $\sigma/\sqrt{75} =$

$n = 75$ $\bar{X} = 0.310$

Find 95% interval for mean

$P(0.025 < z < 0.975)$

95

$P(-1.96 < z < 1.96)$

$\frac{x_1 - 0.31}{0.0015/\sqrt{75}} < z < \frac{x_2 - 0.31}{0.0015/\sqrt{75}}$ $x_1 =$

$\frac{x_1 - 0.31}{0.0015/\sqrt{75}} = -1.96$, $\frac{x_2 - 0.31}{0.0015/\sqrt{75}} = 1.96$

$\boxed{0.3097 < z < 0.3103}$

8) $n = \left(\frac{z_{\frac{\alpha}{2}}\sigma}{e}\right)^2$

$\alpha = 0.05$

$z_{\alpha/2} = 1.96$

$n = \left(\frac{(1.96)(0.0015)}{0.0005}\right)^2 = 34.5744 \approx \boxed{35}$

a) $\bar{X} = \dfrac{(1.01 + 0.97 + 1.03 + 1.04 + 0.99 + 0.98 + 0.99 + 1.01 + 1.03)}{9}$

$= 1.00555\ldots \approx 1.0056$

$s^2 = \frac{1}{9-1}\big((1.01-1.0056)^2 + (0.97-1.0056)^2 + (1.03-1.0056)^2 + (1.04-1.0056)^2 +$
$(0.99-1.0056)^2 + (0.98-1.0056)^2 + (0.99-1.0056)^2 + (1.01-1.0056)^2 +$
$(1.03-1.0056)^2\big) = 0.00060025$

$s = \sqrt{0.00060025} = 0.0245$

$P(0.005 < \mu < 0.995)$ t-distribution

$\downarrow$ -3.355 $\quad$ $\downarrow$ 3.355

$\dfrac{x_1 - 1.0066}{0.0245/3} = -3.355 \quad = x_1 - 1.0056 = -0.02739917$

$x_1 = 0.978$

$\dfrac{x_2 - 1.0056}{0.0245/3} = 3.355 \quad x_2 - 1.0056 = 0.02739917$

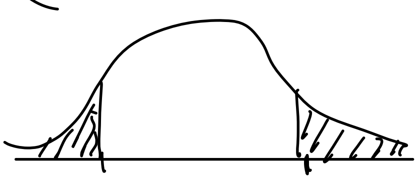
$x_2 = 1.033$

$\boxed{0.978 < \mu < 1.033}$

10) $\mu = 200 \quad n = 9 \quad \sigma = 15$

type I error : H_0 is rejected but it should be H_0

$\left(\dfrac{191-200}{15\sqrt{9}} < Z < \dfrac{209-200}{15\sqrt{9}}\right) \sim \left(\dfrac{-9}{5} < Z < \dfrac{9}{5}\right) = (-1.8 < Z < 1.8)$



$-1.8 \quad 1.8 \approx 0.0359$

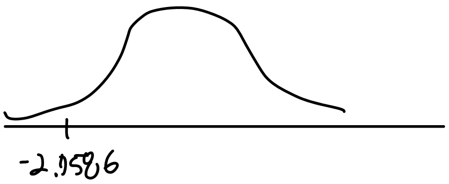
$p = 0.0359 \times 2 = 0.0718$

$\boxed{\therefore 0.0718}$

11) $\mu = 40$ $n = 64$ $\bar{X} = 38$ $\sigma = 5.8$

$H_0 : \mu = 40$
$H_1 : \mu < 40$

$$z = \frac{38-40}{5.8/\sqrt{64}} = \frac{-2}{5.8/8} = \frac{-2}{0.725} = -2.7586$$



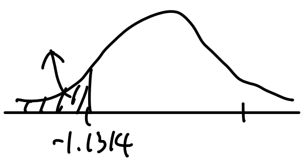
-2.76 p value : 0.0029

$\boxed{\text{p - value is so low reject Ho.}}$

12) $\mu = 3.5$ $\sigma = 0.5$

$H_0 : \mu = 3.5$
$H_1 : \mu \neq 3.5$

$n = 32$ $\bar{X} = 3.4$ $$z = \frac{3.4-3.5}{0.5/\sqrt{32}} = \frac{-0.1}{\frac{0.5}{4\sqrt{2}}} = -1.1314\ldots$$



pvalue = 0.1292 $0.1292 \times 2 = 0.2584$

$\boxed{\text{p value is so big so that the mean lifetime is 3.5}}$