Rambam on Calendar

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Introduction

Latest version of this paper and the code used to produce it are available from http://www.jewish-calendar.org/.

In the "Laws of Sanctification of the Moon", Rambam gives algorithmic description of two calendars: fixed ("arithmetic") and observational ("astronomical"). Both calendars are described in a form ready to be implemented, but the reasons for the calculation steps - especially for the astronomical calendar - are not always given: what are the models - and the formulae resulting from them - that reproduce the numbers? Sometimes Rambam describes the models, so only the formulae need to be reconstructed; sometimes the models are not described, so both the models and the formulae need to be reconstructed.

Goals of this paper are:

- reconstruct models that Rambam uses from the Rambam's text
- describe the models with diagrams and formulae
- reconstruct actual parameters of the models from the numbers given by Rambam
- identify misprints in the traditional editions
- analyze correspondence with the fixed calendar
- verify Rambam's tables and rounding decisions against his models
- analyze impact of various misprints on the sighting calculations
- Verify consistency of Rambam's numbers.
- Translate relevant chapters of Rambam into English. The translation does not have to be literal; literal translations are already available (Yale, Touger). The goal of our translation is to make the text complete and accessible for collaboration by people not proficient enough in Hebrew.
- see how Rambam's calculations of the moon visibility compare to the current models

Approximate timeline of the project:

- In 1984, The Rebbe instructed Jews to learn Rambam every day. As a result, in 1991 I encountered calendar chapters of Rambam, decided to reconstruct the models behind the calculations described in them, and discovered mistakes in the text of one of the laws (15:6). The results were published in "Notices of Temimim and Anash", a weekly of the central Lubavicher Yeshiva 770. Some of the readers were appalled that someone dares to correct the text of Rambam! My point was that the text as published contains mistakes that need to be corrected. Indeed, Frenkel edition [Frenkel], that was already published at the time, corrects all of the mistakes I did. In fact, they were already corrected by Hazon Ish in his commentary on Rambam [[HazonIsh]] - but I did not know it then.

- 2008: I acquired a pile of books on the subject and started looking into it; I also experimented with various ways of publishing the work on the web.
- 2009: coded some of the algorithms to look into the rumors the dates of Birkhat HaHamah [http://dub.podval.org/2019/07/18/sanctification-of-the-sun.html] that were circulating then.
- 2013: code converted to Scala.
- 2017-2018: split-file encoding of the family polymorphism and current representation of the dates and angles were implemented in the supporting code.
- 2018-2019: generation of reading and learning schedules was added to the code.

I want to acknowledge people who contributed to the content of this work - and to the fact of its existence ;) If your name should be on this list but isn't, please forgive me: the omission is not intentional.

- Mordechai Goldin - for providing office space and computer equipment when I, as a Yeshiva student, had neither;
- Ilia Pomansky - for encouraging this work at the early stages, in 1991;
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- Rabbi Michael Koretz - for stimulating discussions during our visit to Israel in 2005, encouragement, and extensive research into customs of Torah/Haftarah reading;
- Dr. Michael Partensky - for encouragement and advise;

- Dr. Peter Ofman - for asking questions that prodded me to revive this project in 2011, after years of hiatus;
- Aron Matskin - for a discussion during his visit on Rosh Chodesh Mar Cheshvan 5772;
- Rabbi Chaim Prus - for causing me to investigate which number does the year of Creation has - 0 or 1;

RoshHaShono corrections

- are there meaningful names for the corrections?
- are corrections explained by the desire to not have Yom Kippur on Sunday etc.?
- calculate third correction from the maximum length of a year and the first correction
- KH 7:8 says that postponement of RoshHaShonoh is done to align the calendar better with the true molad; analyze the statistics of distances between mean and true molad and RoshHashono.

There is an argument [https://he.wikipedia.org/wiki/%D7%9E%D7%97%D7%9C%D7%95%D7%A7%D7%AA_%D7%A8%D7%91_%D7%A1%D7%A2%D7%93%D7%99%D7%94_%D7%92%D7%90%D7%95%D7%9F_%D7%95%D7%91%D7%9F_%D7%9E%D7%90%D7%99%D7%A8] concerning some of the Rosh Hashono delays involving r. Saadia Gaon: Спор о размерах "старого молада", когда он отодвигает Рош ашана на следующий день. По нашей шите от Расага и дальше - 18 часов По шите Аарона Бен Меира - 18 часов и 642/1080 частей часа

make possible calculating the calendar according to the other opinion and compare the results.

Молад года творения человека выходит в пятницу. Это и было Рош а-Шана второго года, ведь тогда принцип ли аду Рош не применялся. Тогда понятней спор Рамбама и Райвада о причине сдвига Рош а-Шана ло аду. Рамбама настаивает, что оно нужно для подсчёта по кибуц эмцаи, а Райвад на него из Талмуда возмущается. Теоретически можно сделать календарь на время до установления календаря без сдвига Рош а-Шана.

Rav Ada's tkufos started a week later than Shmuel's! Isn't it readily observable? Analyze the difference in historic periods...

Solar Theory

Chapter 12, Law 1

It is not clear how Rambam arrived at the values he gives. When value of the movement in one day given by Rambam is multiplied by 29, 100 etc., the result is smaller than that of Rambam. The following table compares printed and calculated values:

days	value	calculated
n	**v(n)**	**v(1)*n**
1	0°59′8″	0°59′8″
10	9°51′23″	9°51′20″
100	98°33′53″	98°33′20″
1000	265°38′50″	985°33′20″
10000	136°28′20″	9855°33′20″
29	28°35′1″	28°34′52″
354	348°55′15″	348°53′12″

Although the value of the movement of the mean solar longitude in one day given by Almagest III 1 (H209) (which, rounded to seconds, becomes Rambam's value) is bigger, it is not big enough, and Rambam's numbers (for more than 10 days) can not be explained by performing calculations with the long value from Almagest and then rounding the results to the precision of the Rambam's values.

days	value	Almagest
n	**v(n)**	***n**
1	0°59′8″	0°59′8″
10	9°51′23″	9°51′23″
100	98°33′53″	98°33′49″
1000	265°38′50″	985°38′7″
10000	136°28′20″	9856°21′10″
29	28°35′1″	28°35′
354	348°55′15″	348°54′54″

From the printed values it is possible to reconstruct the value of the movement in mean solar longitude in one day that Rambam used to calculate each of them:

Tzikuni (p. XXX) gives the algorithm of such reconstruction: add to the remainder as many times 360 as there were full rotations in given time period, and then divide. It also gives a value reconstructed from the printed values for 10,000, 1000 and 100 days: 59′8.33″, or 0.9856472 degrees, and the current "scientific" value of 0.9856473 degrees. It seems that origins of the Rambam's value were questioned by his commentators, including "Pirush". Value that can be derived from the tradition that 19 years = 6939 days 16 hours and 595 parts is 0.9856348. This algorithm can be modified to produce an interval of possible values, taking into account precision of the numbers.

Following table compares the printed values with calculations based on the most "precise" reconstructed value - that for 10000 days.

Values of the movement in one day apparently used by Rambam, if rounded to the seconds, give the "official" Rambam's value. Values calculated from the value reconstructed from the printed 10000 days movement, if rounded, produce the values Rambam published - except for 29 days, 354 days, and 10000 days which we use to calculate the exact value that Rambam used! TODO!

Tzikuni quotes Rambam's value for 354 days as 348°55′15″, but calculated value as 348°55′9″, and notes that this "requires a little thought".

Questions

- Exactification: Tzikuni gives the algorithm of such reconstruction: add to the remainder as many times 360 as there were full rotations in given time period, and then divide... find the page for exact reference; use it to calculate intervals for Rambam's values of the angular velocities

- Exactify all angular velocities.

- Angular movement for 29 days: are Rambam's values reconstructible if value for 29 days is calculated as 3*10-1, not 2*10+9? For any tables other than SunLongitudeMean?

- Convert each year length mentioned in the fixed calendar to angular speed.

- Where and when was the Sun created? Birchat Hachamo?

- make possible calculations for a Moment, not just a Day (at least for the sun);

- Math.findZero(): Finish and test the code; use two separate lengths (precisions)?
- calculate true seasons in SeasonsAstronomical; KH 10:7 says that real vernal equinox is approximately two days before the mean one; add a test.

Lunar Theory

Chapter 14

Law 1: Two Movements of the Moon

Note: The first is "mean" - because it is corrected by the second, giving "true"? Why is the second "mean"? Because of the effects of the sun?

Note: Wrom what and in which direction are the angles measured?

Law 2: Movement in the Mean Lunar Longitude

days	**value**
n	**v(n)**
1	13°10′35″
10	131°45′50″
100	237°38′23″
1000	216°23′50″
10000	3°58′20″
29	22°6′56″
354	344°26′43″

Rambam uses ‘nimzes’ instead of ‘nimzo’ for 10000 days. Also, Rambam says "'ordered' year"; it was rendered as 254 in accordance with Law 12:1.

It is not clear how Rambam arrived at the values he gives. When value of the movement in one day given by Rambam is multiplied by 29, 100 etc., the result is smaller than that of Rambam. The following table compares printed and calculated values:

days	**value**	**calculated**
n	**v(n)**	**v(1)*n**
1	13°10′35″	13°10′35″
10	131°45′50″	131°45′50″
100	237°38′23″	1317°38′20″
1000	216°23′50″	13176°23′20″
10000	3°58′20″	131763°53′20″

days	value	calculated
n	**v(n)**	**v(1)*n**
29	22°6′56″	382°6′55″
354	344°26′43″	4664°26′30″

Moreover, the value of the movement of the mean lunar longitude in one day given by Almagest IV 3 (H278) (which, rounded to seconds, becomes Rambam's value) is even smaller, so Rambam's numbers can not be explained by performing calculations with the long value from Almagest and then rounding the results.

From the printed values it is possible to reconstruct the value of the movement in mean lunar longitude in one day that Rambam used to calculate each of them:

Following table compares the printed values with calculations based on the most "precise" reconstructed value - that for 10000 days.

Values of the movement in one day apparently used by Rambam, if rounded to the seconds, all give the "official" Rambam's value. Values calculated from the value reconstructed from the printed 10000 days movement, if rounded, produce the values Rambam published - except for 354 days movement, which remains unexplained TODO!

Law 3: Movement in the Mean Lunar Anomaly

days	value
n	**v(n)**
1	13°3′54″
10	130°39′
100	226°29′53″
1000	104°58′50″
10000	329°48′20″
29	18°53′4″
354	305°0′13″

Rambam says "'ordered' year"; it was rendered as 254 in accordance with see Law 12:1. Also, this value is actually given in Law 4.

It is not clear how Rambam obtained the values he gives. When value of the movement in one day given by Rambam is multiplied by 29, 100 etc., the result is bigger than what Rambam says! The following table compares printed and calculated values:

days	**value**	**calculated**
n	**v(n)**	**v(1)*n**
1	13°3′54″	13°3′54″
10	130°39′	130°39′
100	226°29′53″	1306°30′
1000	104°58′50″	13065°
10000	329°48′20″	130650°
29	18°53′4″	378°53′6″
354	305°0′13″	4625°0′36″

Although the value of the movement of the mean lunar anomaly in one day given by Almagest IV 3 (H278) (which, rounded to seconds, becomes Rambam's value) is smaller, it is not small enough, and Rambam's numbers can not be explained by performing calculations with the long value from Almagest and then rounding the results (TODO: table).

From the printed values it is possible to reconstruct the value of the movement in mean lunar anomaly in one day that Rambam used to calculate each of them:

Following table compares the printed values with calculations based on the most "precise" reconstructed value - that for 10000 days.

Values of the movement in one day apparently used by Rambam, if rounded to the seconds, all give the "official" Rambam's value. Values calculated from the value reconstructed from the printed 10000 days movement, if rounded, produce the values Rambam published - except for 354 days movement, which remains unexplained TODO!

Here, everything is worse than in 14:2 TODO!!!

Law 4: Position of the Moon on the Epoch

Law 5,6: Mean Lunar Longitude at Observation Time

TODO: Compare with 11:9, about Dli... Something is off!

Chapter 15

Law 1: Doubled Distance

Law 2,3: Corrected Mean Anomaly (Elongation (TODO: ?))

Rambam does not describe the model behind this correction. Because the models Rambam did describe until now are the same as Almagest's models (including numerical parameters), and because this correction is determined by the (doubled) sun-moon elongation, just like Almagest's "improved" model of the moon ("the crank"), I assume that that model was used to calculate this correction. TODO: describe the model!

TODO: Derive the formula! Make the diagram! TODO: How come the fact that sun and moon move in different plains does not affect this calculation?

It seems that Rambam treats "the crank" as just a calculation device - otherwise, table giving visible anomaly from the corrected anomaly should take into account changes in the earth-moon distance depending on the elongation - and it does not! Maybe that is why he does not describe the model behind this correction. And maybe this is why Rambam reiterates - in Law 2 - that only the observability of the new moon needs to be calculated correctly. TODO: How does Almagest calculate visible anomaly?

Law 4,5,6: True Lunar Longitude

Formulae:

$$\tan^{-1}\left(\frac{\sin\alpha}{\cos\alpha+\epsilon}\right)$$

R - radius of the big circle; r - radius of the small circle; $\epsilon = R/r$. A little trigonometry, and we get: visible anomaly = arctg(sin(corrected anomaly)/(cos(corrected anomaly) + e)) and: e = ctg(visible anomaly)*sin(corrected anomaly) - cos(corrected anomaly)

TODO: Make the diagram! Here?

It is clear that values for 150° and 170° are misprinted: they are bigger than the one before them when they should be smaller. Value for 120° is also misprinted, but it is less obvious.

Calculations show that for the precision up to a minute, it is sufficient to know e to up to the second digit after the dot. Below, a column giving the value of e was added to the table. Look at the value of e, and the misprints become obvious: value of e for them is way out of range it is in for the rest of them.

TODO Range of e - graph. Range of possible values for each based on the range of e. This is transcription error: one letter; extra word... Probable correct values.

This is how different sources treat the often misprinted values:

Edition	120°	150°	170°
Vilno	incorrect	incorrect	incorrect
Eshkol	incorrect	incorrect	incorrect
Rambam LaAm	corrected in the notes	incorrect	incorrect
Kapach [a]	incorrect	corrected in the notes	corrected in the notes
Bluming	incorrect	corrected in the notes	corrected in the notes
Frenkel	correct and noted	correct and noted	correct and noted
Keller	not given	not given	not given
Losh [b]	incorrect	incorrect	incorrect

[a] It is not clear how did authoritative Yemenite texts acquire the same misprints as in the ashkenazic editions.

[b] Reprints the text and does not correct any of the misprints - surpising for a textbook on the subject (even introductory). TODO: Link to the scans of the appropriate pages - with the stuff highlighted.

Law 7,8,9:

TODO!!!

Questions

- New moon from 6:7: Rambam mentiones Nisan conjunction on Sunday, 5 hours and 107 units parts after sunrise. He doesn't give the year, and there seems to be no such year...

- Angular speed of the moon = 360 / (1/tropical month + 1/solar year). Moon.meanLunarPeriod - what is it called? Tropical?

- Distance between the mean and true new moons: KH 5:1-2 says that the distance is no more than a day. Calculate this difference for all months; Rambam's epoch - two days after molad?! (Petya Ofman).

- How to find the day of sighting given a month?

Проблемы календаря, который устанавливался по свидетельствам наблюдения новой луны, в отличие от расчитанного:

- основан не на среднем движении солнца и луны, поэтому к молад надо делать поправку, но не путем отодвигания Рош а-шана, а методами последних глав Кидуш а-ходеш

- другой порядок високосных и простых лет

- другой порядок полных и неполных месяцев

- возможно ли рассчитать по Рамбаму начало месяцев по этим принципам и сравнить с завиксированныму в традиции датами и днями недели?

- Пример: выход из Египта - 15 день от освящения месяца, четверг

- Год рождения Моше: спор был ли год високосным или простым. Шита Ребе - год был високосным.

Calculate Meton's cycle

Show why 19-years cycle is the best. Calculate optimal distribution of the leap years.

- 1:2 Solar year is ~11 days longer than the lunar (see chapter 6); when discrepancy accumulates to ~30 days, year is made leap. - 4:1-2 Leap year - additional Adar; Pesach (15th of Nisan) must be not before the vernal equinox (Sun enters Aries, see 9:3) and before the next summer solstice.

Moznaim: Based on Sanhedrin 13a-b, the Ramah (cited in Kessef Mishneh on law 15): "Succos must be after the autumnal equinox"; Rambam doesn't mention this. With vernal equinox on Nissan 15th, autumnal could be on Tishrei 21st, 6 days into Succos. Ohr Sameach: Rambam did say this, printers missed it. Aruch HaShulchan: Rambam holds that that Sanhedrin isn't law; we only care about vernal equinox.

QUESTION: what are the properties of the fixed calendar in this respect (need true seasons)?

- 6:3 Lunar month (between conjunctions) = 29d12h793p. - 6:4 Lunar year (12 lunar months) = 354d8h876p = 354d8h48m40s; Leap lunar year (13 lunar months) = 383d21h589p; Solar year = 365d6h (see also 9:1, 10:6); longer than the lunar year by 10d21h204p. - 6:10 19 year cycle (12+7 leap); because the difference between 19 solar years and the cycle is less than a day: 1h485p. (see 10:1 - with a different year length, the difference is 0)

QUESTION: what other lengths of the cycle have this property?

- 6:11 Leap years: 3,6,8,11,14,17,19.

QUESTION: in what sense is this an optimal correspondence? and for different cycle lengths? It is possible that true Sun calculations are needed to answer those questions...

Table of contents of Rambam's Kiddush HaChodesh

General

Chapter 1

Law 1

Month is lunar; year is solar.

Law 2

Solar year is ~11 days longer than the lunar; when discrepancy accumulates to ~30 days, year is made leap.

Law 3

Moon is not seen for ~1 day before and after the conjunction; first night when the Moon is sighted - Rosh Chodesh; next Rosh Chodesh: if it is sighted on the night of the 30th day - 30th; if not - 31st.

Law 4

Short/full month.

Laws 5-8

Court calculates in a manner _resembling_ the calculations of the astronomers.

Chapter 2

Laws 1-10

Court procedure.

Chapter 3

Laws 1-19

Court procedure; messengers.

Chapter 4

Laws 1-2

Leap year - additional Adar; Pesach (15th of Nisan) must in the spring: not before the day of vernal equinox (Sun enters Aries, see 9:3) and before the next summer solstice.

Laws 3-8

Other reasons for leaping the year; (Succos after the autumnal equinox - not mentioned in Rambam.)

Laws 9-17

Court procedure.

Arithmetic calendar

Chapter 5

Laws 1-2

Introduction; discrepancy with observations: -1...+1 day (+1 - very rare).

Law 2

Fixed Rosh Hodesh: sighting of the Moon can be +/- 1 day; +2 - extremely rare.

Law 3

When started.

Laws 4-12

Second day of festivals.

Law 13

Conclusion.

Chapter 6

Law 1

"Molad" - approximate conjunction (Sun. Moon and Earth on the same line).

Law 2

day = 24 hours; 1 hour = 1080 parts.

Law 3

Lunar month (between conjunctions) = 29d12h793p.

Law 4

Lunar year (12 lunar months) = 354d8h876p = 354d8h48m40s; Leap lunar year (13 lunar months) = 383d21h589p; Solar year = 365d6h (see also 9:1, 10:6); longer than the lunar year by 10d21h204p.

Law 5

Lunar month % 7 = 1d12h793p; lunar year % 7 = 4d8h876p; leap year % 7 = 5d21h589p.

Law 7

if molad Nisan - Sunday, 5h107p after sunrise, next month - Iyar: Tuesday night, 5h900p.

Law 8

First conjunction - Tishrei, year #1: Monday, 5h204p (just subtract % from the next one :)) Conjunction of Rosh Hashono of Adam's creation (year #2, see Shmita and Yovel 10:2): Friday, 6h.

Law 9

We do not need the number of days, just the remainders.

Law 10

19 year cycle (12+7 leap); because the difference between 19 solar years and the cycle is less than a day: 1h485p.

Law 11

Leap years: 3,6,8,11,14,17,19.

Law 12

cycle % 7 = 2d16h595p.

Chapter 7

Law 1

Rosh Hashono gets pushed off from Sunday, Wednesday, Friday.

Law 2

Gets pushed off if molad is after noon (Molad Zoken).

Law 3

ADU applies after noon push-off also.

Law 4

Non-leap year, if molad is at night of the 3rd day after 9h204p: push off to Thursday (or the year willbe too long).

Law 5

Year after leap one, Monday, after 5h589p: push to Tuesday.

Law 7

ADU - why?

Chapter 8

Law 1

-> 6.3

Law 2

Month - 29 or 30 days.

Laws 5-6

Same?

Law 7

TODO

Law 8

TODO

Chapter 9

Law 2

1/4 of the year between seasons (91d 7.5h).

Law 3

Nisan (Spring) - Aries; Tammuz - Cancer; Tishrei - Libra; Teves - Cap.? Vernal equinox of year #1 = 7d9h642p before conjunction of Nisan. Notes 5,7 in Moznaim edition. Every year seasons move 30h; in 28 years - in the same place.

Law 5

Vernal equinox, year 4930: midnight, 5th day, 8th of Nisan.

Laws 6-8

Simplified procedure.

Law 7

Real seasons - 2 days before average ones. Note 5: 1st hour of the 4th day of creation; 1st vernal equinox. Autumnal: Wednesday, 3 hours after daybreak, 1d23h before conjunction of Tishrei.

Chapter 10

Law 1

More precise length of a solar year. Moments.

Law 2

Seasons for this year length.

Law 3

Vernal equinox of year #1 - 9h642p before conjunction of Nisan. (DifIerenbt from 9:3: Rav Ada follows Rabbi Yehoshua - world created in Nisan; Shmuel (9:3) follows Rabbi Eliezer - world created in Tishrei.) Every first year of the 19-year cycle.

Law 4

1/4 of the year between seasons (91d7h519p31m).

Law 5

Simplified procedure.

Law 6

This more precise calculation of seasons was used by Sanhedrin to leap the year because of the spring equinox.

Law 7

Real vernal equinox approximately 2 days before either of these calculations.

Atronomical Calculations

Chapter 11

Laws 1-4

Visibility calculations are deep. Fixed calendar can be appreciated even by school children in 3 or 4 days.

Laws 5-6

Calculations are simplified/approximated to not fluster the inexprerienced; result - visibility of the moon - is not affected.

Laws 7-9

360 degrees; 60 minutes, seconds...; order of the 12 constellations.

Laws 10-12

adding/subtracting angles.

Laws 13-14

Velocities are constant. Earth encircled by all the spheres but isn't in the center. Notes 14,15 in Moznaim.

Law 15

Mean and true motion.

Law 16

Epoch.

Law 17

Location: Jerusalem; see Chapter 17.

Chapter 12

Law 1

Movement of the Sun. Moznaim Note 6: calculations may be accurate only on the first day of the month; Note 7. 59′8.33″ Yale, p.99; Neugeb., p. 388ff

Law 2

Movement of the Sun's apogee. Moznaim Note 10: over 800 years, apogee moved ~12 degrees and is in constellation of Cancer. Note 11: position - at 6PM.

Chapter 13

Laws 1-3

Tue position of the Sun.

Law 4

Course correction table.

Laws 5-10

Examples.

Law 11

True seasons.

Chapter 14

Law 1

Moon: epicycle and deferent; movement as seen from Earth. See Moznaim notes!

Law 2

Deferrent table.

Law 3

Epicycle table.

Law 4

Epoch.

Law 5

Time of sighting correction table: ~20 minutes after sunset.

Chapter 15

Laws 1-2

Double elongation and its limits. See Moznaim notes!

Laws 3-5

Course correction table.

Laws 4-7

Parallax correction rules and table.

Laws 8-9

Examples.

Chapter 16

Laws 1-4

Head movement, table and rules.

Law 5

Example.

Laws 6-18

Moon latitude.

Law 19

Example.

Chapter 17

Chapter 18

Chapter 19

Laws of the Sanctification of the Moon: Translation

There is one positive commandment: to calculate, to know, and to establish on which day each month of the year begins. Explanation of this commandment is in these chapters.

Chapter 1

Law 1

The months of the year are lunar months, as it is said: "[...] the burnt offering of the month in its month" [1] and it is said: "This month shall be for you the head of the months" [2]. This is what the sages said: "The Holy One, blessed be He, showed to Moshe, in a prophetic vision, the image of the Moon, and said to him: '[When] you see [the Moon] like this - sanctify [it].'"[3]

But the years that we reckon are solar years, as it is said: "Keep the month of aviv [spring]". [4]

Law 2

By how much is the solar year longer than the lunar year? By approximately 11 days. Therefore, when this excess accumulates to around 30 days, or a little less, or a little more, a month is added and the year is made to have 13 months - which is called a "pregnant" year. [This is done] because it is impossible to have a year of 12 months and some days, as it is said: "for the months of the year" - you count the months of the year, but not the days of the year. [5]

Law 3

Each month, the moon becomes hidden for about two days, or a little less or more: about one day before its conjunction with the sun at the end of the month, and about one day after its conjunction with the sun, and in the evening it is sightet in the west. The night when it is sighted in the west after being hidden is the beginning of the month. We count

[1]Bamidbar 28:14
[2]Shemos 12:2
[3]Rosh Hashanah 20b; Menachot 29a
[4]Dvarim 16
[5]Megilla 5a; Nazir 7a

29 days from that day; if the moon is sighted on the 30th night, the 30th day is the first day of the [next] month. If it is not sighted, first day of the [next] month is the 31st day, and the 30th day belongs to the previous month. On the 31st night, we do not depend on the moon, regardless if it is visible or not, since lunar month is never longer than 30 days.

Law 4

Month of 29 days, [when] the moon was sighted on the 30th night, is called "lacking". If the moon was not sighted, and the previous month has 30 days, it is called "pregnant" month, or "full" month. The moon sighted on the 30th night is called "moon sighted on time". If it is sighted on the 31st night, but not on the 30th night, it is called "moon sighted on the night of pregnancy".

Law 5

Sighting of the moon is not up to an individual, unlike determination of the Shabbos, where one counts 6 days and observes Shabbos on the 7th, but is given to the court. Only when the court sanctifies and establishes the day as the beginning of the [new] month does it become the beginning of the [new] month. As it is said: "this month shall be for you" - this testimony is entrusted to you.

Law 6

The court calculates, like the astronomers who know positions of the planets and their movements, and look into it until they know if it is possible for the moon to be sighted on time - that is, on the 30th night - or not. If they determined that it is possible for it to be sighted, they sit and wait for withnesses all day - that is, the 30th day. If the witnesses did come and were examined in accordance with the law, and their words were accepted - they sanctify it. If the moon was not sighted and the witnesses did not come, they complete the 30 days and the month will be "pregnant". If they found out from calculations that it is impossible to sight the moon, they do not sit during the 30th day and do not await the witnesses. And if the witnesses did come, it is certain that they are false witnesses or that they saw a likeness of the moon in the clouds and not the real moon.

Law 7

It is a positive commandment of the Torah for the court to calculate and to know whether the moon can be sighted or not, and to examine the witnesses before the month is sanctified, and to send [messengers] to notify the rest of the people which day is the beginning of the month, so that they know on which day are the festivals. As it is said: "which you will call the holy convocations" and as it is said : "keep this statute in its time".

Law 8

Calculation and establishment of the months and declaration of "pregnant" years is done only in the Land of Israel, as it is said: "For out of Zion will come forth Torah, and the word of G-d - from Yerushalaim". But, if there was a man great in wisdom, and ordained in the Land of Israel, who came out of the Land, and there remains no one equal to him in the Land of Israel, he can calculate and establish month, and declares "pregnant" years outside of the Land. If he becomes aware that in the Land of Israel there appeared a man as great as him - or, needless to say, greater than him - he is prohibited to establish and to proclaim outside of the Land, and if he transgressed and did establish or did proclaim, it is as if he didn't do a thing.

Chapter 12

Law 1

The mean rate of movement of the sun in 1 day (that is, 24 hours) is 59′8″. It follows that its movement in 10 days is 9°51′23″. It also follows that its movement in 100 days is 98°33′53″. It also follows that remainder of its movement in 1000 days, after you subtract all [multiples of] 360° (as was explained) is 265°38′50″. It also follows that the remainder of its movement in 10000 days is 136°28′20″.

And in this way you can multiply and calculate its movement for any number [of days] that you want. Similarly, if you want to make known to you values of its movement for 2 days, 3, 4 and so on to 10 - do it. Similarly, if you want for to have known and ready values of its movement for 20 days, 30, 40 and so on to 100 - do it. This is clear and known once you know its movement in 1 day.

And you should have ready and known to you mean movement of the Sun for 29 days and for 354 (which is the number of days in the lunar year when its months are "regular" (TODO link), and it is called "regular year"). The reason is: if you have those movement values ready, this calculations of the visibility of the moon will be easy, because there are 29 complete days from the night of observation to the night of observation of the following month, and so it is every month: no less than 29 days and no more. Since our sole desire in all those calculations is exclusively to determine visibility [of the moon]. And between the night of sighting of this month and night of sighting of the same month next year there is either a regular year or a year and 1 day; and the same every year. Mean movement of the Sun in 29 days is 28°35′1″. Its movement in a regular year is 348°55′15″.

Law 2

There is a point on the Sun's orbit (and on orbits of the other 7 planets) such that when the planet is there it is highest above Earth. This point of the Sun's orbit (and so for other planets, except for the moon) rotates with constant speed. Its movement every 70 years is approximately 1°. This point is called *Sun's apogee*. Its movement in 10 days is one and a half seconds, i.e. 0°0′1″30‴. It follows that its movement in 100 days is 0°0′15″. Its movement in 1000 days is 0°2′30″. Its movement in 10000 days is 0°25′. It also follows that its movement in 29 days is 0°0′4″ and a bit more; and its movement in a regular year is 0°0′53″.

We already said that the epoch that our calculations start from is beginning of the night of the fifth day that is 3rd of Nisan of the year 4938 from the Creation. Position of the Sun in its mean movement at the epoch was 7°3′32″ in the constellation of Ram (Tele). Position of the Sun's apogee at the epoch was 26°45′8″ in the constellation of Twins.

Chapter 14

Law 1

The moon has two mean movements. The moon itself rotates on a small circle that does not surround all of the Earth. Moon's movement on that small circle is called *mean lunar anomaly* (אמצע המסלול). The small circle (epicycle TODO link to Wikipedia) itself rotates on a big circle (deferent TODO link to Wikipedia) that encircles the Earth. The mean movement of the small circle on the big circle that encircles the Earth is called movement of the *mean lunar longitude* (אמצע הירח).

Law 2

Movement in mean lunar longitude in one day is 13°10′35″ [6].

For longer periods, (remainder) of the movement in mean lunar longitude is:

[6]This is part of Law 1.

And in this way you can calculate it for any number of days and years that you want.

Law 3

Movement in mean lunar anomaly in one day is 13°3′54″.

For longer periods, (remainder) of the movement in mean lunar anomaly is:

Law 4

In the beginning of the night to Thursday that is the epoch (TODO measured from what? Chapter 11?) [TODO: link to definition] for these calculations, mean lunar longitude was 1°14′43″ into the constellation of Ram. And mean lunar anomaly on the epoch was 84°28′42″. Now that you know rate of movement in the mean lunar longitude and its value on the epoch to which you add, you'll calculate mean lunar longitude on any day that you want the same way as you did for the mean solar longitude. TODO: reference

Law 5,6

After you calculate mean lunar longitude at the beginning of the night that you want, contemplate the sun (TODO: longitude?) and figure out which constellation (zodiac) is it in [7]. [This is the correction of the mean lunar longitude depending on the mean solar longitude:]

Solar Longitude on the Zodiac	**from**	**to, not including**[a]	**Lunar longitude correction**
middle of Pisces-middle of Taurus [b]	315°	15°	0°
middle of Taurus-beginning of Gemini	15°	30°	0°15′
beginning of Gemini-beginning of Leo	30°	90°	0°15′
beginning of Leo-middle of Virgo	90°	135°	0°15′

[7] This is part of Law 4.

Solar Longitude on the Zodiac	**from**	**to, not including**[a]	**Lunar longitude correction**
middle of Virgo-middle of Libra	135°	165°	0°
middle of Libra-beginning of Sagittarius	165°	210°	-0°15′
beginning of Sagittarius-beginning of Aquarius	210°	270°	-0°30′
beginning of Aquarius-middle of Pisces	270°	315°	-0°15′

[a]My understanding of the word usage in the original.

[b]See Chapter 11, Law 7 on the start of Jewish Zodiac. (TODO reference)

The value of mean [lunar longitude] after you add to it or subtract from it or leave it as it is - that is mean lunar longitude approximately a third of an hour after sunset on the date you are calculating it for. And this is what is called *mean lunar longitude at observation time*. [8]

Chapter 15

Law 1

-uclaƊo find true lunar longitude (הירח האמיתי) on a specific day: First, late mean lunar longitude during observation time of the desired night. Also, calculate mean lunar anomaly and mean solar longitude for the same time. Then, subtract mean solar longitude from the mean lunar longitude and double the result. This value is called *doubled distance* (מרחק הכפול).

Law 2,3

We already stipulated that all the calculations in these chapters are for one purpose only: to know if the moon is visible. And it is not possible for this double distance during the night of observation when the moon

[8]This is Law 6.

is indeed visible to be outside of the interval from 5° to 62°. It cannot be more - nor less. [9]

A correction that depends on the doubled distance is added to the mean lunar anomaly, giving *corrected lunar anomaly* (מסלול הנכון):

doubled distance	**correction of the mean anomaly**
~5°	0°
6°-11°	1°
12°-18°	2°
19°-24°	3°
25°-31°	4°
32°-38°	5°
39°-45°	6°
46°-51°	7°
52°-59°	8°
60°-63°	9°

Law 4,5,6

Then, see how many degrees is corrected lunar anomaly. If it is less than 180°, subtract visible anomaly of this corrected anomaly from the mean lunar longitude the observation time. And if corrected lunar anomaly is more than 180° but less than 360°, add visible anomaly of this corrected anomaly to the mean lunar longitude the observation time. And mean lunar longitude after you add to it or subtract from it is the *true lunar longitude at the observation time*. [10]

If the corrected anomaly is 180° or 360°, there is no visible anomaly, so mean lunar longitude at observation time is also true longitude. [11]

What is the value of the visible anomaly for a given value of the corrected anomaly?

[9] This is Law 2.
[10] This is Law 4.
[11] This is Law 5.

Law 7,8,9:

TODO!!!

Glossary

Astronomical terms

Sun's apogee	Point on the orbit of the Sun where it is highest above the Earth.
mean lunar anomaly (אמצע המסלול)	Mean angular coordinate of the Moon on its epicicle.

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