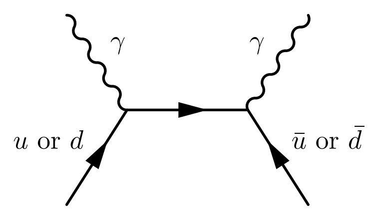
PHYS 5602 - Elementary Particle Physics
Assignment 3

Trevor Burn // 100712473

January 25, 2012

For all Feynman diagrams, time moves bottom to top.

1. A π^0 is a meson given by either $u\bar{u}$ or $d\bar{d}$. Thus, the QED diagram leading to $\pi^0 \to \gamma\gamma$ decay is



2. For this question, we use n^* to be in the centre of mass frame, and n to be in the lab frame.

   (a) In the centre of mass frame, conservation of momentum and energy gets us that $E^*_{\gamma,1} = E^*_{\gamma,2} = m_\pi/2$, with $p^*_{\gamma,1} = -p^*_{\gamma,2} = m_\pi/2$. If we assume that the photons are traveling at an angle θ with respect to the $\hat{\mathbf{x}}$ direction, then $p^*_x = p^* \cos\theta$, and $p^*_y = p^* \sin\theta$. Then taking a Lorentz boost in the $\hat{\mathbf{x}}$ direction at speed β and noting that $E_\pi = \gamma m_\pi$, we get

$$\begin{aligned}
E &= \gamma(E^* - \beta p^*_x) \\
E_{\gamma,1} &= \gamma\left[\frac{m_\pi}{2} - \beta\frac{m_\pi}{2}\cos\theta\right] \\
&= \frac{m_\pi}{2}\gamma(1 - \beta\cos\theta) \\
&= \frac{E_\pi}{2}(1 - \beta\cos\theta) \\
E_{\gamma,2} &= \gamma\left[\frac{m_\pi}{2} + \beta\frac{m_\pi}{2}\cos\theta\right] \\
&= \frac{m_\pi}{2}\gamma(1 + \beta\cos\theta) \\
&= \frac{E_\pi}{2}(1 + \beta\cos\theta)
\end{aligned}$$

   (b) The ratio is given by

$$\begin{aligned}
R &= \frac{E_{\gamma,1}}{E_{\gamma,2}} \\
&= \frac{\frac{E_\pi}{2}(1 - \beta\cos\theta)}{\frac{E_\pi}{2}(1 + \beta\cos\theta)} \\
&= \frac{1 - \beta\cos\theta}{1 + \beta\cos\theta}
\end{aligned}$$

   (c) For ultra relativistic particles, $\beta \approx 1$, and hence $R \approx (1 - \cos\theta)/(1 + \cos\theta)$.

3. (a) The minimum energy particle B can have is when B is at rest, and hence $E_{\text{min}} = m_B$. The maximum energy particle B can have is when the other particles are at rest, and their energies are m_C, m_D. Then the available energy is

$$\begin{aligned} E_{\text{max}} &= m_A - m_B - m_C - m_D + m_B \\ &= m_A - m_C - m_D \end{aligned}$$

And hence $m_B \leq E_B \leq m_A - m_C - m_D$. Note that all of this assumes that $m_A \geq m_B + m_C + m_D$.

(b) Taking neutrino mass to be zero, we therefore get $m_e \leq E_e \leq m_\mu$, or $511\,\text{keV} \leq E_e \leq 106\,\text{MeV}$.