Topics: Probability Distribution

1. The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable X that has the density function

$$f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \le x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that over a period of one year, a family runs their vacuum cleaner

   (a) fewer than 110 hours;

   (b) between 60 and 100 hours.

2. The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with a cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ 1 - e^{-4x}, & x \ge 0. \end{cases}$$

Find the probability of waiting fewer than 12 minutes between successive speeders

   (a) using the cumulative distribution function of X;

   (b) using the probability density function of X.

3. The time to failure in hours of an important piece of electronic equipment used in a DVD player has the density function

$$f(x) = \begin{cases} \frac{1}{1000} exp(-x/1000), & x \ge 0, \\ 0, & x < 0. \end{cases}$$

   (a) Find F(x).

   (b) Determine the probability that the component (and thus the DVD player) lasts more than 1000 hours before the component needs to be replaced.

   (c) Determine the probability that the component fails before 2000 hours.

4. Magnetron tubes are produced on an automated assembly line. A sampling plan is used periodically to assess quality of the lengths of the tubes. This measurement is subject to uncertainty. It is thought that the probability that a random tube meets length specification is 0.99. A sampling plan is used in which the lengths of 5 random tubes are measured.

(a) Show that the probability function of Y , the number out of 5 that meet length specification, is given by the following discrete probability function:

$$f(y) = \frac{5!}{y!(5-y)!}(0.99)^y(0.01)^{5-y}$$

for y = 0,1,2,3,4,5.

(b) Suppose random selections are made off the line and 3 are outside specifications. Use f(y) above either to support or to refute the conjecture that the probability is 0.99 that a single tube meets specifications.

5. If the joint probability distribution of X and Y is given by

$$f(x,y) = \frac{x+y}{30}, \text{ for } x = 0,1,2,3; \quad y = 0,1,2;$$

(a) $P(X \le 1, Y = 1)$;

(b) $P(X > 1, Y \le 1)$;

(c) $P(X \le Y)$;

(d) $P(X + Y = 2)$.

6. Each rear tire on an experimental airplane is supposed to be filled to a pressure of 40 pounds per square inch (psi). Let X denote the actual air pressure for the right tire and Y denote the actual air pressure for the left tire. Suppose that X and Y are random variables with the joint density function

$$f(x,y) = \begin{cases} k(x^2+y^2), & 30 \le x < 50, 30 \le y \le 50, \\ 0, & \text{elsewhere.} \end{cases}$$

Find

(a) Find k.

(b) Find $P(30 \le X \le 40 \text{ and } 40 \le Y < 50)$

(c) Find the probability that both tires are underfilled.

7. The joint density function of the random variables X and Y is

$$f(x,y) = \begin{cases} 6x, & 0 < x < 1, 0 < y < 1-x, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Show that X and Y are not independent.

(b) Find $P(X > 0.3 | Y = 0.5)$.

8. Let the number of phone calls received by a switchboard during a 5-minute interval be a random variable X with probability function

$$f(x) = \frac{e^{-2}2^x}{x!}, \qquad \text{for } x = 0,1,2,\cdots$$

(a) Determine the probability that X equals 0, 1, 2, 3, 4, 5, and 6.

(b) Graph the probability mass function for these values of x.

(c) Determine the cumulative distribution function for these values of X.

9. A chemical system that results from a chemical reaction has two important components among others in a blend. The joint distribution describing the proportions X_1 and X_2 of these two components is given by

$$f(x_1, x_2) = \begin{cases} 2, & 0 < x_1 < x_2 < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Give the marginal distribution of X_1 .
   (b) Give the marginal distribution of X_2 .
   (c) What is the probability that component proportions produce the results $X_1 < 0.2$ and $X_2 > 0.5$?
   (d) Give the conditional distribution $f_{X_1|X_2}(x_1|x_2)$.

10. Consider the random variables X and Y that represent the number of vehicles that arrive at two separate street corners during a certain 2-minute period. These street corners are fairly close together so it is important that traffic engineers deal with them jointly if necessary. The joint distribution of X and Y is known to be

$$f(x, y) = \frac{9}{16} \cdot \frac{1}{4^{x+y}}$$

for $x = 0, 1, 2, \cdots$ and $y = 0, 1, 2, \cdots$

   (a) Are the two random variables X and Y independent? Explain why or why not.
   (b) What is the probability that during the time period in question less than 4 vehicles arrive at the two street corners?