Probability and Statistics COSE 112 Homework 1

Topics: probability

1. In how many different ways can a true-false test consisting of 10 question be answered?

true-false → 2ways $\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2} = 2^{10}$ ∴ 1024 ways

2. A California study concluded that following 7 simple health rules can extended a mans life by 11 years on the average and a womans life by 7 years. These 7 rules are as follows: no smoking, get regular exercise, use alcohol only in moderation, get 7 to 8 hours of sleep, maintain proper weight, eat breakfast, and do not eat between meals. In how many ways can a person adopt 5 of these rules to follow

(a) if the person presently violates all 7 rules?

5 rules out of 7 rules, distinct objects

$$_7C_5 = \frac{7!}{5!(7-5)!} = \frac{7!}{5!\,2!} = \frac{7\times6}{2\times1} = 21$$

(b) if the person never drinks and always eat breakfast?

person choose 2 out of 5 to follow, so only needs three out of 5 rest choices $(7-2=5)$

$$_5C_3 = \frac{5!}{3!\,2!} = \frac{5\times\cancel{4}^{2}\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}\times\cancel{2\times1}} = 10$$

∴ 10 ways

3. (a) How many three-digit numbers can be formed from the digits 0, 1, 2, 3, 4, 5, and 6 if each digit can be used only once?

$$\underline{6} \times \underline{6} \times \underline{5} = 180$$

0 cannot be here

so 1, 2, 3, 4, 5, 6 / 6 numbers

one number from 1, 2, 3, 4, 5, 6 is gone but 0 can be here

so, 6 numbers

∴ 180

(b) How many of these are odd numbers?

Odd numbers end with odd numbers.

_ _ 1 ends with 1

5 ways 5 ways → 25 ways

_ _ 3 also 25 ways

5 ways 5 ways

_ _ 5 → also 25 ways

5 5

→ 75 ways

∴ 75 ways

(c) How many are greater than 330?

1) three digits starts with 4, 5, 6.

3 6 5 → 90 ways

4, 5, 6

2) 1 3 5 → 15 ways

$\boxed{3}$ greater than 3.

→ 4, 5, 6

90 + 15 = 105

∴ 105 ways

Mint_138

4. A box contains 500 envelops, of which 50 contain \$100 in cash, 150 contain \$25, and 300 contain \$10. An envelope may be purchased for \$25. What is the sample space for the different amounts of money? Assign probabilities to the sample points and then find the probability that the first envelope purchased contains less than \$100.

→ possible outcome : \$ amount

Sample space $\{100, 25, 10\}$

$P(\{\$100\}) = \frac{50}{500} = \frac{1}{10}$ $\quad P(\{\$25\}) = \frac{150}{500} = \frac{3}{10}$ $\quad P(\{\$10\}) = \frac{300}{500} = \frac{6}{10}$

less than \$100 → \$25, \$10

$P(\{25, 10\}) = P(\{25\}) + P(\{10\}) = \frac{3}{10} + \frac{6}{10} = \frac{9}{10} \therefore 0.9$

5. A pair of fair dice is tossed. Find the probability of getting

(a) a total of 10 $\quad |S| = 36$ (6×6)

$\left.\begin{matrix}(4,6)\\(5,5)\\(6,4)\end{matrix}\right\} 3$ $\quad \therefore \frac{3}{36} = \frac{1}{12}$

(b) at most a total of 4

$|S| = 36$

$\underbrace{\begin{matrix}(1,1) & (2,1) & (3,1)\\(1,2) & (2,2) & \\(1,3) & & \end{matrix}}_{6}$ $\quad \therefore \frac{6}{36} = \frac{1}{6}$

6. It is common in many industrial areas to use a filling machine to fill boxes full of product. This occurs in the food industry as well as other areas in which the product is used in the home, for example, detergent. These machines are not perfect, and indeed they may A, fill to specification, B, underfill, and C, overfill. Generally, the practice of underfilling is that which one hopes to avoid Let $P(B) = 0.001$ while $P(A) = 0.990$.

(a) Give $P(C)$.

$P(A) + P(B) + P(C)$ should be 1

so $0.990 + 0.001 + P(C) = 1$

$= 0.991 + P(C) = 1$ $\therefore P(C) = 0.009$

(b) what is the probability that the machine does not underfill?

$1 - P(B) = 1 - 0.001 = 0.999.$ $\therefore 0.999$

(c) what is the probability that the machine either overfills or underfills?

$P(C) + P(B) = 0.009 + 0.001 = 0.010$

$\therefore 0.01$

7. A random sample of 200 adults are classified below by sex and their level of education attained.

Education	Male	Female
Elementary	38	45
Secondary	28	50
College	22	17

If a person is picked at random from this group, find the probability that

(a) the person is a male, given that the person has a secondary education

$$P(\text{male} \mid \text{secondary}) = \frac{P(\text{male} \cap \text{secondary})}{P(\text{secondary})}$$

$$= \frac{28}{\cancel{200}} \times \frac{\cancel{200}}{78} = \frac{14}{39} \qquad \therefore \frac{14}{39}$$

Mint_140

(b) the person does not have a college degree, given that the person is a female

$P(\text{college}' / \text{female})$ $\dfrac{P(\text{college}' \cap \text{female})}{P(\text{female})}$

$= \dfrac{95}{200} \times \dfrac{200}{112}$

$= \dfrac{95}{112}$ $\therefore \dfrac{95}{112}$

8. The probability that a married man watches a certain television show is 0.4, and the probability that a married woman watches the show is 0.5. The probability that a man watches the show, given that his wife does, is 0.7. Find the probability that

(a) a married couple watch the show;

$P(\text{man}) = 0.4$ $P(w) = 0.5$

$P(\text{man} / \text{his wife}) = 0.7$

$\dfrac{P(\text{man} \cap \text{his wife})}{P(\underset{\text{women}}{\text{his wife}})} = 0.7$ $\dfrac{x}{0.5} = 0.7$ $x = 0.7 \times 0.5$

$= 0.35$

$\therefore 0.35$

(b) a wife watches the show, given that her husband does;

$P(\text{wife} / \text{her husband})$

$= \dfrac{P(\text{wife} \cap \text{husband})}{P(\underset{\text{man}}{\text{husband}})} = \dfrac{0.35}{0.4} = \dfrac{35}{100} \times \dfrac{10}{4} = \dfrac{35}{40} = \dfrac{7}{8}$

$\therefore \dfrac{7}{8}$

(c) at least one member of a married couple will watch the show.

1 - (no member of married couple watch the show)

$P(\text{man} \cup \text{women}) = P(m) + P(w) - P(m \cap w)$

$= 0.4 + 0.5 - 0.35$

$= 0.55$ $1 - 0.55 = 0.45$

$0.4 - 0.35$

$\boxed{\therefore 0.45}$

Mint_141

체인졸

1. the probability that a vehicle entering the Luray Caverns has Canadian license plates is 0.12; the probability that it is a camper is 0.28; and the probability that it is a camper with Canadian license plates is 0.09. What is the probability that

(a) a camper entering the Luray Caverns has Canadian license plates?

Camper = C
Canadian license = CN

$P(CN) = 0.12$ $\qquad P(C \cap CN) = 0.09$
$P(C) = 0.28$

$$P(CN / C) = \frac{P(CN \cap C)}{P(C)} = \frac{9}{\cancel{100}} \times \frac{\cancel{100}}{28} = \frac{9}{28}$$

(b) a vehicle with Canadian license plates entering the Luray Caverns is a Camper?

$$P(C / CN) = \frac{P(C \cap CN)}{P(CN)} = \frac{9}{\cancel{100}} \times \frac{\cancel{100}}{12} = \frac{3}{4}$$

(c) a vehicle entering the Luray Caverns does Not have Canadian plates or it is not a camper?

$$P(C' \cup CN') = P(C') + P(CN') - P(C' \cap CN')$$
$$= P(C \cap CN)'$$
$$1 - P(C \cap CN) = 1 - 0.09 = \boxed{0.91}$$

$\therefore 0.91$

10. Police plan to enforce speed limits by using radar traps at four different locations within the city limits. The radar traps at each of the locations L1, L2, L3, and L4 will operated 40%, 30%, 20%, and 30% of the time. If a person who is speeding on her way to work has probabilities of 0.2, 0.1, 0.5, and 0.2, respectively, of passing through these locations, what is the probability that she will receive a speeding ticket?

$P(t) = P(t \cap L_1) + P(t \cap L_2) + P(t \cap L_3) + P(t \cap L_4)$

$P(t) = P(L_1) \times P(\omega_1) + P(L_2) \times P(\omega_2) + P(L_3) \times P(\omega_3) + P(L_4) \times P(\omega_4)$

$= 0.4 \times 0.2 + 0.3 \times 0.1 + 0.2 \times 0.5 + 0.3 \times 0.2$

$= 0.08 + 0.03 + 0.1 + 0.06$

$= 0.27$

$\therefore 0.27$

11. A paint-store chain produces and sells latex and semigloss paint. Based on long-range sales, the probability that a customer will purchase latex paint is 0.75. Of those that purchase latex paint, 60% also purchase rollers. But only 30% of semigloss paint buyers purchase rollers. If randomly selected buyer purchases a roller and a can of paint. what is the probability that the paint is latex?

Buy latex paint = l

Buy rollers = R

Buy semigloss paint = S

$P(l) = 0.75$

$P(R \mid l) = 0.6$

$P(R \mid S) = 0.3$

$\frac{P(R \cap l)}{P(l)} = \frac{x}{0.75} = 0.6$

$x = 2 \times \frac{6}{10} \times \frac{75}{100}$

$P(R \cap l) = 0.8$

$P(R \cap \text{paint})$

$\downarrow$

A

$P(l \mid A)$?

$\hookrightarrow \frac{P(l) \cdot P(R \cap S)}{P(R \cap l) + P(R \cap S)}$

$P(A) = P(R \cap l) + P(R \cap S) = 0.525$

$= P(l) \cdot P(R \mid l) + P(S) \cdot P(R \mid S)$

$= 0.75 \times 0.6 + 0.25 \times 0.3$

$= 0.45 + 0.075 = 0.525$

$\frac{0.45}{0.525}$

$= \frac{45}{100} \times \frac{1000}{525} = \boxed{\frac{6}{7}}$

$\boxed{\therefore \frac{6}{7}}$