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Complete Powerpoint Lecture Notes for
Computer Systems: A Programmer's Perspective (CS:APP)

Randal E. Bryant and David R. O'Hallaron

http://csapp.cs.cmu.edu/public/lectures.html

The book is used explicitly in CS 2505 and CS 3214 and as a reference in CS 2506.

Locality Example (1)

Claim: Being able to look at code and get a qualitative sense of its locality is a key skill for a professional programmer.

Question: Which of these functions has good locality?

```
int sumarrayrows(int a[M][N])
{
   int i, j, sum = 0;

   for (i = 0; i < M; i++)
      for (j = 0; j < N; j++)
         sum += a[i][j];
   return sum;
}
```

```
int sumarraycols(int a[M][N])
{
   int i, j, sum = 0;

   for (j = 0; j < N; j++)
      for (i = 0; i < M; i++)
         sum += a[i][j];
   return sum;
}
```

Layout of C Arrays in Memory

C arrays allocated in contiguous memory locations with addresses ascending with the array index:

```
int32_t A[20] = {0, 1, 2, 3, 4, ..., 8, 9};
```

80430000	0
80430004	1
80430008	2
8043000C	3
80430010	4
...	...
80430048	8
8043004C	9

Layout of C Arrays in Memory

Two-dimensional C arrays allocated in *row-major order* - each row in contiguous memory locations:

```
int32_t A[3][5] =
 { { 0,  1,  2,  3,  4},
   {10, 11, 12, 13, 14},
   {20, 21, 22, 23, 24},
 };
```

80430000	0
80430004	1
80430008	2
8043000C	3
80430010	4
80430014	10
80430018	11
8043001C	12
80430020	13
80430024	14
80430028	20
8043002C	21
80430030	22
80430034	23
80430038	24

Layout of C Arrays in Memory

```
int32_t A[3][5] =
 { { 0,  1,  2,  3,  4},
   {10, 11, 12, 13, 14},
   {20, 21, 22, 23, 24},
 };
```

Stepping through columns in one row:

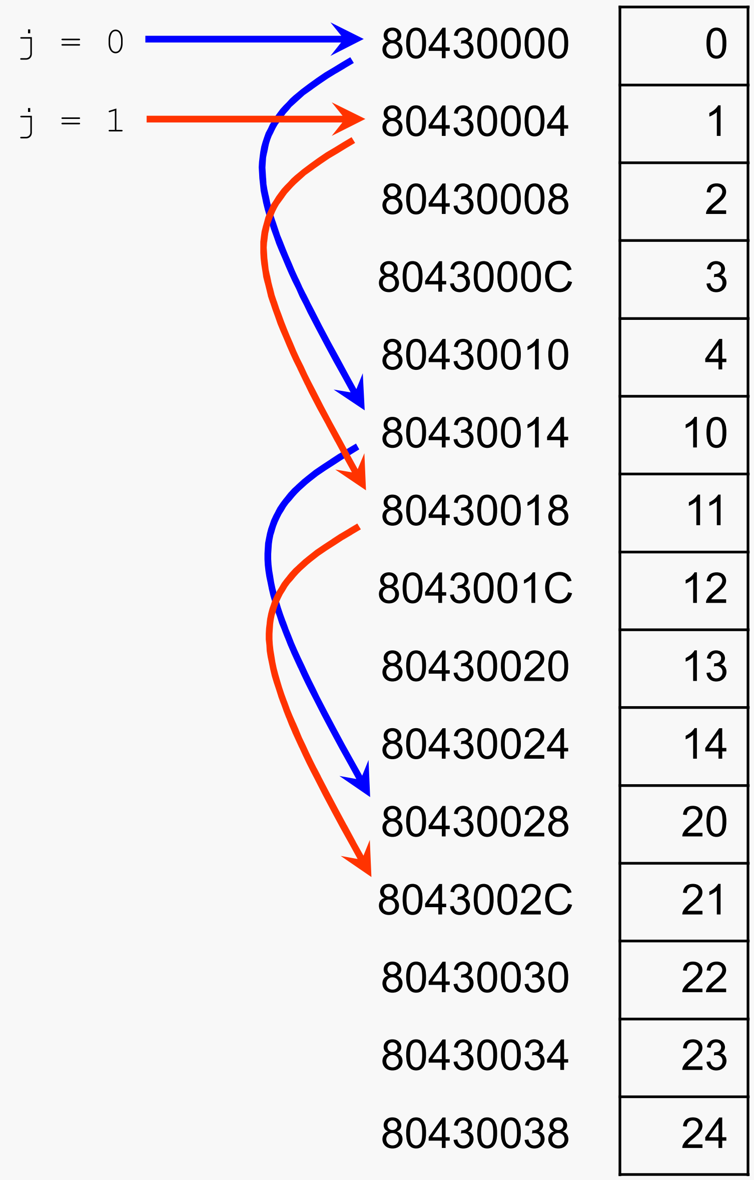
```
for (i = 0; i < 3; i++)
   for (j = 0; j < 5; j++)
      sum += A[i][j];
```

- accesses successive elements in memory
- if cache block size B > 4 bytes, exploit spatial locality
  compulsory miss rate = 4 bytes / B

	80430000	0
	80430004	1
`i = 0`	80430008	2
	8043000C	3
	80430010	4
	80430014	10
	80430018	11
`i = 1`	8043001C	12
	80430020	13
	80430024	14
	80430028	20
	8043002C	21
`i = 2`	80430030	22
	80430034	23
	80430038	24

Layout of C Arrays in Memory

```
int32_t A[3][5] =
 { { 0,  1,  2,  3,  4},
   {10, 11, 12, 13, 14},
   {20, 21, 22, 23, 24},
 };
```



Stepping through rows in one column:

```
for (j = 0; i < 5; i++)
    for (i = 0; i < 3; i++)
        sum += a[i][j];
```

accesses distant elements

no spatial locality!

compulsory miss rate = 1 (i.e. 100%)

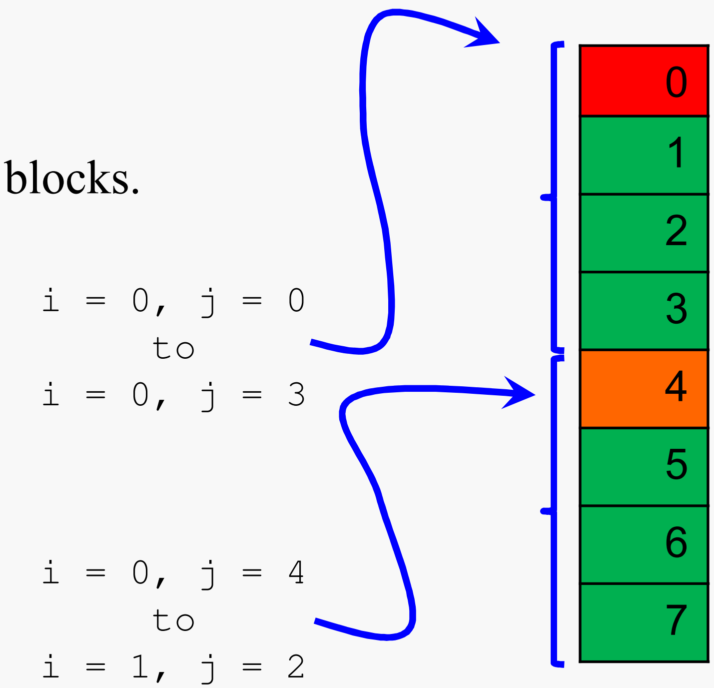
Writing Cache Friendly Code

Repeated references to variables are good (temporal locality)

Stride-1 reference patterns are good (spatial locality)

Assume an initially-empty cache with 16-byte cache blocks.

```
int sumarrayrows(int a[M][N])
{
   int i, j, sum = 0;

   for (i = 0; i < M; i++)
      for (j = 0; j < N; j++)
         sum += a[i][j];
   return sum;
}
```



Miss rate = 1/4 = 25%

Writing Cache Friendly Code

"Skipping" accesses down the rows of a column do not provide good locality:

```
int sumarraycols(int a[M][N])
{
   int i, j, sum = 0;

   for (j = 0; j < N; j++)
      for (i = 0; i < M; i++)
         sum += a[i][j];
   return sum;
}
```

Miss rate = 100%

(That's actually somewhat pessimistic... depending on cache geometry.)

Locality Example (2)

Question: Can you permute the loops so that the function scans the 3D array `a[]` with a stride-1 reference pattern (and thus has good spatial locality)?

```
int sumarray3d(int a[M][N][N])
{
    int i, j, k, sum = 0;

    for (i = 0; i < M; i++)
        for (j = 0; j < N; j++)
            for (k = 0; k < N; k++)
                sum += a[k][i][j];
    return sum
}
```

Layout of C Arrays in Memory

It's easy to write an array traversal and see the addresses at which the array elements are stored:

```
int A[5] = {0, 1, 2, 3, 4};

for (i = 0; i < 5; i++)
   printf("%d:  %X\n",
            i, (unsigned)&A[i]);
```

We see there that for a 1D array, the index varies in a stride-1 pattern.

```
i   address
-----------
0:    28ABE0
1:    28ABE4
2:    28ABE8
3:    28ABEC
4:    28ABF0
```

stride-1 : addresses differ by the size of an array cell (4 bytes, here)

```
int B[3][5] = { ... };
for (i = 0; i < 3; i++)
   for (j = 0; j < 5; j++)
      printf("%d %3d:  %X\n",
                i, j, (unsigned)&B[i][j]);
```

We see that for a 2D array, the second index varies in a stride-1 pattern.

```
i-j order:

i    j     address
----------------
0    0:     28ABA4
0    1:     28ABA8
0    2:     28ABAC
0    3:     28ABB0
0    4:     28ABB4
1    0:     28ABB8
1    1:     28ABBC
1    2:     28ABC0
```

} stride-1

But the first index does not vary in a stride-1 pattern.

```
j-i order:

i    j     address
----------------
0    0:   28CC9C
1    0:   28CCB0
2    0:   28CCC4
0    1:   28CCA0
1    1:   28CCB4
2    1:   28CCC8
0    2:   28CCA4
1    2:   28CCB8
```

} stride-5 (0x14/4)

Layout of C Arrays in Memory

```
int C[2][3][5] = { ... };

for (i = 0; i < 2; i++)
   for (j = 0; j < 3; j++)
      for (k = 0; k < 5; k++)
         printf("%3d  %3d  %3d: %d\n",
                    i, j, k, (unsigned)&C[i][j][k]);
```

We see that for a 3D array, the <u>third</u> index varies in a stride-1 pattern:

```
i-j-k order:

i   j   k     address
------------------
0   0   0:    28CC1C
0   0   1:    28CC20
0   0   2:    28CC24
0   0   3:    28CC28
0   0   4:    28CC2C
0   1   0:    28CC30
0   1   1:    28CC34
0   1   2:    28CC38
```

But… if we change the order of access, we no longer have a stride-1 pattern:

```
k-j-i order:

i   j   k     address
------------------
0   0   0:    28CC24
1   0   0:    28CC60
0   1   0:    28CC38
1   1   0:    28CC74
0   2   0:    28CC4C
1   2   0:    28CC88
0   0   1:    28CC28
1   0   1:    28CC64
```

```
i-k-j order:

i   j   k     address
------------------
0   0   0:    28CC24
0   1   0:    28CC38
0   2   0:    28CC4C
0   0   1:    28CC28
0   1   1:    28CC3C
0   2   1:    28CC50
0   0   2:    28CC2C
0   1   2:    28CC40
```

Locality Example (2)

Question: Can you permute the loops so that the function scans the 3D array `a[]` with a stride-1 reference pattern (and thus has good spatial locality)?

```
int sumarray3d(int a[M][N][N])
{
    int i, j, k, sum = 0;

    for (i = 0; i < M; i++)
        for (j = 0; j < N; j++)
            for (k = 0; k < N; k++)
                sum += a[k][i][j];
    return sum
}
```

This code does not yield good locality at all.

The inner loop is varying the <u>first</u> index, worst case!

Locality Example (3)

Question: Which of these two exhibits better spatial locality?

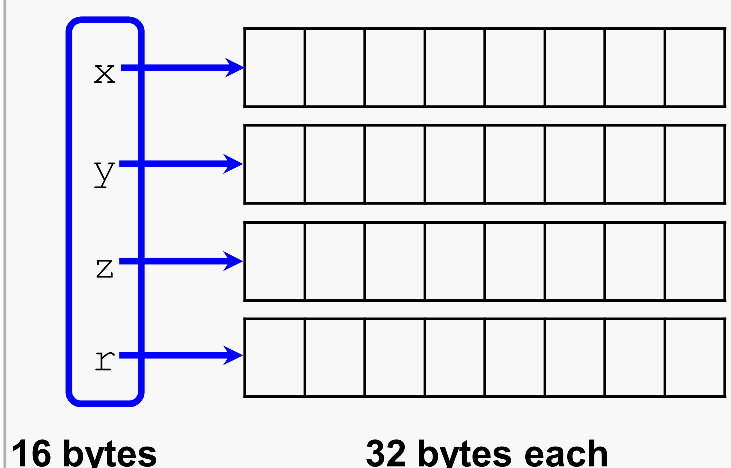
```
// struct of arrays
struct soa {
  float *x;
  float *y;
  float *z;
  float *r;
};

compute_r(struct soa s) {
  for (i = 0; …) {
    s.r[i] = s.x[i] * s.x[i]
           + s.y[i] * s.y[i]
           + s.z[i] * s.z[i];
  }
}
```

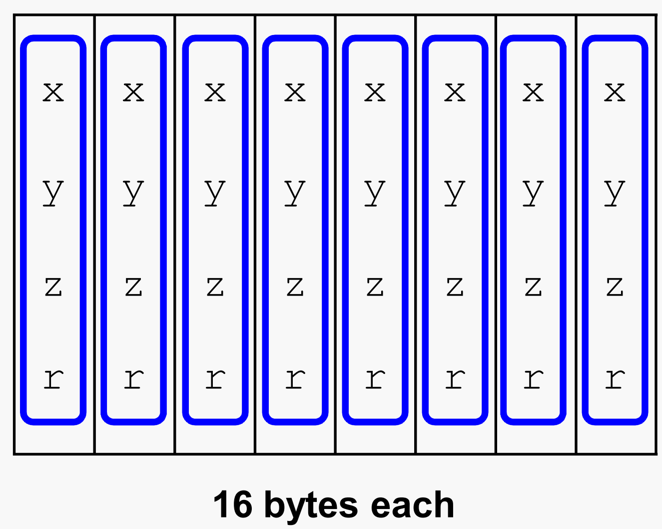
```
// array of structs
struct aos {
  float x;
  float y;
  float z;
  float r;
};

compute_r(struct aos *s) {
  for (i = 0; …) {
    s[i].r = s[i].x * s[i].x
           + s[i].y * s[i].y
           + s[i].z * s[i].z;
  }
}
```

Locality Example (3)

```
// struct of arrays
struct soa {
  float *x;
  float *y;
  float *z;
  float *r;
};
struct soa s;
s.x = malloc(8*sizeof(float));
...
```

```
// array of structs
struct aos {
  float x;
  float y;
  float r;
};

struct aos s[8];
```



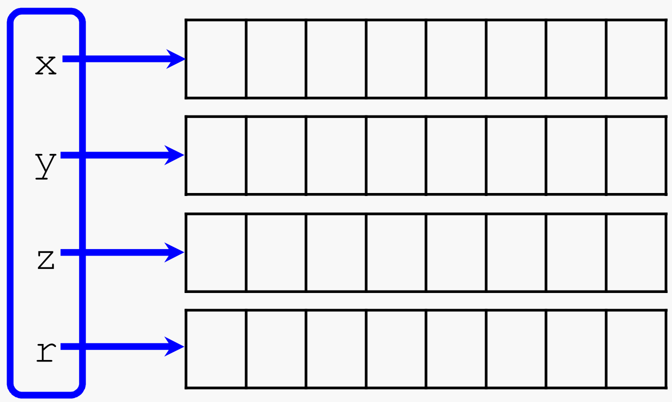


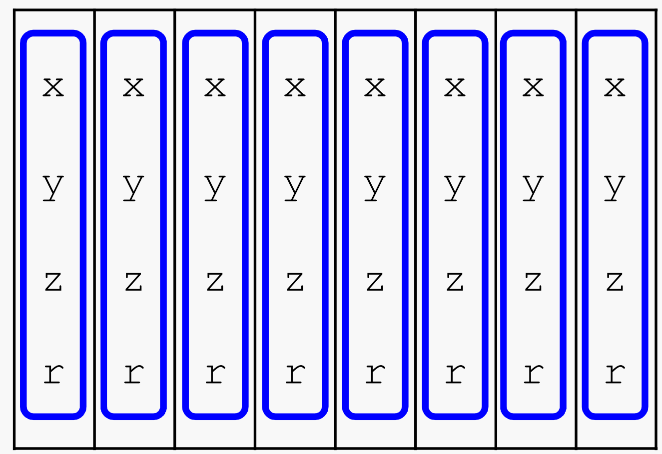
Locality Example (3)

Question: Which of these two exhibits better spatial locality?

```
// struct of arrays
compute_r(struct soa s) {
  for (i = 0; …) {
    s.r[i] = s.x[i] * s.x[i]
           + s.y[i] * s.y[i]
           + s.z[i] * s.z[i];
  }
}
```

```
// array of structs
compute_r(struct aos *s) {
  for (i = 0; …) {
    s[i].r = s[i].x * s[i].x
           + s[i].y * s[i].y
           + s[i].z * s[i].z;
  }
}
```



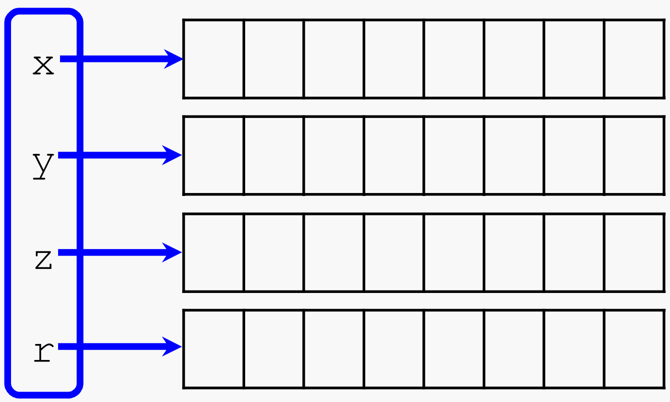


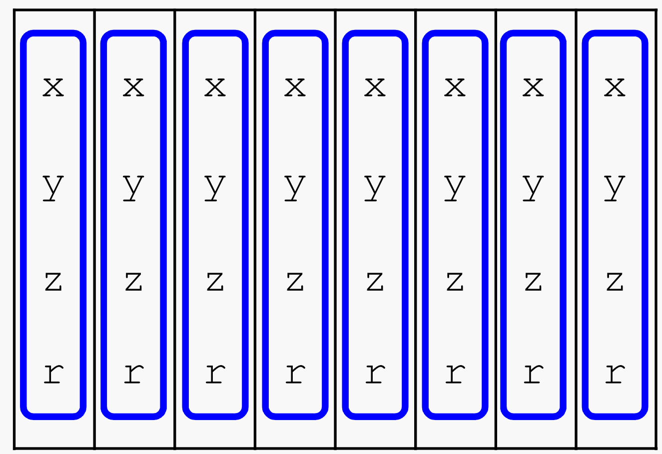
Locality Example (4)

Question: Which of these two exhibits better spatial locality?

```
// struct of arrays
sum_r(struct soa s) {
  sum = 0;
  for (i = 0; …) {
    sum += s.r[i];
  }
}
```

```
// array of structs
sum_r(struct aos *s) {
  sum = 0;
  for (i = 0; …) {
    sum += s[i].r;
  }
}
```





Locality Example (5)

QTP: How would this compare to the previous two?

```
// array of pointers to structs
struct aos {
  float x;
  float y;
  float z;
  float r;
};

struct aops[8];

for (i = 0; i < 8; i++)
   apos[i] = malloc(sizeof(struct aops));
```


Writing Cache Friendly Code

Make the common case go fast

- Focus on the inner loops of the core functions

Minimize the misses in the inner loops

- Repeated references to variables are good (temporal locality)
- Stride-1 reference patterns are good (spatial locality)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories.

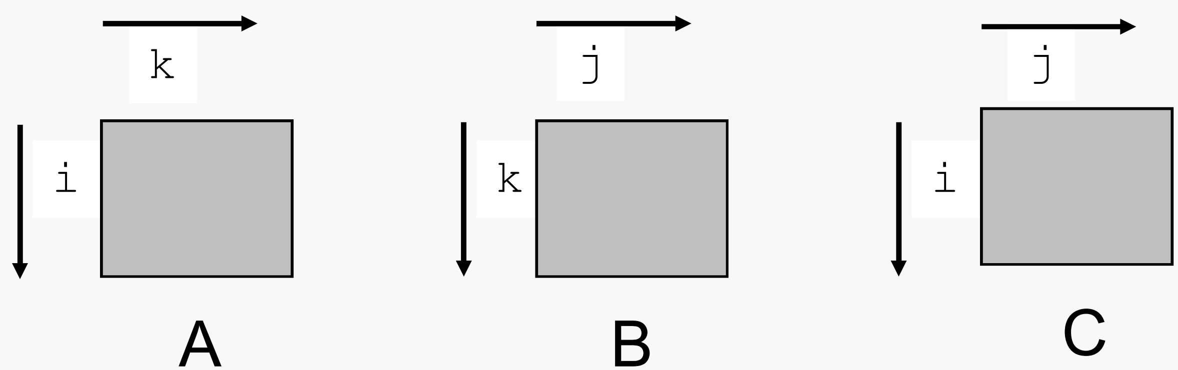
Miss Rate Analysis for Matrix Multiply

Assume:

- Line size = 32B (big enough for four 64-bit words)
- Matrix dimension (N) is very large
  - Approximate 1/N as 0.0
- Cache is not even big enough to hold multiple rows

Analysis Method:

- Look at access pattern of inner loop



Matrix Multiplication Example

Description:

- Multiply N x N matrices
- $O(N^3)$ total operations
- N reads per source element
- N values summed per destination

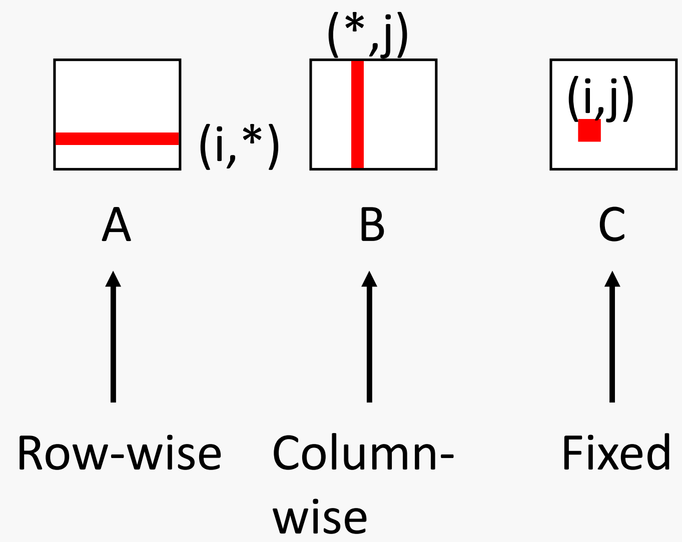
Variable `sum` *held in register*

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Matrix Multiplication (ijk)


```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



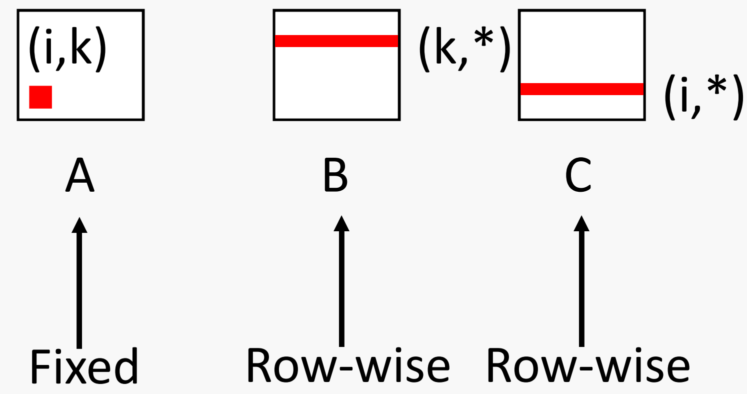
<u>Misses per inner loop iteration:</u>

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



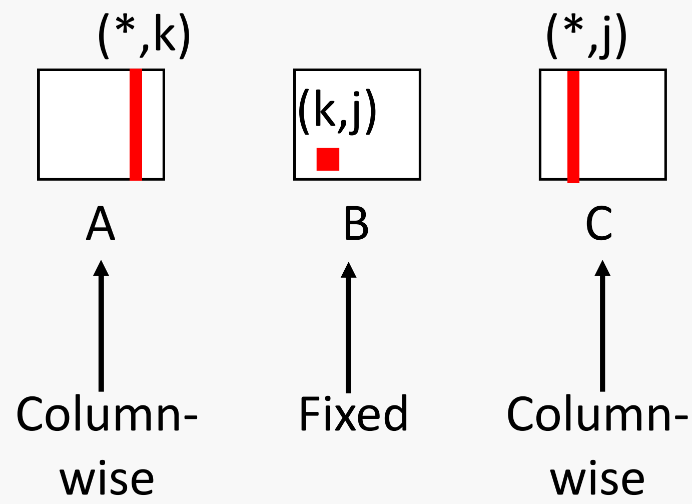
Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)


```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



<u>Misses per inner loop iteration:</u>

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

ijk (& jik):
- 2 loads, 0 stores
- misses/iter = 1.25

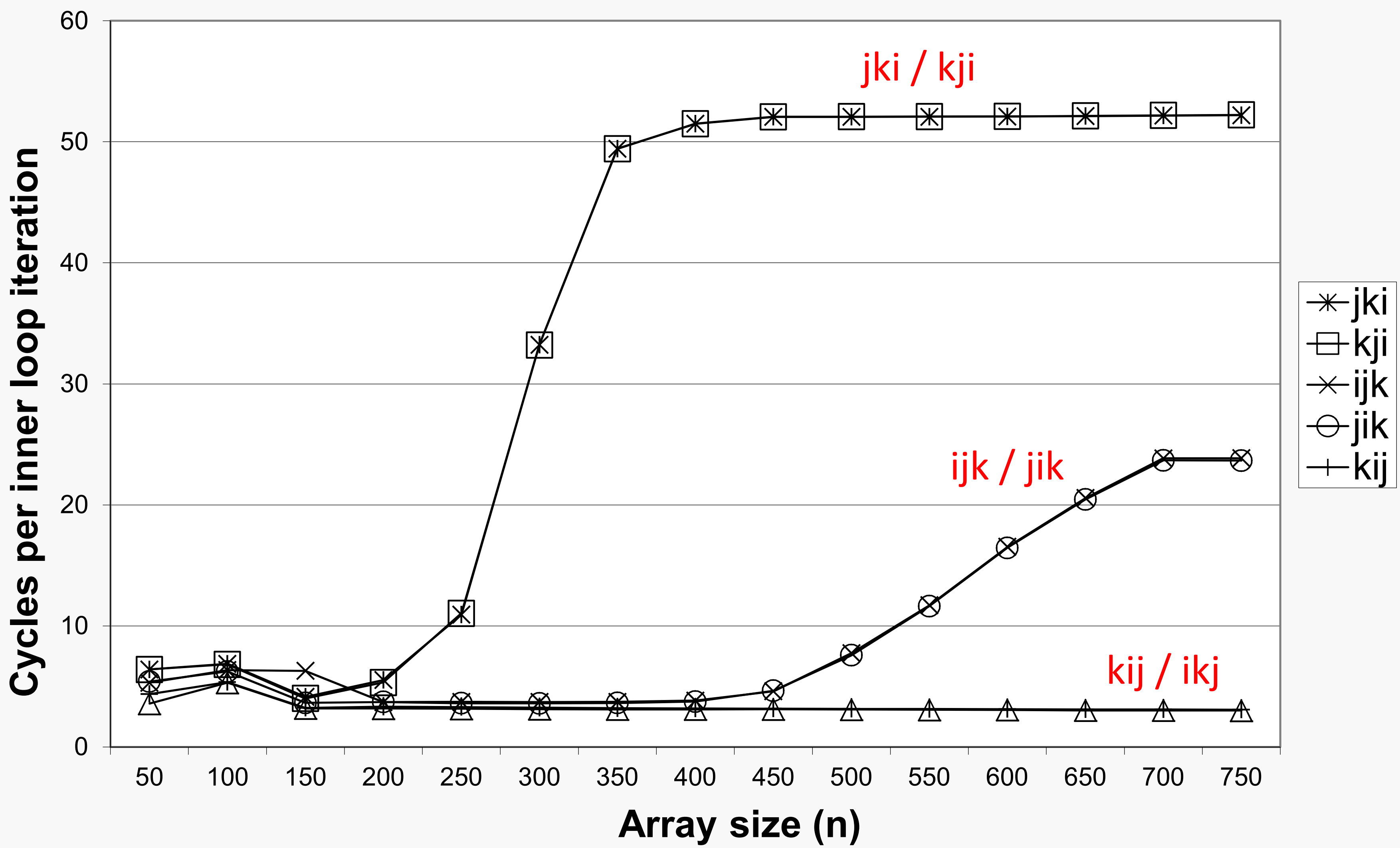
```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):
- 2 loads, 1 store
- misses/iter = 0.5

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

jki (& kji):
- 2 loads, 1 store
- misses/iter = 2.0

Core i7 Matrix Multiply Performance



Concluding Observations

- Programmer can optimize for cache performance
  - How data structures are organized
  - How data are accessed
    - Nested loop structure
    - Blocking is a general technique

- All systems favor "cache friendly code"
  - Getting absolute optimum performance is very platform specific
    - Cache sizes, line sizes, associativities, etc.
  - Can get most of the advantage with generic code
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)