Rambam on Calendar

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Introduction

In the "Laws of Sanctification of the Moon", Rambam (TODO: where?) gives algorithmic description of two calendars: fixed ("arithmetic") and observational ("astronomical"). Both calendars are described in the form ready to be implemented, but the reasons for the calculation steps are not always given. Especially for the astronomical calendar, the following questions arise: where do the numbers come from? What are the models - and the formulae resulting from them - that reproduce the numbers? Sometimes Rambam describes the models, so only the formulae need to be reconstructed; sometimes the models are not described, so both the models and the formulae need to be reconstructed.

Goals

- Translate appropriate chapters of Rambam into English. The translation does not have to be literal; literal translations are already available (Yale, Touger). The goal of our translation is to make the text complete and accessible for collaboration by people not proficient enough in Hebrew.
- Reconstruct the models and formulae implied in the text.
- Implement the algorithms.
- Verify consistency of Rambam's numbers.
- Compare Rambam's models with contemporary astronomical models.

History

In 1984, The Rebbe instructed the Jews to learn Rambam every day. As a result, in 1991 I encountered calendar chapters of Rambam, decided to reconstruct the models behind the calculations described in them, and discovered mistakes in the text of one of the laws (15:6). The results were published in "Notices of Temimim and Anash", a weekly of the central Lubavicher Yeshiva 770. Some of the readers were appalled that someone dares to correct the text of Rambam! My point was that the text as published contains mistakes that need to be corrected. Indeed, Frenkel edition [Frenkel], that was already published at the time, corrects all of the mistakes I did. In fact, they were already corrected by Hazon Ish in his commentary on Rambam [[HazonIsh]] - but I did not know it then.

In 2005, during a visit to Israel, I discussed the subject with Rabbi Michael Koretz.

In 2008 I acquired a pile of books on the subject and started looking into it; I also experimented with various ways of publishing the work on the web.

In 2009 I coded some of the algorithms to look into the rumors the dates of Birkhat HaHamah that were circulating then.

In 2011 I realized that the work needs to be published before it is finished - so that others could help finish it ;)

Acknowledgements

I want to acknowledge people who contributed to the content of this work - and to the fact of its existence ;) If your name should be on this list but isn't, please forgive me: the omission is not intentional.

Mordechai Goldin - for providing office space and computer equipment when I, as a Yeshiva student, had neither; Ilia Pomansky - for encouraging this work at the early stages, in 1991; my wife Nina - for listening to my wild ideas, for her patience, and for help with math; my daughter Menucha Rochel - for assistance with the translation; Rabbi Michael Koretz - for interesting descussions and help in obtaining texts on the subject; Dr. Michael Partensky - for encouragement and advise; Peter Ofman - for asking questions that prodded me to revive this project in 2011, after years of hiatus; Aron Matskin - for a discussion during his visit on Rosh Chodesh Mar Cheshvan 5772; Rabbi Chaim Prus - for causing me to investigate which number does the year of Creation has - 0 or 1.

Solar Theory

Chapter 12

Law 1

The mean rate of movement of the sun in one day - that is, twenty four hours - is 59'8".

For longer periods, (remainder) of the movement in mean lunar longitude is:

days	**value**
n	**v(n)**
1	0°59'8"
10	9°51'23"
29	28°35'1"
100	98°33'53"
354	348°55'15"
1000	265°38'50"
10000	136°28'20"

And in this way you can calculate it for any number of days and years that you want. And so if you want to calculate movement in 2, 3, 4 - up to 10 days - you can do it. And so if you want to calculate movement in 20, 30, 40 - up to 100 days - you can do it. And it is obvious that once you know the movement in one day. Regular lunar year is 354 days.

Between two consecutive nights of observation there is 29 full days. Between night of observation of a given month and the night of observation of the same month next year is in a regular year - or one day more. Our goal in all these calculations is solely to determine visibility of the moon.

Discussion

It is not clear how Rambam arrived at the values he gives. When value of the movement in one day given by Rambam is multiplied by 29, 100 etc., the result is smaller than that of Rambam. The following table compares printed and calculated values:

days	value	calculated
n	**v(n)**	**v(1)*n**
1	0°59'8"	0°59'8"
10	9°51'23"	9°51'20"
29	28°35'1"	28°34'52"
100	98°33'53"	98°33'20"
354	348°55'15"	348°53'12"
1000	265°38'50"	265°33'20"
10000	136°28'20"	135°33'20"

Although the value of the movement of the mean solar longitude in one day given by Almagest III 1 (H209) (which, rounded to seconds, becomes Rambam's value) is bigger, it is not big enough, and Rambam's numbers (for more than 10 days) can not be explained by performing calculations with the long value from Almagest and then rounding the results.

days	value	Alamgest
n	**v(n)**	***n**
1	0°59'8"	0°59'8"17'"13,12,31
10	9°51'23"	9°51'22"52'"12,5,10
29	28°35'1"	28°35'0"19'"23,2,59
100	98°33'53"	98°33'48"42'"0,51,40
354	348°55'15"	348°54'53"35'"55,50,54
1000	265°38'50"	265°38'7"0'"8,36,40
10000	136°28'20"	136°21'10"1'"26,6,40

From the printed values it is possible to reconstruct the value of the movement in mean solar longitude in one day that Rambam used to calculate each of them:

days	**value**	**reconstructed 1-day movement**
n	**v(n)**	**r(n)**
1	0°59'8"	0°59'7"59'"59,58,59
10	9°51'23"	0°59'8"17'"59,58,59
29	28°35'1"	0°59'8"18'"37,14,0
100	98°33'53"	0°59'8"19'"47,58,59
354	348°55'15"	0°59'8"20'"50,49,59
1000	265°38'50"	0°59'8"19'"48,58,59
10000	136°28'20"	0°59'8"19'"48,58,59

Tzikuni (p. XXX) gives the algorithm of such reconstruction: add to the remainder as many times 360 as there were full rotations in given time period, and then divide. It also gives a value reconstructed from the printed values for 10,000, 1000 and 100 days: 59'8.33", or 0.9856472 degrees, and the current "scientific" value of 0.9856473 degrees. It seems that origins of the Rambam's value were questioned by his commentators, including "Pirush". Value that can be derived from the tradition that 19 years = 6939 days 16 hours and 595 parts is 0.9856348. This algorithm can be modified to produce an interval of possible values, taking into account precision of the numbers.

Following table compares the printed values with calculations based on the most "precise" reconstructed value - that for 10000 days.

days	**value**	**recalculated**
n	**v(n)**	**r(10000)*n**
1	0°59'8"	0°59'8"19'"48,58,59
10	9°51'23"	9°51'23"18'"9,49,50
29	28°35'1"	28°35'1"34'"40,30,31

days	value	recalculated
n	**v(n)**	**r(10000)*n**
100	98°33'53"	98°33'53"1'"38,18,20
354	348°55'15"	348°55'8"55'"0,0,6
1000	265°38'50"	265°38'50"16'"23,3,20
10000	136°28'20"	136°28'22"43'"50,33,20

days	value	recalculated
n	**v(n)**	**r(10)*n**
1	0°59'8"	0°59'8"19'"48,58,59
10	9°51'23"	9°51'23"18'"0,0,0
29	28°35'1"	28°35'1"34'"11,59,59
100	98°33'53"	98°33'53"0'"0,0,0
354	348°55'15"	348°55'8"49'"12,0,0
1000	265°38'50"	265°38'49"59'"59,59,59
10000	136°28'20"	136°28'19"59'"59,59,59

Values of the movement in one day apparently used by Rambam, if rounded to the seconds, give the "official" Rambam's value. Values calculated from the value reconstructed from the printed 10000 days movement, if rounded, produce the values Rambam published - except for 29 days, 354 days, and 10000 days which we use to calculate the exact value that Rambam used! TODO!

Tzikuni quotes Rambam's value for 354 days as 348°55'15", but calculated value as 348°55'9", and notes that this "requires a little thought".

Law 2

There is a point on the Sun's orbit (and on orbits of the other 7 planets) where Sun is the highest above Earth. This point of the Sun's orbit (and so for other planets, except for the moon) rotates with constant speed. Movement of this point is approximately 1° every 70 years. This point is called Sun's apogee (gva hashemesh).

We already said that the epoch that our calculations are based on is beginning of the night of the fifth day that is 3rd of Nisan of the year 4938 from the Creation. And mean longitude of the Sun was at the epoch 7°3'32" in the constelation of Ram (tele). And apogee of the Sun was 26°45'8" in the constellation of Twins.

Chapter 13

Law 1

Law 2

Law 3

Law 4

Law 5

Law 6

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Law 8

Law 9

Law 10

Law 11

Lunar Theory

Chapter 14

Law 1: Two Movements of the Moon

The moon has two mean movements. The moon itself rotates on a small circle that does not surround all of the Earth. Moon's movement on that small circle is called *mean lunar anomaly* (אמצע המסלול). The small circle (epicycle TODO link to Wikipedia) itself rotates on a big circle (deferent TODO link to Wikipedia) that encircles the Earth. The mean movement of the small circle on the big circle that encircles the Earth is called movement of the *mean lunar longitude* (אמצע הירח).

Note: The first is "mean" - because it is corrected by the second, giving "true"? Why is the second "mean"? Because of the effects of the sun?

Note: Wrom what and in which direction are the angles measured?

Law 2: Movement in the Mean Lunar Longitude

Movement in mean lunar longitude in one day is 13°10'35" [1].

For longer periods, (remainder) of the movement in mean lunar longitude is:

days	value
n	v(n)
1	13°10'35"
10	131°45'50"
29	22°6'56"
100	237°38'23"
354	344°26'43"

[1] This is part of Law 1.

days	value
n	v(n)
1000	216°23'50"
10000	3°58'20"

And in this way you can calculate it for any number of days and years that you want.

Discussion

Rambam uses ‘nimzes’ instead of ‘nimzo’ for 10000 days. Also, Rambam says "'ordered' year"; it was rendered as 254 in accordance with Law 12:1.

It is not clear how Rambam arrived at the values he gives. When value of the movement in one day given by Rambam is multiplied by 29, 100 etc., the result is smaller than that of Rambam. The following table compares printed and calculated values:

days	value	calculated
n	v(n)	v(1)*n
1	13°10'35"	13°10'35"
10	131°45'50"	131°45'50"
29	22°6'56"	22°6'55"
100	237°38'23"	237°38'20"
354	344°26'43"	344°26'30"
1000	216°23'50"	216°23'20"
10000	3°58'20"	3°53'20"

Moreover, the value of the movement of the mean lunar longitude in one day given by Almagest IV 3 (H278) (which, rounded to seconds, becomes Rambam's value) is even smaller, so Rambam's numbers can not be explained by performing calculations with the long value from Almagest and then rounding the results.

From the printed values it is possible to reconstruct the value of the movement in mean lunar longitude in one day that Rambam used to calculate each of them:

days	**value**	**reconstructed 1-day movement**
n	**v(n)**	**r(n)**
1	13°10'35"	13°10'35"0'"0,58,59
10	131°45'50"	13°10'35"0'"0,58,59
29	22°6'56"	13°10'35"2'"4,8,0
100	237°38'23"	13°10'35"1'"47,59,59
354	344°26'43"	13°10'35"2'"12,12,0
1000	216°23'50"	13°10'35"1'"47,59,59
10000	3°58'20"	13°10'35"1'"47,59,59

Following table compares the printed values with calculations based on the most "precise" reconstructed value - that for 10000 days.

days	**value**	**recalculated**
n	**v(n)**	**r(10000)*n**
1	13°10'35"	13°10'35"1'"47,59,59
10	131°45'50"	131°45'50"17'"59,59,50
29	22°6'56"	22°6'55"52'"11,59,31
100	237°38'23"	237°38'22"59'"59,58,20
354	344°26'43"	344°26'40"37'"11,54,6
1000	216°23'50"	216°23'49"59'"59,43,20
10000	3°58'20"	3°58'19"59'"57,13,20

Values of the movement in one day apparently used by Rambam, if rounded to the seconds, all give the "official" Rambam's value. Values calculated from the value reconstructed from the printed 10000 days movement, if rounded, produce the values Rambam published - except for 354 days movement, which remains unexplained TODO!

Law 3: Movement in the Mean Lunar Anomaly

Movement in mean lunar anomaly in one day is 13°3'54".

For longer periods, (remainder) of the movement in mean lunar anomaly is:

days	value
n	**v(n)**
1	13°3'54"
10	130°39'0"
29	18°53'4"
100	226°29'53"
354	305°0'13"
1000	104°58'50"
10000	329°48'20"

Discussion

Rambam says "'ordered' year"; it was rendered as 254 in accordance with see Law 12:1. Also, this value is actually given in Law 4.

It is not clear how Rambam obtained the values he gives. When value of the movement in one day given by Rambam is multiplied by 29, 100 etc., the result is bigger than what Rambam says! The following table compares printed and calculated values:

days	value	calculated
n	**v(n)**	**v(1)*n**
1	13°3'54"	13°3'54"
10	130°39'0"	130°39'0"
29	18°53'4"	18°53'6"
100	226°29'53"	226°30'0"
354	305°0'13"	305°0'36"
1000	104°58'50"	105°0'0"
10000	329°48'20"	330°0'0"

Although the value of the movement of the mean lunar anomaly in one day given by Almagest IV 3 (H278) (which, rounded to seconds, becomes

Rambam's value) is smaller, it is not small enough, and Rambam's numbers can not be explained by performing calculations with the long value from Almagest and then rounding the results (TODO: table).

From the printed values it is possible to reconstruct the value of the movement in mean lunar anomaly in one day that Rambam used to calculate each of them:

days	**value**	**reconstructed 1-day movement**
n	**v(n)**	**r(n)**
1	13°3'54"	13°3'53"59'''59,58,59
10	130°39'0"	13°3'54"0'''0,0,0
29	18°53'4"	13°3'53"55'''51,43,0
100	226°29'53"	13°3'53"55'''48,58,59
354	305°0'13"	13°3'53"56'''6,6,0
1000	104°58'50"	13°3'53"55'''48,58,59
10000	329°48'20"	13°3'53"55'''48,58,59

Following table compares the printed values with calculations based on the most "precise" reconstructed value - that for 10000 days.

days	**value**	**recalculated**
n	**v(n)**	**r(10000)*n**
1	13°3'54"	13°3'53"55'''48,58,59
10	130°39'0"	130°38'59"18'''9,49,50
29	18°53'4"	18°53'3"58'''40,30,31
100	226°29'53"	226°29'53"1'''38,18,20
354	305°0'13"	305°0'11"19'''0,0,6
1000	104°58'50"	104°58'50"16'''23,3,20
10000	329°48'20"	329°48'22"43'''50,33,20

Values of the movement in one day apparently used by Rambam, if rounded to the seconds, all give the "official" Rambam's value. Values calculated from the value reconstructed from the printed 10000 days movement, if rounded, produce the values Rambam published - except for 354 days movement, which remains unexplained TODO!

Here, everything is worse than in 14:2 TODO!!!

Law 4: Position of the Moon on the Epoch

In the beginning of the night to Thursday that is the epoch (TODO measured from what? Chapter 11?) [TODO: link to definition] for these calculations, mean lunar longitude was 1°14'43" into the constellation of Ram. And mean lunar anomaly on the epoch was 84°28'42". Now that you know rate of movement in the mean lunar longitude and its value on the epoch to which you add, you'll calculate mean lunar longitude on any day that you want the same way as you did for the mean solar longitude. TODO: reference

Law 5,6: Mean Lunar Longitude at Observation Time

After you calculate mean lunar longitude at the beginning of the night that you want, contemplate the sun (TODO: longitude?) and figure out which constellation (zodiac) is it in [2]. [This is the correction of the mean lunar longitude depending on the mean solar longitude:]

Solar Longitude on the Zodiac	**from**	**to, not including**[a]	**Lunar longitude correction**
middle of Pisces-middle of Taurus [b]	315°	15°	0°
middle of Taurus-beginning of Gemini	15°	30°	0°15'
beginning of Gemini-beginning of Leo	30°	90°	0°15'

[2]This is part of Law 4.

Solar Longitude on the Zodiac	**from**	**to, not including**[a]	**Lunar longitude correction**
beginning of Leo-middle of Virgo	90°	135°	0°15'
middle of Virgo-middle of Libra	135°	165°	0°
middle of Libra-beginning of Sagittarius	165°	210°	-0°15'
beginning of Sagittarius-beginning of Aquarius	210°	270°	-0°30'
beginning of Aquarius-middle of Pisces	270°	315°	-0°15'

[a]My understanding of the word usage in the original.
[b]See Chapter 11, Law 7 on the start of Jewish Zodiac. (TODO reference)

TODO: Compare with 11:9, about Dli... Something is off!

The value of mean [lunar longitude] after you add to it or subtract from it or leave it as it is - that is mean lunar longitude approximately a third of an hour after sunset on the date you are calculating it for. And this is what is called *mean lunar longitude at observation time*. [3]

Chapter 15

Law 1: Doubled Distance

To find true lunar longitude (הירח האמיתי) on a specific day: First, calculate mean lunar longitude during observation time of the desired night. Also, calculate mean lunar anomaly and mean solar longitude for the same time. Then, subtract mean solar longitude from the mean lunar

[3]This is Law 6.

longitude and double the result. This value is called *doubled distance* (מרחק הכפול).

Law 2,3: Corrected Mean Anomaly (Elongation (TODO: ?))

We already stipulated that all the calculations in these chapters are for one purpose only: to know if the moon is visible. And it is not possible for this double distance during the night of observation when the moon is indeed visible to be outside of the interval from 5° to 62°. It cannot be more - nor less. [4]

A correction that depends on the doubled distance is added to the mean lunar anomaly, giving *corrected lunar anomaly* (מסלול הנכון):

doubled distance	**correction of the mean anomaly**
~5°	0°
6°-11°	1°
12°-18°	2°
19°-24°	3°
25°-31°	4°
32°-38°	5°
39°-45°	6°
46°-51°	7°
52°-59°	8°
60°-63°	9°

Discussion

Rambam does not describe the model behind this correction. Because the models Rambam did describe until now are the same as Almagest's models (including numerical parameters), and because this correction is determined by the (doubled) sun-moon elongation, just like Almagest's

[4] This is Law 2.

"improved" model of the moon ("the crank"), I assume that that model was used to calculate this correction. TODO: describe the model!

TODO: Derive the formula! Make the diagram! TODO: How come the fact that sun and moon move in different plains does not affect this calculation?

It seems that Rambam treats "the crank" as just a calculation device - otherwise, table giving visible anomaly from the corrected anomaly should take into account changes in the earth-moon distance depending on the elongation - and it does not! Maybe that is why he does not describe the model behind this correction. And maybe this is why Rambam reiterates - in Law 2 - that only the observability of the new moon needs to be calculated correctly. TODO: How does Almagest calculate visible anomaly?

Law 4,5,6: True Lunar Longitude

Then, see how many degrees is corrected lunar anomaly. If it is less than 180°, subtract visible anomaly of this corrected anomaly from the mean lunar longitude the observation time. And if corrected lunar anomaly is more than 180° but less than 360°, add visible anomaly of this corrected anomaly to the mean lunar longitude the observation time. And mean lunar longitude after you add to it or subtract from it is the *true lunar longitude at the observation time*. [5]

If the corrected anomaly is 180° or 360°, there is no visible anomaly, so mean lunar longitude at observation time is also true longitude. [6]

What is the value of the visible anomaly for a given value of the corrected anomaly?

corrected anomaly	**visible anomaly**
mca	**mva**
10°	0°50'
20°	1°38'

[5]This is Law 4.
[6]This is Law 5.

corrected anomaly	**visible anomaly**
mca	**mva**
30°	2°24'
40°	3°6'
50°	3°44'
60°	4°16'
70°	4°41'
80°	5°0'
90°	5°5'
100°	5°8'
110°	4°59'
120°	4°40'
130°	4°14'
140°	3°33'
150°	2°48'
160°	1°56'
170°	0°59'

Discussion

Formulae:

$$\tan^{-1}\left(\frac{\sin\alpha}{\cos\alpha+\varepsilon}\right) \qquad (1)$$

R - radius of the big circle; r - radius of the small circle; e = R/r. A little trigonometry, and we get: visible anomaly = arctg(sin(corrected anomaly)/(cos(corrected anomaly) + e)) and: e = ctg(visible anomaly)*sin(corrected anomaly) - cos(corrected anomaly)

TODO: Make the diagram! Here?

It is clear that values for 150° and 170° are misprinted: they are bigger than the one before them when they should be smaller. Value for 120° is also misprinted, but it is less obvious.

Calculations show that for the precision up to a minute, it is sufficient to know e to up to the second digit after the dot. Below, a column giving the value of e was added to the table. Look at the value of e, and the misprints become obvious: value of e for them is way out of range it is in for the rest of them.

corrected anomaly	**visible anomaly**	**e**
mca	**mva**	
10°	0°50'	10.95
20°	1°38'	11.05
30°	2°24'	11.06
40°	3°6'	11.1
50°	3°44'	11.1
60°	4°16'	11.11
70°	4°41'	11.13
80°	5°0'	11.08
90°	5°5'	11.24
100°	5°8'	11.14
110°	4°59'	11.12
120°	4°40'	11.11
130°	4°14'	10.99
140°	3°33'	11.13
150°	2°48'	11.09
160°	1°56'	11.07
170°	0°59'	11.1

TODO Range of e - graph. Range of possible values for each based on the range of e. This is transcription error: one letter; extra word... Probable correct values.

This is how different sources treat the often misprinted values:

Edition	**120°**	**150°**	**170°**
Vilno	incorrect	incorrect	incorrect

Edition	**120°**	**150°**	**170°**
Eshkol	incorrect	incorrect	incorrect
Rambam LaAm	corrected in the notes	incorrect	incorrect
Kapach [a]	incorrect	corrected in the notes	corrected in the notes
Bluming	incorrect	corrected in the notes	corrected in the notes
Frenkel	correct and noted	correct and noted	correct and noted
Keller	not given	not given	not given
Losh [b]	incorrect	incorrect	incorrect

[a] It is not clear how did authoritative Yemenite texts acquire the same misprints as in the ashkenazic editions.

[b] Reprints the text and does not correct any of the misprints - surpising for a textbook on the subject (even introductory). TODO: Link to the scans of the appropriate pages - with the stuff highlighted.

Law 7,8,9:

TODO!!!

Glossary

mean lunar anomaly (אמצע המסלול)	Mean angular coordinate of the Moon on its epicicle.

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