Magnetometer Offset Cancellation: Theory and Implementation, revisited

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This is a re-examination of a problem discussed by this author in a previous paper, with a new solution.

Problem: Magnetometers have non-zero offsets from a combination of effects, including local magnetic fields in the body frame, and offsets in the electronics. It is desired to have a simple way to determine the offsets, and subtract them out.

Solution: Use the apparent variation of the magnitude of the magnetic field to determine offsets.

Theory:

It is assumed that the magnetometer's gains are calibrated, it is just the offsets we are worried about. For example, both the HMC5843 and the HMC5883L 3-Axis magnetometers have a self-test feature that determines calibration, but it does not have any way to determine the offsets.

It is possible to compensate for the offsets in flight.

In a previous paper the approach was based on using the rotation information in the direction cosine matrix to compute and remove magnetometer offsets. However, during recent work to compensate for magnetometer misalignment, I discovered that misalignment causes errors in the previously reported method for removing offsets. It is possible to compensate for misalignment, and there will soon be a separate report on that subject. By compensating for misalignment, it is possible to restore the accuracy of the offset removal. However, if you prefer not to compensate for misalignment, there is another method for removing offsets that works well in the face of misalignment.

An alternate approach is to focus on the magnitude of the magnetic measurements. Assuming both a magnetic offset vector and a misalignment rotation matrix, the magnetic measurement in the body frame can be expressed as follows:

$$\mathbf{b_B} = \mathbf{b_0} + \mathbf{A} \cdot \mathbf{R^T} \cdot \mathbf{b_E}$$

$$\mathbf{b_E} = \text{actual magnetic field in the earth frame}$$

$$\mathbf{b_B} = \text{magnetic field measured in body frame} \qquad \text{equation 1}$$

$$\mathbf{R} \text{ and } \mathbf{R^T} = \text{ the direction cosine matrix and its transpose}$$

$$\mathbf{A} = \text{rotation matrix representing misalignment}$$

Equation 1 can be rewritten as:

$$\mathbf{b_B} - \mathbf{b_0} = \mathbf{A} \cdot \mathbf{R^T} \cdot \mathbf{b_E} \qquad \text{equation 2}$$

The square of the magnitude of a vector is found by taking its dot product with itself, which can be expressed in matrix notation as multiplying the transpose of the vector by itself:

$$\begin{aligned}\left|\mathbf{b_B} - \mathbf{b_0}\right|^2 &= \left(\mathbf{A}\cdot\mathbf{R^T}\cdot\mathbf{b_E}\right)^{\mathbf{T}}\cdot\left(\mathbf{A}\cdot\mathbf{R^T}\cdot\mathbf{b_E}\right)\\ &= \left|\mathbf{b_E}\right|^2\end{aligned} \qquad \text{equation 3}$$

In other words, because both matrices in equation 3 are rotation matrices, their transpose is equal to their inverse, and they preserve vector magnitudes. Therefore, the magnitude of the difference between each magnetic measurement minus the magnetic offset must be a constant.

Expand the left side of equation 3. We find:

$$\left|\mathbf{b_B}\right|^2 + \left|\mathbf{b_0}\right|^2 - \mathbf{2}\cdot\mathbf{b_B^T}\cdot\mathbf{b_0} = \left|\mathbf{b_E}\right|^2 \qquad \text{equation 4}$$

Now, take measurements in two different attitudes, put them into two instances of equation 4, subtract and rearrange. We find:

$$\left|\mathbf{b_{B2}}\right|^2 - \left|\mathbf{b_{B1}}\right|^2 = 2\cdot\left(\mathbf{b_{B2}^T} - \mathbf{b_{B1}^T}\right)\cdot\mathbf{b_0} \qquad \text{equation 5}$$

Equation 5 does not tell us what the offset vector is equal to. But we do not need to know exactly what it is in order to gradually cancel it. What we can do is gradually refine our estimate of the offset. Equation 5 gives us enough information to compute the refinements.

Equation 5 defines a plane in 3 dimensions. We can tell from equation 5 on what side of the plane the offset vector must be. A reasonable refinement is in a direction perpendicular to the plane. In that case, we find a good guess at a step in the right direction is given by:

$$\begin{aligned}&\hat{\mathbf{b}}_\mathbf{0} = \hat{\mathbf{b}}_\mathbf{0} + gain\cdot\frac{\left(\mathbf{b_{B2}} - \mathbf{b_{B1}}\right)}{\left|\mathbf{b_{B2}} - \mathbf{b_{B1}}\right|}\cdot\left(\left|\mathbf{b_{B2}}\right| - \left|\mathbf{b_{B1}}\right|\right)\\ &\hat{\mathbf{b}}_\mathbf{0} = \text{estimate of the offset}\end{aligned} \qquad \text{equation 6}$$

The gain parameter is selected to strike a reasonable balance between numerical stability and speed of convergence. I used a value of 1 for gain in my implementation, but you might want to experiment with other values.

What equation 6 says to do is to divide the difference vector between two magnetic measurements by its magnitude (in other words, normalize the vector), multiply by the difference of the magnitudes of the two measurements, and multiply by the convergence gain. Then, add the refinement to the estimate of the offset. The offset estimate is then removed from each measurement.

The way that the method works is that each refinement is in the approximate correct direction. As the estimate improves, the difference in the second factor goes to zero, and the estimate must converge to the correct value. Each time the aircraft rotates a little bit, the magnetic measurement will change. If there is any change in the magnitude of the measurement, that is an indication of an offset. Application of equation 6 gradually completely removes the offset.

Some of you might be worried by the division by the magnitude of the difference of the measurements. In practice, that is not a problem, provided you do not do the division if the denominator is zero. In that case, the second factor in equation 6 will be zero anyway, so the computed refinement will come out to be zero.

I have performed extensive tests of offset removal using equation 6, and found convergence to be rapid and accurate. A typical plot of the computed offsets during a spin test at 78 RPM is shown in Figure 1. The computed values are converging to the values that I measured manually.

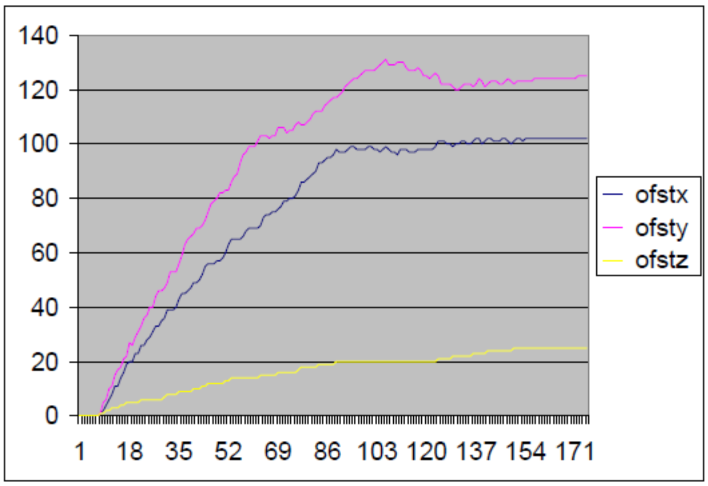
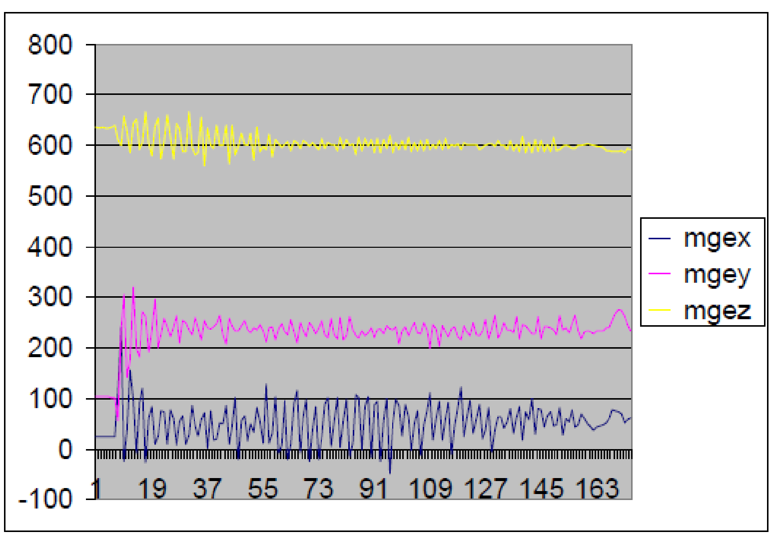


Figure 1 Computed offsets using equation 6

The measured magnetic field vectors with offsets removed and rotated back into the earth frame of reference while the IMU and magnetometer were spinning at 78 RPM are shown in Figure 2.



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Implementation

Implementation is trivial. Here is the section of the MatrixPilot code that updates the offset vector estimate:

```
magFieldBodyMagnitude = vector3_mag( udb_magFieldBody[0],
      udb_magFieldBody[1], udb_magFieldBody[2] ) ;
VectorSubtract( 3, vectorBuffer , udb_magFieldBody ,
      magFieldBodyPrevious ) ;
vector3_normalize( vectorBuffer , vectorBuffer ) ;
VectorScale( 3 , offsetEstimate , vectorBuffer ,
      magFieldBodyMagnitude - magFieldBodyMagnitudePrevious ) ;

udb_magOffset[0] = udb_magOffset[0]
      + ( ( offsetEstimate[0] + 2 ) >> 2 ) ;
udb_magOffset[1] = udb_magOffset[1]
      + ( ( offsetEstimate[1] + 2 ) >> 2 ) ;
udb_magOffset[2] = udb_magOffset[2]
      + ( ( offsetEstimate[2] + 2 ) >> 2 ) ;
```

The routines that are called do the following:
vector3_mag computes the magnitude of a vector.
VectorSubtract subtracts two vectors.
Vector3_normalize normalizes a vector to have magnitude equal to 1.
VectorScale multiplies a vector by a scalar.