1. (a) $x \times 100 = 110 \quad x = 1.1$

$P(0 < x < 1)$

$\int_0^1 x\,dx = \int_0^1 \frac{1}{2}x^2 = \frac{1}{2} = 0.5$ ①

$P(1 \le x < 1.1)$

$\int_1^{1.1} (2-x)\,dx = \int_1^{1.1} (2x - \frac{1}{2}x^2)$

$= 2.2 - 0.605 - 2 + 0.5$

$= 0.2 - 0.105$

$= 0.095$ ②

$0.5 + 0.095 = 0.595$

$\boxed{\therefore 0.595}$

(b) $60 < 100x < 100$

$0.6 < x < 1$

$\int_{0.6}^1 x\,dx = \int_{0.6}^1 \frac{x^2}{2} = \frac{1}{2} - 0.18 = 0.32$

$\boxed{\therefore 0.32}$

2. (a) 12mi $= \frac{12}{60} = \frac{1}{5} = 0.2$

$F(0.2) = 1 - e^{-4(0.2)} = 1 - e^{-0.8} = 1 - 0.4493\ldots$

$= 0.55067$

$\boxed{\therefore 0.5507}$

(b) $F(x)'$ for $0 \le x$

$= \frac{d}{dx}(1 - e^{-4x})$

$= \frac{d}{dx} \cdot 1 - \frac{d}{dx}(e^{-4x})$

$= 0 - e^{-4} \cdot \frac{d}{dx}(-4x)$

$= 4 \cdot e^{-4x}$

$= 4e^{-4x}$

$\int_0^{0.2} 4e^{-4x}\, dx$

$= 4\int_0^{0.2} e^{-4x}\, dx$

$= \Big[-e^{-4x}\Big]_0^{0.2} = -e^{-4(0.2)} + e^0$

$= -e^{-0.8} + 1 = 1 - e^{-0.8}$

$= 1 - 0.499\ldots$

$= \boxed{0.5507}$

3. (a) $P(0 \le x)$

$$= \int_0^x \frac{1}{1000} \exp(-x/1000)\, dx$$

$$= \int_0^x \frac{1}{1000} e^{(-x/1000)}\, dx$$

$$= \int_0^x -e^{-\frac{x}{1000}} + C$$

$$= -e^{-\frac{x}{1000}} + 1$$

$$= 1 - e^{-\frac{x}{1000}} \text{ for } x \geq 0$$

(b) $x = 1000$

$$f(1000) = \frac{1}{1000} e^{(-\frac{1000}{1000})}$$

$$= \frac{1}{1000} e^{-1}$$

$$= \frac{1}{1000e} = 0.0003678\ldots$$

$$= \dot{=}\ 3.678 \times 10^{-4}$$

(c) $F(2000)$

from (a) for all $x \geq 0$ $F(x) = 1 - e^{-\frac{x}{1000}}$

$$F(2000) = 1 - e^{-\frac{2000}{1000}}$$

$$= 1 - e^{-2}$$

$= 1 - 0.1353\ldots$

$= 0.8646\ldots$

$\boxed{\therefore\ 0.8647}$

4) $P(t) = 0.99$

(a) for $Y=0$ $\quad f(0) = \frac{5!}{0!(5)!}(0.99)^0 \cdot (0.01)^5 = (0.01)^5$

for $Y=1$ $\quad f(0) = \frac{5!}{1!\,4!} \cdot (0.99)^1 (0.01)^4 = 5 \cdot (0.99) \cdot (0.01)^4$

for $Y=2$ $\quad f(2) = \frac{5!}{2!3!} \cdot (0.99)^2 (0.01)^3 = 10 \cdot (0.99)^2 \cdot (0.01)^3$

for $Y=3$ $\quad f(3) = \frac{5!}{3!2!} (0.99)^3 (0.01)^2 = 10 \cdot (0.99)^3 \cdot (0.01)^2$

for $Y=4$ $\quad f(4) = \frac{5!}{4!\,1!} \cdot (0.99)^4 (0.01)^1 = 5 \cdot (0.99)^4 \cdot (0.01)$

for $Y=5$ $\quad f(5) = \frac{5!}{5!0!} \cdot (0.99)^5 \cdot (0.01)^0 = (0.99)^5$

$= (0.01)^5 + 5(0.99)(0.01)^4 + 10(0.99)^2(0.01)^3 + 10(0.99)^3(0.01)^2$
$+ 5(0.99)^4(0.01) + (0.99)^5$

$0.01 = x \quad 0.99 = y$

$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

$= (x+y)^5$

$\therefore (0.99 + 0.01)^5 = 1^5 = 1$ $\quad\quad$ $\therefore$ A vaild density fcenction

(b) 3 tubes are outside specifications

$\therefore$ $Y \leq 2$ are right fit

$Y=1$ when we know $Y \le 2$ are correct

$$P(Y=1 \mid Y \le 2) = \frac{P((Y=1) \cap (Y \le 2))}{P(Y \le 2) \to P(0)+P(1)+P(2)} \qquad P(Y=1) = \frac{5!}{1!4!} \cdot (0.99)^1 (0.01)^4$$

$$= \frac{\frac{5!}{1!4!} \cdot (0.99)^1 (0.01)^4}{\frac{5!}{0!5!} \cdot (0.99)^0 (0.01)^5 + \frac{5!}{1!4!} (0.99)^1 (0.01)^4 + \frac{5!}{2!3!} (0.99)^2 (0.01)^3}$$

$$= \frac{5 \cdot (0.99)(0.01)^4}{(0.01)^5 + (5 \cdot (0.99)(0.01)^4) + 10 \cdot (0.99)^2 \cdot (0.01)^3}$$

$$= \frac{0.0000000495}{0.00000098506} = 0.00502\ldots \boxed{0.005 \text{ Not true}}$$

5

$$f(x,y) = \frac{x+y}{30}, \text{ for } x = 0,1,2,3 \quad y = 0,1,2;$$

(a) $P(X \le 1, Y = 1)$;

$$\sum_{x \le 1} \frac{x+1}{30}$$

$$\begin{cases} (0,1) \\ (1,1) \end{cases}$$

$$(0,1) = \frac{1}{30}$$

$$(1,1) = \frac{2}{30}$$

$$\frac{1}{30} + \frac{2}{30} = \frac{3}{30} = \boxed{\frac{1}{10}}$$

(b) $P(X > 1, Y \le 1)$

$x = 2, 3$

$y = 0, 1$

$$(2,0) = \frac{2}{30}$$

$$(2,1) = \frac{3}{30}$$

$$(3,0) = \frac{3}{30}$$

$$(3,1) = \frac{4}{30}$$

$$\frac{2+3+3+4}{30} = \frac{\cancel{12}}{\cancel{30}} = \frac{\cancel{6}}{15} = \boxed{\frac{2}{5}}$$

(c) $P(X \le Y)$

$(0,0)(0,1),(0,2)$
$(1,1)(1,2)$
$(2,2)$

$(0,0)=\frac{0}{30}$
$(0,1)=\frac{1}{30}$
$(0,2)=\frac{2}{30}$
$(1,1)=\frac{2}{30}$
$(1,2)=\frac{3}{30}$
$(2,2)=\frac{4}{30}$

$$\frac{0}{30}+\frac{1}{30}+\frac{2}{30}+\frac{2}{30}+\frac{3}{30}+\frac{4}{30}=\frac{12}{30}=\boxed{\frac{2}{5}}$$

(D) $P(X+Y=2)$

$$(0,2)(1,1)(2,0) = \frac{2}{30}\times 3 = \frac{2}{10}=\boxed{\frac{1}{5}}$$

6.
(a) $\int_{30}^{50}\int_{30}^{50} k(x^2+y^2)\,dx\,dy$ suppose to be 1

$$k\int_{30}^{50} \frac{x^3}{3}+xy^2\cdot dy\Big|_{30}^{50} \quad \frac{(50)^3}{3}+50y^2-\frac{(30)^3}{3}-30y^2 = \frac{98000}{3}+20y^2$$

$$k\int_{30}^{50}\left(\frac{98000}{3}+20y^2\right)dy$$

$$= k\cdot\left(\frac{98000}{3}y+\frac{20}{3}y^3\right)\Big|_{30}^{50}$$

$$= k \cdot \left(\frac{98000}{3}(50) + \frac{20}{3}(50)^3 - \frac{98000}{3}(30) - \frac{20}{3}(30)^3 \right)$$

$$= 1 = k \cdot \left(\frac{4900000 + 2500000 - 2940000 - 540000}{3} \right)$$

$$1 = k \cdot \left(\frac{3920000}{3} \right)$$

$$k = \frac{3}{3920000} \approx 7.65 \times 10^{-7}$$

(b) $$\int_{30}^{40} \int_{40}^{50} k(x^2+y^2)\, dx\, dy$$

$$= k \cdot \int_{30}^{40} x^2 y + \frac{y^3}{3} \Big|_{40}^{50}$$

$$\begin{array}{r} 125000 \\ -64000 \\ \hline 61000 \end{array}$$

$$50x^2 + \frac{2500}{3} - 40x^2 - \frac{1600}{3}$$

$$= 10x^2 + \frac{61000}{3}$$

$$k \cdot \left(10x^2 + \frac{61000}{3} \right) \cdot dx \Big|_{30}^{40}$$

$$= k \cdot \left(\frac{10}{3}x^3 + \frac{61000}{3}x \right) \Big|^{40}$$

$(\qquad 1130$

$$= k \cdot \left(\frac{370000}{3} + \frac{610000}{3} \right)$$

$$= k \cdot \left(\frac{980000}{3} \right)$$

$$= \frac{\cancel{3}}{3420000} \times \frac{980000}{\cancel{3}} = \frac{98\cancel{0000}}{342\cancel{0000}}$$

$$= \frac{\cancel{98}}{\cancel{342}} \quad \frac{49}{196}$$

$$= \frac{7}{28} = \frac{1}{4}$$

$$= 0.25$$

(c) $k \cdot \int_{30}^{40} \int_{30}^{40} (x^2 + y^2)\, dx\, dy$

$$= k \cdot \int_{30}^{40} x^2 y + \frac{y^3}{3} \Big|_{30}^{40}$$

$$= k \cdot \int_{30}^{40} \left(10x^2 + \frac{37000}{3} \right) dx$$

$$= k \left(\frac{10}{3} x^3 + \frac{37000}{3} x \right) \Big|_{30}^{40}$$

$$= k \cdot \left(\frac{10}{3} (40)^3 + \frac{37000}{3} (40) - \frac{10}{3} (30)^3 - \frac{37000}{3} (30) \right)$$

$$= k\left(\frac{640000-270000}{3} + \frac{370000}{3}\right)$$

$$= k\left(\frac{740000}{3}\right) = \frac{\cancel{8}}{3920000} \times \frac{740000}{\cancel{3}}$$

$$= \frac{74}{392} = \frac{37}{196}$$

$$\boxed{\therefore \frac{37}{196}}$$

≈ 0.189

7: $f(x,y) = \begin{cases} 6x, & 0<x<1, 0<y<1-x, \\ 0, & \text{elsewhere} \end{cases}$

(a) $g(x) = \int_0^{1-x} 6x \cdot dy$

$$= 6xy \Big|_0^{1-x} = 6(x-x^2)$$

$$= \underbrace{6x-6x^2}\quad \text{for } 0<x<1$$

$h(y) = \int_0^1 6x \cdot dx$, $y<1-x$

$= x<1-y$

$$= 3x^2 \Big|_0^{1-y} = 3(1-y)^2$$

$$g(x)h(y) = 3(1-y)^2(6x-6x^2) \neq 6x$$

$$\boxed{g(x)\,h(y) \neq f(x,y)}$$

(b) $P(X > 0.3 \mid Y = 0.5)$

inorder to Y be 0.5

$0 < y < 1-x$

$1-x$ has to be bigger than 0.5

$1-x > 0.5$

$0.5 > x$ $\therefore$

$0.3 < x < 0.5$

$$\int_{0.3}^{0.5} 6x \, dx = 3x^2 \Big|_{0.3}^{0.5}$$

$$= 3(0.5)^2 - 3(0.3)^2$$

$$= 3\left(\frac{25}{100} - \frac{9}{100}\right)$$

$$= \frac{48}{100} = 0.48$$

8. $f(0) = \dfrac{e^{-2}2^0}{0!} = \dfrac{e^{-2}}{1} = e^{-2}$

$f(1) = \dfrac{e^{-2}2^1}{1!} = \dfrac{2e^{-2}}{1} = 2e^{-2}$

$$f(2) = \frac{e^{-2}2^2}{2!} = \frac{e^{-2}\times \cancel{4}}{\cancel{2\times 1}} = 2e^{-2}$$

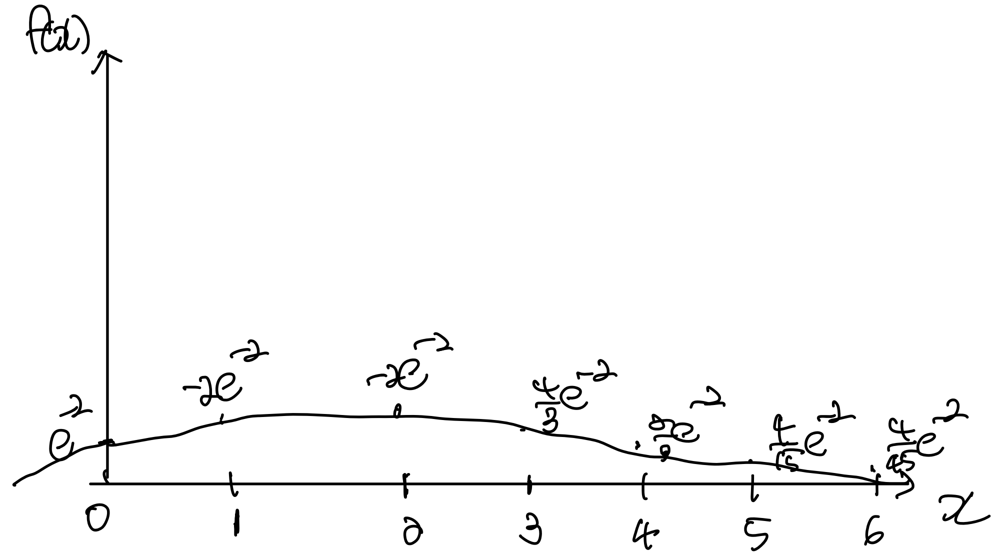
$$f(3) = \frac{e^{-2}2^3}{3!} = \frac{e^{-2}\cdot 8}{3\times 2\times 1} = \frac{4}{3}e^{-2}$$

$$f(4) = \frac{e^{-2}\cdot 2^4}{4!} = \frac{e^{-2}\cdot 16}{4\times 3\times 2\times 1} = \frac{2e^{-2}}{3}$$

$$f(5) = \frac{e^{-2}\cdot 2^5}{5!} = \frac{e^{-2}\,\cancel{32}\,\cancel{16}\,4}{5\times \cancel{4}\times 3\times \cancel{2}\times 1} = \frac{4}{15}e^{-2}$$

$$f(6) = \frac{e^{-2}\cdot 2^6}{6!} = \frac{e^{-2}\cdot \cancel{64}\,\cancel{32}\,\cancel{8}\,4}{\cancel{36}\times 5\times \cancel{4}\times \cancel{3}\times \cancel{2}\times \cancel{1}} = \frac{4}{45}e^{-2}$$

(b)



(c) $P(X \le 0) = P(X=0) = e^{-2}$

$P(X \le 1) = P(X=0) + P(X=1) = e^{-2} + 2e^{-2} = 3e^{-2}$

$P(X \le 2) = P(X \le 1) + P(X=2) = 3e^{-2} + 2e^{-2} = 5e^{-2}$

$P(X \le 3) = P(X \le 2) + P(X=3) = 5e^{-2} + \frac{4}{3}e^{-2} = \frac{19}{3}e^{-2}$

$P(X \le 4) = P(X \le 3) + P(X=4) = \frac{19}{3}e^{-2} + \frac{2}{3}e^{-2} = \frac{21}{3}e^{-2}$

$$P(X \le 5) = P(X \le 4) + P(X = 5) = \frac{21}{3}e^{-2} + \frac{4}{15}e^{-2} = \frac{109}{15}e^{-2}$$

$$P(X \le 6) = P(X \le 5) + P(X = 6) = \frac{109}{15}e^{-2} + \frac{4}{45}e^{-2}$$

$$= \frac{327+4}{45}e^{-2}$$

$$= \frac{331}{45}e^{-2}$$

9.

$$f(x_1, x_2) = \begin{cases} 2, & 0 < x_1 < x_2 < 1, \\ 0 & \text{elsewhere} \end{cases}$$

(a)

$$g(x_1) = \int_{x_1}^{1} 2 \cdot dx_2$$

$$= 2x_2 \Big|_{x_1}^{1}$$

$$= 2 - 2x_1$$

(b)

$$h(x_2) = \int_{0}^{x_2} 2 \cdot dx_1$$

$$= 2x_1 \Big|_{0}^{x_2}$$

$$= 2x_2$$

(c) $P(X_1 < 0.2, X_2 > 0.5)$

$$\int_0^{0.2}\int_{0.5}^{1} 2 \, dx\, dy$$

$$= \int_0^{0.2} 2y \Big|_{0.5}^{1}$$

$$= \int_0^{0.2} 1 \, dx$$

$$= x \Big|_0^{0.2} = 0.2$$

(D) $f_{X_1|X_2}(x_1|x_2)$ for $0 < x_1 < x_2 < 1$

$$= \frac{f(x_1, x_2)}{h(x_2)}$$

$h(x_2) = 2x_2$

$$\frac{f(x_1, x_2)}{2x_2} = \frac{\cancel{2}}{\cancel{2}x_2} = \frac{1}{x_2}$$

otherwise, 0

10.

$$g(x) = \sum_{y=0}^{\infty} \left(\frac{9}{16} \cdot \frac{1}{4^{x+y}} \right)$$

$$= \sum_{y=0}^{\infty} \frac{9}{16(4^{x+y})}$$

$$= \sum_{y=0}^{\infty} \left(\frac{9}{4^x \cdot 16} \times \frac{1}{4^y} \right)$$

$$= \frac{9}{4^x \cdot 16} \sum_{y=0}^{\infty} \left(\frac{1}{4^y} \right) -$$

$$\frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$= \frac{\cancel{9}\,3}{\cancel{4}^{x+1}} \cdot \left(\frac{\cancel{4}}{\cancel{3}} \right)$$

$$= \frac{3}{4^{x+1}}$$

$$h(y) = \sum_{x=0}^{\infty} \left(\frac{9}{16} \times \frac{1}{4^{x+y}} \right)$$

$$= \sum_{x=0}^{\infty} \frac{9}{16(4^{x+y})}$$

$$= \frac{9}{4^y \cdot 16} \sum_{x=0}^{\infty} \left(\frac{1}{4^x} \right)$$

$$= \frac{\cancel{9}}{4^{\cancel{y+2}}} \times \frac{\cancel{4}}{\cancel{3}} = \frac{3}{4^{y+1}}$$

$g(x) \cdot h(y)$ suppose to be $f(x,y)$

$$\frac{3}{4^{x+1}} \cdot \frac{3}{4^{y+1}} = \frac{9}{4^{x+y+2}} = \frac{9}{16 \cdot 4^{x+y}}$$

$$= \frac{9}{16} \cdot \frac{1}{4^{x+y}}$$

(b) $P(x+y<4)$

$(0,0), (0,1), (0,2), (0,3)$
$(1,0), (1,1), (1,2)$
$(2,0), (2,1)$
$(3,0)$

$$P(0,0) + P(0,1) + P(0,2) \ldots + P(3,0)$$

$$= \frac{9}{16} \cdot \left(\frac{1}{4^{0+0}} + \frac{1}{4^{0+1}} \cdots + \frac{1}{4^{3+0}} \right)$$

$$= \frac{9}{16} \cdot \left(\frac{1}{1} + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^3} \right)$$

$$= \frac{9}{16} \times \left(\frac{64+16+4+1+16+4+1+4+1+1}{\;} \right)$$

20 64 20 8 40

$$= \frac{9}{16} \times \left(\frac{112}{64} \right)$$

$$= \frac{9}{\cancel{16}\ \cancel{8}\ \cancel{4}\ \cancel{2}} \times \frac{\cancel{112}\ \cancel{56}\ \cancel{28}\ \cancel{14}\ 7}{64} = \frac{63}{64}$$