EE488C Special Topics in EE <Deep Learning and AlphaGo> September 21, 2016
EE, KAIST, Fall 2016 Due: 1:00pm, September 28

Problem Set #1

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0. (Reading assignment) DL Book Chapters $1 \sim 6$

1. (Independent random variables) Find an example of three binary random variables X, Y and Z such that they are pairwise independent but not mutually independent, i.e., $p(x,y) = p(x)p(y)$, $p(x,z) = p(x)p(z)$, $p(y,z) = p(y)p(z)$ for all $(x,y,z) \in \{0,1\}^3$, but $p(x,y,z) \neq p(x)p(y)p(z)$ for some $(x,y,z) \in \{0,1\}^3$.

2. (Saddle points)

(a) Construct a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that its saddle points are given by $\{(n, 2m+1-n) | n \in \mathbb{Z}, m \in \mathbb{Z}\}$, where $\mathbb{Z}$ is the set of integers.

(b) Construct a function $f : \mathbb{R}^n \to \mathbb{R}$ such that it has roughly 2^n times more saddle points than local minima in $[0, 100]^n$, where $[a, b]$ is the closed interval between a and b.

3. (No free lunch theorem) You are given a binary random variable X and are told to estimate the value of another binary random variable Y. There can be many learning algorithms producing an estimate $\hat{Y}$ of Y based on the observation X, e.g., $\hat{Y} = 0$, $\hat{Y} = 1$, $\hat{Y} = X$, or $\hat{Y} = 1 - X$, i.e., the output of the algorithm is "always 0", "always 1", "same as the first outcome", and "different from the first outcome", respectively. There can also be randomized algorithms, e.g., set $\hat{Y} = 1$ randomly with probability p or set $\hat{Y} = 1$ with probability p and set $\hat{Y} = X$ with probability $1 - p$.

(a) Assume X and Y are i.i.d. Bernoulli(1/2). Show that no learning algorithm can perform any better than random guessing, i.e., producing $\hat{Y}$ as Bernoulli(1/2) regardless of X.

(b) Can there be another joint distribution $p(x,y)$ on (X,Y) such that no learning algorithm can perform any better than random guessing?