PHYS 5602 - Elementary Particle Physics
Assignment 2

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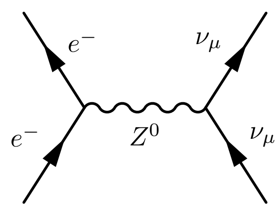
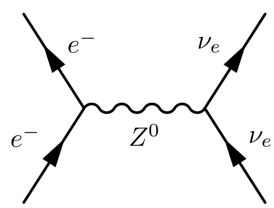
January 23, 2012

For all Feynman diagrams, time moves bottom to top.

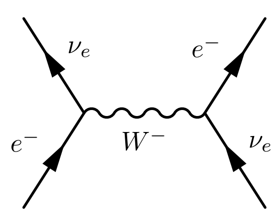
1. For $\nu_\mu + e^- \to \nu_\mu + e^-$, the Feynman diagram is



and for $\nu_e + e^- \to \nu_e + e^-$, the Feynman diagram is



The first interaction is a more unequivocal demonstration of weak neutral currents. This is because there is actually a second possible diagram for the second interaction:



This also conserves lepton number and charged, but demonstrates weak *charged* current, as opposed to weak *neutral* current.

2. (a) $\nu_\mu + p \to \mu^+ + n$
   Lepton number is not conserved, since $L(\nu_\mu) = +1$ and $L(\mu^+) = -1$. Therefore, this interaction is forbidden.

   (b) $\Sigma^0 \to \pi^+ + e^- + \bar{\nu}_e$
   Since Σ^0 is a baryon, and π^+ is a meson, baryon number is not conserved. Therefore, this interaction is forbidden.

(c) $\pi^0 \to \gamma + \gamma$

Baryon number is conserved. Lepton number is conserved. Charge is conserved. Mass will be conserved since the photons will take all the energy. Momentum will be conserved, for the same reason. Therefore, this is a possible reaction.

(d) $p \to n + e^+ + \nu_e$

The mass of the proton is 938.27 MeV and the mass of the neutron is 939.57 MeV. Thus, this will only occur when the proton has sufficient energy to create the final particles. All other conservation laws are obeyed.

(e) $\tau^+ \to e^+ + e^- + e^+$

Lepton number is not conserved, and hence this is a forbidden interaction.

(f) $p + \bar{p} \to \pi^+ + \pi^- + \pi^0$

Charge is conserved. Lepton number is conserved. Baryon number is conserved (since the baryon number of an anti-baryon is negative one). The mass of the proton/anti-proton is 939.27 MeV. The mass of each charged pion is 139.57 MeV, and the uncharged pion's mass is 134.98 MeV. Thus, mass can be conserved, as can momentum. Therefore, this is a possible reaction.

(g) $\pi^+ \to e^+ + \nu_e$

Charge, lepton number, and baryon number are all conserved. Mass will be conserved, since $m_{\pi^+} > m_{e^+}$, and so will momentum. Thus, this is a possible reaction.

(h) $K^+ \to \pi^+ + \pi^- + \gamma$

Charge is not conserved, and hence this reaction is impossible.

(i) $\nu_e + \bar{\nu}_e \to g$

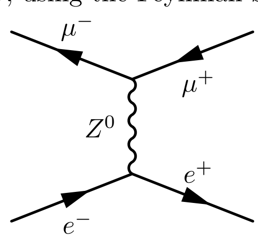
Since leptons do not carry color, this is an impossible reaction.

(j) $Z^0 \to t + \bar{t}$

This is forbidden by conservation of mass, unless the Z^0 has enough energy to create the top and anti-top particles. Namely, the Z^0 has a mass of 91.2 GeV, whereas the t quark (and hence the $\bar{t}$) has a mass of 172.9 GeV.

3. We are given that $E \gg m$ for all particles involved, and hence $p \gg m$.

(a) The Feynman diagram is given below, using the Feynman-Stückelberg interpreation.



(b) When the electron is at rest, we have a momentum of zero and an electron energy of m_e. The positron, however, will have a momentum of p and energy $E = \sqrt{p^2 + m_e}^2$. The Z^0 will, (by conservation of momentum), have momentum of p and energy $E = \sqrt{p^2 + m_Z^2}$. This would consitute a violation of conservation of energy, but since $\Delta E \Delta t \approx \hbar = 1$ (in natural units), we can have small violations. We also have that $\Delta x = \Delta t$, and so $\Delta x \approx 1/\Delta E$. Then

$$\begin{aligned} \Delta E &= m_e + \sqrt{p^2 + m_e^2} - \sqrt{p^2 + m_Z^2} \\ &\approx m_e \end{aligned}$$

since we have that $p \gg m_e$ and $p \gg m_Z$. Thus $\Delta x \approx 1/m_e$.

(c) In the centre of mass frame, the electron has momentum p and energy $E = \sqrt{p^2 + m_e^2}$. Similarly, the positrion has momentum $-p$ and energy $E = \sqrt{p^2 + m_e^2}$. Then by conservation of momentu, the Z_0 has momentum p and energy m_Z. As before, $\Delta x \approx 1/\Delta E$, and

$$\begin{aligned}\Delta E &= \sqrt{p^2 + m_e^2} + \sqrt{p^2 + m_e^2} - m_Z \\ &\approx E - m_Z \\ &\approx E\end{aligned}$$

as before, with $E \gg m_e, m_Z$. Then $\Delta x \approx 1/E$. Then, since $E = \gamma m_e$, and under the lorentz contraction, $d' = d/\gamma$, we get that $\Delta x' \approx 1/m_e$, which is consistent.