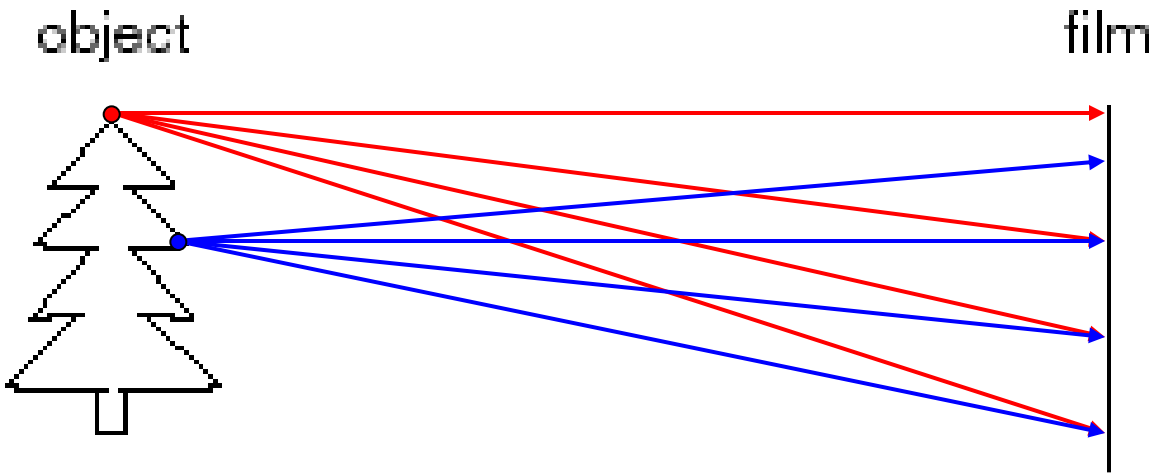
Understanding image formation

slides adapted from Svetlana Lazebnik

Overview

- Pinhole projection model
  - Qualitative properties
  - Perspective projection matrix
- Cameras with lenses
  - Lens aberrations
- Digital sensors

Let’s design a camera

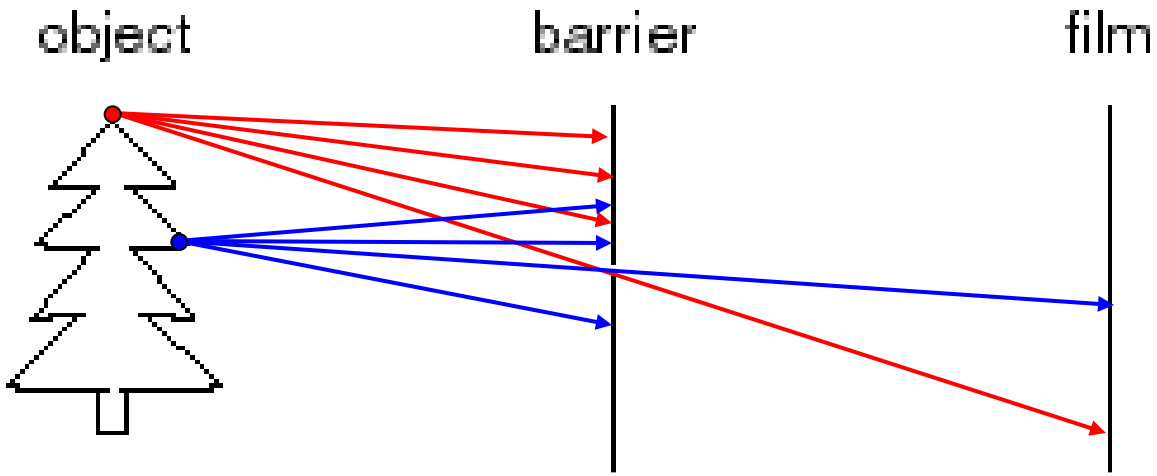


Idea 1: Put a piece of film in front of an object

Do we get a reasonable image?

Slide by Steve Seitz

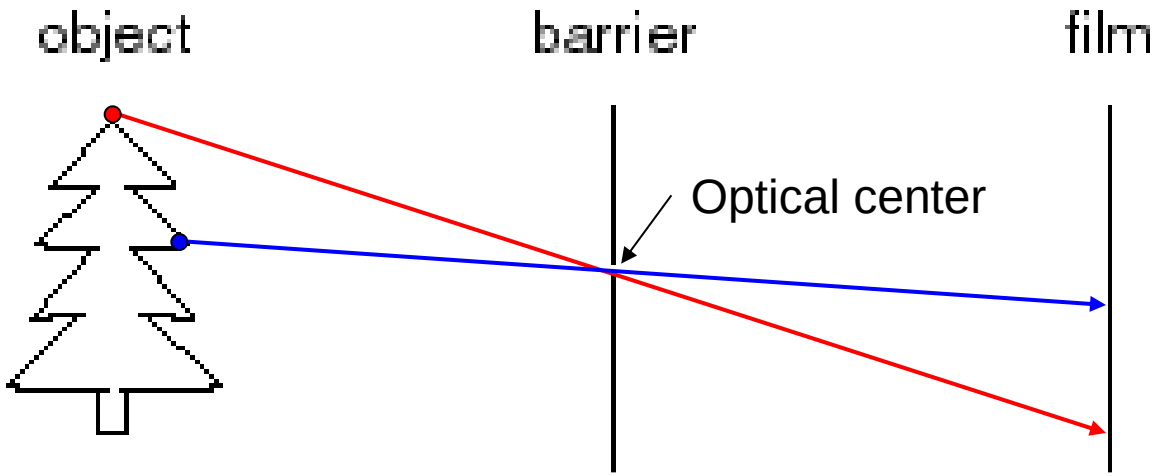
Pinhole camera



Add a barrier to block off most of the rays

Slide by Steve Seitz

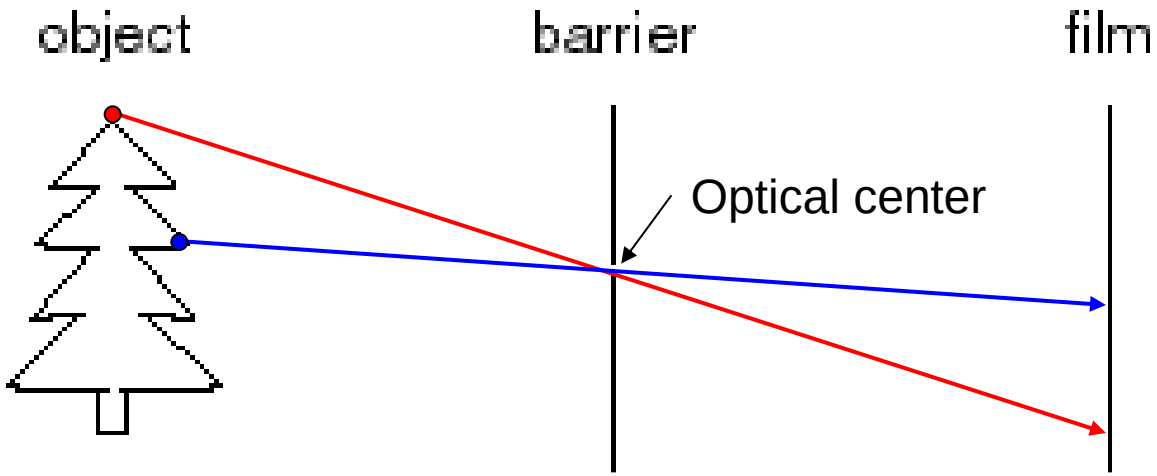
Pinhole camera



- Captures **pencil of rays** – all rays through a single point: **optical center**
- The width of the “hole” is called **aperture**.
- How does the aperture affect the image formation?

Slide by Steve Seitz

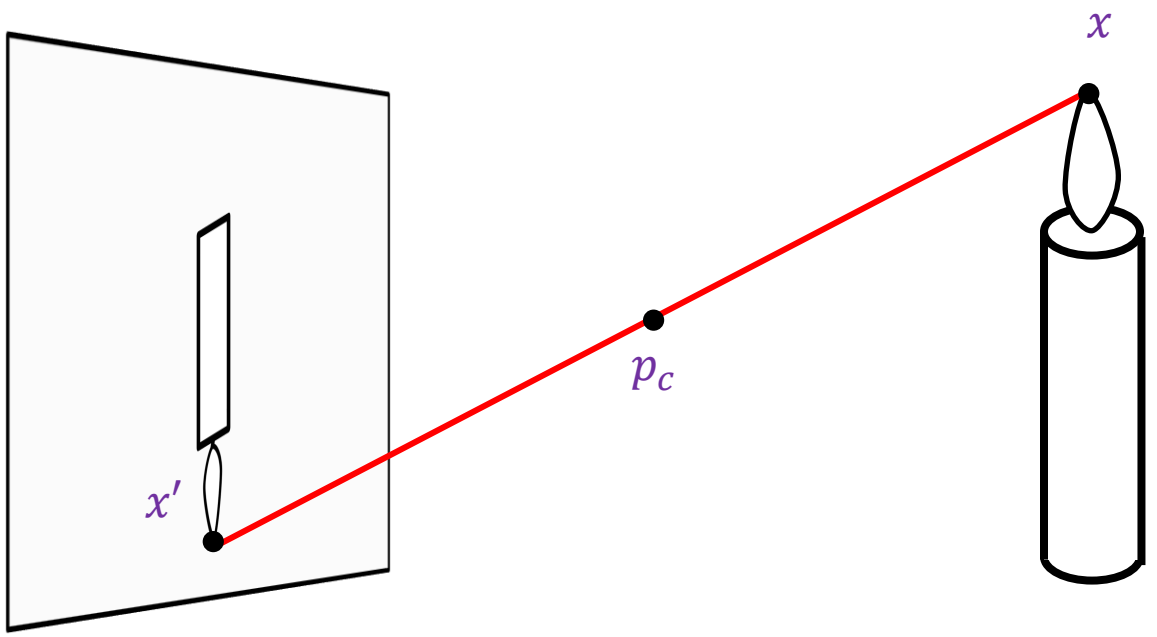
Pinhole camera



- Captures **pencil of rays** – all rays through a single point: **optical center**
- The width of the “hole” is called **aperture**.
- Larger the aperture, brighter but blurrier the image!

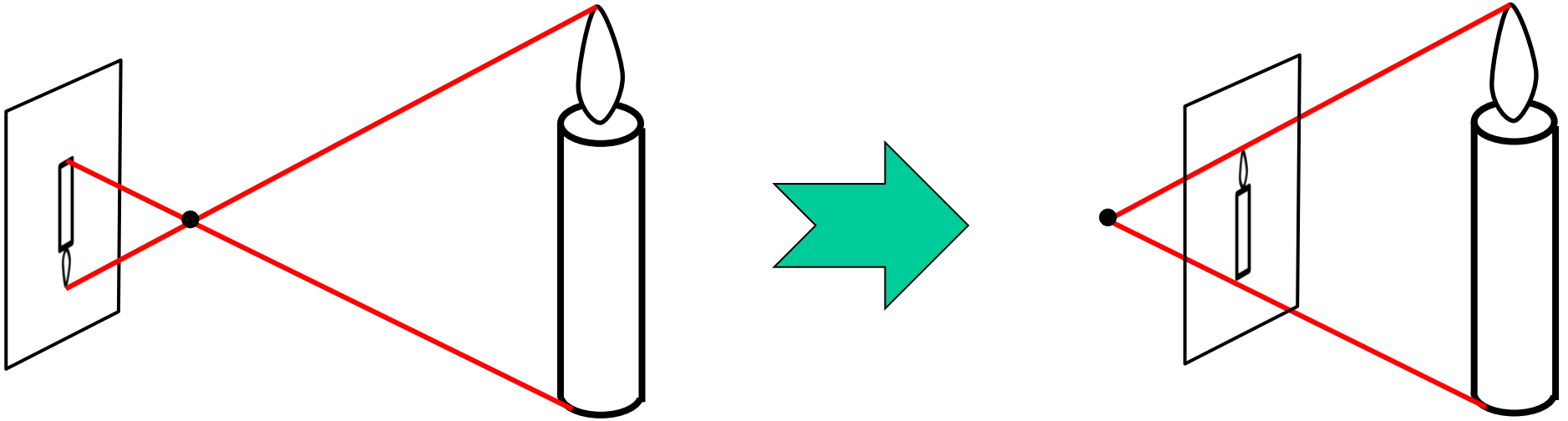
Slide by Steve Seitz

Modeling projection



- How do we find the projection x' of a scene point x?
  - Form the **visual ray** connecting x to the camera center p_c and find where it intersects the image plane
- All scene points that lie on this visual ray have the same projection in the image
- Are there scene points for which this projection is undefined?

Modeling projection



Note: instead of dealing with an image that is upside down, most of the time we will pretend that the image plane is *in front* of the camera center

Properties of projection

- Real-world sizes (lengths) are *not* preserved in projection
  - What other properties are/are not preserved?

Piero della Francesca, *Flagellation of Christ*, 1455-1460

slido

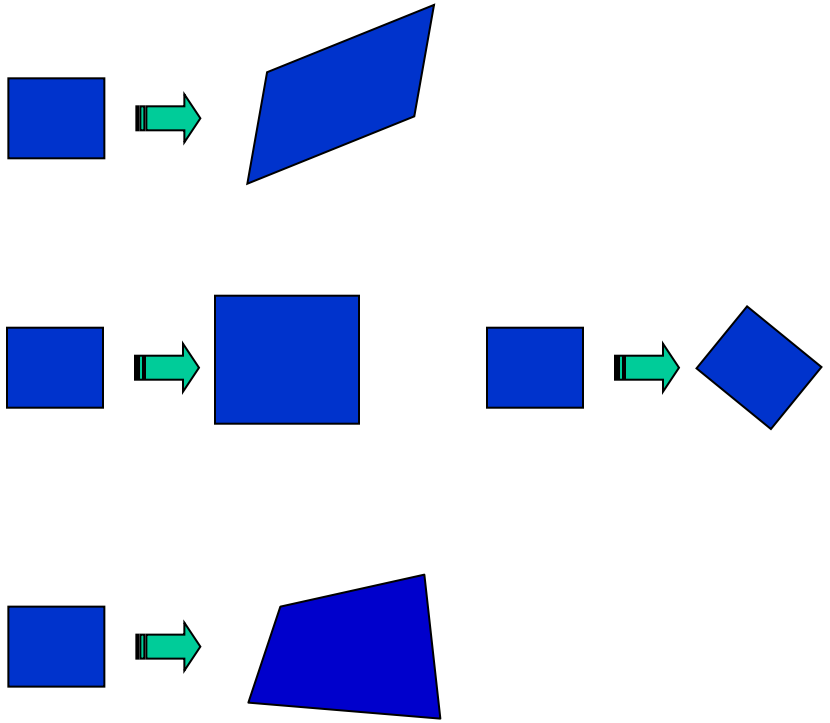
Please download and install the Slido app on all computers you use

What else is not preserved in perspective projection?

ⓘ Start presenting to display the poll results on this slide.

2D transformation models

- Affine
  - Similarity (translation, scale, rotation)
- Projective (homography)



Projection of lines

- What happens to parallel lines in projection?

Piero della Francesca, *Flagellation of Christ*, 1455-1460

Projection of lines

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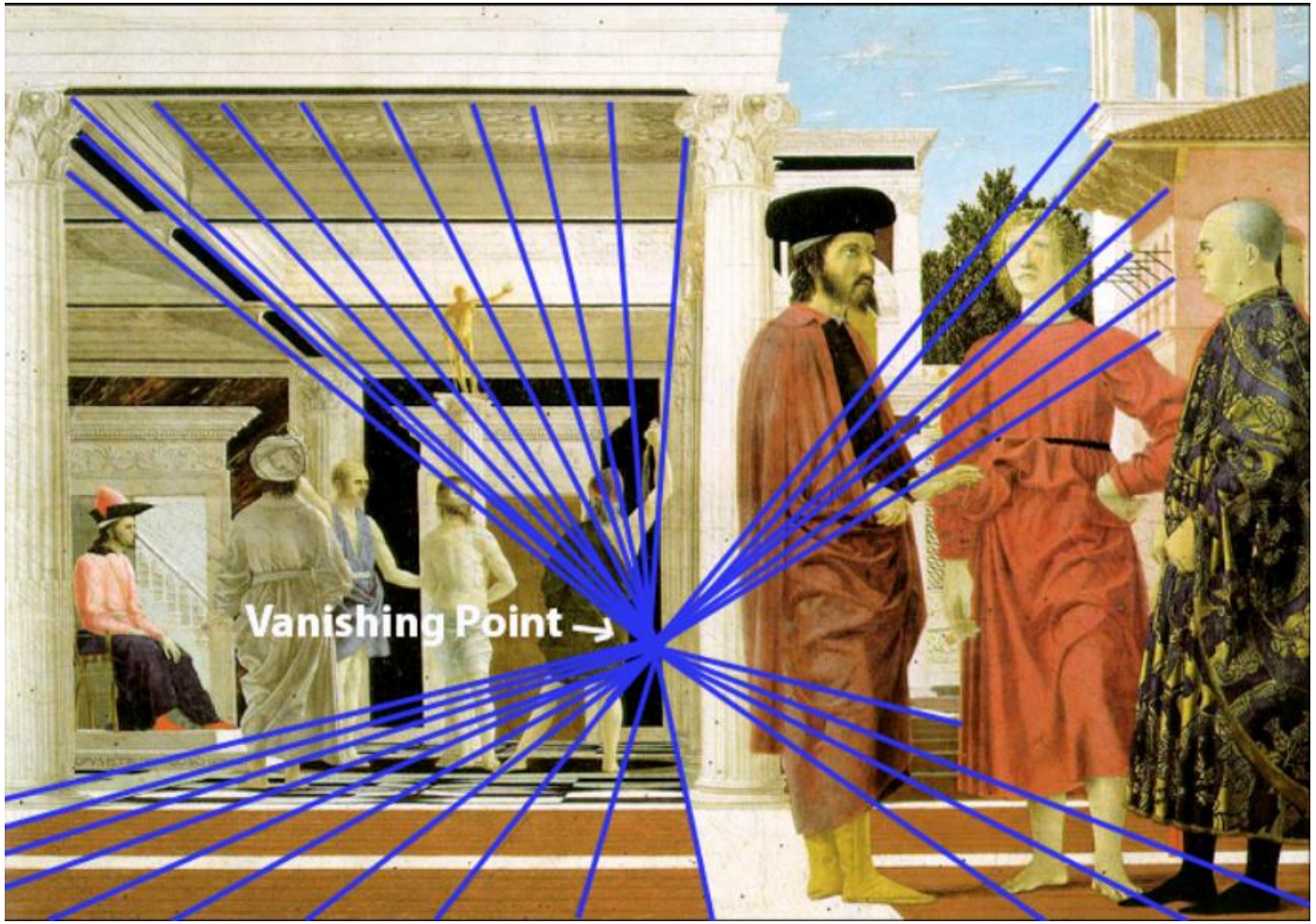
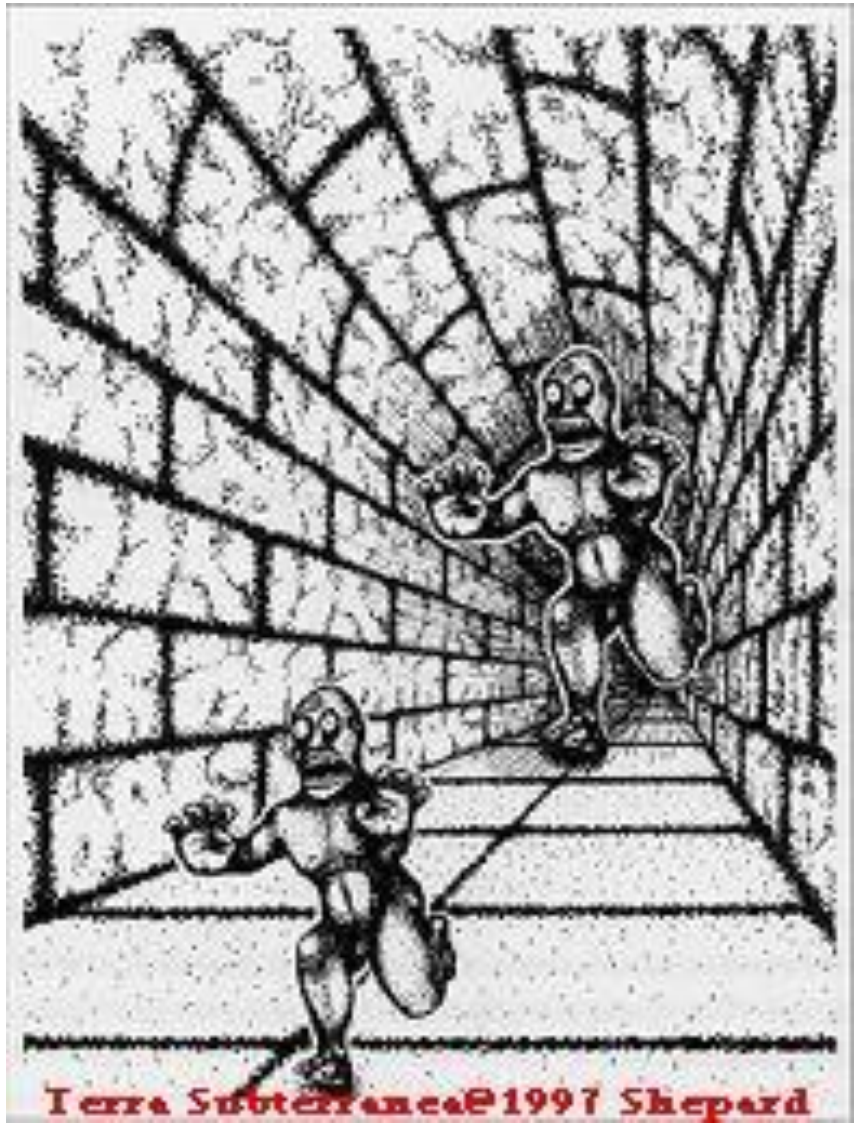


Piero della Francesca, *Flagellation of Christ*, 1455-1460

Image source

Converging lines are a powerful perspective cue



Slide by Steve Seitz

Projection of 3D shapes

- What is the shape of the projection of a sphere under ideal conditions?

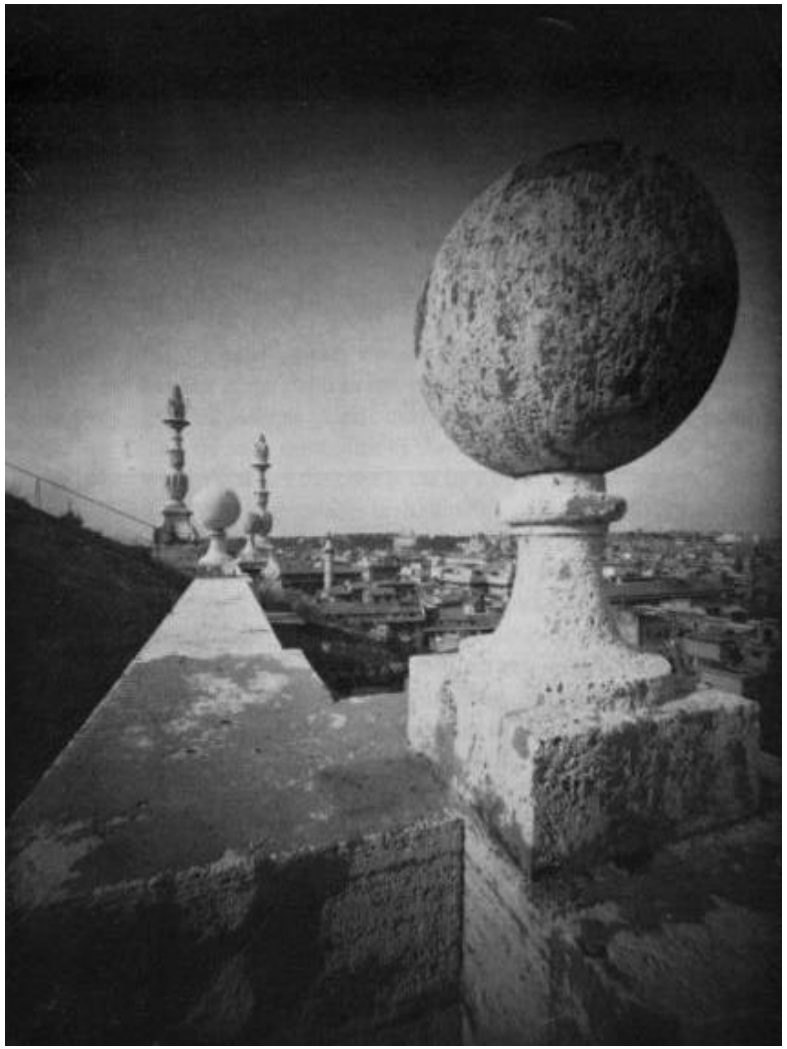


Image source: F. Durand

Overview

- Pinhole projection model
  - Qualitative properties
  - Perspective projection matrix

Homogeneous coordinates

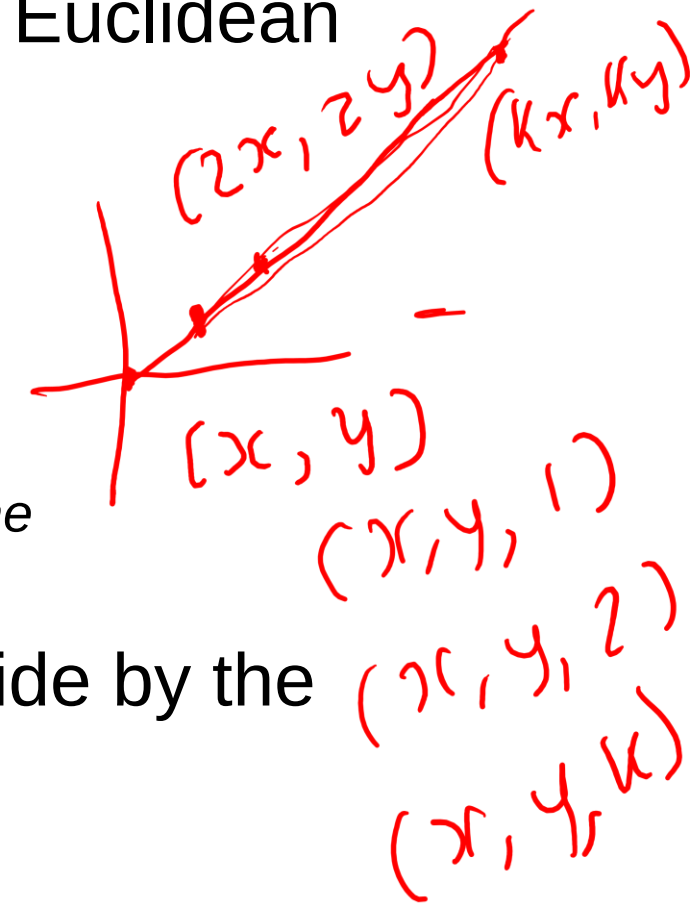
- To form homogeneous coordinates from normal Euclidean coordinates, append 1 as the last entry:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous *image* coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous *scene* coordinates



- To convert *from* homogeneous coordinates, divide by the last entry:

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

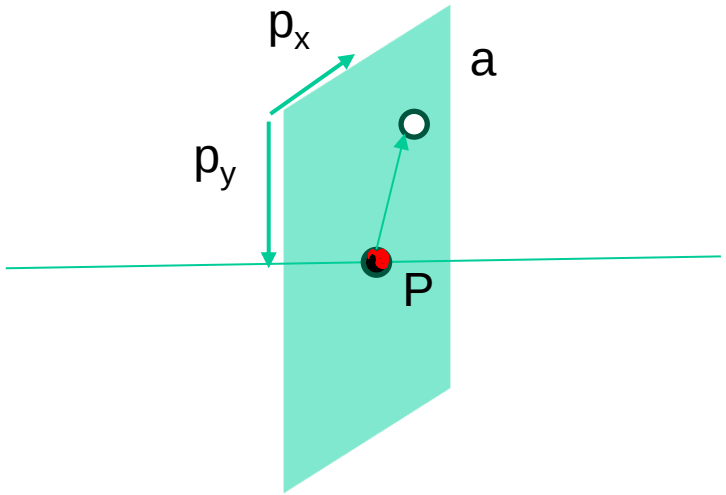
$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

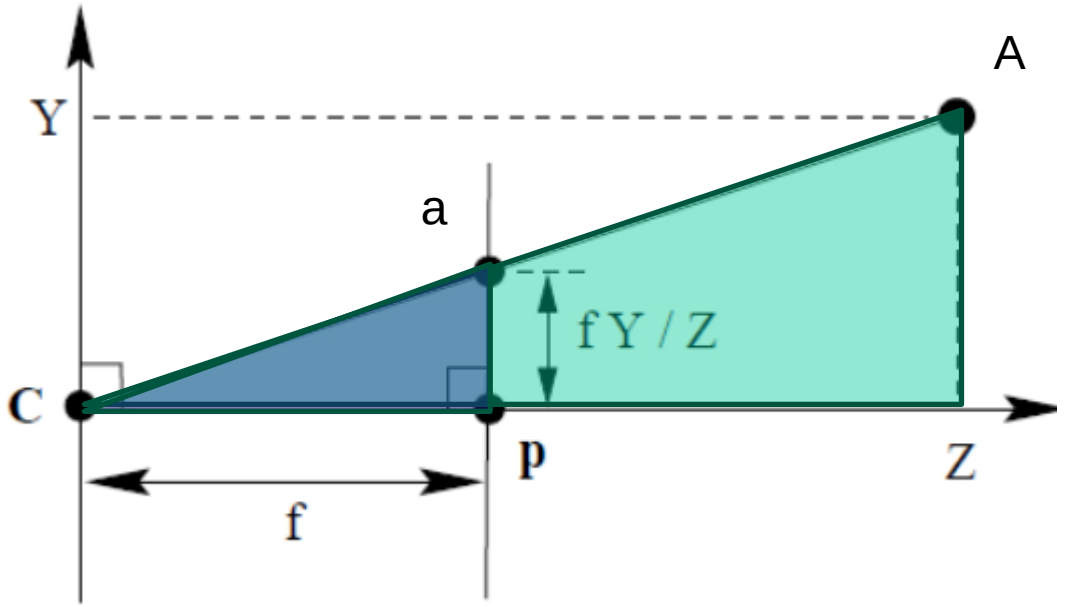
In homogeneous coordinates, all scalar multiples represent the same point!

Perspective Projection

$$\frac{y}{f} = \frac{Y}{Z}$$

$$\therefore y = \frac{fY}{Z}$$





Similarly,

$$x = \frac{fX}{Z}$$

C = camera center
P = principal point (Image center)
f = focal length
A = point in 3D at (X,Y,Z)
a = point on image plane at (x,y)

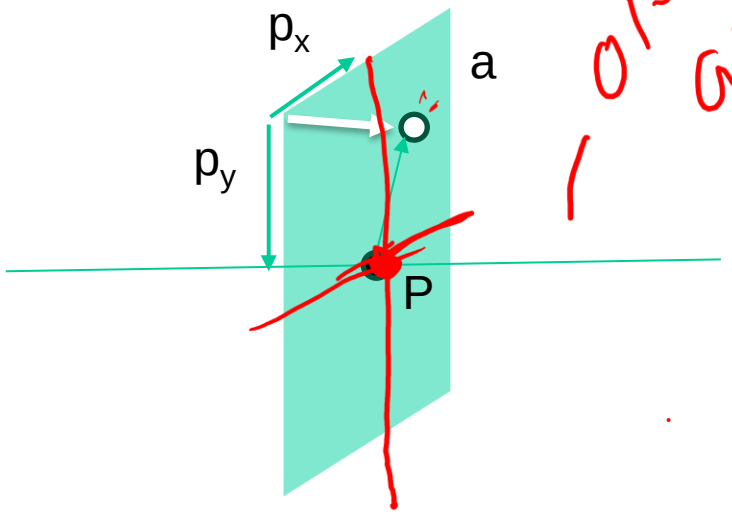
Perspective Projection

$$\frac{y}{f} = \frac{Y}{Z}$$

$$\therefore y = \frac{fY}{Z} + p_y$$

Similarly,

$$x = \frac{fX}{Z} + p_x$$



$$\frac{y}{f} = \frac{Y}{Z}$$

A

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C = camera center
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f = focal length
A = point in 3D at (X,Y,Z)
a = point on image plane at (x,y)

Perspective Projection Matrix

- Projection is a matrix multiplication using homogeneous coordinates:

$$= \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \Rightarrow \left(f_x \frac{x}{z} + px, f y \frac{y}{z} + py \right)$$

divide by the third coordinate

$$\begin{pmatrix} \text{2D point (3x1)} \\ \lambda \boldsymbol{x} \end{pmatrix} = \begin{pmatrix} \text{Camera to pixel coord. trans. matrix (3x3)} \\ \boldsymbol{K} \end{pmatrix} \begin{pmatrix} \text{Perspective projection matrix (3x4)} \end{pmatrix} \begin{pmatrix} \text{World to camera coord. trans. matrix (4x4)} \\ \begin{bmatrix} \boldsymbol{R} & \boldsymbol{t} \\ \boldsymbol{0}^T & 1 \end{bmatrix} \end{pmatrix} \begin{pmatrix} \text{3D point (4x1)} \\ \boldsymbol{X} \end{pmatrix}$$

***Intrinsic* camera parameters**: principal point, scaling factors

***Extrinsic* camera parameters**: rotation, translation

Perspective Projection Matrix

- Projection is a matrix multiplication using homogeneous coordinates:

$$\begin{bmatrix} f_x & 0 & p_x & 0 \\ 0 & f_y & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} f_x x + pxz \\ f_y y + pyz \\ z \end{bmatrix} \Rightarrow \left(f_x \frac{x}{z} + px, f y \frac{y}{z} + py \right)$$

divide by the third coordinate

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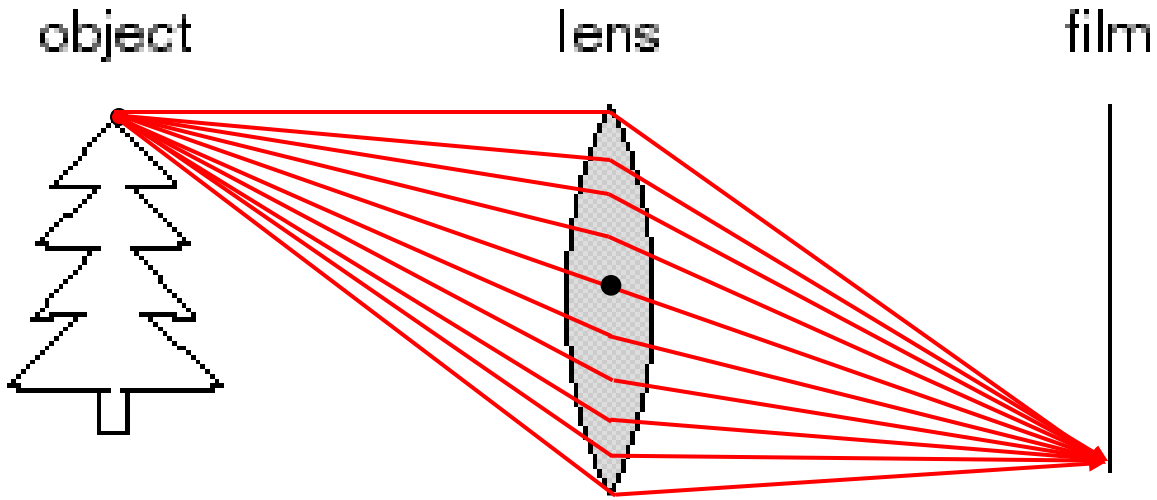
Overview

- Pinhole projection model
  - Qualitative properties
  - Perspective projection matrix
- Cameras with lenses

Adding a lens

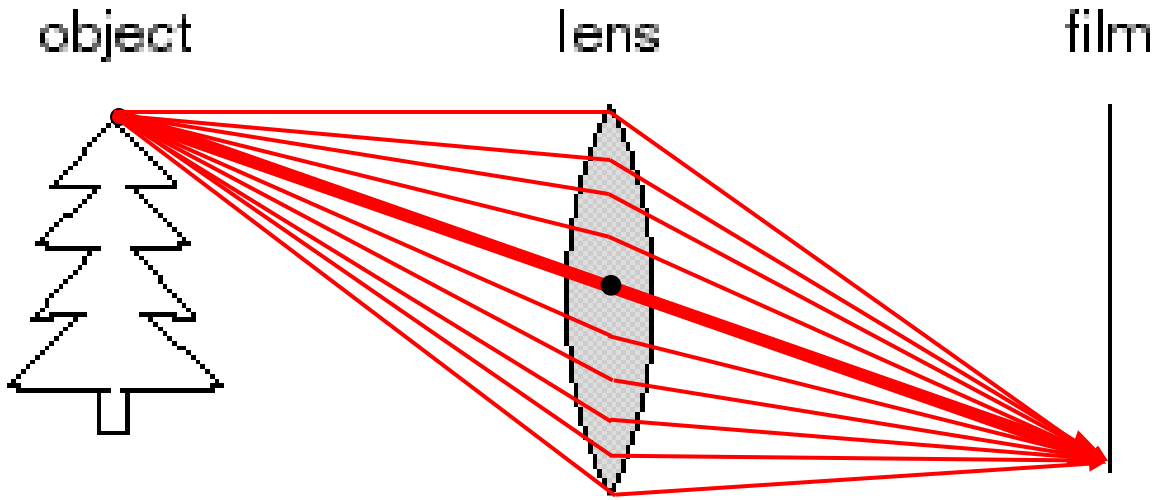


A lens focuses light onto the film

- Thin lens model:

Slide by Steve Seitz

Adding a lens

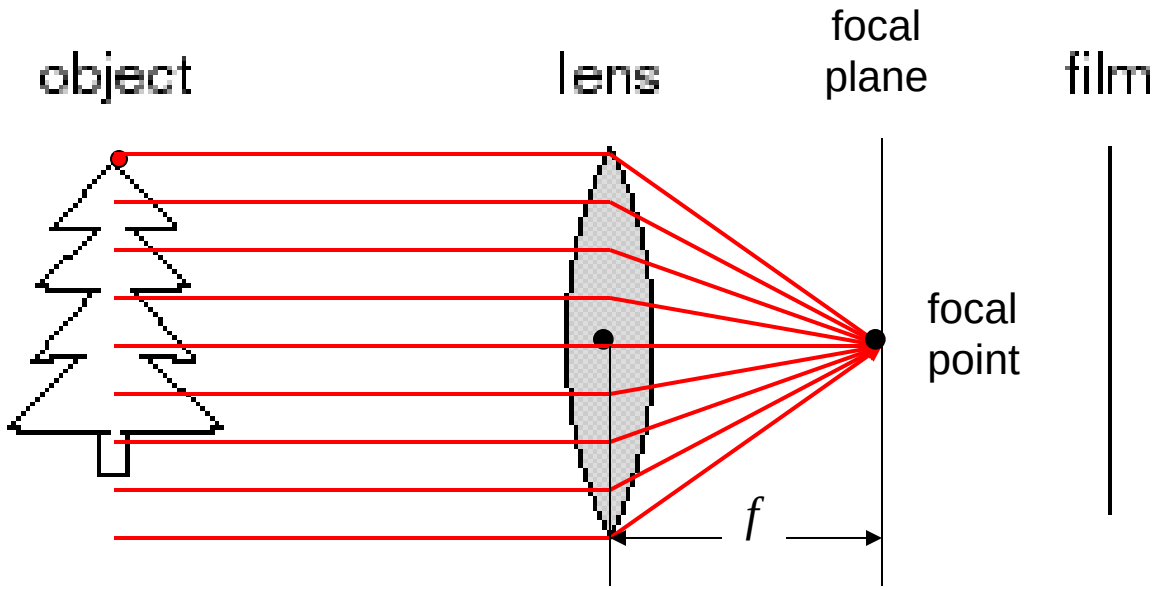


A lens focuses light onto the film

- Thin lens model:
  - Rays passing through the center are not deviated (pinhole projection model still holds)

Slide by Steve Seitz

Adding a lens



A lens focuses light onto the film

- Thin lens model:
  - Rays passing through the center are not deviated (pinhole projection model still holds)
  - All rays parallel to the optical axis pass through the *focal point*
  - All parallel rays converge to points on the *focal plane*

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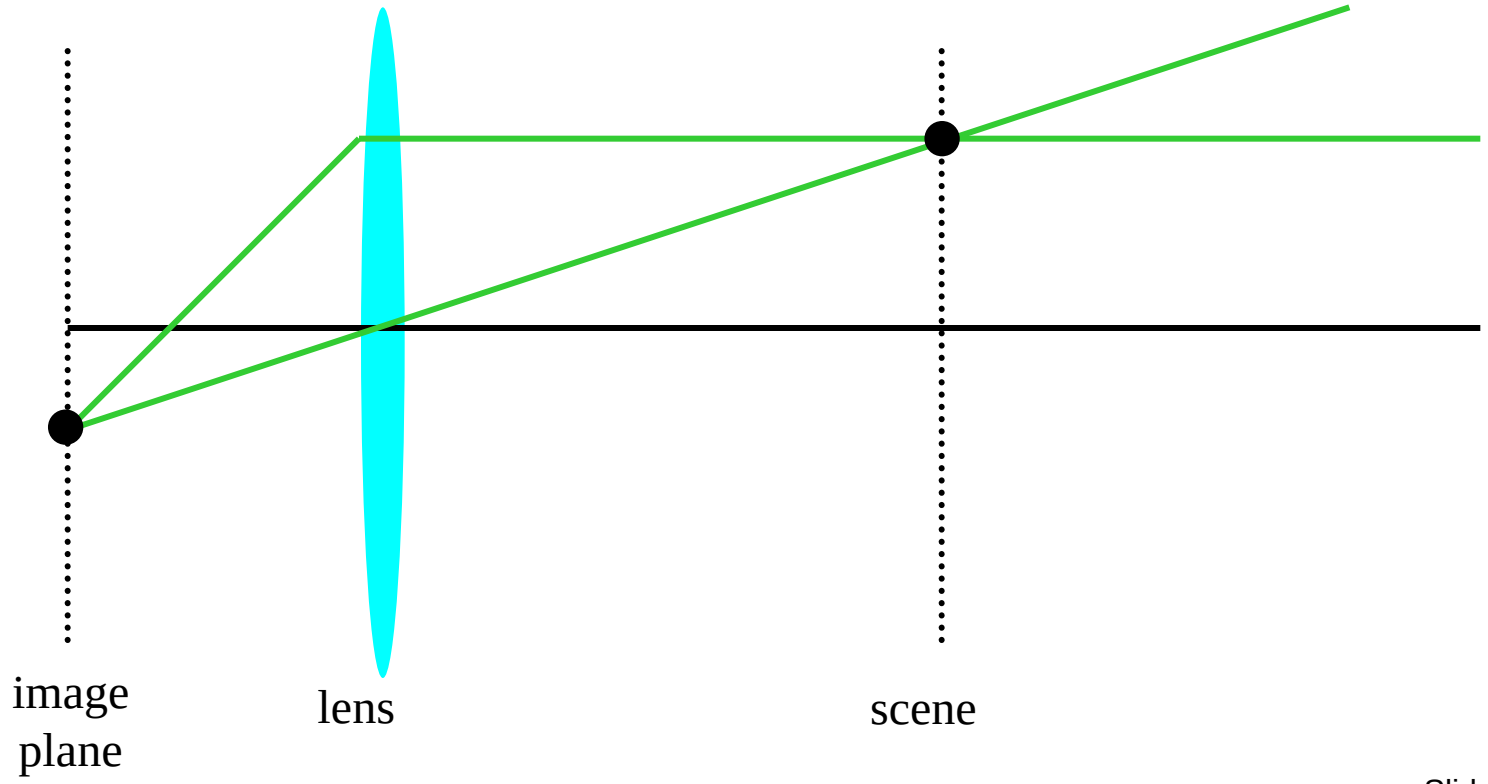
Thin lens formula

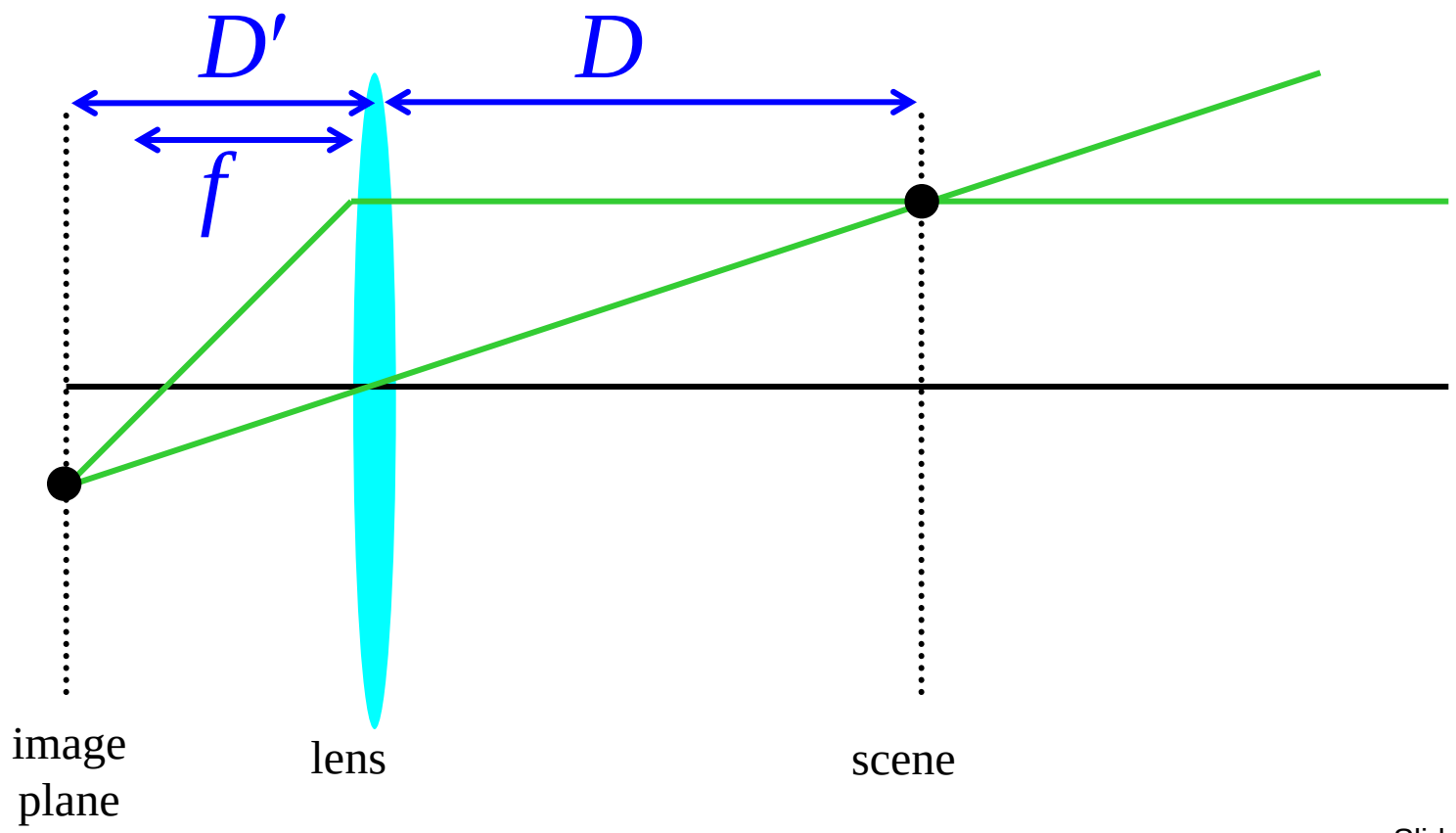
- Where does the lens focus the rays coming from a given point in the scene?



Slide by Frédo Durand

Thin lens formula

- What is the relation between the focal length (f),
  the distance of the object from the optical center (D),
  and the distance at which the object will be in focus (D')?



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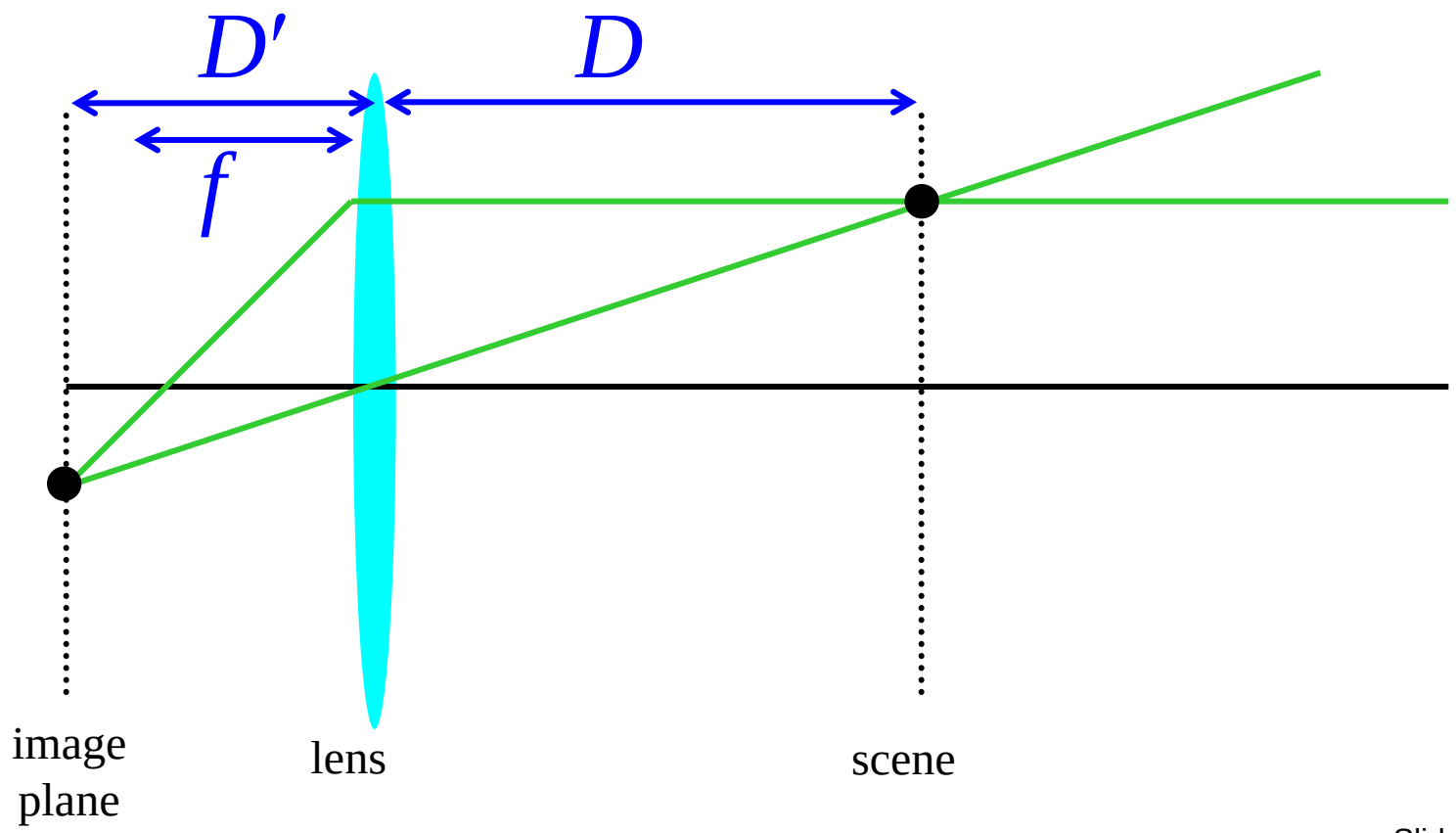
Thin lens formula

$$\frac{1}{D'} + \frac{1}{D} = \frac{1}{f}$$

Any point satisfying the thin lens equation is in focus.



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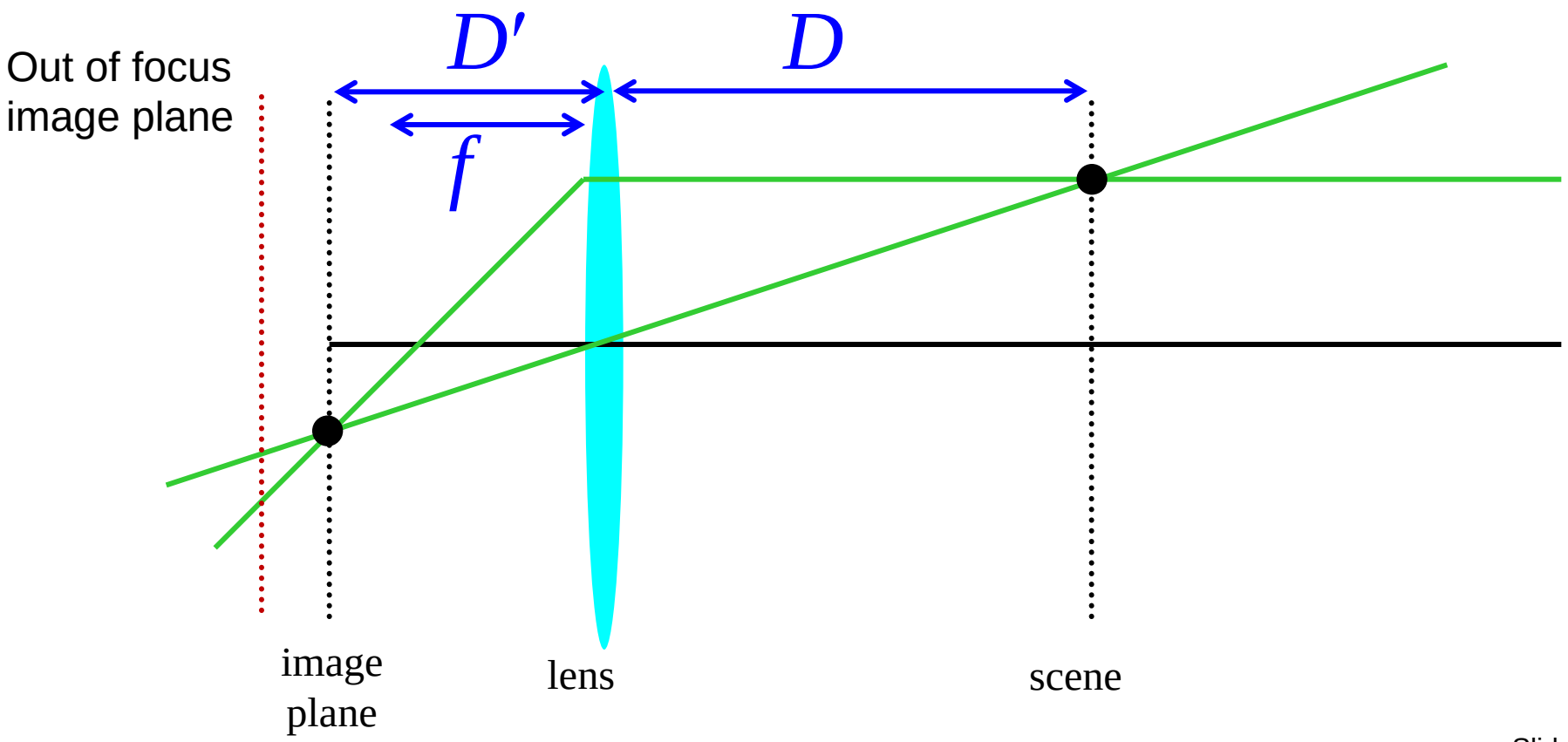

Slide by Frédo Durand

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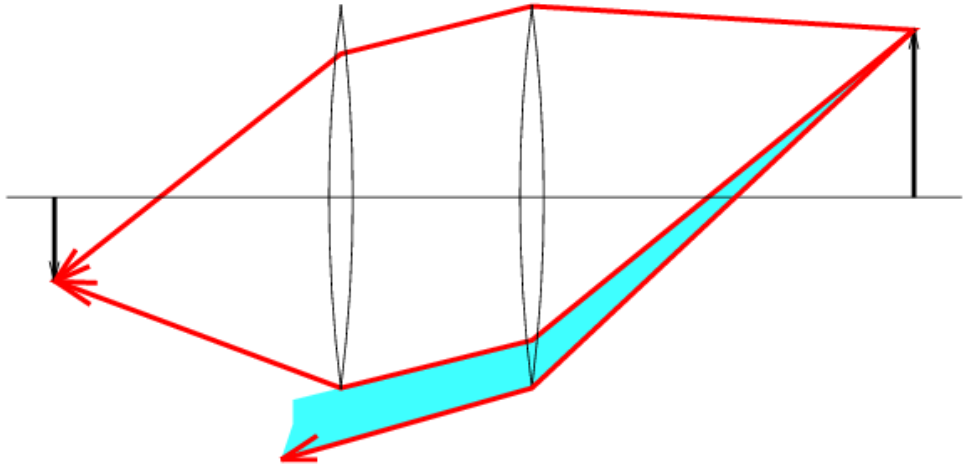
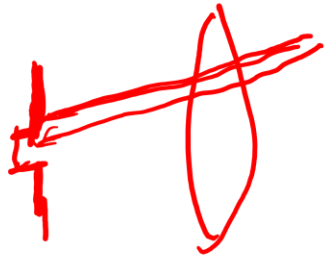
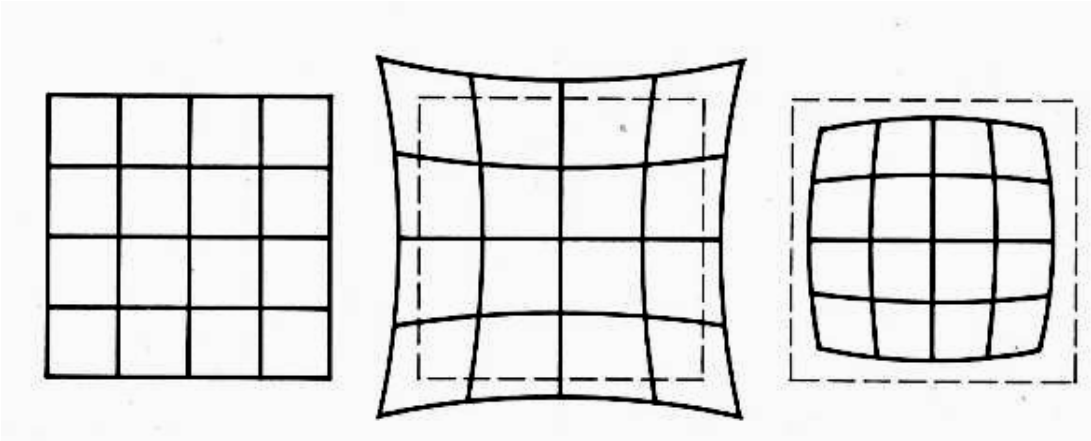
Lens flaws: Vignetting



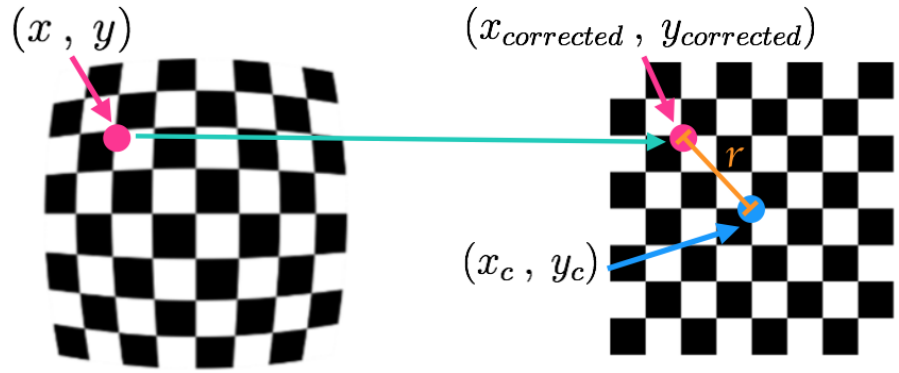


Lens flaws: Radial distortion

- Caused by imperfect lenses
- Deviations are most noticeable near the edge of the lens



No distortion Pin cushion Barrel

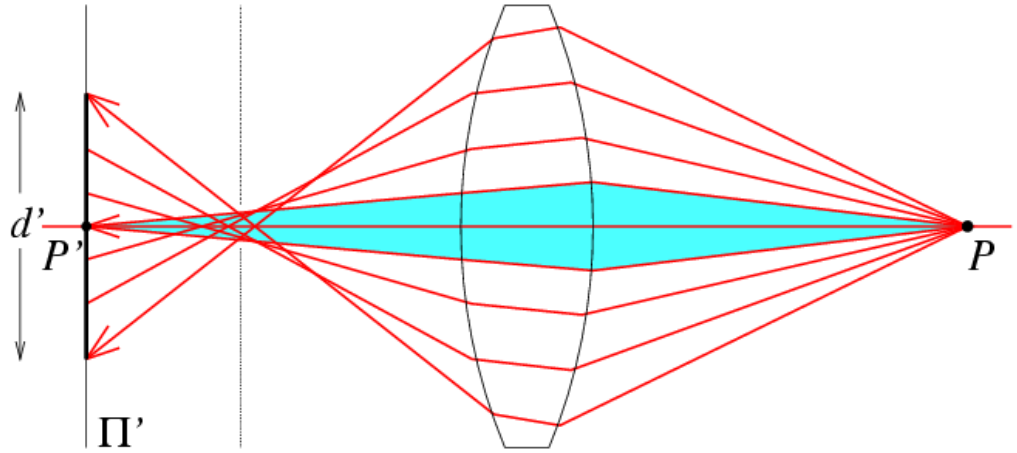


source

Lens flaws: Spherical aberration

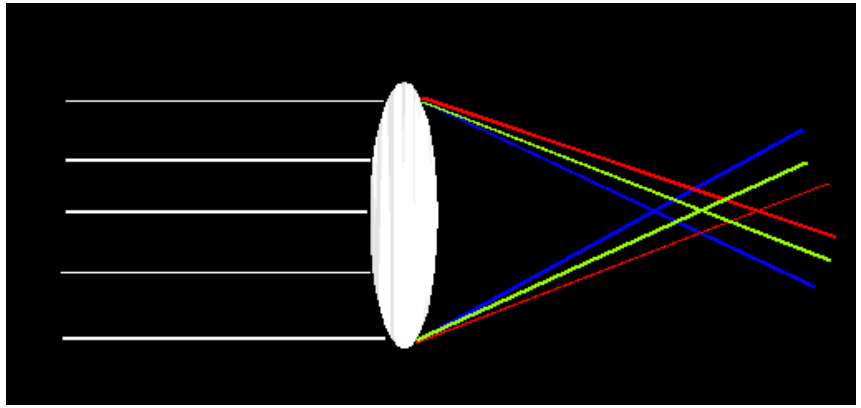
Spherical lenses don’t focus light perfectly

Rays farther from the optical axis focus closer



Lens flaws: Chromatic aberration

Lens has different refractive indices for different wavelengths causes color fringing



Near Lens Center

Near Lens Outer Edge

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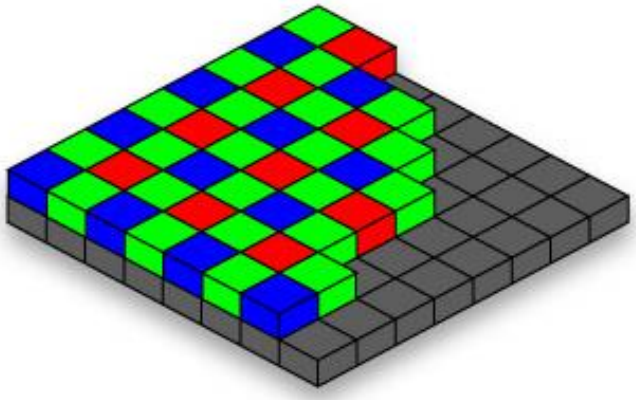
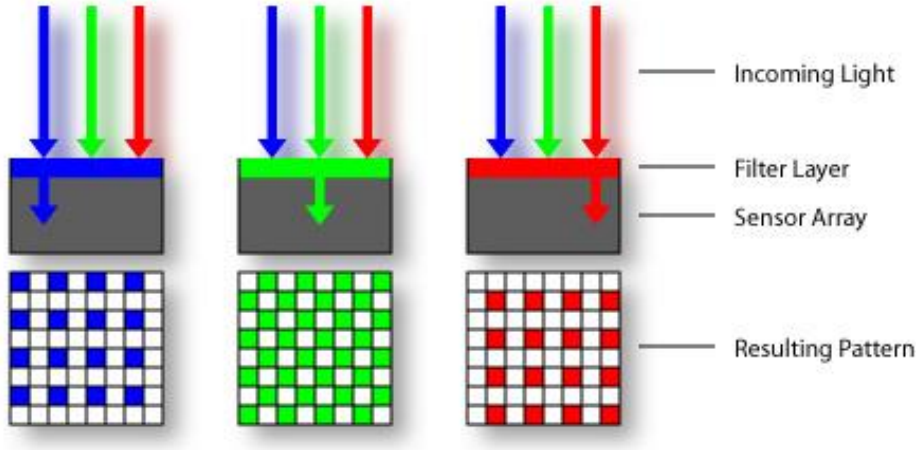
Color Filter Arrays

Bayer grid (1976)

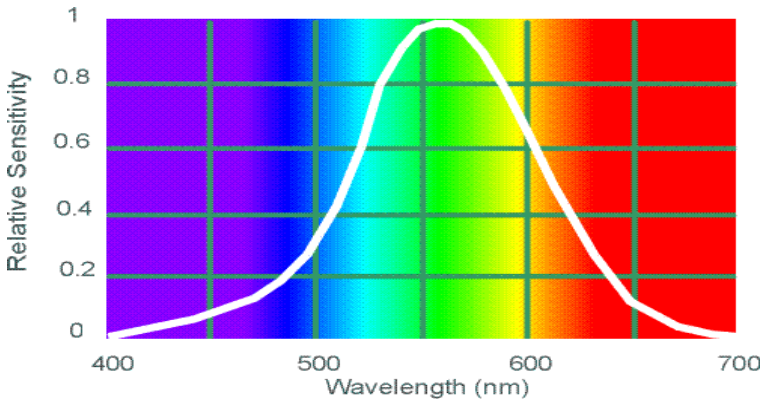


Demosaicing:

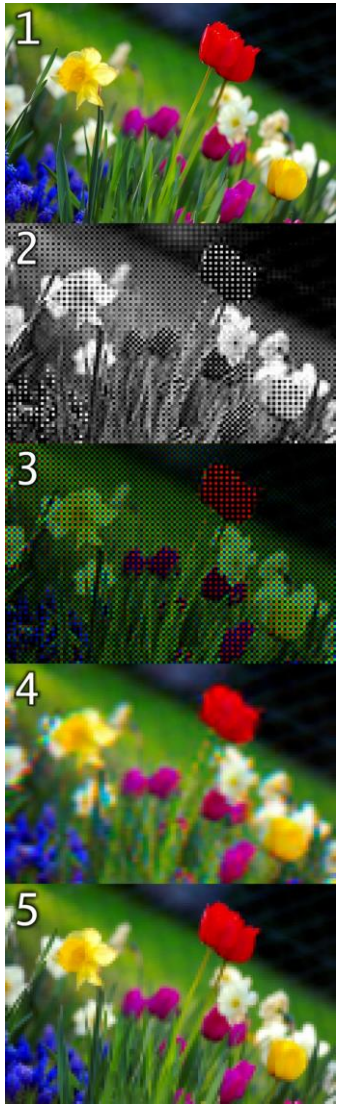
Estimation of missing components from neighboring values



Why more green?



Human Luminance Sensitivity Function



Source: Steve Seitz

Color Filter Arrays

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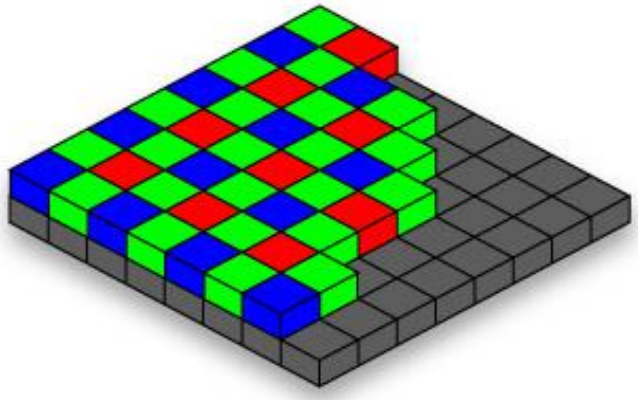

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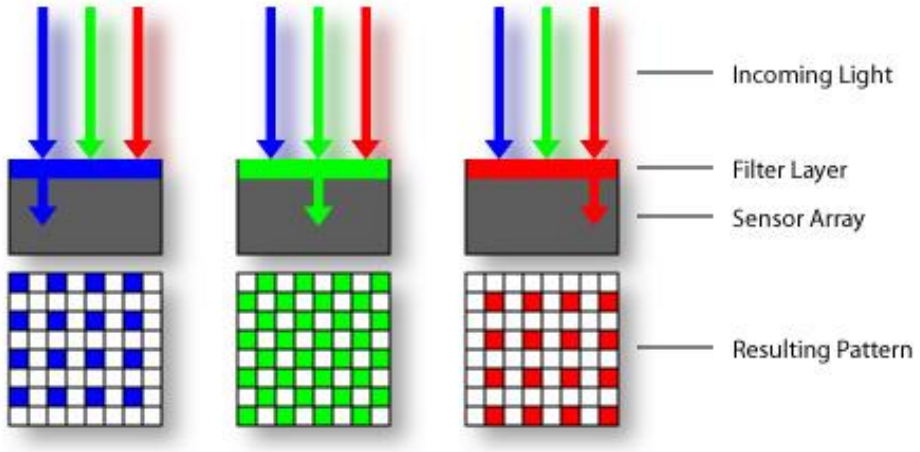
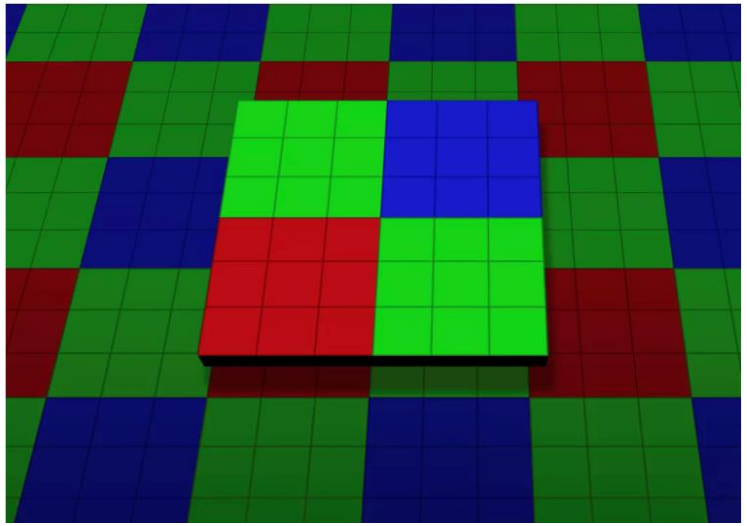
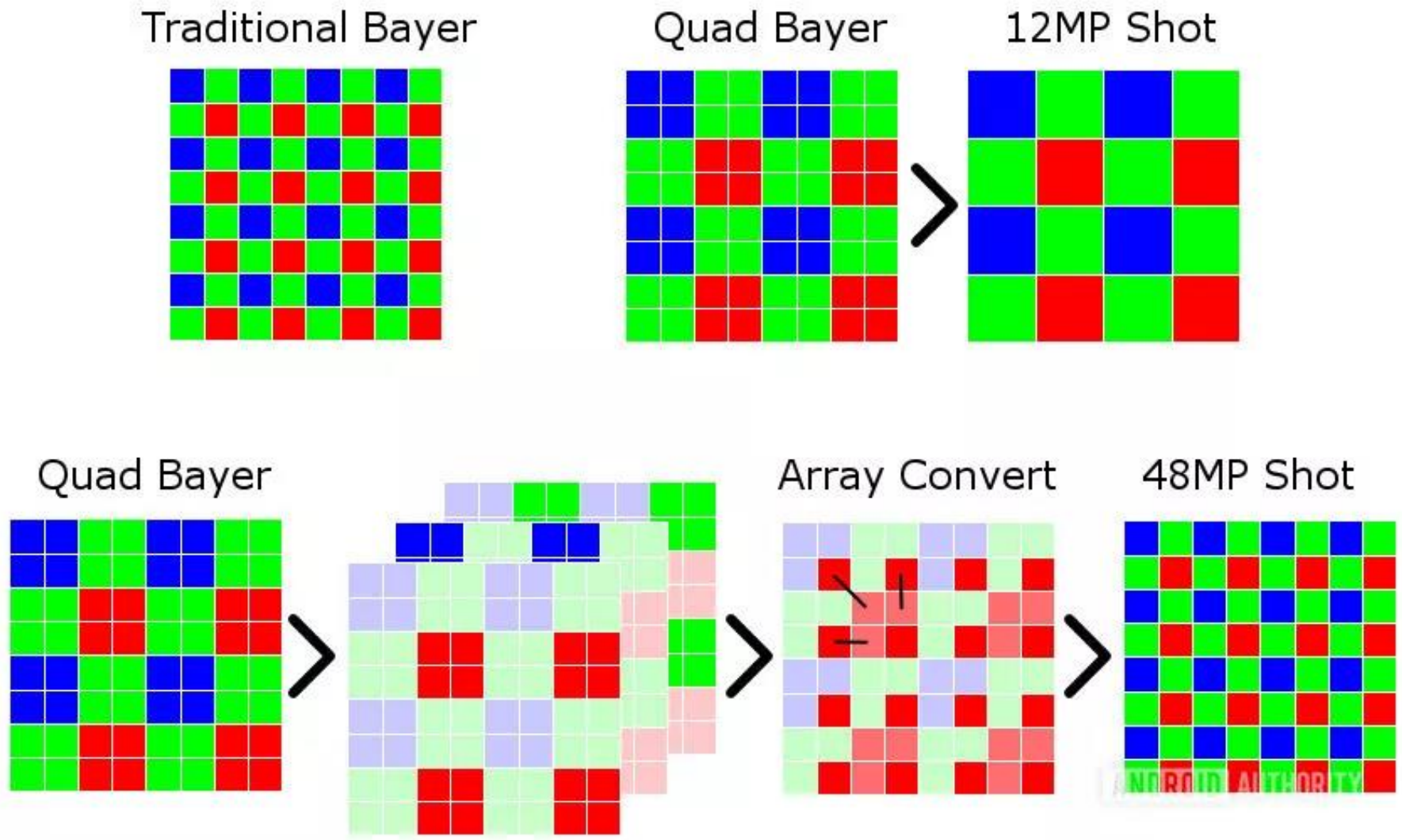




Recent camera-phone technology: pixel binning

Color Filter Arrays



source

Misc. digital camera artifacts

Noise

- low light is where you most notice noise
- light sensitivity (ISO) / noise tradeoff
- stuck pixels

In-camera processing

- oversharpening can produce halos

Compression

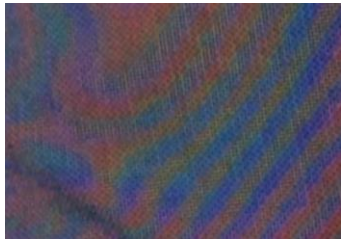
- JPEG artifacts, blocking

Blooming

- CCD charge overflowing into neighboring pixels

Color artifacts

- Color moire
- Purple fringing from microlenses



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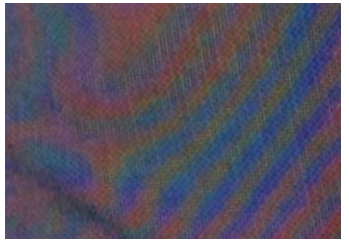

Next Class

Image processing I: Colors, light, linear filtering

Reading: SZ Ch 2.2.1, 2.2.2, 2.3.2, 3.1, 3.2