OpenBarter

central limit order book of fungible values

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abstract

The model described implements a central limit order book mechanism independent of any monetary standard. Based on barter, it's liquidity is nearly equivalent to a regular market by allowing exchange cycles with more than two partners. It can be used to build market-based instruments providing a regulation mean for sharing a diversity of scarce ecological resources.

Day-to-day experience shows that the difficulty of driving a system grows as its complexity. For a system whose stability is controlled by a command signal, cybernetics would state that the number of dimensions of this command increases as the complexity of the system. Since the dollar value is mono-dimensional and financial markets are based on dollar, it is not surprising that the traditional market-based instruments fail to address efficiently the urgent needs of ecological economics.

The physical model of greenhouse gases for example depends on a diversity of gas spices including many trace gases whose abundances need to be accurately monitored[1]. Unfortunately, the urgency of building a bridge between this complex model and the market economy imposed a simplification of the command signal to a single dimension, the carbon and the dollar. But it is the physical reality of natural processes that should drive adaptation of market instruments in order to adapt allocation of resources to the reality of natural processes.

The primary requirement of adaptation is the multidimensionality of resources. For the model openBarter, a resource is a value by itself measured independently of any external value standard. It is a couple **(quantity,quality)**, where quantity is an integer, and quality a name.

An order is as a statement – the commitment of an owner - to provide a value in exchange of an other. The ratio ω between provided and required quantities measures the effort to provide compared to the pleasure to receive. It is used instead of the price to implement the mechanism equivalent to a price, but it can be used by the owner to define the limit of its order as does a limit price on a regular market.

If we consider an exchange cycle where each partner provides a quantity and receives an other, the product of ratios ω between provided and required quantities is a non-dimensional number equal to 1. The basic principle of this paper is based on this observation.

We will consider the bilateral case between a buyer and a seller, and will extend it to a multilateral exchange cycle.

1 Bilateral exchange

Using the regular definition of price for an exchange between a seller providing 10 Kg of apples in

1 W.C. WANG and others, 'Greenhouse Effects Dues to Man-Made Perturbations of Traces Gases', *SCIENCES*, 194 (1976), 685,690.

exchange of 20 pounds, the price is 2 pounds/Kg. Likewise, if a seller provides a quantity g of a stock to a buyer in exchange of a quantity m of money, the regular definition of price is: $p=\frac{m}{g}$.

Buyer and seller have usually different ideas of prices, We note the buyer price p_b and the seller price p_s . An agreement between the buyer and the seller is the result of a barter between these prices.

If we consider now the ratio between quantities provided and received, it depends on the view-point.

this ratio is ω_b for the buyer and such as $\omega_b=\frac{m}{g}$

and ω_s for the seller such as $\omega_s=\frac{g}{m}$.

We can observe that $\omega_b=p_b$ and $\omega_s=\frac{1}{p_s}$

1.1 Agreement

Agreement on price between the buyer and the seller exists when their respective prices $\boldsymbol{p_b}$ and $\boldsymbol{p_s}$ are equal: $p_b=p_s$. An equivalent statement using expressions ω_b and ω_s of price is:

[1] $$\omega_b=\frac{1}{\omega_s}\Rightarrow\omega_b*\omega_s=1$$

1.2 Best matching

Using the regular definition of price, preference of price depends on the point of view:

- For the buyer, $p_{b1}\geq p_{b2}$ means p_{b2} is better than p_{b1} .
- For the seller, $p_{s1}\leq p_{s2}$ means p_{s2} is better than p_{s1} .

These rules are called the best price rule by both of them.

1.3 Compromise

Compromise is required when buyer and seller do not agree on their prices; that is when $p_b\neq p_s$. If we choose the geometric mean of p_b and p_s: $p'=\sqrt{p_b*p_s}$ as the result of the compromise we can observe that $\frac{p_b}{p'}=\frac{p'}{p_s}$. Since an increase for one side should be compared to a decrease for the other these ratios $\frac{p_b}{p'}$ and $\frac{p'}{p_s}$ measure the negotiation effort of the buyer and the seller respectively. Such a rule that balance efforts of partners is the reason why we propose to choose this geometric mean as the compromise between prices.

Expressed using ω, this barter provides new values ω'_s and ω'_b such as:

$$\omega_s' = \frac{\omega_s}{\sqrt[2]{\omega_s * \omega_b}}$$
$$\omega_b' = \frac{\omega_b}{\sqrt[2]{\omega_s * \omega_b}}$$

We can verify that their product is 1 as expected by [1].

The result of this compromise has the same form for both partners:

[2] $$\omega' = \frac{\omega}{\sqrt[2]{\omega_s * \omega_b}}$$

It is used to compute the quantity of money and of stock finally exchanged between the buyer and the seller.

1.4 Central limit order book (CLOB)

The process performed by a central limit order book when a new order (bid or ask) is submitted can be presented as follows:

A. The list of orders matching this new order is obtained from the order book. If this list is empty, the new order is stored in the order book and the process stops.
B. The best order is found from this list according to the best price rule,
C. The execution produces a couple of movements between partners that reduces the provided and required quantities of the couple of matching orders – bid and ask. Among these two orders, at least one is completed. If the new order is not complete, the process is repeated from A. Otherwise the process stops.

The matching is possible when the quality of the stock provided by the bid is the same as the quality required by the ask. The matching also requires a compromise between the couple of orders, since prices are rarely equal, in order to find a common price for both sides and finally execute their orders[2]. Due to the “limit price”, bids are rejected if the compromise is higher than a limit defined by the buyer, and asks are rejected if the compromise is lower than a limit defined by the seller.

2 Multilateral exchange

2.1 Agreement

Let's consider a list of ***n*** orders where the quality offered by one equals the quality required by the following, and where the quality offered by the last equals the quality required by the first order. This list forms a potential multilateral exchange cycle. An order ***i*** with $i \in [0, n-1]$ offers a quantity p_i and requires a quantity r_i. Let $\vec{\omega}$ be the vector with coordinates $\boldsymbol{\omega_i}$: .

$$\omega_i = \frac{p_i}{r_i}$$

2 It is a meaning of the word 'barter' different from an economic exchange that do not require any currency.

Let's suppose an agreement exists between partners, and that each partner ***i***, with $i \in [0, n-1]$, of the cycle provides a given quantity $\boldsymbol{q_i}$ to an other partner. Let $\vec{\omega}'$ be the vector with coordinates $\boldsymbol{\omega'_i}$ ratio between quantity provided and quantity received by a partner ***i*** :

$$\omega'_i = \frac{q_i}{q_{i-1}} \quad for \quad i \in [1, n-1]$$

$$\omega'_0 = \frac{q_0}{q_{n-1}}$$

If we make the product of those ***n*** expression,

we obtain: $$\prod_{i=0}^{n-1} \omega'_i = \frac{q_0}{q_{n-1}} * \prod_{j=1}^{n-1} \frac{q_i}{q_{i-1}} = 1$$

We see that when an agreement exists we have:

[3] $$\prod_{i=0}^{n-1} \omega'_i = 1$$

We also see that [1] is a special case of [3].

If the product of $\boldsymbol{\omega_i}$ is 1, we just choose $\vec{\omega} = \vec{\omega}'$ in order to verify this necessary condition. Otherwise, $\vec{\omega}$ need to be adjusted to obtain a compromise $\vec{\omega}'$.

2.2 Best matching

When a new order is added to the order book, the search for coincidence provides a set of cycles containing this new order that will be transformed into agreements using a priority order that should comply with the best price rule of the bilateral case.

A potential agreement is defined by a cycle C of orders and corresponding ω_i with $i \in [0, n-1]$.

For a cycle C, we note Ω the product $\prod_{i=0}^{n-1} \omega_i$.

We propose to define a total order $.\preccurlyeq.$ on the set of cycles found as follows:

[4] $$C_1 \preccurlyeq C_2 \Leftrightarrow \Omega_1 \leq \Omega_2$$

Since this order is total, it defines a maximum cycle on the set of cycles.

Let's consider the best price rule from the view-point of buyers and sellers.

A buyer compares cycles C_1 and C_2 when a common order belongs to both – it is an ask in this case. It's price is p_b, and prices of corresponding bids are p_{s1} and p_{s2}. He chooses C_2 when $p_{s2} \leq p_{s1}$. Since $\omega_s = \frac{1}{p_s}$ we have: $p_{s2} \leq p_{s1} \Rightarrow \omega_{s1} \leq \omega_{s2} \Rightarrow \omega_b * \omega_{s1} \leq \omega_b * \omega_{s2} \Rightarrow C_1 \preccurlyeq C_2$

A seller compares cycles C_1 and C_2 when a common order belongs to both – it is a bid in this case. It's price is p_s, and prices of corresponding asks are p_{b1} and p_{b2}. He chooses C_2 when $p_{b1} \leq p_{b2}$. Since $\omega_b = p_b$ we have: $p_{b1} \leq p_{b2} \Rightarrow \omega_{b1} \leq \omega_{b2} \Rightarrow \omega_{b1} * \omega_s \leq \omega_{b2} * \omega_s \Rightarrow C_1 \preccurlyeq C_2$

Hence, we can state that the order [4] comply with the best price rule of bilateral exchanges.

2.3 Compromise

We propose to extend [2] to a cycle C by choosing ω'_i as :

$$\omega_i' = \omega_i * \left\{ \frac{1}{\Omega} \right\}^{\frac{1}{n}}$$

Since Ω is equally shared between partners, this is a fair compromise when all partners of the cycle of bids are distinct. But several orders on a cycle may belong to the same partner. This can be an opportunity for him to take advantage of the situation[3].

The compromise is hence modified by sharing first Ω between partners, then by sharing the results between orders of each partner. Formally let m be the number of partners, such as $m \leq n$ and let $\boldsymbol{o_i}$ be the number of orders of the author of the order i , such as $\sum_{i=0}^{m-1} o_i = n$. The compromise is:

$$\omega_i' = \omega_i * \left\{ \left[\frac{1}{\Omega} \right]^{\frac{1}{m}} \right\}^{\frac{1}{o_i}}$$

or :

[5] $$\omega_i' = \omega_i * \Omega^{-\frac{1}{m * o_i}}$$

that verify the equality [3] . This compromise remains fair even if more than one order belongs to the same partner in the cycle.

Once ω_i' are obtained from [5], the next step is to find a vector $\vec{Q}$ of integers representing quantities to be exchanged by the draft agreement from quantities offered by orders.

A vector $\vec{q}$ of real numbers is first computed as the maximum flow circulation through the cycle with the constraints of ω_i' and of quantities offered by orders (see annexe I).

This vector $\vec{q}$ of real numbers needs to be rounded to obtain the vector $\vec{Q}$ of integers. There are several solutions since each coordinate can be rounded to lower or upper bound. We exclude from them those where some coordinate of $\vec{Q}$ is null because a draft agreement where some partners don't provide anything would be unfair. Among remaining candidates, we choose the one minimizing a distance between the two vectors $\vec{q}$ and $\vec{Q}$, chosen as the angle between the two vectors.

This computation finally provides $\vec{Q}$ representing quantities of the draft agreement defining a fair compromise between orders of the cycle.

3 This can never occur on bilateral exchanges since when several orders belong to the same partner, the partner is alone to exchange.

2.4 CLOB

The process performed by this CLOB when a new order is submitted is similar to a bilateral exchange:

A. The list of cycles matching the new order is obtained from the order book. Each cycle is itself a list of orders including the new order. If this list is empty, the new order is stored in the order book and the process stops.
B. The best cycle is found from this list according to [4] of §2.2,
C. A compromise is found between orders of this best cycle according to [5] of §2.3 to find an agreement between partners and finally execute these orders. The execution produces a set of movements between partners that reduces the provided quantities of the orders of the cycle. Among the orders of the cycle, at least one is complete. If the new order is not complete, the process is repeated from A. Otherwise the process stops.

The matching is possible when the quality provided by an order is the same as the quality required by the following. Cycles are rejected if the compromise is higher than the limit ω defined by the owner.

3 Implementation of the CLOB

This is by far the most time-consuming part of the work for the following reasons:

- Integrity of data is a primary concern of users of this CLOB. Hence, the whole insertion process of a single order must be wrapped in a single atomic transaction that can be rolled back in case of technical failure.
- Security of external access to the CLOB is an other primary requirement.
- Graph algorithms that explores a large amount of data are far more time-consuming than regular CLOB that provide a result in a single millisecond.

This is the reasons why the following work were made:

- Choice of a tool that provides both security and integrity and that can be easily customized. PostgreSQL was good candidate among existing databases due to its maturity, extensibility, open source licence, and to the quality and size of the worldwide community that developed it.
- Development of an adaptation of the Bellman-Ford algorithm to find cycles with Ω maximum. This algorithm required the graph to be acyclic.
- Reduce the graph exploration to cycles having a limited number of nodes, while maintaining the acyclic constraint for those cycles.
- Adaptation of this algorithm that tolerates cycles in the order book.
- Integration of this algorithm in PostgreSQL that maintain its original security and integrity properties.
- Sharing of this algorithm between standards primitives of PostgreSQL and fast subroutines

using the C native language and extensibility mechanisms of the database.

The result of this work has been published under open-source licence:

http://olivierch.github.com/openBarter

This software was developed so that complicate calculations on a limited set of data are performed in C language, and that traversal of larger sets of data are performed with search primitives of the database.

With this purpose, a special data type representing a path of orders was developed, and special functions was developed to add an order to this path, test if this path was a cycle, and finding in a set of cycles the maximum according to [4] using an aggregate function. Likewise, a single C function provides the vector of quantities of the agreement produced by a cycle. These functions allowed the implementation of the largest graph traversal in a single SQL recursive query that could be submitted globally to the PostgreSQL optimization process.

Some primitives were developed using this work:

- Get a quote by providing (owner, quality and quantity provided, quality and quantity required),
- Execute a quote,
- Insert an order defined by (owner, quality and quantity provided, quality and quantity required).
- Remove an order.

The results of these primitives are movements that swap round the property of values made available by orders.

3.1 Benchmark

Order has been submitted continuously for 10 hours 47 minutes. 3405 orders was inserted; the mean time of execution was 11,4 seconds.
599 exchange cycles was produced described by 1986 account movements.
The illustration 1 represents of the 925 active orders remaining at the end of activity. Nodes represent 20 distinct qualities exchanged, and directed arcs the 925 orders. Their colours represent the author of the order also owner of the quantity offered.

599 exchange cycles was produced, described by 1986 account movements.

Illustration 1: graph representation of the order book

Cycles contained at most 8 movements, The distribution of the length of cycles presented on illustration 2. Not surprisingly it shows that cycles remain short when the diversity of qualities is low.

If the maximum length of cycles was reduced to 6 it would not decrease a lot the chance of matching and increase performance.

The study of the distribution of the length of cycles in a relation with the number of orders contained by the order book and the variety of qualities could bring an interesting response to such optimisation.

The experience showed that unwanted orders tend to accumulate and decrease performance when the book grows.

number of exchanges	**cycles**
2	203
3	175
4	110
5	59
6	45
7	6
8	1
Total	**599**

Illustration 2: distribution of the length of cycles

The software has been improved in order to increase performance:

- Various protocols were used to limit the life time of orders in the book. The life time measurement was chosen in order not to penalize orders containing uncommon qualities. An order older than this maximum life time is translated into a single movement of the value provided by the order to the author of the order.
- The maximum number of valid cycle candidates for competition between cycles was limited to 1024 with a very low impact on search completeness.
- Index was built on search keys.

Cycles with a maximum of 64 orders using 100 distinct qualities show that cycles are never longer than 15.

A benchmark in November 2012 was performed on this software with 1 000 000 orders, random quantities and 100 distinct qualities. The mean time of execution of an order was 300 milliseconds, this time including all required integrity and security mechanisms offered by PostgreSQL.

4 Use as market-based instrument

openBarter can be seen as a simple back box accepting orders from owners of the form:

“I provide a value for an other value”

and producing movements of values between authors of orders.

It can be used as market-based instrument to share a limited quantity of scarce ecological resources between market participants independently of any financial market.

ANNEXE I – Maximum flow

From $\vec{\omega}$ and the vector $\vec{a}$ of quantities provided by orders we describe how the maximum flow $\vec{q}$ of real numbers is obtained.

If integers a_i with $i \in [0, n-1]$ define the quantities provided by orders then the flow $\vec{q}$ should verify the following constraints:

$$\frac{q_i}{q_{i-1}} = \omega_i \quad \mathit{for} \quad i \in [1, n-1]$$
$$\frac{q_0}{q_{n-1}} = \omega_0 \quad \mathit{for} \quad i = 0$$
$$q_i \leq a_i \quad \mathit{for} \quad i \in [0, n-1]$$

To find this $\vec{q}$ let's first consider the ratios $\frac{q_i}{q_0} = \pi_i$. They are such as:

$$\pi_0 = 1$$
$$\pi_i = \prod_{j=0}^{i-1} \omega_j \quad \mathit{for} \quad i \in [1, n-1]$$

We can observe that: $$q_i \leq a_i \Rightarrow \frac{q_i}{\pi_i} \leq \frac{a_i}{\pi_i} \Rightarrow q_0 \leq \frac{a_i}{\pi_i} \mathit{for}\, i \in [0, n-1]$$

When π_i are computed, we choose $\vec{q}$ such as $q_0 = min(\frac{a_i}{\pi_i})$ and then $q_i = q_0 * \pi_i$. We can see that $\vec{q}$ verify all constraints.