PHYS 5602 - Elementary Particle Physics
Assignment 3

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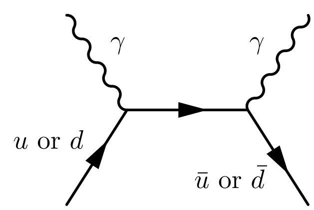
For all Feynman diagrams, time moves bottom to top.

1. A π^0 is a meson given by either $u\bar{u}$ or $d\bar{d}$. Thus, the QED diagram leading to $\pi^0 \to \gamma\gamma$ decay is



2. For this question, we use n^* to be in the centre of mass frame, and n to be in the lab frame.

   (a) In the centre of mass frame, conservation of momentum and energy gets us that $E^*_{\gamma,1} = E^*_{\gamma,2} = m_\pi/2$, with $p^*_{\gamma,1} = -p^*_{\gamma,2} = m_\pi/2$. If we assume that the photons are traveling at an angle θ with respect to the $\hat{\mathbf{x}}$ direction, then $p^*_x = p^* \cos\theta$, and $p^*_y = p^* \sin\theta$. Then taking a Lorentz boost in the $\hat{\mathbf{x}}$ direction at speed β and noting that $E_\pi = \gamma m_\pi$, we get

$$\begin{aligned}
E &= \gamma(E^* - \beta p^*_x) \\
E_{\gamma,1} &= \gamma\left[\frac{m_\pi}{2} - \beta\frac{m_\pi}{2}\cos\theta\right] \\
&= \frac{m_\pi}{2}\gamma(1-\beta\cos\theta) \\
&= \frac{E_\pi}{2}(1-\beta\cos\theta) \\
E_{\gamma,2} &= \gamma\left[\frac{m_\pi}{2} + \beta\frac{m_\pi}{2}\cos\theta\right] \\
&= \frac{m_\pi}{2}\gamma(1+\beta\cos\theta) \\
&= \frac{E_\pi}{2}(1+\beta\cos\theta)
\end{aligned}$$

   (b) The ratio is given by

$$\begin{aligned}
R &= \frac{E_{\gamma,1}}{E_{\gamma,2}} \\
&= \frac{\frac{E_\pi}{2}(1-\beta\cos\theta)}{\frac{E_\pi}{2}(1+\beta\cos\theta)} \\
&= \frac{1-\beta\cos\theta}{1+\beta\cos\theta}
\end{aligned}$$

   (c) For ultra relativistic particles, $\beta \approx 1$, and hence $R \approx (1-\cos\theta)/(1+\cos\theta)$.

3. (a) The minimum energy particle B can have is when B is at rest, and hence $E_{\text{min}} = m_B$. The maximum energy particle B can have is when the other particles are at rest, and their energies are m_C, m_D. Then the available energy is

$$\begin{aligned} E_{\text{max}} &= m_A - m_B - m_C - m_D + m_B \\ &= m_A - m_C - m_D \end{aligned}$$

And hence $m_B \leq E_B \leq m_A - m_C - m_D$. Note that all of this assumes that $m_A \geq m_B + m_C + m_D$.

(b) Taking neutrino mass to be zero, we therefore get $m_e \leq E_e \leq m_\mu$, or $511\,\text{keV} \leq E_e \leq 106\,\text{MeV}$.

4. (a) We have that the CKM matrix is a unitary matrix, with

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

where

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Then since V is unitary, we have the following

$$\begin{aligned} 1 &= VV^\dagger \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \end{aligned}$$

This gives us

- $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$
- $|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$
- $V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$
- $V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0$
- $V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0$

Note: there would be three more, but these are simply the complex conjugates of the last 3, and hence provide no information.

We also have

$$\begin{aligned} 1 &= V^\dagger V \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} &= \begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \end{aligned}$$

This gives us

- $|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1$
- $|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1$
- $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$
- $V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$
- $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$
- $V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$

As before, the other three relationships are just the complex conjugates.

(b) We then have

$$\begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix} = V^* \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}$$

where V^* is the complex conjugate. Then

$$\begin{aligned}
&d'\bar{d}'Z + s'\bar{s}'Z + b'\bar{b}'Z \\
=&\left(V_{ud}d + V_{us}s + V_{ub}b\right)\left(V_{ud}^*\bar{d} + V_{us}^*\bar{s} + V_{ub}^*\bar{b}\right)Z \\
&+\left(V_{cd}d + V_{cs}s + V_{cb}b\right)\left(V_{cd}^*\bar{d} + V_{cs}^*\bar{s} + V_{cb}^*\bar{b}\right)Z \\
&+\left(V_{td}d + V_{ts}s + V_{tb}b\right)\left(V_{td}^*\bar{d} + V_{ts}^*\bar{s} + V_{tb}^*\bar{b}\right)Z \\
=&d\bar{d}Z\left(|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2\right) + s\bar{s}Z\left(|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2\right) \\
&+ b\bar{b}Z\left(|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2\right) + d\bar{s}Z\left(V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^*\right) \\
&+ d\bar{b}Z\left(V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^*\right) + s\bar{d}Z\left(V_{us}V_{ud}^* + V_{cs}V_{cd}^* + V_{ts}V_{td}^*\right) \\
&+ s\bar{b}Z\left(V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^*\right) + b\bar{d}Z\left(V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^*\right) \\
&+ b\bar{s}Z\left(V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^*\right) \\
=&d\bar{d}Z + s\bar{s}Z + b\bar{b}Z
\end{aligned}$$

using the relationships above.