PHYS 5602 - Elementary Particle Physics
Assignment 1

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1. We want that $\mathbf{F} = 0$, so that the positron will travel in a straight line in the positive $\hat{\mathbf{x}}$ direction. Thus

$$0 = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

with $\mathbf{E} = -E\hat{\mathbf{y}}$, $\mathbf{B} = -B\hat{\mathbf{z}}$, and $\mathbf{v} = v\hat{\mathbf{x}}$. Then $\mathbf{v} \times \mathbf{B} = vB\mathbf{y}$. Thus

$$\begin{aligned} 0 &= -qE + qvB \\ 0 &= E - vB \\ vB &= E \\ v &= \frac{E}{B} \end{aligned}$$

Then in the mass spectrometer, we have centripetal acceleration with a magnetic field strength B_0 and radius $r = d/2$.

$$\begin{aligned} F &= qvB_0 \\ ma &= qvB_0 \\ m\frac{v^2}{r} &= qvB_0 \\ \frac{m}{q} &= \frac{rB_0}{v} \\ \frac{m_e}{e} &= \frac{dBB_0}{2E} \end{aligned}$$

2. In 1928, Dirac suggested that it should be possible for electrons to exist with positive charge and negative energy. In 1932, Carl Anderson detected a particle with the same charge to mass ratio as an electron, but bent in the opposite direction under a magnetic field, thus having positive charge.

3. (a) The cylcotron frequency will be given by

$$\omega_c = \frac{v}{r}$$

In a cylcotron, the motion is given by

$$\frac{mv^2}{r} = qvB_0$$

and so

$$\frac{v}{r} = \frac{q}{m}B_0$$

Therefore

$$\omega_c = \frac{q}{m}B_0$$

(b) At radius r, the particle will have velocity $v = \frac{q}{m} B_0 r$, and hence will have kinetic energy

$$\begin{aligned} T &= \frac{1}{2} m v^2 \\ &= \frac{1}{2} m \frac{q^2}{m^2} B_0^2 r^2 \\ &= \frac{1}{2} \frac{q^2}{m} B_0^2 r^2 \end{aligned}$$

(c) When the relativistic equation has to be used, we get that the velocity no longer increases as $\sqrt{T}$, but is given by

$$\begin{aligned} T &= (\gamma - 1) m c^2 \\ \frac{T}{mc^2} + 1 &= \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \\ \sqrt{1 - \left(\frac{v}{c}\right)^2} &= \frac{1}{\frac{T}{mc^2} + 1} \\ 1 - \frac{v^2}{c^2} &= \left(\frac{1}{\frac{T}{mc^2} + 1}\right)^2 \\ \frac{v^2}{c^2} &= 1 - \left(\frac{1}{\frac{T}{mc^2} + 1}\right)^2 \\ v &= c \sqrt{1 - \left(\frac{1}{\frac{T}{mc^2} + 1}\right)^2} \end{aligned}$$

Then, since $\omega_c \propto v$, and v is no longer proportional to r, the cylcotron constant must vary with v.