- 10/17/Mon - midnight

ch15

Q

the

1. longest common subsequence problem.
(LCS)

an optimal substructure property

Notions (개념)

① a sequence → a string over Σ = 문 3 숫자의 열 (pos · length)

ex) Σ = {a, b, c} abba abacc b

② a subsequence → a sequence of S - a seq consists of symbols $s_1, s_2, s_3, \dots s_n$ of S
such that s_1 preceeds $s_2, s_3, \dots s_n$
s_2 " $s_3, s_4 \dots s_n$
⋮
s_n " s_n

Ex) S = abbabaccb
subsequences - abb (o)
- aaacb (o)

③ a common subsequence (of S_1, S_2)

Ex) S_1 = abbaab (bab)
S_2 = babbb

ab
ab → bab

LCS subsequens 2개를 받았을때 common subsequence 중 가장 긴것.

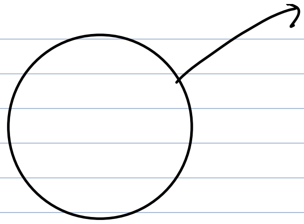
S_1, S_2

A

optimal structure problems

claim X = ⟨0, 0, ⋯ 0⟩ (m)

LCS



$Y = \langle 0, 0, 0, 0 \rangle$ n

LCS: $Z = \langle \underbrace{0, 0}_{}, 0 \rangle$ k, k-1

① if
②
③

3조건이 맞으면 optimal propetery

아니면 contradiction

$X = \langle x_1, x_2, x_3, \ldots x_m \rangle$

① if $x_m = Y_n$ then (1) $x_m = Y_n = Z_k$

(2) $\underbrace{Z_{k-1}}$ is a LCS of X_{m-1}, Y_{n-1}

② if $x_m \neq Y_n$ then $Z_k = x_m$

③ if $x_m \neq Y_n$ then $Z_k = Y_n$

m = 8

Ex) $X = \langle A, B, C, B, D, A, B, C \rangle$ n = 7

$Y = \langle B, D, C, A, B, A, C \rangle$

$Z = \langle B, C, B, A, C \rangle$ k = 5

$Z_4 = \langle B, C, B, A \rangle$

prove ①

1) $\boxed{x_m = Y_n}$ but $x_m \neq Z_k$ contradiction

LCS

x_m, Y_n(마지막)이 같으면

1. LCS

LCS의 마지막 인덱스는 항상 이어야한다.

2. $\boxed{\quad X}$

X 3. $\boxed{}$

prove ②

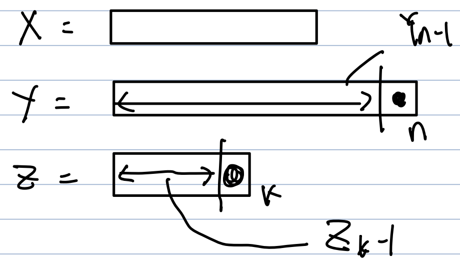
x' $\qquad x_m = B \quad m=7$

Ex) $x = \langle A, B, C, B, D, A, B \rangle \qquad n=6$

$Y = \langle B, D, C, A, B, A \rangle \quad y_n = A$

Y'

$Z = \langle B, C, B, A \rangle \quad z_k = A \quad k=4$



Z is LCS of X_{m-1} and Y

① $x_m \neq y_n$

② $z_k = y_n$

Not Z_{k-1} is an LCS of X and Y_{n-1}

Q

2) Designing an algorithm

input: $\#2^m$ 개의 distinct integers, $m \geq 1$, $2 = n$

output: max, min

$\#$ of comparisons $= \boxed{\frac{3}{2}n - 2}$

5 1 6 2 $n=4$

4

max $\boxed{3}$ $n-1$

min $n-2$

$2n-3$

A

Find min, max in a list that has 2^k #'s $k \geq 1$

$\#$ of comparisons $= \frac{3}{2}n - 2$

max min (S)

if $|S| = 2$

let $S = \{a_1, a_2\}$

return $(\max(a_1, a_2), \min(a_1, a_2))$ ← 1 comparison

else

divide S into two sets of equal size S_1, S_2

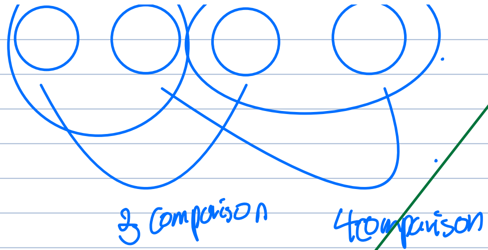
$(\max 1, \min 1) \leftarrow \text{maxmin}(S_1)$

$(\text{Max}2, \text{Min}2) \leftarrow \text{MaxMin}(S_2)$

return $(\text{Max}(\text{Max}1, \text{Max}2), \min(\min 1, \min 2))$ ← 1 comparison

1 comparison

1 comparison 2 comparison



$$T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + 2 & n \geq 4 \\ 1 & n = 2 \end{cases}$$

$$T(n) = \begin{cases} 1 & \text{if } n = 2 \\ 2T\left(\frac{n}{2}\right) + 2 & \text{if } n \geq 4 \end{cases}$$

$T(n) = 2T\left(\frac{n}{2}\right) + 2$ $\qquad T(n) = 2^2 \cdot T\left(\frac{n}{4}\right) + 2^2 + 2 = 2^3\left(T\frac{n}{2^3}\right) + 2^3 + 2^2 + 2$

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + 2$

$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + 2$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + \underbrace{2^k + 2^{k-1} \dots + 2}_{k}$$

$T(2) = 1$ $\qquad \frac{n}{2^k} = 2$ $\quad$ since $n = 2^k$ $\qquad \frac{n}{2^{k-1}} = 2$

$n = 2$

$2^k = \frac{n}{2}$ $\qquad 2^{k-1} = \frac{n}{2}$

$k = \log_2 \frac{n}{2}$ $\qquad k = \log_2 \frac{n}{2} + 1$

$$= 2^{\log_2 \frac{n}{2}} \cdot T(2) + \frac{2\left(2^{\log_2 \frac{n}{2}} - 1\right)}{2 - 1}$$

$$= \frac{n}{2} + 2 \cdot \left(\frac{n}{2} - 1\right)$$