Topics: Estimation and Test with Normal Distribution

1. The average contents of saturated fat in eight bars of a certain brand of low-fat cereal selected at random are measured as follows: 0.65, 0.72, 0.45, 0.55,0.58, 0.39, 0.68, and 0.52 grams. Calculate
   (a) the means;
   (b) the variance.
2. The heights of 1000 students are approximately normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Suppose 200 random samples of size 25 are drawn from this population and the means recorded to the nearest tenth of a centimeter. Determine
   (a) the mean and standard deviation of the sampling distribution of $\bar{X}$;
   (b) the number of sample means that fall between 172.5 and 175.8 centimeters inclusive;
   (c) the number of sample means falling below 172.0 centimeters.
3. In a chemical process, the amount of a certain type of impurity in the output is difficult to control and is thus a random variable. Speculation is that the population mean amount of the impurity is 0.20 gram per gram of output. It is known that the standard deviation is 0.1 gram per gram. An experiment is conducted to gain more insight regarding the speculation that $\mu = 0.2$. The process is run on a lab scale 50 times and the sample average $\bar{x}$ turns out to be 0.23 gram per gram. Comment on the speculation that the mean amount of impurity is 0.20 gram per gram. Make use of the Central Limit Theorem in your work.
4. The price quotations for a home appliance is collected randomly from five different retail shops in a city. These are \$305, \$312, \$296, \$304, and \$307. Find the variance of the quoted prices.
5. The breaking strength X of a certain rivet used in a machine engine has a mean 5000 psi and standard deviation 400 psi. A random sample of 36 rivets is taken. Consider the distribution of $\bar{X}$, the sample mean breaking strength.
   (a) What is the probability that the sample mean falls between 4800 psi and 5200 psi?
   (b) What sample n would be necessary in order to have $P(4900 < \bar{X} < 5100) = 0.99$?
6. Consider the situation of Review Exercise 8.62. If the population from which the sample was taken has population mean $\mu = 53,000$ kilometers, does the sample information here seem to support that claim? In your answer, compute

$$t = \frac{\bar{x} - 53,000}{s/\sqrt{10}}$$

   and determine from Table A.4 (with 9 d.f.) whether the computed $t-$value is reasonable or appears to be a rare event.
7. Many cardiac patients wear an implanted pacemaker to control their heartbeat. A plastic connector module mounts on the top of the pacemaker. Assuming a standard deviation of 0.0015 inch and an approximately normal distribution, find a 95% confidence interval for the mean of the depths of all connector modules made by a certain manufacturing company. A random sample of 75 modules has an average depth of 0.310 inch.

8. How large a sample is needed in Exercise 9.3 if we wish to be 95% confident that our sample mean will be within 0.0005 inch of the true mean?

9. A machine produces metal pieces that are cylindrical in shape. A sample of pieces is taken, and the diameters are found to be 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, and 1.03 centimeters. Find a 99% confidence interval for the mean diameter of pieces from this machine, assuming an approximately normal distribution.

10. A soft-drink machine at a steak house is regulated so that the amount of drink dispensed is approximately normally distributed with a mean of 200 milliliters and a standard deviation of 15 milliliters. The machine is checked periodically by taking a sample of 9 drinks and computing the average content. If $\bar{x}$ falls in the interval $191 < \bar{x} < 209$, the machine is thought to be operating satisfactorily; otherwise, we conclude that $\mu \neq 200$ milliliters. Find the probability of committing a type I error when $\mu = 200$ milliliters.

11. In a research report, Richard H. Weindruch of the UCLA Medical School claims that mice with an average life span of 32 months will live to be about 40 months old when 40% of the calories in their diet are replaced by vitamins and protein. Is there any reason to believe that $\mu < 40$ if 64 mice that are placed on this diet have an average life of 38 months with a standard deviation of 5.8 months? Use a $P-$value in your conclusion.

12. An automobile industry uses a particular brand of automobile batteries that have their lifetimes approximately normally distributed, with a mean of 3.5 years and standard deviation of 0.5 years. Test the hypothesis that $\mu = 3.5$ years against the alternative, $\mu \neq 3.5$ years, if a random sample of 32 batteries has an average life of 3.4 years. Use a $P-$value in your answer.