Rambam on Calendar

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Introduction

Latest version of this paper and the code used to produce it are available from http://www.jewish-calendar.org/.

In the "Laws of Sanctification of the Moon", Rambam gives algorithmic description of two calendars: fixed ("arithmetic") and observational ("astronomical"). Both calendars are described in a form ready to be implemented, but the reasons for the calculation steps - especially for the astronomical calendar - are not always given: what are the models - and the formulae resulting from them - that reproduce the numbers? Sometimes Rambam describes the models, so only the formulae need to be reconstructed; sometimes the models are not described, so both the models and the formulae need to be reconstructed.

Goals of this paper are:

- reconstruct models that Rambam uses from the Rambam's text
- describe the models with diagrams and formulae
- reconstruct actual parameters of the models from the numbers given by Rambam
- identify misprints in the traditional editions
- analyze correspondence with the fixed calendar
- verify Rambam's tables and rounding decisions against his models
- analyze impact of various misprints on the sighting calculations
- Verify consistency of Rambam's numbers.
- Translate relevant chapters of Rambam into English. The translation does not have to be literal; literal translations are already available (Yale, Touger). The goal of our translation is to make the text complete and accessible for collaboration by people not proficient enough in Hebrew.
- see how Rambam's calculations of the moon visibility compare to the current models

Approximate timeline of the project:

- In 1984, The Rebbe instructed Jews to learn Rambam every day. As a result, in 1991 I encountered calendar chapters of Rambam, decided to reconstruct the models behind the calculations described in them, and discovered mistakes in the text of one of the laws (15:6). The results were published in "Notices of Temimim and Anash", a weekly of the central Lubavicher Yeshiva 770. Some of the readers were appalled that someone dares to correct the text of Rambam! My point was that the text as published contains mistakes that need to be corrected. Indeed, Frenkel edition [Frenkel], that was already published at the time, corrects all of the mistakes I did. In fact, they were already corrected by Hazon Ish in his commentary on Rambam [[HazonIsh]] - but I did not know it then.

- 2008: I acquired a pile of books on the subject and started looking into it; I also experimented with various ways of publishing the work on the web.
- 2009: coded some of the algorithms to look into the rumors the dates of Birkhat HaHamah [http://dub.podval.org/2019/07/18/sanctification-of-the-sun.html] that were circulating then.
- 2013: code converted to Scala.
- 2017-2018: split-file encoding of the family polymorphism and current representation of the dates and angles were implemented in the supporting code.
- 2018-2019: generation of reading and learning schedules was added to the code.

I want to acknowledge people who contributed to the content of this work - and to the fact of its existence ;) If your name should be on this list but isn't, please forgive me: the omission is not intentional.

- Mordechai Goldin - for providing office space and computer equipment when I, as a Yeshiva student, had neither;
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- Aron Matskin - for a discussion during his visit on Rosh Chodesh Mar Cheshvan 5772;
- Rabbi Chaim Prus - for causing me to investigate which number does the year of Creation has - 0 or 1;

Part I: Some Remarks

New Year corrections

- are there meaningful names for the corrections?
- are corrections explained by the desire to not have Yom Kippur on Sunday etc.?
- calculate third correction from the maximum length of a year and the first correction
- KH 7:8 says that postponement of RoshHaShonoh is done to align the calendar better with the true molad; analyze the statistics of distances between mean and true molad and RoshHashono.

There is an argument [https://he.wikipedia.org/wiki/%D7%9E%D7%97%D7%9C%D7%95%D7%A7%D7%AA_%D7%A8%D7%91_%D7%A1%D7%A2%D7%93%D7%99%D7%94_%D7%92%D7%90%D7%95%D7%9F_%D7%95%D7%91%D7%9F_%D7%9E%D7%90%D7%99%D7%A8] concerning some of the Rosh Hashono delays involving r. Saadia Gaon: Спор о размерах "старого молада", когда он отодвигает Рош ашана на следующий день. По нашей шите от Расага и дальше - 18 часов По шите Аарона Бен Меира - 18 часов и 642/1080 частей часа

make possible calculating the calendar according to the other opinion and compare the results.

Молад года творения человека выходит в пятницу. Это и было Рош а-Шана второго года, ведь тогда принцип ли аду Рош не применялся. Тогда понятней спор Рамбама и Райвада о причине сдвига Рош а-Шана ло аду. Рамбама настаивает, что оно нужно для подсчёта по кибуц эмцаи, а Райвад на него из Талмуда возмущается. Теоретически можно сделать календарь на время до установления календаря без сдвига Рош а-Шана.

Rav Ada's tkufos started a week later than Shmuel's! Isn't it readily observable? Analyze the difference in historic periods...

Solar Theory

Model (TODO)

Figure out from the text directions of Sun's movements and what are the position angles from (look at the parallax correction stuff: subtract mean apogee from the mean Sun...).

TODO diagrams for the correction in Moznaim's Rambam.

Exactification (TODO)

It is not clear how Rambam arrived at the values he gives. When value of the movement in one day given by Rambam is multiplied by 29, 100 etc., the result is smaller than that of Rambam. The following table compares printed and calculated values: TODO

Although the value of the movement of the mean solar longitude in one day given by Almagest III 1 (H209) (which, rounded to seconds, becomes Rambam's value) is bigger, it is not big enough, and Rambam's numbers (for more than 10 days) can not be explained by performing calculations with the long value from Almagest and then rounding the results to the precision of the Rambam's values.

Tzikuni (TODO p. XXX) gives the algorithm of such reconstruction: add to the remainder as many times 360 as there were full rotations in given time period, and then divide. It also gives a value reconstructed from the printed values for 10,000, 1000 and 100 days: 59′8.33″, or 0.9856472 degrees, and the current "scientific" value of 0.9856473 degrees. It seems that origins of the Rambam's value were questioned by his commentators, including "Pirush". Value that can be derived from the tradition that 19 years = 6939 days 16 hours and 595 parts is 0.9856348.

This algorithm can be modified to produce an interval of possible values, taking into account precision of the numbers.

From the printed values it is possible to reconstruct the value of the movement in mean solar longitude in one day that Rambam used to calculate each of them:

Use it to calculate intervals for Rambam's values of the angular velocities.

Tzikuni quotes Rambam's value for 354 days as 348°55′15″, but calculated value as 348°55′9″, and notes that this "requires a little thought".

TODO About apogee, Rambam says "the same way", but doesn't give value for 1 day... Exactification requires extra attention, since there is no value for 1 day. SunApogee rambamValue and almagestValue.

Angular movement for 29 days (TODO)

Are Rambam's values reconstructible if value for 29 days is calculated as 3*v(10)-v(1), not 2*v(10)+9*v(1), nor 29*vexact(1)? For any tables other than SunLongitudeMean?

Values of the movement in one day apparently used by Rambam, if rounded to the seconds, give the "official" Rambam's value. Values calculated from the value reconstructed from the printed 10000 days movement, if rounded, produce the values Rambam published - except for 29 days, 354 days, and 10000 days which we use to calculate the exact value that Rambam used! TODO!

Zeros (TODO)

Math.findZero(): Finish and test the code; use two separate lengths (precisions)?

Sun for a precise time (TODO)

make possible calculations for a Moment, not just a Day (at least for the sun);

Sun at Creation (TODO)

Where and when was the Sun created? (Birchat Hachamo is on a different opinion.) Does it let us get more precise values of Sun's angular velocities?

True seasons (TODO)

calculate true seasons in SeasonsAstronomical; KH 10:7 says that real vernal equinox is approximately two days before the mean one; add a test.

SeasonsAstronomical.seasonForYear: should this always be Nisan?

Year lengths (TODO)

Convert each year length mentioned in the fixed calendar to angular speed.

What year length does value for mean anomaly velocity given by Moznaim correspond?

Lunar Theory

Law 1: Two Movements of the Moon: The first is "mean" - because it is corrected by the second, giving "true"? Why is the second "mean"? Because of the effects of the sun?

Note: From what and in which direction are the angles measured?

Anomaly (and head?): opposite direction!

TODO MoonHeadMean rambamValue and almagestValue

Rambam uses ‘nimzes’ instead of ‘nimzo’ for 10000 days. Also, Rambam says "'ordered' year"; it was rendered as 254 in accordance with Law 12:1.

Rambam says "'ordered' year"; it was rendered as 254 in accordance with see Law 12:1. Also, this value is actually given in Law 4.

14: 5,6 TODO: Compare with 11:9, about Dli… Something is off!

15:2,3: Corrected Mean Anomaly (Elongation (TODO: ?)). Rambam does not describe the model behind this correction. Because the models Rambam did describe until now are the same as Almagest's models (including numerical parameters), and because this correction is determined by the (doubled) sun-moon elongation, just like Almagest's "improved" model of the moon ("the crank"), I assume that that model was used to calculate this correction. TODO: describe the model!

TODO: Derive the formula! Make the diagram! TODO: How come the fact that sun and moon move in different plains does not affect this calculation?

It seems that Rambam treats "the crank" as just a calculation device - otherwise, table giving visible anomaly from the corrected anomaly should take into account changes in the earth-moon distance depending on the elongation - and it does not! Maybe that is why he does not describe the model behind this correction. And maybe this is why Rambam reiterates - in Law 2 - that only the observability of the new moon needs to be calculated correctly. TODO: How does Almagest calculate visible anomaly?

Law 4,5,6: True Lunar Longitude. Formulae:

$$\tan^{-1}\left(\frac{\sin\alpha}{\cos\alpha + \epsilon}\right)$$

R - radius of the big circle; r - radius of the small circle; $\epsilon = R/r$. A little trigonometry, and we get: visible anomaly = arctg(sin(corrected anomaly)/(cos(corrected anomaly) + e)) and: e = ctg(visible anomaly)*sin(corrected anomaly) - cos(corrected anomaly)

TODO: Make the diagram! Here?

It is clear that values for 150° and 170° are misprinted: they are bigger than the one before them when they should be smaller. Value for 120° is also misprinted, but it is less obvious.

Calculations show that for the precision up to a minute, it is sufficient to know e to up to the second digit after the dot. Below, a column giving the value of e was added to the table. Look at the value of e, and the misprints become obvious: value of e for them is way out of range it is in for the rest of them.

TODO Range of e - graph. Range of possible values for each based on the range of e. This is transcription error: one letter; extra word... Probable correct values.

This is how different sources treat the often misprinted values:

Edition	**120°**	**150°**	**170°**
Vilno	incorrect	incorrect	incorrect
Eshkol	incorrect	incorrect	incorrect
Rambam LaAm	corrected in the notes	incorrect	incorrect
Kapach [a]	incorrect	corrected in the notes	corrected in the notes
Bluming	incorrect	corrected in the notes	corrected in the notes
Frenkel	correct and noted	correct and noted	correct and noted
Keller	not given	not given	not given
Losh [b]	incorrect	incorrect	incorrect

[a] It is not clear how did authoritative Yemenite texts acquire the same misprints as in the ashkenazic editions.

[b] Reprints the text and does not correct any of the misprints - surpising for a textbook on the subject (even introductory). TODO: Link to the scans of the appropriate pages - with the stuff highlighted.

Questions

New moon from 6:7

Rambam mentiones Nisan conjunction on Sunday, 5 hours and 107 units parts after sunrise. He doesn't give the year, and there seems to be no such year...

Moznaim Rambam, KH 13:2f4

(13, 3, 53, 53) - as explanation of the value for 100 days (7 missing seconds)

Moon's mean period from astronomical models

Angular speed of the moon = 360 / (1/tropical month + 1/solar year). Moon.meanLunarPeriod - what is it called? Tropical?

14:5 Moon Longitude Adjustment for Time of Seeing

See notes in Moznaim Rambam Sun longitude: mean or true? In 3rd and 7th lines, most editions incorrectly have 15.

TODO Moznaim Rambam, KH 15:1f2

double elongation = distance between moon's mean and apogee Moon longitude adjustment for double elongation: Moznaim Rambam, KH 15:2f3: discrepancy in the limits; encode the limits

MoonSightable.forSightingLimits

is that what Rambam is saying?

Distance between mean and true new moon

KH 5:1-2 says that the distance is no more than a day. Calculate this difference for all months; Rambam's epoch - two days after molad?! (Petya Ofman).

Differences between arithmetic and observational calendars

- основан не на среднем движении солнца и луны, поэтому к молад надо делать поправку, но не путем отодвигания Рош а-шана, а методами последних глав Кидуш а-ходеш
- другой порядок високосных и простых лет
- другой порядок полных и неполных месяцев
- возможно ли рассчитать по Рамбаму начало месяцев по этим принципам и сравнить с завиксированныму в традиции датами и днями недели?
- Пример: выход из Египта - 15 день от освящения месяца, четверг
- Год рождения Моше: спор был ли год високосным или простым. Шита Ребе - год был високосным.

TimeToRotation.calculate()

without the '29' case, mean sun longitude for 4938/Iyar/2 is not what Rambam quotes in KH 15:8-9 (see test); see if variations in this algorithms are logical: e.g., for 600, add for 1000 and subtract 4*for 100? see if the end result is stable when Rambam's "real" value is used with straight multiplication and rounding (abstract away the calculation mechaninsm).

TODO Example in 15:8-9

moonAnomalyTrue - printing error in standard editions, 180 instead of 108; still 108°21′46″, which doesn't round to 108°21... moonLongitudeTrueRaw - printing error in standard editions, 33 instead of 35.

TODO Example 17:13-14

"this longitude is in Taurus" - but longitude3 isn't, so I get 1/6 instead of 1/5...

TODO crescent calculations: KH 18-19!

Meton's cycle

Show why 19-years cycle is the best. Calculate optimal distribution of the leap years.

- 1:2 Solar year is ~11 days longer than the lunar (see chapter 6); when discrepancy accumulates to ~30 days, year is made leap. - 4:1-2 Leap year - additional Adar; Pesach (15th of Nisan) must be not before the vernal equinox (Sun enters Aries, see 9:3) and before the next summer solstice.

Moznaim: Based on Sanhedrin 13a-b, the Ramah (cited in Kessef Mishneh on law 15): "Succos must be after the autumnal equinox"; Rambam doesn't mention this. With vernal equinox on Nissan 15th, autumnal could be on Tishrei 21st, 6 days into Succos. Ohr Sameach: Rambam did say this, printers missed it. Aruch HaShulchan: Rambam holds that that Sanhedrin isn't law; we only care about vernal equinox.

QUESTION: what are the properties of the fixed calendar in this respect (need true seasons)?

- 6:3 Lunar month (between conjunctions) = 29d12h793p. - 6:4 Lunar year (12 lunar months) = 354d8h876p = 354d8h48m40s; Leap lunar year (13 lunar months) = 383d21h589p; Solar year = 365d6h (see also 9:1, 10:6); longer than the lunar year by 10d21h204p. - 6:10 19 year cycle (12+7 leap); because the difference between 19 solar years and the cycle is less than a day: 1h485p. (see 10:1 - with a different year length, the difference is 0)

QUESTION: what other lengths of the cycle have this property?

- 6:11 Leap years: 3,6,8,11,14,17,19.

QUESTION: in what sense is this an optimal correspondence? and for different cycle lengths? It is possible that true Sun calculations are needed to answer those questions...

Rambam: Summary

General

Chapter 1: Introduction.

Law 1: Month is lunar; year is solar.
Law 2: Solar year is ~11 days longer than the lunar; when discrepancy accumulates to ~30 days, year is made leap.
Law 3: Moon is not seen for ~1 day before and after the conjunction; first night when the Moon is sighted - Rosh Chodesh; next Rosh Chodesh: if it is sighted on the night of the 30th day - 30th; if not - 31st.
Law 4: Short/full month.
Laws 5-8: Court calculates in a manner **resembling** the calculations of the astronomers.

Chapter 2: Court procedure.

Laws 1-10: Court procedure.

Chapter 3: Court procedure.

Laws 1-19: Court procedure; messengers.

Chapter 4: Leap Year.

Laws 1-2: Leap year - additional Adar; Pesach (15th of Nisan) must in the spring: not before the day of vernal equinox (Sun enters Aries, see 9:3) and before the next summer solstice.
Laws 3-8: Other reasons for leaping the year; (Succos after the autumnal equinox - not mentioned in Rambam.)
Laws 9-17: Court procedure.

Arithmetic calendar.

Chapter 5: Introduction.

Laws 1-2: Introduction; discrepancy with observations: -1...+1 day (+1 - very rare).
Law 2: Fixed Rosh Hodesh: sighting of the Moon can be +/- 1 day; +2 - extremely rare.

Law 3: When started.
Laws 4-12: Second day of festivals.
Law 13: Conclusion.

Chapter 6: Conjunction. Lunar month and year. Leap year cycle.

Law 1: "Molad" - approximate conjunction (Sun. Moon and Earth on the same line).
Law 2: day = 24 hours; 1 hour = 1080 parts.
Law 3: Lunar month (between conjunctions) = 29d12h793p.
Law 4: Lunar year (12 lunar months) = 354d8h876p = 354d8h48m40s; Leap lunar year (13 lunar months) = 383d21h589p; Solar year = 365d6h (see also 9:1, 10:6); longer than the lunar year by 10d21h204p.
Law 5: Lunar month % 7 = 1d12h793p; lunar year % 7 = 4d8h876p; leap year % 7 = 5d21h589p.
Law 7: if molad Nisan - Sunday, 5h107p after sunrise, next month - Iyar: Tuesday night, 5h900p.
Law 8: First conjunction - Tishrei, year #1: Monday, 5h204p (just subtract % from the next one :)) Conjunction of Rosh Hashono of Adam's creation (year #2, see Shmita and Yovel 10:2): Friday, 6h.
Law 9: We do not need the number of days, just the remainders.
Law 10: 19 year cycle (12+7 leap); because the difference between 19 solar years and the cycle is less than a day: 1h485p.
Law 11: Leap years: 3,6,8,11,14,17,19.
Law 12: cycle % 7 = 2d16h595p.

Chapter 7: New Year corrections.

Law 1: Rosh Hashono gets pushed off from Sunday, Wednesday, Friday.
Law 2: Gets pushed off if molad is after noon (Molad Zoken).
Law 3: ADU applies after noon push-off also.
Law 4: Non-leap year, if molad is at night of the 3rd day after 9h204p: push off to Thursday (or the year willbe too long).
Law 5: Year after leap one, Monday, after 5h589p: push to Tuesday.
Law 7: ADU - why?

Chapter 8: TODO

Law 1: -> 6.3
Law 2: Month - 29 or 30 days.
Laws 5-6: Same?

Law 7: TODO
Law 8: TODO

Chapter 9: Seasons.

Law 2: 1/4 of the year between seasons (91d 7.5h).
Law 3: Nisan (Spring) - Aries; Tammuz - Cancer; Tishrei - Libra; Teves - Cap.? Vernal equinox of year #1 = 7d9h642p before conjunction of Nisan. Notes 5,7 in Moznaim edition. Every year seasons move 30h; in 28 years - in the same place.
Law 5: Vernal equinox, year 4930: midnight, 5th day, 8th of Nisan.
Laws 6-8: Simplified procedure.
Law 7: Real seasons - 2 days before average ones. Note 5: 1st hour of the 4th day of creation; 1st vernal equinox. Autumnal: Wednesday, 3 hours after daybreak, 1d23h before conjunction of Tishrei.

Chapter 10: More precise length of the year. Seasons.

Law 1: More precise length of a solar year. Moments.
Law 2: Seasons for this year length.
Law 3: Vernal equinox of year #1 - 9h642p before conjunction of Nisan. (Differenbt from 9:3: Rav Ada follows Rabbi Yehoshua - world created in Nisan; Shmuel (9:3) follows Rabbi Eliezer - world created in Tishrei.) Every first year of the 19-year cycle.
Law 4: 1/4 of the year between seasons (91d7h519p31m).
Law 5: Simplified procedure.
Law 6: This more precise calculation of seasons was used by Sanhedrin to leap the year because of the spring equinox.
Law 7: Real vernal equinox approximately 2 days before either of these calculations.

Atronomical Calculations.

Chapter 11. Approximation. Angles. Mean and true motions. Epoch and location.

Laws 1-4: Visibility calculations are deep. Fixed calendar can be appreciated even by school children in 3 or 4 days.

Laws 5-6: Calculations are simplified/approximated to not fluster the inexprerienced; result - visibility of the moon - is not affected.
Laws 7-9: 360 degrees; 60 minutes, seconds...; order of the 12 constellations.
Laws 10-12: adding/subtracting angles.
Laws 13-14: Velocities are constant. Earth encircled by all the spheres but isn't in the center. Notes 14,15 in Moznaim.
Law 15: Mean and true motion.
Law 16: Epoch.
Law 17: Location: Jerusalem; see Chapter 17.

Chapter 12: Sun and its apogee.

Law 1: Movement of the Sun. Moznaim Note 6: calculations may be accurate only on the first day of the month; Note 7. 59′8.33″ Yale, p.99; Neugeb., p. 388ff
Law 2: Movement of the Sun's apogee. Moznaim Note 10: over 800 years, apogee moved ~12 degrees and is in constellation of Cancer. Note 11: position - at 6PM.

Chapter 13: True position of the sun. True seasons.

Laws 1-3: Tue position of the Sun.
Law 4: Course correction table.
Laws 5-10: Examples.
Law 11: True seasons.

Chapter 14

Law 1: Moon: epicycle and deferent; movement as seen from Earth. See Moznaim notes!
Law 2: Deferrent table.
Law 3: Epicycle table.
Law 4: Epoch.
Law 5: Time of sighting correction table: ~20 minutes after sunset.

Chapter 15

Laws 1-2: Double elongation and its limits. See Moznaim notes!
Laws 3-5: Course correction table.
Laws 4-7: Parallax correction rules and table.
Laws 8-9: Examples.

Chapter 16

Laws 1-4: Head movement, table and rules.
Law 5: Example.
Laws 6-18: Moon latitude.
Law 19: Example.

Chapter 17 (TODO)

Chapter 18 (TODO)

Chapter 19 (TODO)

Part II: The Text

Laws of the Sanctification of the Moon

There is one positive commandment: to calculate, to know, and to establish on which day each month of the year begins. Explanation of this commandment is in these chapters.

Chapter 1

Law 1

The months of the year are lunar months, as it is said: "[...] the burnt offering of the month in its month" [1] and it is said: "This month shall be for you the head of the months" [2]. This is what the sages said: "The Holy One, blessed be He, showed to Moshe, in a prophetic vision, the image of the Moon, and said to him: '[When] you see [the Moon] like this - sanctify [it].'"[3]

But the years that we reckon are solar years, as it is said: "Keep the month of aviv [spring]". [4]

Law 2

By how much is the solar year longer than the lunar year? By approximately 11 days. Therefore, when this excess accumulates to around 30 days, or a little less, or a little more, a month is added and the year is made to have 13 months - which is called a "pregnant" year. [This is done] because it is impossible to have a year of 12 months and some days, as it is said: "for the months of the year" - you count the months of the year, but not the days of the year. [5]

Law 3

Each month, the moon becomes hidden for about two days, or a little less or more: about one day before its conjunction with the sun at the end of the month, and about one day after its conjunction with the sun, and in the evening it is sightet in the west. The night when it is sighted in the west after being hidden is the beginning of the month. We count 29 days from that day; if the moon is sighted on the 30th night, the 30th day is the first day of the [next] month. If it is not sighted, first day of the [next] month is the 31st day, and the 30th day belongs to the previous month. On the 31st night, we do not depend on the moon, regardless if it is visible or not, since lunar month is never longer than 30 days.

[1] Bamidbar 28:14
[2] Shemos 12:2
[3] Rosh Hashanah 20b; Menachot 29a
[4] Dvarim 16
[5] Megilla 5a; Nazir 7a

Law 4

Month of 29 days, [when] the moon was sighted on the 30th night, is called "lacking". If the moon was not sighted, and the previous month has 30 days, it is called "pregnant" month, or "full" month. The moon sighted on the 30th night is called "moon sighted on time". If it is sighted on the 31st night, but not on the 30th night, it is called "moon sighted on the night of pregnancy".

Law 5

Sighting of the moon is not up to an individual, unlike determination of the Shabbos, where one counts 6 days and observes Shabbos on the 7th, but is given to the court. Only when the court sanctifies and establishes the day as the beginning of the [new] month does it become the beginning of the [new] month. As it is said: "this month shall be for you" - this testimony is entrusted to you.

Law 6

The court calculates, like the astronomers who know positions of the planets and their movements, and look into it until they know if it is possible for the moon to be sighted on time - that is, on the 30th night - or not. If they determined that it is possible for it to be sighted, they sit and wait for withnesses all day - that is, the 30th day. If the witnesses did come and were examined in accordance with the law, and their words were accepted - they sanctify it. If the moon was not sighted and the witnesses did not come, they complete the 30 days and the month will be "pregnant". If they found out from calculations that it is impossible to sight the moon, they do not sit during the 30th day and do not await the witnesses. And if the witnesses did come, it is certain that they are false witnesses or that they saw a likeness of the moon in the clouds and not the real moon.

Law 7

It is a positive commandment of the Torah for the court to calculate and to know whether the moon can be sighted or not, and to examine the witnesses before the month is sanctified, and to send [messengers] to notify the rest of the people which day is the beginning of the month, so that they know on which day are the festivals. As it is said: "which you will call the holy convocations" and as it is said : "keep this statute in its time".

Law 8

Calculation and establishment of the months and declaration of "pregnant" years is done only in the Land of Israel, as it is said: "For out of Zion will come forth Torah, and the word of G-d - from Yerushalaim". But, if there was a man great in wisdom, and ordained in the Land of Israel, who came out of the Land, and there remains no one equal to him in the Land of Israel, he can calculate and establish month, and declares "pregnant" years outside of the Land. If he becomes aware that in the Land of Israel there appeared a man as great as him - or, needless to say, greater than him - he is prohibited to establish and to proclaim outside of the Land, and if he transgressed and did establish or did proclaim, it is as if he didn't do a thing.

Chapter 4

Law 1

TODO

Law 2

TODO

Chapter 11

Law 1

TODO

Law 2

TODO

Law 3

TODO

Law 4

TODO

Law 5

TODO

Law 6

TODO

Law 7

TODO

Law 8

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Law 9

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Law 10

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Law 11

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Law 12

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Law 13

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Law 14

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Law 15

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Law 16

TODO

Law 17

TODO

Chapter 12

Law 1

The mean rate of movement of the sun in one day (that is, twenty four hours) is fifty nine minutes and eight seconds, in symbols - 59′8″. It follows that its movement in ten days is nine degrees fifty one minutes and twenty three seconds, in symbols - 9°51′23″. It also follows that its movement in a hundred days is ninety eight degrees thirty three minutes and fidty three seconds, in symbols - 98°33′53″. It also follows that remainder of its movement in a thousand days, after you subtract all [multiples of] 360° (as was explained) is two hundred sixty five degrees thirty eight minutes and fifty seconds, in symbols - 265°38′50″. It also follows that the remainder of its movement in ten thousand days is one hundred thirty six degrees twenty eight minutes and twenty seconds, in symbols - 136°28′20″.

And in this way you can multiply and calculate its movement for any number [of days] that you want. Similarly, if you want to make known to you values of its movement for two days, three, four and so on to ten - do it. Similarly, if you want for to have known and ready values of its movement for twenty days, thirty, fourty and so on to a hundred - do it. This is clear and known once you know its movement in 1 day.

And you should have ready and known to you mean movement of the Sun for twenty nine days and for three hundred and fifty four (which is the number of days in the lunar year when its months are "regular" (TODO link), and it is called "regular year"). The reason is: if you have those movement values ready, this calculations of the visibility of the moon will be easy, because there are twenty nine complete days from the night of observation to the night of observation of the following month, and so it is every month: no less than twenty nine days and no more. Since our sole desire in all those calculations is exclusively to determine visibility [of the moon]. And between the night of sighting of this month and night of sighting of the same month next year there is either a regular year or a year and 1 day; and the same every year. Mean movement of the Sun in twenty nine days is twenty eight degrees thirty five minutes and one second, in symbols - 28°35′1″. Its movement in a regular year is three hundred forty eight degrees, fifty five minutes and fifteen seconds, in symbols - 348°55′15″.

Law 2

There is a point on the Sun's orbit (and on orbits of the other seven planets) such that when the planet is there it is highest above Earth. This point of the Sun's orbit (and so for other planets, except for the moon) rotates with constant speed. Its movement every seventy years is approximately one degree. This point is called *Sun's apogee*. Its movement in ten days is one and a half seconds, half a second being thirty thirds. It follows that its movement in a hundred days is fifteen seconds. Its movement in a thousand days is two minutes and thirty seconds. Its movement in ten thousand days is twenty five minutes. It also follows that its movement in twenty nine days is a bit more (TO-DO!) than four seconds; and its movement in a regular year is fifty three seconds.

We already said that the epoch that our calculations start from is beginning of the night of the fifth day that is 3rd of Nisan of the year four thousand nine hundred thirty eight from the Creation. Position of the Sun in its mean movement at the epoch was seven degrees three minutes and thirty two seconds in the constellation of Ram (Tele), in symbols - 7°3′32″. Position of the Sun's apogee at the epoch was twenty six degrees fourty five minutes and eight seconds in the constellation of Twins, in symbols - 26°45′8″.

When you want to know position of the Sun in its mean movement at any time you want, take the number of days from the epoch to the day that you want, and find out its mean movement during those days from the values we mentioned, add all of it to the epoch, adding each kind [of unit] with its kind, and the result is the position of the Sun in its mean movement on that day.

For example, if we wanted to know Sun's mean position at the beginning of the night of Shabbat whose day is the fourteenth of Tammuz of this year, which is the year of the epoch - we find the number of days from the day of the epoch to the beginning of the day that we want to find Sun's position on to be a hundred days. We take its mean movement over a hundred days - which is 98°33′53″ - and add it to the epoch - which is 7°3′32″. Result of this calculation: one hundred and five degrees thrity seven minutes and twenty five seconds. In symbols - 105°37′25″. So its position in its mean movement at the beginning of this night is in the constellation of Cancer, at fifteen degrees in it, and thirty seven minutes of the sixteenth degree.

Mean position thus calculated is sometimes exactly at the beginning of the night, sometimes an hour before sunset, and sometimes - an hour after sunset. This fact is not important for the Sun in the calculations of the observation [of the new Moon] since we compensate for this approximation when calculating mean Moon.

Do it the same way for whatever time you want whatsoever, even after a thousand years: when you add up all the reminders and add to the epoch, result you get is the the mean position. And the same you do for the mean Moon and mean of every planet: once you know what is the movement in one day and what is the epoch that we start from, and you sum up its movement in the years and days that you want, and add it to the epoch - you get its position in mean movement.

Do the same for the Sun's apogee: add its movement in those days or years to the epoch, and you get position of the Sun's apogee for the day that you want.

Also, if you want to make a different epoch from which to start, other that the epoch that we start from (which is this year), so that epoch is at the beginning of the year of a known cycle or at the beginning of a hundred-year period - you can do it. And if yuo want the epoch from which you start to be in the years that already passed before this epoch or many years after it - the way is known.

What is this way? You already know movement of the Sun in an regular year, and its movement in twenty nine days, and its movement in one day. And it is known that year whose months are full is longer than the regular year by one day, and the year whose months are lacking is shorter than the regular year by one day. And the leap year: if its months are regular, it will be longer than the regular year by thirty days, and if its months are full, it will be longer than the regular one by thirty one day, and if its months are lacking, it will be longer than the regular one by twenty nine days. Once you know all those facts, calculate Sun's mean movement for the years and days that you want, and add to the epoch that we made, and you'll get its mean position for the day that you want from the coming years - and that you make into an epoch. Or subtract mean [movement] that you calculated form the epoch that we made, and you get an epoch for the day that you want from the years that passed, and make that mean [position] an epoch. And the same you shall do for the mean Moon and the rest of the planets if [their movements] will be known to you. And it should be clear to you from the idea of our words that the same way you can find out mean Sun for any day you want from the coming days, you can calculate its mean for any day you want from the days that passed.

Chapter 13

Law 1

If you want to find out true position of the Sun for any day that you want, first calculate its mean position for that day the way we explained, and calculate position of the Sun's apogee. Subtract position of the Sun's apogee from the Sun's mean position; what is left is called *maslul of the Sun*.

Law 2

See how many degrees is the maslul of the Sun; if maslul was less than one hundred and eighty degrees, subtract parallax correction from the position of the mean Sun; if maslul was more than one hundred and eighty degrees up to three hundred and sixty, add parallax correction to the position of the mean Sun; that what it is after you add to it or subtract from it is the true position.

Law 3

Know that if maslul is exactly one hundred and eighty or exactly three hundred and sixty, it doesn't have parallax correction; rather, the mean position is the true position

Law 4

And what is the value of the parallax correction? If maslul will be ten degrees, its parallax correction will be twenty seconds; and if it will be twenty degrees, its parallax correction will be fourty minutes; and if it will be thirty degrees, its parallax correction will be fifty eight minutes; and if it will be fourty degrees, its parallax correction will be one degree and fifteen minutes; and if it will be fifty degrees, its parallax correction will be one degree and twenty nine minutes; and if it will be sixty degrees, its parallax correction will be one degree and fourty one minutes; and if it will be seventy degrees, its parallax correction will be one degree and fifty one minutes; and if it will be eighty degrees, its parallax correction will be one degree and fifty seven minutes; and if it will be ninety degrees, its parallax correction will be one degree and fifty nine minutes; and if it will be one hundred degrees, its parallax correction will be one degree and fifty eight minutes; and if it will be one hundred and ten degrees, its parallax correction will be one degree and fifty three minutes; and if it will be one hundred and twenty degrees, its parallax correction will be one degree and fourty five minutes; and if it will be one hundred and thirty degrees, its parallax correction will be one degree and thirty three minutes; and if it will be one hundred and forty degrees, its parallax correction will be one degree and nineteen minutes; and if it will be one hundred and fifty degrees, its parallax correction will be one degree and one minute; and if it will be one hundred and sizty degrees, its parallax correction will be fourty two minutes; and if it will be one hundred and seventy degrees, its parallax correction will be twenty one minutes; and if it will be one hundred and eighty degrees exactly, it doesn't have parallax correction, as we explained, rather Sun's mean position is its true position.

Law 5

If maslul was more that one hundred and eighty degrees, subtract it from three hundred and sixty degrees and find out its parallax correction. For example, if maslul was two hundred degrees, subtract it from three hundred and suxty, and one hundred and sixty degrees will be left. And we already made known that parallax correction for one hundred and sixty degrees is fourty two minutes, and so is the parallax correction for two hundred - fourty two minutes.

Law 6

Also, if the maslul was three hindred degrees, subtract it from three hundred and sixty - and sixty will remain. And you already know that parallax correction for sixty degrees [is] one degree and fouty one minutes, and so is the parallax correction for three hundred degrees. And the same way - for every number.

Law 7

TODO

Law 8

TODO

Law 9

TODO

Law 10

TODO

Law 11

TODO

Chapter 14

Law 1

The moon has two mean movements. The moon itself rotates on a small circle that does not surround all of the Earth. Moon's movement on that small circle is called *mean lunar anomaly* (אמצע המסלול). The small circle (epicycle TODO link to Wikipedia) itself rotates on a big circle (deferent TODO link to Wikipedia) that encircles the Earth. The mean movement of the small circle on the big circle that encircles the Earth is called movement of the *mean lunar longitude* (אמצע הירח).

Law 2

Movement in mean lunar longitude in one day is 13°10′35″ [1].

For longer periods, (remainder) of the movement in mean lunar longitude is:

And in this way you can calculate it for any number of days and years that you want.

Law 3

Movement in mean lunar anomaly in one day is 13°3′54″.

For longer periods, (remainder) of the movement in mean lunar anomaly is:

Law 4

In the beginning of the night to Thursday that is the epoch (TODO measured from what? Chapter 11?) [TODO: link to definition] for these calculations, mean lunar longitude was 1°14′43″ into the constellation of Ram. And mean lunar anomaly on the epoch was 84°28′42″. Now that you know rate of movement in the mean lunar longitude and its value on the epoch to which you add, you'll calculate mean lunar longitude on any day that you want the same way as you did for the mean solar longitude. TODO: reference

[1] This is part of Law 1.

Law 5

After you calculate mean lunar longitude at the beginning of the night that you want, contemplate the sun (TODO: longitude?) and figure out which constellation (zodiac) is it in [2]. [This is the correction of the mean lunar longitude depending on the mean solar longitude:]

Law 6

Solar Longitude on the Zodiac	**from**	**to, not including**[a]	**Lunar longitude correction**
middle of Pisces-middle of Taurus [b]	315°	15°	0°
middle of Taurus-beginning of Gemini	15°	30°	0°15′
beginning of Gemini-beginning of Leo	30°	90°	0°15′
beginning of Leo-middle of Virgo	90°	135°	0°15′
middle of Virgo-middle of Libra	135°	165°	0°
middle of Libra-beginning of Sagittarius	165°	210°	-0°15′
beginning of Sagittarius-beginning of Aquarius	210°	270°	-0°30′
beginning of Aquarius-middle of Pisces	270°	315°	-0°15′

[a] My understanding of the word usage in the original.

[b] See Chapter 11, Law 7 on the start of Jewish Zodiac. (TODO reference)

[2] This is part of Law 4.

The value of mean [lunar longitude] after you add to it or subtract from it or leave it as it is - that is mean lunar longitude approximately a third of an hour after sunset on the date you are calculating it for. And this is what is called *mean lunar longitude at observation time*.

Chapter 15

Law 1

To find *true lunar longitude* on a specific day: First, calculate mean lunar longitude during observation time of the desired night. Also, calculate mean lunar anomaly and mean solar longitude for the same time. Then, subtract mean solar longitude from the mean lunar longitude and double the result. This value is called *doubled distance* .

Law 2

We already stipulated that all the calculations in these chapters are for one purpose only: to know if the moon is visible. And it is not possible for this double distance during the night of observation when the moon is indeed visible to be outside of the interval from 5° to 62°. It cannot be more - nor less.

Law 3

A correction that depends on the doubled distance is added to the mean lunar anomaly, giving *corrected lunar anomaly* (מסלול הנכון):

doubled distance	**correction of the mean anomaly**
~5°	0°
6°-11°	1°
12°-18°	2°
19°-24°	3°
25°-31°	4°
32°-38°	5°
39°-45°	6°
46°-51°	7°
52°-59°	8°
60°-63°	9°

Law 4

Then, see how many degrees is corrected lunar anomaly. If it is less than 180°, subtract visible anomaly of this corrected anomaly from the mean lunar longitude the observation time. And if corrected lunar anomaly is more than 180° but less than 360°, add visible anomaly of this corrected anomaly to the mean lunar longitude the observation time. And mean lunar longitude after you add to it or subtract from it is the *true lunar longitude at the observation time*.

Law 5

If the corrected anomaly is 180° or 360°, there is no visible anomaly, so mean lunar longitude at observation time is also true longitude. [1]

Law 6

What is the value of the visible anomaly for a given value of the corrected anomaly?

Law 7

TODO

Law 8

TODO

Law 9

TODO

[1] This is Law 5.

Glossary

Astronomical terms

Sun's apogee	Point on the orbit of the Sun where it is highest above the Earth.
mean lunar anomaly (אמצע המסלול)	Mean angular coordinate of the Moon on its epicycle.
true lunar longitude (הירח האמיתי)	???
doubled distance (מרחק הכפול)	???

Bibliography

Laws of Sanctification of the Moon by Rambam

[Vilno] Rambam. *Mishne Torah*. Widow and Brothers Romm.

[Rambam LaAm] Rambam. *Mishne Torah*. Rambam LaAm.

[Eshkol] Rambam. *Mishne Torah*. Eshkol. 1987.

[Kapach] Rambam. *Mishne Torah*. Rabbi. Kapach.

[Frenkel] Rambam. *Mishne Torah*. Rabbi. Frenkel.

[Bluming] Rambam. *Mishne Torah*. Rambam HaAruch. Rabbi. Y Bluming. 2005.

[Yale] Rambam. Solomon Gandz. Julian Obermann. *Mishne Torah*. *Code of Maimonides, Book 3, Treatise 8: Sanctification of the New Moon (Yale Judaica Series, Vol. 11)*. Yale. 1967.

[Touger] Rambam. *Mishne Torah*. Rabbi. Touger.

[HazonIsh] Hazon Ish.

Jewish Writings on Calendar

[Losh] *Sefer HaTchunah LeMatchil*. Yosef Yitzchok Losh. 1988.

[Keller] *Tchunat HaShamaim*. Yosef Yitzchok Keller. 1988.

[Glitzenshtein] *Kidush HaChodesh LeHaRambam*. Menachem Mendel Glitzenshtein. 1993.

[Tzikuni] *חזון שמים (Hazon Shamaim)*. Eitan Tzikuni. 2001.

Hazon Ish and the sine tables. http://www.biu.ac.il/JH/Parasha/pekudey/lev.html.

Greeks on Astronomy

[Neugebauer] *The Exact Sciences in Antiquity*. Otto Neugebauer. 2nd. Dover Publications. 1969. http://isbn.nu/0486223329.

Astronomy and History Selected Essays. Otto Neugebauer. Springer. August 8, 1983. http://isbn.nu/0387908447.

[Crowe] *Theories of the World from Antiquity to the Copernican Revolution*. Michael J. Crowe. 2nd. Dover Publications. 2001. http://isbn.nu/0486414442.

[Evans] *The History & Practice of Ancient Astronomy*. James Evans. Oxford University Press. 1998. http://isbn.nu/0195095391.

[Thurston] *Early Astronomy*. Hugh Thurston. Springer Verlag. 1996. http://isbn.nu/0387948228.

[Almagest] *Ptolemy's Almagest*. G. J. Toomer. Princeton University Press. 1998. http://isbn.nu/0691002606.

[Campanus] *Campanus of Novara & Medieval Planetary Theory*. G. J. Toomer, Jr and Francis S. Benjamin. University of Wisconsin Press. 1972. http://isbn.nu/029905960X.

Ancient Planetary Model Animations. http://people.sc.fsu.edu/~dduke/models.

A Modern Almagest: An updated version of Ptolemy's Almagest. http://farside.ph.utexas.edu/Books/Syntaxis/Syntaxis.html.

Summary of Ptolemaic Astronomy. http://www.clas.ufl.edu/users/ufhatch/HIS-SCI-STUDY-GUIDE/0034_summaryPtolemiacAstron.html.

Contemporary Models

[Meeus] *Astronomical Algorithms*. Jean Meeus. 2nd. Willmann-Bell. 1998. http://isbn.nu/0943396611.

[Montenbruck] *Astronomy on the Personal Computer*. Oliver Montenbruck and Thomas Pfleger. 4th. Springer Verlag. 2000. http://isbn.nu/3540672214.

[Dershowitz] *Calendrical Calculations*. Nachum Dershowitz and Edward M. Reingold. 3rd. Cambridge University Press. 2008. http://isbn.nu/0521702386.

[Dermott] *Solar System Dynamics*. S. F. Dermott and Carl D. Murray. Cambridge University Press. 2000. http://isbn.nu/0521575974.

How to compute planetary positions. http://stjarnhimlen.se/comp/ppcomp.html.

Sky View. http://www.sky-view.co.il/.

Israeli New Moon Society. https://sites.google.com/site/moonsoc/.

EarthTools. http://www.earthtools.org/webservices.htm#usage.

NOAA Solar Calculator. http://www.esrl.noaa.gov/gmd/grad/solcalc/.

Naval Oceanography Portal. http://aa.usno.navy.mil/data/index.php.

Zmanim. http://www.myzmanim.com/search.aspx.

MikvahCalendar. https://www.mikvahcalendar.com/.

On Jewish Calendar

Understanding the Jewish Calendar. Nathan Bushwick. Moznaim. February 1, 1989. http://isbn.nu/0940118173.

The Hebrew Calendar. Dr. Irv Bromberg. University of Toronto, Canada. http://www.sym454.org/hebrew.

[Feldman] W. M. Feldman. *Rabbinical Mathematics and Astronomy*. Judaic Studies Library; No. Shp4. Intellectbooks. January 1, 1931. 3. http://isbn.nu/0872030261.

Belenkiy. http://u.cs.biu.ac.il/~belenka/calendar.html.

Bromberg. http://individual.utoronto.ca/kalendis/.

Remy Landau. *Hebrew Calendar Science and Myths*. http://hebrewcalendar.tripod.com/.

Sacha Stern. *Calendar and Community: A History of the Jewish Calendar, 2nd Century BCE to 10th Century CE*.

The Reality of Epicycles and Eccentrics Denied. http://homepages.wmich.edu/~mcgrew/maimonides.htm.