$\boxed{1.}$ $\mu = \sum_{x=0}^{5} x \cdot f(x)$

1) $x=0, \quad f(0) = \binom{5}{0}\left(\frac{1}{4}\right)^{x}\left(\frac{3}{4}\right)^{5-x}$

$$= \left(\frac{5!}{5!}\right)\left(\frac{1}{4}\right)^{0}\left(\frac{3}{4}\right)^{5} = 1 \cdot 1 \cdot \frac{3^5}{4^5} = \frac{3^5}{4^5}$$

2) $x=1, \quad f(1) = \binom{5}{1}\left(\frac{1}{4}\right)^{1}\left(\frac{3}{4}\right)^{5-1}$

$$= \left(\frac{5!}{1!\,4!}\right)\left(\frac{1}{4}\right)\left(\frac{3^4}{4^4}\right) = \left(\frac{5\times\cancel{4\times3\times2\times1}}{\cancel{4\times3\times2\times1}}\right)\left(\frac{1}{4}\right)\left(\frac{3^4}{4^4}\right)$$

$$= \frac{5\cdot3^4}{4^5}$$

3) $x=2, \quad f(2) = \binom{5}{2}\left(\frac{1}{4}\right)^{2}\left(\frac{3}{4}\right)^{5-2} = \left(\frac{5!}{2!\,3!}\right)\left(\frac{1}{4^2}\right)\left(\frac{3^3}{4^3}\right)$

$$= \left(\frac{5\times\overset{2}{\cancel{4}}\times\cancel{3\times2\times1}}{\cancel{2\times1}\times\cancel{3\times2\times1}}\right)\left(\frac{1}{4^2}\right)\left(\frac{3^3}{4^3}\right)$$

$$= \frac{10\times3^3}{4^5}$$

4) $x=3, \quad f(3) = \binom{5}{3}\left(\frac{1}{4}\right)^{3}\left(\frac{3}{4}\right)^{5-3}$

$$= \left(\frac{5!}{3!\,2!}\right)\left(\frac{1}{4^3}\right)\left(\frac{3^2}{4^2}\right) = \left(\frac{5\times\overset{2}{\cancel{4}}\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}\times\cancel{2\times1}}\right)\left(\frac{1}{4^3}\right)\left(\frac{3^2}{4^2}\right)$$

$$= \frac{10\cdot3^2}{4^5}$$

5) $x=4, \quad f(4) = \binom{5}{4}\left(\frac{1}{4}\right)^{4}\left(\frac{3}{4}\right)^{5-4}$

$$= \left(\frac{5!}{4!\,1!}\right)\left(\frac{1}{4^4}\right)\left(\frac{3}{4}\right) = \left(\frac{5\times\cancel{4\times3\times2\times1}}{\cancel{4\times3\times2\times1}\times1}\right)\left(\frac{1}{4^4}\right)\left(\frac{3}{4}\right)$$

$$= \frac{15}{4^5}$$

6) $x=5$, $f(5) = \binom{5}{5}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^{5-5}$

$$= \left(\frac{5!}{5!}\right)\left(\frac{1}{4^5}\right)\left(\frac{3}{4}\right)^0 = \frac{1}{4^5}$$

$$\mu = 0\cdot\left(\frac{3^5}{4^5}\right) + 1\cdot\left(\frac{5\cdot 3^4}{4^5}\right) + 2\cdot\left(\frac{10\times 3^3}{4^5}\right) + 3\cdot\left(\frac{10\times 3^2}{4^5}\right) + 4\left(\frac{5}{4^5}\right)$$

$$+ 5\cdot\left(\frac{1}{4^5}\right)$$

$$= \frac{5\cdot 3^4}{4^5} + \frac{20\cdot 3^3}{4^5} + \frac{10\cdot 3^3}{4^5} \cdot \frac{60}{4^5} + \frac{5}{4^5}$$

$$= \frac{5(3^4 + 4\cdot 3^3 + 2\cdot 3^3 \cdot 2^2\cdot 3 + 1)}{4^5}$$

$$= \frac{5(3^4 + 6\cdot 3^3 + 2^2\cdot 3 + 1)}{4^5}$$

$$= \frac{5(81 + 162 + 12 + 1)}{4^5} = \frac{\cancel{580}}{\cancel{4^6}}$$

$$= \frac{5\cdot(256)}{4^4} = \frac{1280}{1024} = \frac{640}{512} = \frac{320}{256}$$

$$= \frac{160}{128} = \frac{80}{64} = \frac{40}{32} = \boxed{\frac{5}{4}}$$

$\boxed{2.}$ average profit per automibile = Expected value of x

$$E[x] = \int_0^1 x \cdot 2(1-x)\, dx$$

$$= \int_0^1 (2x - 2x^2)\, dx = x^2 - \frac{2}{3}x^3 \Big|_0^1$$

$$= 1 - \frac{2}{3} = \frac{1}{3} \rightarrow \text{Expected value}$$

$$\therefore \$ \frac{5000}{3}$$

$\boxed{3.}$ $z = \sqrt{x^2+y^2}$

$$E[z] = \int_0^1 \int_0^1 z \cdot f(x,y)\, dx\, dy$$

$$= \int_0^1 \int_0^1 \sqrt{x^2+y^2} \cdot 4xy\, dx\, dy$$

$$= \int_0^1 \int_0^1 \sqrt{x^2+y^2} \cdot 4xy\, dx\, dy$$

$u = x^2+y^2 \qquad du = 2x\, dx \qquad dx = \frac{1}{2x} du$

$$= \int_0^1 \int_0^1 \sqrt{u} \cdot \cancel{4xy} \cancel{\frac{1}{2x}}\, du\, dy$$

$$= \int_0^1 \int_0^1 \sqrt{u} \cdot 2y \, du \, dy$$

$$= \int_0^1 \int_0^1 u^{\frac{1}{2}} \cdot 2y \, du \, dy$$

$(1)^2 - (0)^2$

$$= \int_0^1 \int_0^1 \frac{2}{3} u^{\frac{3}{2}} \cdot 2y \, dy$$

$$= \int_0^1 \frac{2}{3} (x^2 + y^2)^{\frac{3}{2}} \cdot 2y \cdot dy \Big|_0^1$$

$$= \int_0^1 \left(\frac{2}{3} (1 + y^2)^{\frac{3}{2}} - \frac{2}{3} (0 + y^{\cancel{2}})^{\cancel{\frac{3}{2}}} \right) \cdot 2y \cdot dy$$

$$= \int_0^1 \left(\frac{2}{3} (1 + y^2)^{\frac{3}{2}} - \frac{2}{3} (y^3) \right) \cdot 2y \cdot dy$$

$$= \frac{2}{3} \int_0^1 \left((1 + y^2)^{\frac{3}{2}} \cdot \underline{2y} - 2y^4 \right) \cdot dy$$

$$= \frac{2}{3} \int_0^1 2 \left(y (1 + y^2)^{\frac{3}{2}} - y^4 \right) dy$$

$$= \frac{4}{3} \int_0^1 \left(y (1 + y^2)^{\frac{3}{2}} - y^4 \right) dy$$

$$= \frac{4}{3}\left(\int_0^1 y(1+y^2)^{\frac{3}{2}} \cdot dy - \int_0^1 y^4 \, dy \right)$$

$$= \frac{4}{3}\left(\int_0^1 y(1+y^2)^{\frac{3}{2}} \, dy - \left. \frac{y^5}{5} \right|_0^1 \right)$$

$$= \frac{4}{3}\left(\int_0^1 y(1+y^2)^{\frac{3}{2}} \, dy - \frac{1}{5} \right)$$

$$u = 1+y^2 \qquad \frac{du}{dy} = 2y \qquad dy = \frac{1}{2y} du$$

$$= \frac{4}{3}\left(\int_0^1 \cancel{y}(u)^{\frac{3}{2}} \frac{1}{2\cancel{y}} du - \frac{1}{5} \right)$$

$$= \frac{4}{3}\left(\frac{1}{2}\int_0^1 u^{\frac{3}{2}} du - \frac{1}{5} \right) = \frac{4}{3}\left(\frac{1}{2} \left. \frac{2}{5} u^{\frac{5}{2}} \right|_0^1 - \frac{1}{5} \right)$$

$$= \frac{4}{3}\left(\left. \frac{1}{5} (1+y^2)^{\frac{5}{2}} \right|_0^1 - \frac{1}{5} \right)$$

$$= \frac{4}{3}\left(\frac{1}{5}(2)^{\frac{5}{2}} - \frac{1}{5} - \frac{1}{5} \right) \qquad 2$$

$$= \frac{4}{3}\left(\frac{(2)^{\frac{5}{2}} - 2}{5} \right)$$

$$= \frac{4 \cdot 2(2^{\frac{3}{2}} - 1)}{15} = \boxed{\frac{8}{15}(2^{\frac{3}{2}} - 1)}$$

4. $\sigma^2 = E[x^2] - (E[x])^2$

$E[x] = -2(0.3) + 3(0.3) + 5(0.4)$

$= -0.6 + 0.9 + 2.0$

$= 2.3$

$E[x^2] = (-2)^2 \cdot (0.3) + (3)^2 \cdot (0.3) + (5)^2 \cdot (0.4)$

$= 1.2 + 2.7 + 10.0$

$= 13.9$

$\therefore \sigma^2 = 13.9 - (2.3)^2$

$= 13.9 - 5.29$

$= 8.61$

$\sqrt{8.61} = 2.934\ldots$ $\boxed{\therefore 2.934}$

5.

① $E[Y]$

[illegible]

$$= \int_0^{\infty} (3x-2)\cdot\left(\frac{1}{4}e^{-\frac{x}{4}}\right) dx$$

$$= \frac{1}{4}\int_0^{\infty} (3x-2)\cdot\left(e^{-\frac{x}{4}}\right) dx$$

$$= \frac{1}{4}\int_0^{\infty} \left(3e^{-\frac{x}{4}}x - 2e^{-\frac{x}{4}}\right) dx$$

$$= \frac{3}{4}\int_0^{\infty} e^{-\frac{x}{4}} x\, dx - \frac{1}{2}\int_0^{\infty} e^{-\frac{x}{4}} dx$$

$$f = x \quad df = dx$$

$$dg = e^{-\frac{x}{4}} dx, \quad g = -4e^{-\frac{x}{4}}$$

$$= \left(-3e^{-\frac{x}{4}} x\right)\Big|_0^{\infty} + \frac{5}{2}\int_0^{\infty} e^{-\frac{x}{4}} dx$$

$$= \lim_{x\to\infty} -3e^{-\frac{x}{4}}\cdot x + \frac{5}{2}\int_0^{\infty} e^{-\frac{x}{4}} dx$$

$$= 0 + \frac{5}{2}\int_0^{\infty} e^{-x/4} dx$$

$u = -\frac{x}{4} \quad du = -\frac{1}{4}dx$

$$= -10\int_0^{-\infty} e^{u}\, du$$

$$= 10e^{u}\Big|_{-\infty}^{0}$$

$$= 10$$

$$Var[x] = E[x^2] - (E[x])^2$$

$$E[x^2]$$

$$= \int_0^{\infty} (3x-2)^2 \left(\frac{1}{4}e^{-\frac{x}{4}}\right) \cdot dx$$

$$= \frac{1}{4}\int_0^{\infty} e^{-\frac{x}{4}} (3x-2)^2\, dx$$

$$= \frac{1}{4}\int_0^{\infty} (9x^2 e^{-\frac{x}{4}} - 12e^{-\frac{x}{4}}x + 4e^{-\frac{x}{4}})\, dx$$

$$= \frac{9}{4}\int_0^{\infty} (x^2 e^{-\frac{x}{4}})\, dx - 3\int_0^{\infty} e^{-\frac{x}{4}} x\, dx + \int_0^{\infty} e^{-\frac{x}{4}}\, dx$$

$$f = x^2 \quad df = 2x\,dx \quad dx = \frac{1}{2x}df$$

$$dg = e^{-\frac{x}{4}}dx \quad g = -4e^{-\frac{x}{4}}$$

$$= (-9e^{\frac{x}{4}}x^2)\Big|_0^{\infty} + 15\int_0^{\infty} e^{-\frac{x}{4}} x\, dx + \int_0^{\infty} e^{-\frac{x}{4}}\, dx$$

$$= \lim_{x\to\infty} \underbrace{-4e^{-\frac{x}{4}}x^2}_{=0} + 15\int_0^\infty e^{-\frac{x}{4}}x\,dx + \int_0^\infty -e^{-\frac{x}{4}}\,dx$$

$f=x,\ df=dx \quad dg=e^{-\frac{x}{4}}dx \quad g=-4e^{-\frac{x}{4}}$

$$= 0 + \left(-60e^{-\frac{x}{4}}\right)\Big|_0^\infty + 61\int_0^\infty -e^{-\frac{x}{4}}\,dx$$

$$= \lim_{x\to\infty} \underbrace{-60e^{-\frac{x}{4}}x}_{=0} + 61\int_0^\infty e^{-\frac{x}{4}}\,dx$$

$$= 61\int_0^\infty e^{x/4}\,dx$$

$$= -244e^{-\frac{x}{4}}\Big|_0^\infty$$

$$= 244$$

$$\therefore \sigma^2 = 244 - (10)^2 = 244 - 100$$

$$\boxed{=144}$$

$\boxed{6.}$ $f(x) = \begin{cases} 3x^2, & 0 < x < 1, \\ 0, & \text{elsewhere} \end{cases}$

variance $\sigma^2 = E[x^2] - (E[x])^2$

$E[x] = \int_0^1 x \cdot 3x^2 \cdot dx$

$= \int_0^1 3x^3 \, dx$

$= 3\frac{x^4}{4} \Big|_0^1 = \frac{3}{4}$

$E[x^2] = \int_0^1 x^2 \cdot 3x^2 \cdot dx$

$= \int_0^1 3x^4 \, dx$

$= 3\frac{x^5}{5} \Big|_0^1 = \frac{3}{5}$

$E[x^2] - (E[x])^2$

$$= \frac{3}{5} - \left(\frac{3}{4}\right)^2 = \frac{3}{5} - \frac{9}{16}$$

$$= \frac{48-45}{80} = \boxed{\frac{3}{80}}$$

$$6 = \boxed{\sqrt{\frac{3}{80}}} \approx 0.194\ldots.$$

$\boxed{17.}$ (a) $E(2X-3Y)$

$$= E[2X] - E[3Y]$$

$$E[2X] = (2\cdot 2)\cdot(0.18+0.3+0.12)$$

0.32

$$+ \; (2\cdot 4)\cdot(0.12+0.20+0.08)$$

$$= 4(0.6) + 8(0.4)$$

$$= 5.6$$

$$E[3Y] = (3\cdot 1)\cdot(0.18+0.12)$$

$$+ \; (3\cdot 3)\cdot(0.30+0.20)$$

$+$

$(3\cdot 5)\cdot(0.12+0.08)$

$= 3(0.3) + 9(0.5) + 15(0.2)$

$= 0.9 + 4.5 + 3.0$

$= 8.4$

$E[2X-3Y] = 5.6 - 8.4 = -2.8$

(b) $E(XY)$

Since X and Y are independent random variables

$E(XY) = E[X]\cdot E[Y]$

$E[X] = 2\cdot(0.18+0.30+0.12)$

$+$

$4\cdot(0.12+0.20+0.08)$

$= 2\cdot(0.6) + 4(0.4)$

$= 1.2 + 1.6 = 2.8$

$E[Y] = 1\cdot(0.18+0.12)$

$+$

$3\cdot(0.30+0.20)$

$+$

$$5(0.12+0.08)$$

$$= 0.3 + 1.5 + 1.0$$

$$= 2.8$$

$$E[XY] = 2.8 \times 2.8 = \boxed{7.84}$$

8.

$$E[g]$$

$$= \int_0^1 \int_1^2 \left(\frac{x}{y^4} + x^2 y\right)\left(\frac{2}{7}(x+2y)\right) dx\,dy$$

$$= \frac{2}{7}\int_0^1 \int_1^2 \left(\frac{x^2}{y^4} + \frac{2x}{y^3} + x^3 y + 2x^2 y^2\right) dx\,dy$$

$$= \frac{2}{7}\int_1^2 \frac{x^3}{3y^4} + \frac{x^2}{y^3} + \frac{x^4 y}{4} + \frac{2x^3 y^2}{3} \Big|_0^1$$

$$= \frac{2}{7}\int_1^2 \left(\frac{1}{3y^4} + \frac{1}{y^3} + \frac{y}{4} + \frac{2y^2}{3}\right) dy$$

$$= \frac{2}{7}\left(-\frac{1}{9y^3} + -\frac{1}{2y^2} + \frac{y^2}{8} + \frac{2y}{9} \right)\Bigg|_1^2$$

$$= \frac{2}{7}\left\{ \frac{-1}{72} + \frac{-1}{8} + \frac{4}{8} + \frac{16}{9} - \left(\frac{-1}{9} + \frac{-1}{2} + \frac{1}{8} + \frac{2}{9}\right)\right\}$$

$$= \frac{2}{7}\left\{ \frac{-1 - 9 + 36 + 128 + 8 + 36 - 9 - 16}{72} \right\}$$

$$= \frac{2}{7}\left\{ \frac{173}{72} \right\} = \frac{173}{7 \cdot 36} = \boxed{\frac{173}{504}} \approx 0.6865$$

9.)

$$\text{mean} = \sum_{n=1}^{K} x_n \cdot f(x)$$

$$\mu = \sum_{n=1}^{K} x_n \cdot \frac{1}{K}$$

$$\boxed{= \frac{1}{K} \sum_{n=1}^{K} x_n} \rightarrow \text{1 to the K sum}$$

variance

$$E[x^2] - \mu^2$$

$$E[x^2] = \sum_{n=1}^{k} (x_n)^2 \cdot \frac{1}{k}$$

$$\text{variance} = \boxed{\frac{1}{k} \sum_{n=1}^{k} (x_n - \mu)^2}$$

10.

(a) $\binom{5}{2} (0.75)^2 (0.25)^3$

$$= \left(\frac{5!}{2!\,3!}\right) (0.75)^2 (0.25)^3$$

$$= \frac{5 \times \cancel{4}^{2} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{2 \times 1 \times \cancel{3} \times \cancel{2} \times 1} (0.5625)(0.015625)$$

$$= \boxed{0.08789}$$

(b) at most 3

4 resulted

$\binom{5}{\ } (0.75)^4 (0.25)^1$

$\binom{\ \ }{4}(0.75)(0.25)$

$= \left(\frac{5!}{4!\,1!}\right) \cdot (0.75)^4 \cdot (0.25)$

$= 5 \cdot (0.3164\ldots)(0.25)$

$= 5 \cdot (0.0791\ldots)$

$= 0.3955$

5 resulted

$= \binom{5}{5}(0.75)^5$

$= 0.2373\ldots$

$1 - 0.3955 - 0.2373$

$= \boxed{0.3672}$

$\boxed{11}$ (a) $\binom{5}{0}(0.60)^5(0.40)^0$

$= 1 \cdot (0.6)^5 \cdot 1$

$$= \boxed{0.07776}$$

(b) $\binom{5}{1}(0.60)^4(0.40)^1$

$$= \frac{5!}{1!4!}(0.60)^4(0.40)^1$$

$$= 5 \cdot (0.1296)(0.4)$$

$$= 0.2592$$

$$0.2592 + 0.07776 = \boxed{0.3369}$$

(c) 1) $\binom{5}{4}(0.60)^1(0.40)^4$

$$= \left(\frac{5!}{4!1!}\right)(0.60)(0.40)^4$$

$$= 5 \cdot (0.6)(0.4)^4$$

$$= 5 \cdot (0.6)(0.0256)$$

$$= 0.0768$$

2) $\binom{5}{\ }(0.60)^0(0.40)^5$

$C_5 \, (0.0 \ldots$

$= 1 \cdot 1 \cdot (0.40)^5$

$= .01024$

$0.1024 + 0.0768 = \boxed{0.08704}$

$\boxed{12}$ (a) $b(7, 3, \frac{1}{2})$

$$= \binom{6}{2} (0.5)^3 (0.5)^4$$

$$= \left(\frac{6!}{2! \, 4!}\right) (0.5)^7$$

$$= \frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4 \times 3 \times 2 \times 1}}{\cancel{2} \times 1 \times \cancel{4 \times 3 \times 2 \times 1}} (0.5)^7$$

$$= 15 \cdot (0.5)^7$$

$$= \boxed{0.1172}$$

(6) $b(4, 1, \frac{1}{2})$

$$= \binom{3}{0} (0.5)^1 (0.5)^{4-1}$$

$$= 1 \cdot (0.5)^4$$

$$= \boxed{0.0625}$$

$\boxed{13}$ a) $\lambda = 5$

$$\text{poisson}(5) = P(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$1 - P(x \leq 5)$

$$P(0) = e^{-\lambda}$$

$$P(1) = \lambda \cdot e^{-\lambda}$$

$$P(2) = \frac{\lambda^2}{2} e^{-\lambda}$$

$$P(3) = \frac{\lambda^3}{3} e^{-\lambda}$$

$$P(4) = \frac{\lambda^4}{4} e^{-\lambda}$$

$$p(5) = \frac{\lambda^5}{5} e^{-\lambda}$$

$$p(0) + p(1) + \cdots p(5)$$

$$= e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{3} + \frac{\lambda^4}{4} + \frac{\lambda^5}{5}\right)$$

$$= e^{-5}\left(1 + 5 + \frac{25}{2} + \frac{125}{3} + \frac{625}{4} + \frac{5^5}{5}\right)$$

$$= 0.61596$$

$$1 - 0.61596 = \boxed{0.3840}$$

(b) $p(0) = e^{-5}$

$$= \boxed{6.7379 \times 10^{-3}}$$

14.

$X \sim \text{poisson}(1.2)$

millimeter

$P(0, 5\lambda)$

$\lambda = 1.2$

$$P(0) = \frac{(5\lambda)^0}{0!} e^{-5\lambda}$$

$$= \frac{1.2^0}{1} e^{-1.2 \times 5}$$

$$= e^{-1.2 \times 5}$$

$$= e^{-6}$$

$$= 2.4788 \times 10^{-3}$$

$$= \boxed{2.479 \times 10^{-3}}$$