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1 CS585 Problem Set 1 (Total points: 50 + 5 bonus)¶

Assignment adapted from A. Efros and Daniel McKee.

1.0.1 Instructions¶

1. Assignment is due at **11:59 PM on Tuesday Feb 11th 2025**.
2. Submission instructions:
   A. A single `.pdf` report that contains your work for Q1, Q2 and Q3. For Q1 and Q2 you can either type out your responses in LaTeX, or any other word processing software. You can also hand write them on a tablet, or scan in hand-written answers. If you hand-write, please make sure they are neat and legible. If you are scanning, make sure that the scans are legible. Lastly, convert your work into a `PDF`.

   For Q3 your response should be electronic (no handwritten responses allowed). You should respond to the questions 3.1, 3.2 and 3.3 individually and include images as necessary. Your response to Q3 in the PDF report should be self-contained. It should include all the output you want us to look at. You will not receive credit for any results you have obtained, but failed to include directly in the PDF report file.

   PDF file should be submitted to Gradescope (https://www.gradescope.com) under `PS1`. Please tag the reponses in your PDF with the Gradescope questions outline as described in Submitting an Assignment (https://youtu.be/u-pK4GzpId0).
   B. You also need to submit Pyhton code for Q3 in the form of a single `.py` file that includes all your code, all in the same directory. You can convert this notebook to Python code by downloading the `.ipynb` file as Python(`.py`). Code should also be submitted to Gradescope (https://www.gradescope.com) under `PS1-Code`. *Not submitting your code will lead to a loss of 100% of the points on Q3.*
   C. We reserve the right to take off points for not following submission instructions. In particular, please tag the reponses in your PDF with the Gradescope questions outline as described in Submitting an Assignment (https://youtu.be/u-pK4GzpId0).

1.0.2 Problems¶

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1. ****Linear Algebra Review [10 pts total, 5 parts].** Answer the following questions about matrices. Show the calculation steps (as applicable) to get full credit.

a) Matrix Multiplication [2 pts]. Let $V = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$. Compute $V \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $V \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. What does matrix multiplication Vx do to x?

b) Matrix Transpose [2 pts]. Verify that $V^{-1} = V^\top$. What does $V^\top x$ do?

c) Diagonal Matrix [2 pts]. Let $\Sigma = \begin{bmatrix} \sqrt{8} & 0 \\ 0 & 2 \end{bmatrix}$. Compute $\Sigma V^\top x$ where $x = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$ respectively These are 4 corners of a square. What is the shape of the result points?

d) Matrix Multiplication II [2 pts]. Let $U = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$. What does Ux do? (Rotation matrix wiki (https://en.wikipedia.org/wiki/Rotation_matrix))

e) Geometric Interpretation [2 pts]. Compute $A = U\Sigma V^T$. From the above questions, we can see a geometric interpretation of Ax: (1) V^T first rotates point x, (2) Σ rescales it along the coordinate axes, (3) then U rotates it again. Now consider a general squared matrix $B \in \mathbb{R}^{n \times n}$. How would you obtain a similar geometric interpretation for Bx?

a) Matrix Multiplication [2 pts]. Let $V = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$. Compute $V \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $V \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. What does matrix multiplication Vx do to x?

$$V \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \cdot 1 + -\frac{1}{\sqrt{2}} \cdot 0 \\ \frac{1}{\sqrt{2}} \cdot 1 + -\frac{1}{\sqrt{2}} \cdot 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$V \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \cdot 0 + -\frac{1}{\sqrt{2}} \cdot 1 \\ \frac{1}{\sqrt{2}} \cdot 0 + -\frac{1}{\sqrt{2}} \cdot 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}.$$

The rotation matrix is $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$.

$\frac{1}{\sqrt{2}} = \sin 135^{\backslash \text{degree}}$, $-\frac{1}{\sqrt{2}} = \cos 135^{\backslash \text{degree}}$. This matrix, V rotates vectors 135 degree to counter clockwise.

b) Matrix Transpose [2 pts]. Verify that $V^{-1} = V^{\top}$. What does $V^{\top}x$ do?

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The Inverse Matrix of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \frac{1}{\det(A)} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

The inverse Matrix of V is

$$V^{-1} = \frac{1}{(-\frac{1}{\sqrt{2}} \times -\frac{1}{\sqrt{2}}) - (-\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}})} \cdot \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{1} \cdot \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

This is the same as V^T. I showed they are the same.

$$V^T \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \cdot 1 + \frac{1}{\sqrt{2}} \cdot 0 \\ -\frac{1}{\sqrt{2}} \cdot 1 + -\frac{1}{\sqrt{2}} \cdot 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$V^T \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \cdot 0 + \frac{1}{\sqrt{2}} \cdot 1 \\ -\frac{1}{\sqrt{2}} \cdot 0 + -\frac{1}{\sqrt{2}} \cdot 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

I can also observe the rotation matrix. The inverse and transpose of matrix V, is the rotation matrix when the degree θ is $-135^{\backslash \text{degree}}$. which mean V^T rotate 135 degrees clockwise.

c) Diagonal Matrix [2 pts]. Let $\Sigma = \begin{bmatrix} \sqrt{8} & 0 \\ 0 & 2 \end{bmatrix}$. Compute $\Sigma V^\top x$ where $x = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$ respectively These are 4 corners of a square. What is the shape of the result points?

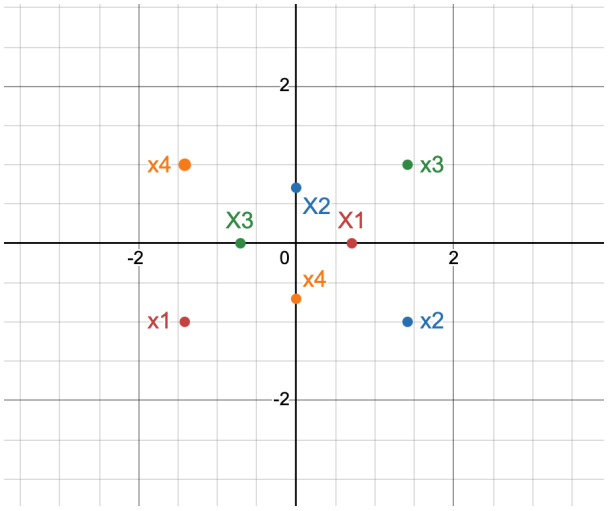
$$\Sigma \cdot V^T \cdot X_1 = \begin{bmatrix} \sqrt{8} & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{8}}{2} \\ -1 \end{bmatrix}$$

$$\Sigma \cdot V^T \cdot X_2 = \begin{bmatrix} \sqrt{8} & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{8}}{2} \\ -1 \end{bmatrix}$$

$$\Sigma \cdot V^T \cdot X_3 = \begin{bmatrix} \sqrt{8} & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{8}}{2} \\ 1 \end{bmatrix}$$

$$\Sigma \cdot V^T \cdot X_4 = \begin{bmatrix} \sqrt{8} & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{8}}{2} \\ 1 \end{bmatrix}$$

When I draw this in the x and y coordinates:



I can see the each x values rotate 135 degree clockwise and transform. The result x looks like a rectangle.

d) Matrix Multiplication II [2 pts]. Let $U = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$. What does Ux do? (Rotation matrix wiki (https://en.wikipedia.org/wiki/Rotation_matrix))

From the previous question, I know this matrix U is a rotation matrix. In order to find the degree, I can use the information $\cos\theta = -\frac{\sqrt{3}}{2}$, $\sin\theta = -\frac{1}{2}$. The θ value should be $210^{\backslash\text{degree}} = \frac{7}{6}\pi$ to get this two sin and cos values.

So this matrix U rotates $210^{\backslash\text{degree}}$ counter clockwise.

e) Geometric Interpretation [2 pts]. Compute $A = U\Sigma V^T$. From the above questions, we can see a geometric interpretation of Ax: (1) V^T first rotates point x, (2) Σ rescales it along the coordinate axes, (3) then U rotates it again. Now consider a general squared matrix $B \in \mathbb{R}^{n\times n}$. How would you obtain a similar geometric interpretation for Bx?

$$A = U\Sigma V^T$$

$$A = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} \sqrt{8} & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

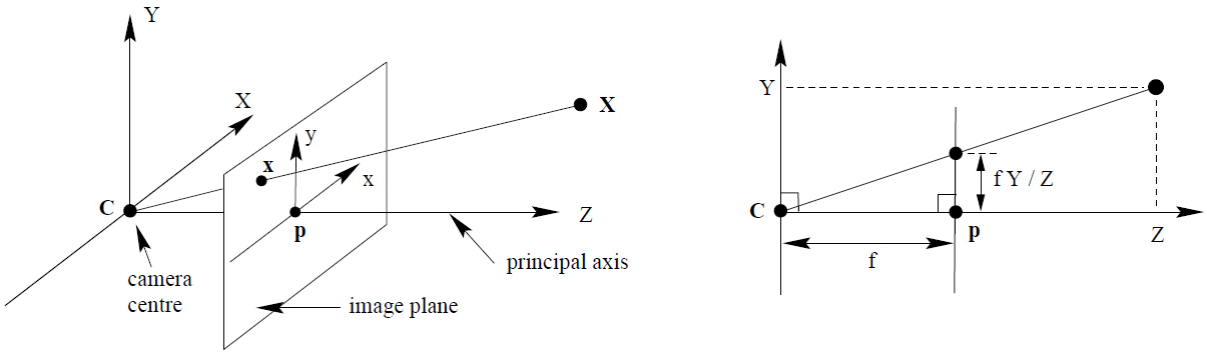
$$A = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} -\frac{\sqrt{8}}{\sqrt{2}} & \frac{\sqrt{8}}{\sqrt{2}} \\ -\frac{2}{\sqrt{2}} & -\frac{2}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} -2 & 2 \\ -\sqrt{2} & -\sqrt{2} \end{bmatrix}$$

$$A = \begin{bmatrix} \sqrt{3} - \frac{\sqrt{2}}{2} & -\sqrt{3} - \frac{\sqrt{2}}{2} \\ 1 + \frac{\sqrt{6}}{2} & -1 + \frac{\sqrt{6}}{2} \end{bmatrix}$$

I can decompose a general squared matrix B into the $U\Sigma V^T$. The SVD decomposition. Then the matrix B will be composed of two rotational matrix, and a transform matrix. By multiplying each matrices one by one. It will be the same result as using one matrix B.

2. **Pinhole Camera [10 pts total, 4 parts].**

A scene point at coordinates (400, 600, 1200) in millimeters is projected in a pinhole camera with focal length 72mm.



a) [3 pts] Assuming that the aspect ratio of the pixel is 1, what are the image plane coordinates assuming their origin is at the camera's principal point? (Recall, the aspect ratio is defined as the ratio between the width and the height of a pixel.)

b) [3 pts] Assuming that the image center is at (320,240), given in mm, what are the new image plane coordinates in pixels?

c) [2 pts] Assuming the aspect ratio is not 1 and the same scene point projects instead to image coordinate (24, 24), given in pixels, what is the aspect ratio of the pixels in this camera?

d) [2 pts] List and describe in a single sentence two types of flaws images can have due to the lens they use.

a) [3 pts] Assuming that the aspect ratio of the pixel is 1, what are the image plane coordinates assuming their origin is at the camera's principal point? (Recall, the aspect ratio is defined as the ratio between the width and the height of a pixel.)

The central projection mapping from 3d to 2d coordinate is $(X, Y, Z)^T = (\frac{fX}{Z}, \frac{fY}{Z})^T$ Assuming the origin is at the camera's principal point, the image plane coordiate at camera is $(\frac{fX}{Z}, \frac{fY}{Z}) = (\frac{72 \cdot 400}{1200}, \frac{72 \cdot 600}{1200}) = (24, 36)$

b) [3 pts] Assuming that the image center is at (320,240), given in mm, what are the new image plane coordinates in pixels?

It is given that the image center is now at (320, 240). The new coordinates are $x_{pixel} = x_{\text{new image center}} + x^{'}$, $y_{pixel} = y_{\text{new image center}} + y^{'}$.

The new image plane coordinates in pixels are $(320 + 24, 240 + 36) = (344, 276)$

c) [2 pts] Assuming the aspect ratio is not 1 and the same scene point projects instead to image coordinate (24, 24), given in pixels, what is the aspect ratio of the pixels in this camera?

Since the pixel ratio is not 1:1, the original pixel (24, 36) changed into (24, 24). The x pixel coordinate stays the same, but the y coordinates changed from 36 to 24. The pixel x side ratio is $\frac{24}{24} = 1$, the y side ratio is $\frac{276}{344} = \frac{69}{86} \approx 0.802$. So the pixel ration is $0.802 : 1$.

d) [2 pts] List and describe in a single sentence two types of flaws images can have due to the lens they use.

1. Vignetting is a gradual darkening of an image towards the edges or corners due to limitations in how lenses capture and distribute light.
2. Radial distortion is the warping of straight lines into curves due to the way light bends when passing through a lens just like a fish eyes.

3. **Colorizing images [30 points total, 3 parts].**

In this problem we will learn to work with images by taking the digitized Prokudin-Gorskii glass plate images (https://www.loc.gov/exhibits/empire/gorskii.html) and automatically producing a color image with as few visual artifacts as possible. In order to do this, you will need to extract the three color channel images, place them on top of each other, and align them so that they form a single RGB color image.

a) Read images [5 pts]. We'll start simple. Our first task is to read the file 00351v.jpg (https://drive.google.com/file/d/11fwxjlZkDOApoVZx0Pr4am1ClA6qdNaY/view?usp=sharing), extract the three color channel images and display each of them. Note that the filter order from top to bottom is BGR.

In [1]:
```
from PIL import Image
import numpy as np
import matplotlib.pyplot as plt

def crop_bgr_from_img(img_path: str)->[np.array,np.array,np.array]:
    """
     Read the file as a numpy array and crop the three color channel images.
     Note that the filter order from top to bottom is BGR.

     :param img_path: the path to the image file, which is a string.

     :return: blue, green and red color channel images. Each of them is a 341x396 numpy array.
    """

    img=np.array(Image.open(img_path),dtype=np.uint16)

    # Renormalize the pixel value to be in range [0,255].
    img=255*(img*1.0-np.min(img))/(np.max(img)-np.min(img))
    img=np.uint8(img)

    h,w=img.shape

    # Extract the three color channel images from top to bottom.
    seg_h=h//3

    # ADD YOUR CODE HERE (5 pts)
    blue=img[0:seg_h, 0:w]
    green=img[seg_h:seg_h*2, 0:w]
    red=img[seg_h*2:seg_h*3, 0:w]

    # Make sure the three color channel images have the same shape
    assert blue.shape==green.shape==red.shape

    return blue,green,red
```

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In [2]:
```
blue,green,red=crop_bgr_from_img('00351v.jpg')

# Display three color channel images
plt.figure(figsize=(20,20))
for i,img_arr in enumerate([blue,green,red]):
    img=Image.fromarray(img_arr)
    plt.subplot(1,3,i+1)
    plt.axis('off')
    plt.imshow(img,cmap='gray')
```

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b)** **Basic alignment [10 pts].** Now that we've divided the image into three channels, the next thing to do is to align two of the channels to the third. The easiest way to align the parts is to exhaustively search over a window of possible displacements (say [-15,15] pixels independently for the x and y axis), score each one using some image matching metric, and take the displacement with the best score. We can use normalized cross-correlation (NCC) as the image matching metric, which is simply the dot product between the two images normalized to have zero mean and unit norm.

$$NCC(A,B) = \frac{(A-\hat{A})\dot{(}B-\hat{B})}{\|A-\hat{A}\|\|B-\hat{B}\|}$$

In [3]:
```
def ncc(img_a: np.array, img_b: np.array)->float:
    """
     Compute the normalized cross-correlation between two color channe
l images
     and return the matching score.

     :param img_a: the first image, which is a 341x396 numpy array.
     :param img_b: the second image, which is a 341x396 numpy array.

     :return: the normalized cross-correlation score.
    """

    ncc=0

    # ADD YOUR CODE HERE (5 pts)
    a_normalized = img_a - np.mean(img_a)
    b_normalized = img_b - np.mean(img_b)

    numerator = np.dot(a_normalized.flatten(), b_normalized.flatten())
    denominator = np.linalg.norm(a_normalized) * np.linalg.norm(b_norm
alized) # || A - A^{hat} || * || B - B^{hat}||

    ncc = numerator / denominator if denominator != 0 else 0

    return ncc
```


Then, we align two color channel images by exhaustively searching over a window of possible displacements, score each one using NCC, and take the displacement with the best score. (Hint: you can use np.roll function to shift the entire image by the specified displacement.

In [4]:
```
def align_imga_to_imgb(img_a:np.array,img_b:np.array, wd_size:int=15)->(np.array,(int,int)):
    """
     Align two color channel images. Return the aligned image_a and its displacement.

     :param img_a: the image to be shifted, which is a 341x396 numpy array.
     :param img_b: the image that is fixed, which is a 341x396 numpy array.

     :return: a tuple of (aligned_a, displacement_of_a). ''aligned_a'' is the aligned image_a,
     which is a 341x396 numpy array.''displacement_of_a'' is the displacement vector of img_a, which is
     a tuple of (row displacement, column displacement).
    """

    # Initialize the image matching score.
    score=float('-inf')

    # Initialize the aligned image A.
    aligned_a=None

    #Initialize the displacement vector.
    displacement=(0,0)

    # Shift image A whithin range [-wd_size, wd_size], score each shifted image
    # and take the one with the best score.
    for i in range(-wd_size, wd_size):
        for j in range(-wd_size, wd_size):
            # Shift img_a's rows by i pixels, columns by j pixels
            shifted_a= np.roll(img_a, shift=(i, j), axis=(0, 1)) # np.roll shifts an array along a specified axis without changing its shape.

            new_score=ncc(shifted_a,img_b)

            if new_score>score:
                score=new_score
                aligned_a=shifted_a
                displacement=[i,j]
    return  aligned_a,displacement
```


Finally, we can display the colorized output and the (x,y) displacement vector that were used to align the channels.

In [5]:
```
def colorize_image(b:np.array,g:np.array,r:np.array)->(np.array,list):
    """
     Align the three color channel images. Return the colored image
     and a list of the displacement vector for each channel.

     :param b: the blue channel image, which is a 341x396 numpy array.
     :param g: the greeb channel image, which is a 341x396 numpy arra
y.
     :param r: the red channel image, which is a 341x396 numpy array.

     :return: a tuple of (colored_image, displacements). ''colored_ima
ge'' is a 341x396x3 numpy array.
     ''displacements'' is a list of the displacement vector for each c
hannel.
    """

    # Align the red and blue channels to the green channel.
    aligned_r,dis_r = align_imga_to_imgb(r,g)
    aligned_b,dis_b = align_imga_to_imgb(b,g)

    aligned_g,dis_g=g,(0,0)

    # Combine the aligned channels to a color image.
    colored_img=np.stack([aligned_r,aligned_g,aligned_b],axis=2)

    return colored_img,[dis_r,dis_g,dis_b]

colored_img,displacements=colorize_image(blue,green,red)
print('Displacement of aligning the red channel to the green channe
l:',displacements[0])
print('Displacement of aligning the green channel to the green channe
l:',displacements[1])
print('Displacement of aligning the blue channel to the green channe
l:',displacements[2])
plt.figure(figsize=(5,5))
plt.axis('off')
plt.imshow(colored_img)
```

```
Displacement of aligning the red channel to the green channel: [9, 0]
Displacement of aligning the green channel to the green channel: (0,
0)
Displacement of aligning the blue channel to the green channel: [24,
0]
```

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Out[5]: <matplotlib.image.AxesImage at 0x10dfc4b50>

c) Multiscale alignment [15 pts]. Now let's try colorizing the high-resolution image 01047u.tif (https://drive.google.com/file/d/1HbmTOLAw_f64wurxJyorOraKGX6EIree/view?usp=sharing). This image is of size 9656 x 3741. Therefore, exhaustive search over all possible displacements will become prohibitively expensive. To deal with this case, we can implement a faster search procedure using an image pyramid. An image pyramid represents the image at multiple scales (usually scaled by a factor of 2) and the processing is done sequentially starting from the coarsest scale (smallest image) and going down the pyramid, updating your estimate as you go. It is very easy to implement by adding recursive calls to your original single-scale implementation. The running time of your implementation should be less than 1 minute.

In [6]:
```
import cv2

def find_best_window(img_a:np.array, img_b:np.array, target_img:np.arr
ay, wd_size:int=15, a_dx:int=0, a_dy:int=0, b_dx:int=0, b_dy:int=0):
    best_a_dx, best_a_dy, best_b_dx, best_b_dy, best_a_score, best_b_s
core = 0, 0, 0, 0, float("-inf"), float("-inf")

    for i in range(-wd_size, wd_size):
        for j in range(-wd_size, wd_size):
            shifted_a = np.roll(img_a, shift=(a_dx + i, a_dy + j), axi
s=(0, 1))
            shifted_b = np.roll(img_b, shift=(b_dx + i, b_dy + j), axi
s=(0, 1))

            a_score = ncc(shifted_a, target_img)
            b_score = ncc(shifted_b, target_img)

            if a_score > best_a_score:
                best_a_score = a_score
                best_a_dx, best_a_dy = i, j

            if b_score > best_b_score:
                best_b_score = b_score
                best_b_dx, best_b_dy = i, j

    return best_a_dx + a_dx, best_a_dy + a_dy, best_b_dx + b_dx, best_
b_dy + b_dy

def align_image_recursively(img_a:np.array, img_b:np.array, target_im
g:np.array, wd_size:int=15):
    if (min(target_img.shape) < 100):
        return find_best_window(img_a, img_b, target_img, wd_size)

    img_a_small = cv2.pyrDown(img_a) # shrink it by half
    img_b_small = cv2.pyrDown(img_b)
    target_img_small = cv2.pyrDown(target_img) # shrink by half
    new_wd_size = min(wd_size*2, 20)

    a_dx, a_dy, b_dx, b_dy = align_image_recursively(img_a_small, img_
b_small, target_img_small, new_wd_size)

    a_dx, a_dy, b_dx, b_dy = 2 * a_dx, 2 * a_dy, 2 * b_dx, 2 * b_dy  #
multiply 2 to adjust back to the 2X bigger images

    return find_best_window(img_a, img_b, target_img, wd_size, a_dx, a
_dy, b_dx, b_dy)

def colorize_image_recursively(b:np.array,g:np.array,r:np.array)->(np.
array, list):
    """
     Align the high-resolution three color channel images. Return the
colored image
     and a list of the displacement vector for each channel.

     :param b: the high-resolution blue channel image, which is a 3218
x3741 numpy array.
     :param g: the high-resolution greeb channel image, which is a 321
```

```
8x3741 numpy array.
    :param r: the high-resolution red channel image, which is a 3218x
3741 numpy array.

    :return: a tuple of (colored_image, displacements). ''colored_ima
ge'' is a 3218x3741x3 numpy array.
    ''displacements'' is a list of the displacement vector for each c
hannel.
    """
    colored_img=None # np.stack([displaced_r, displaced_g, displaced_
b], axis=2)
    displacements=[] # [[red], (green), [blue]]

    # ADD YOUR CODE HERE (15 pts)
    dx_red, dy_red, dx_blue, dy_blue = align_image_recursively(r, b,
g, 3)

    aligned_r = np.roll(r, shift=(dx_red, dy_red), axis=(0, 1))
    aligned_b = np.roll(b, shift=(dx_blue, dy_blue), axis=(0, 1))
    aligned_g = g

    colored_img=np.stack([aligned_r, aligned_g, aligned_b],axis=2)

    displacements = [[dx_red, dy_red], (0, 0), [dx_blue, dy_blue]]

    return colored_img, displacements
```

Table of Contents Expand

In [7]:
```
blue,green,red=crop_bgr_from_img('01047u.tif')
hrs_colored_img, hrs_displacements=colorize_image_recursively(blue,gre
en,red)
print('Displacement of aligning the red channel to the green channe
l:',hrs_displacements[0])
print('Displacement of aligning the green channel to the green channe
l:',hrs_displacements[1])
print('Displacement of aligning the blue channel to the green channe
l:',hrs_displacements[2])
plt.figure(figsize=(40,40))
plt.axis('off')
plt.imshow(hrs_colored_img)
```



```
Displacement of aligning the red channel to the green channel: [48, 1
2]
Displacement of aligning the green channel to the green channel: (0,
0)
Displacement of aligning the blue channel to the green channel: [-26,
-2]
```

Out[7]: <matplotlib.image.AxesImage at 0x12c694ac0>

d) Improve the alignment [5 bonus pts]. The borders of the photograph will have strange colors since the three channels won't exactly align. See if you can devise an automatic way of cropping the border to get rid of the bad stuff. One possible idea is that the information in the good parts of the image generally agrees across the color channels, whereas at borders it does not.

Table of Contents Expand

In [ ]:
```
def crop_border(img:np.array)->np.array:
    """
     Crop the border to get rid of strange colors in the image.

     :param img: the colorized image, which is a 341x396x3 numpy arra
y.
     :return: the improved image, which is a HxWx3 numpy array.

    """
    new_img=None

    # ADD YOUR CODE HERE

    return new_img
```

In [ ]:
```
new_img=crop_border(colored_img)
plt.figure(figsize=(5,5))
plt.axis('off')
plt.imshow(new_img)
```